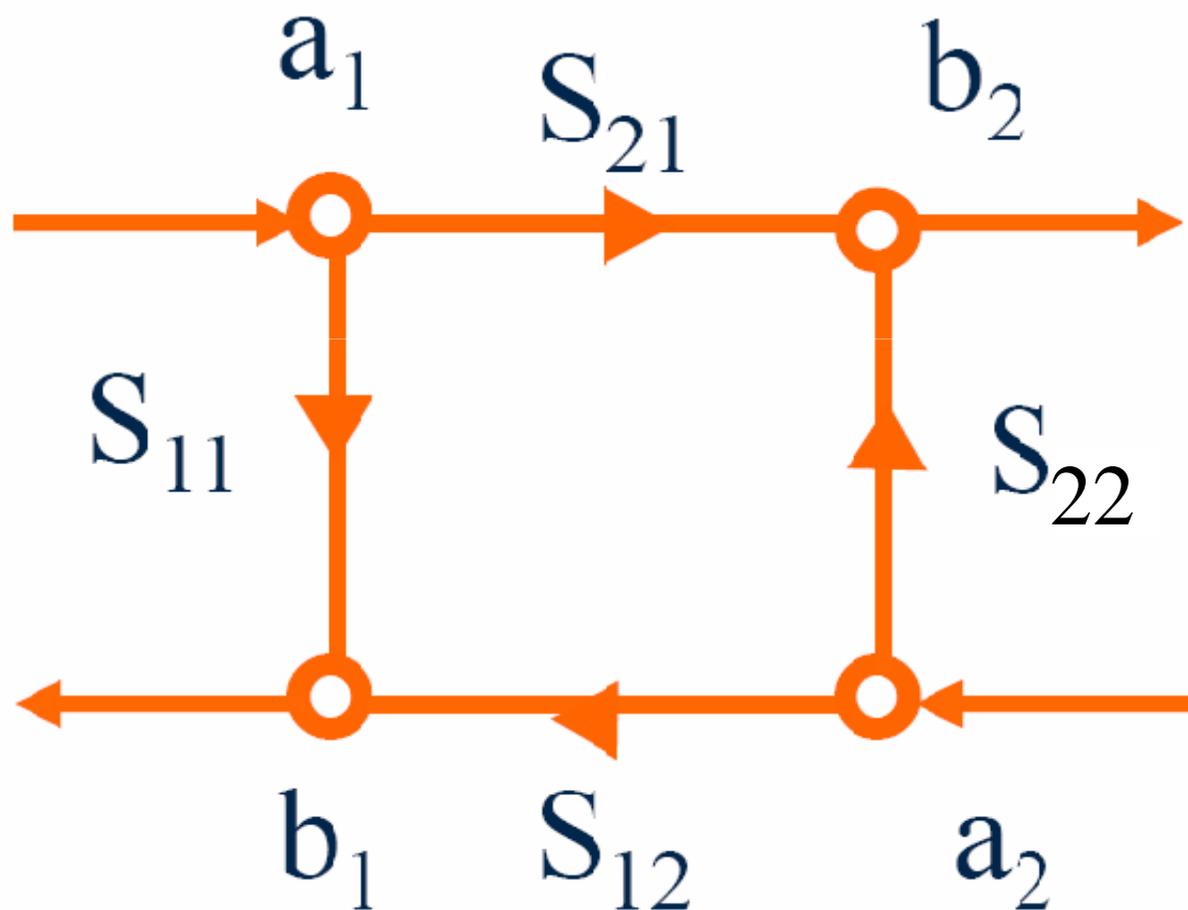


# Matrika $S$ in smerni grafi za mikrovalovno prakso



Mobitel d.d.,  
izobraževanje

6. 11. 2009,  
predavanje 26

Prof. dr. Jožko  
Budin

# Vsebina

1. Grafična predstavitev linearnih enačb s smernimi grafi:
2. Predstavitev signalnega toka 4, 6 in 8-polnih vezij z grafom
  - Primeri 4, 6, in 8-polnih vezij z grafi
3. Načini reševanja grafov:
  - Dekompozicija, redukcija
  - Masonovo pravilo nedotikajočih se zank
4. Smerni grafi običajnih in kaskadnih vezi
5. Primeri reševanja grafov

# Poimenovanja in terminologija

## Valovi:

Potujoči, stojni

Vpadni (napredujoči)

Povratni (odbiti)

Vhodni (vstopajoči)

Izhodni (izstopajoči)

## Grafi:

Graf signalnega toka

Signalni graf oz. graf

Smerni graf

## S parametri:

Matrika [S]

Matrika porazdelitve

Matrika sipanja

Matrika razpršitve

## Elementi grafov:

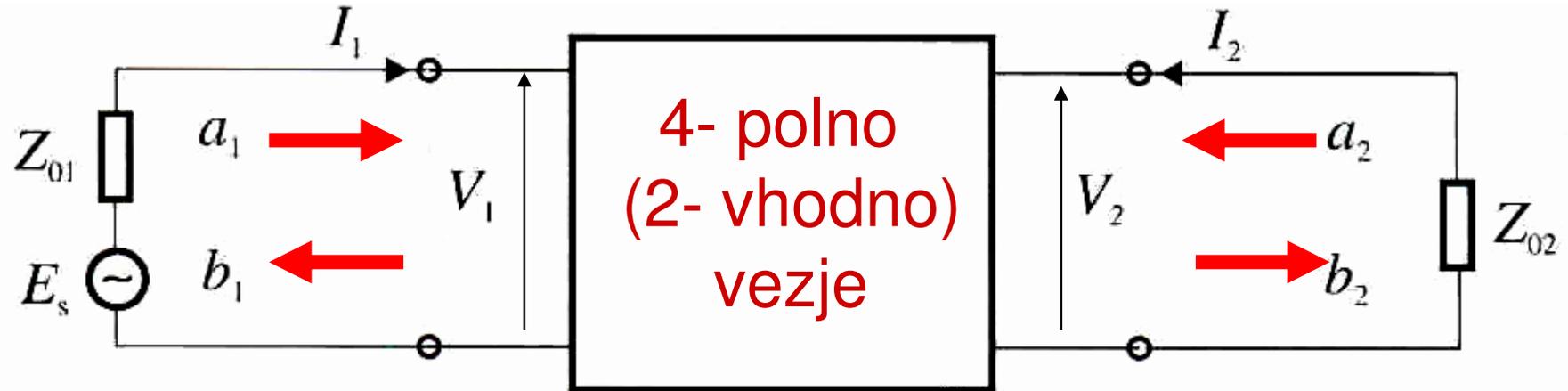
Vir

Vozel

Zanka, red zanke

Nedotikajoča se zanka

# Definicija valov



$$V_n = \sqrt{Z_{0n}}(a_n + b_n)$$

$$I_n = \frac{1}{\sqrt{Z_{0n}}}(a_n - b_n)$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = \frac{V_1/\sqrt{Z_{01}} - \sqrt{Z_{01}}I_1}{V_1/\sqrt{Z_{01}} + \sqrt{Z_{01}}I_1}$$

$V$  in  $I$  pomenita amplitudi.

Moč na  $n$ -tem vhodu:

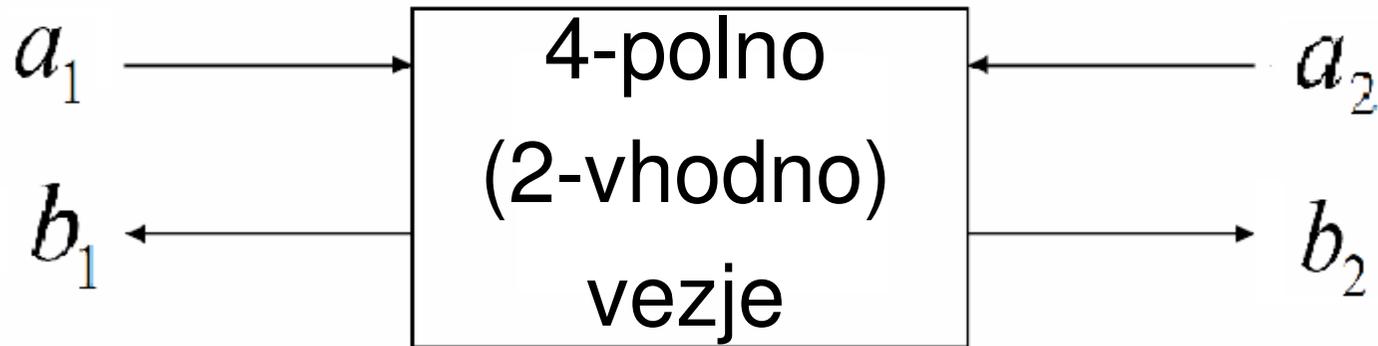
$$a_n = \frac{1}{2} \left( \frac{V_n}{\sqrt{Z_{0n}}} + \sqrt{Z_{0n}} I_n \right)$$

$$P_n = \frac{1}{2} \operatorname{Re}(V_n \cdot I_n^*) = \frac{1}{2} (a_n a_n^* - b_n b_n^*)$$

$$b_n = \frac{1}{2} \left( \frac{V_n}{\sqrt{Z_{0n}}} - \sqrt{Z_{0n}} I_n \right)$$

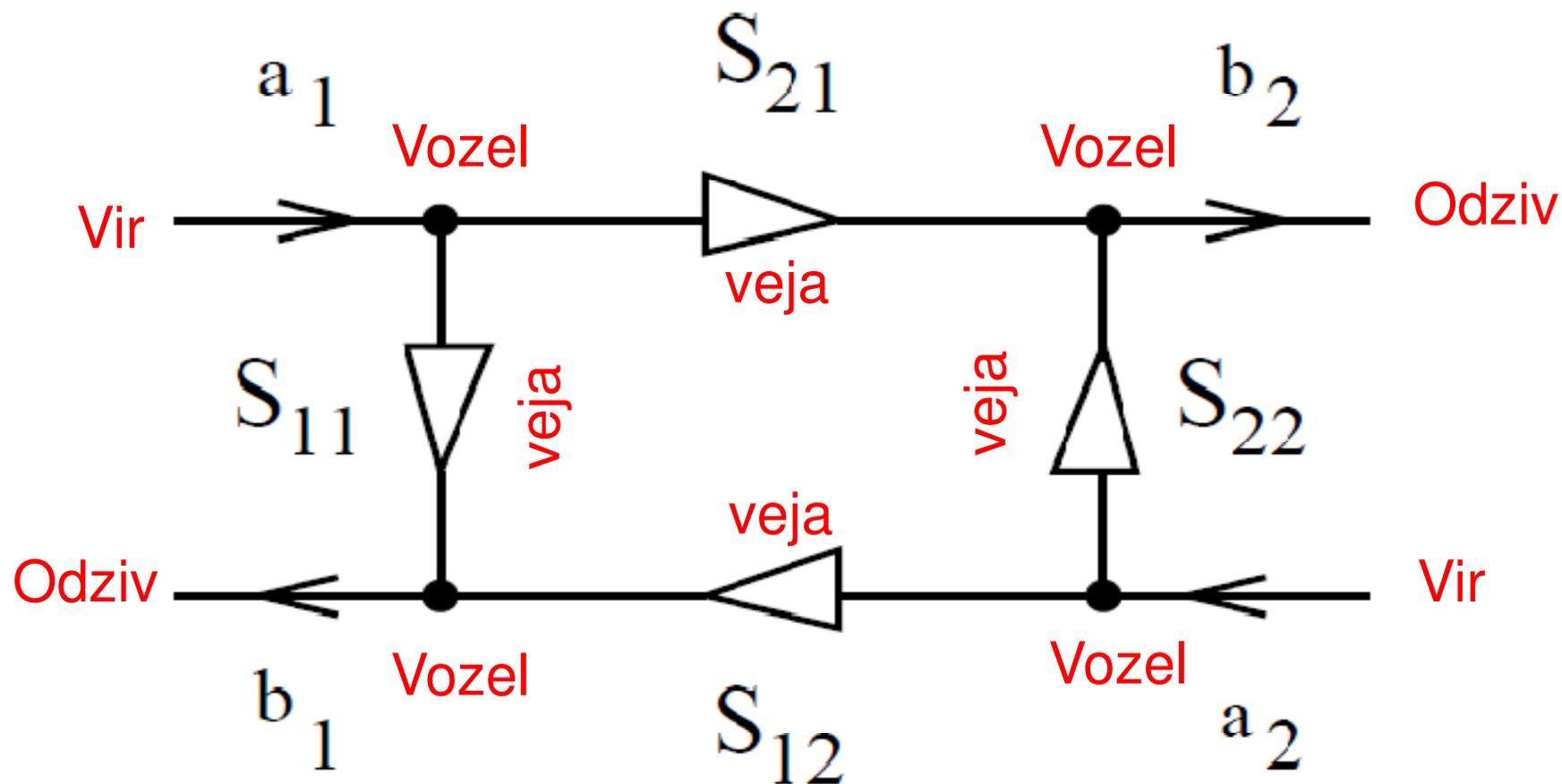
Kvadrat absolutne vrednosti potujočega vala predstavlja njegovo moč.

# S-parametri 4-polnega vezja



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

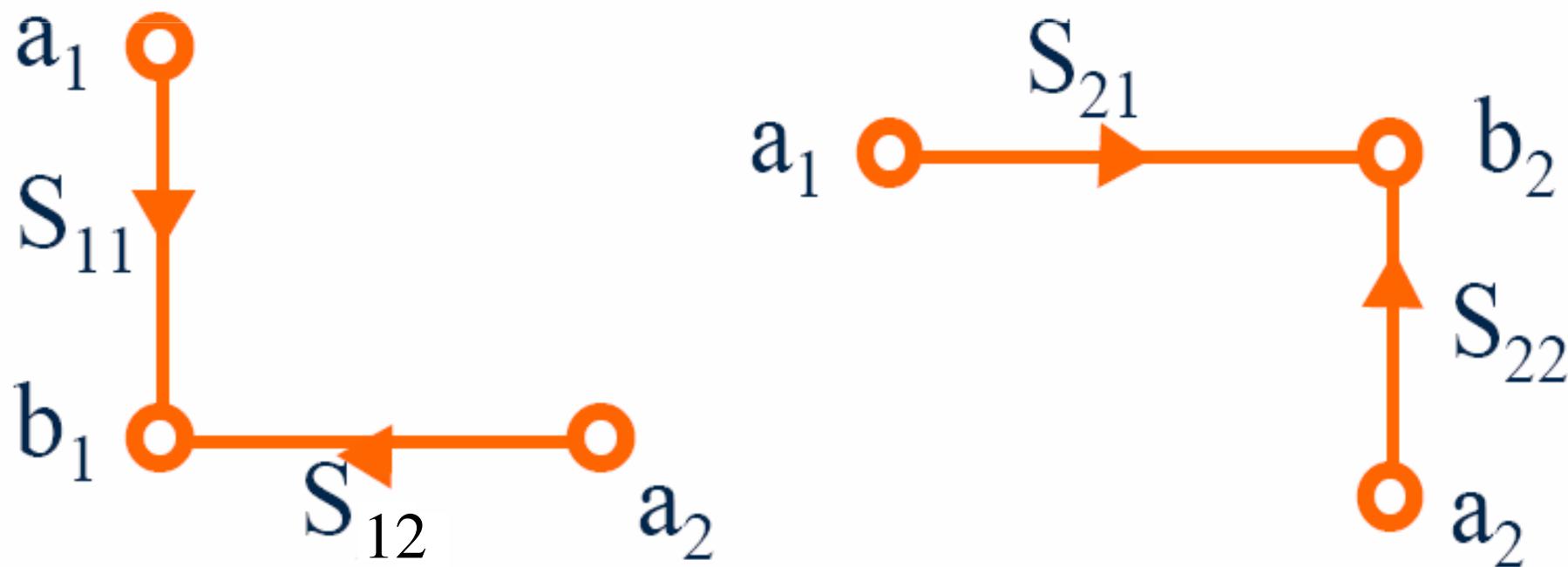
# Elementi smernega grafa štiripolnega vezja



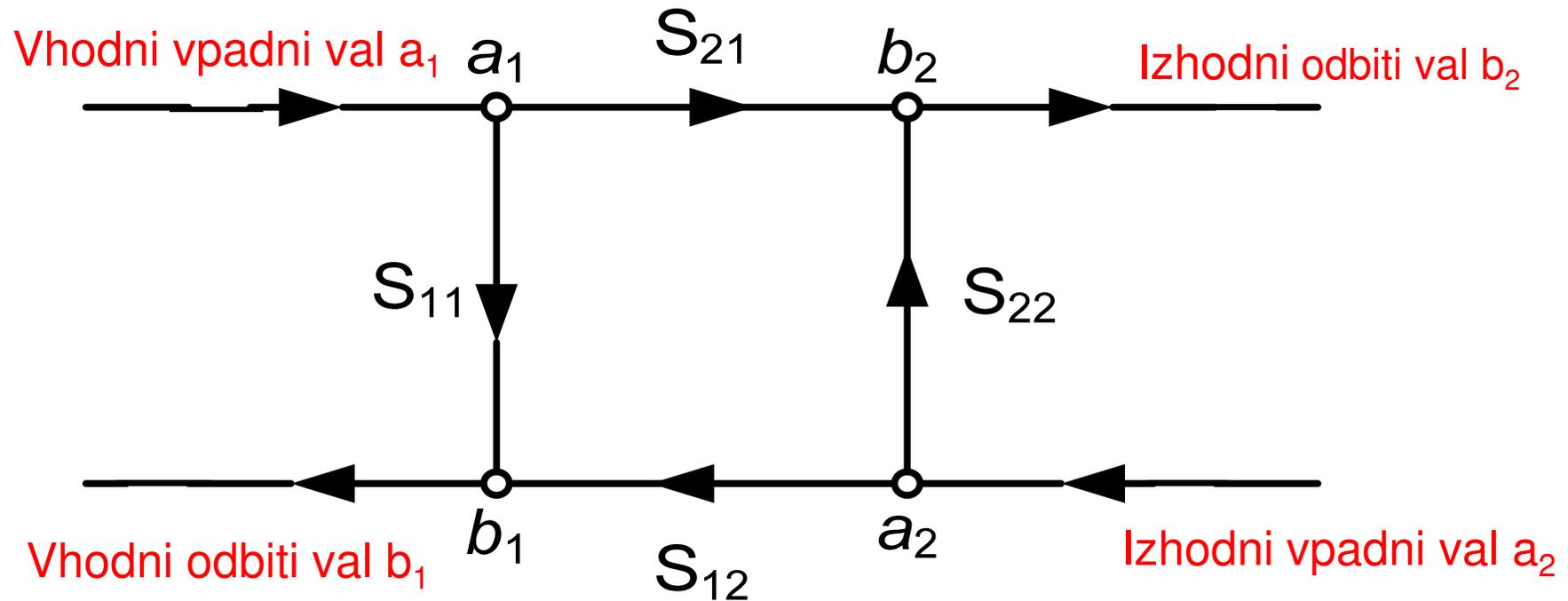
# Grafična predstavitev linearnih enačb

$$b_1 = a_1 S_{11} + a_2 S_{12}$$

$$b_2 = a_1 S_{21} + a_2 S_{22}$$



# 4-polno vezje, graf in enačbe



$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

# Definicije parametrov matrike S

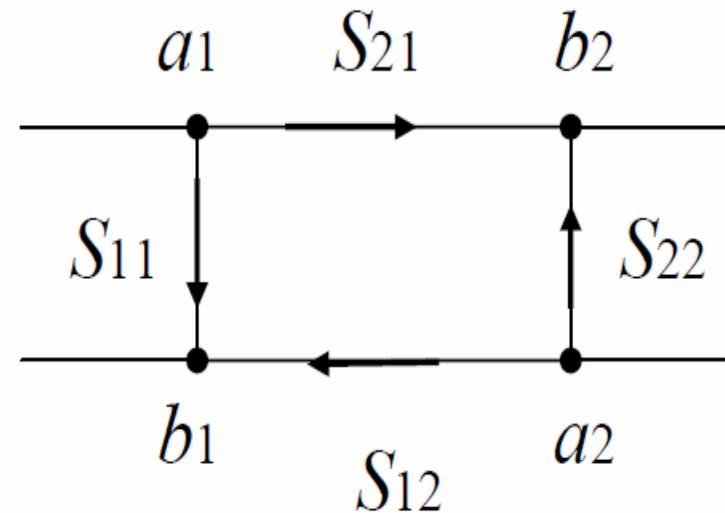
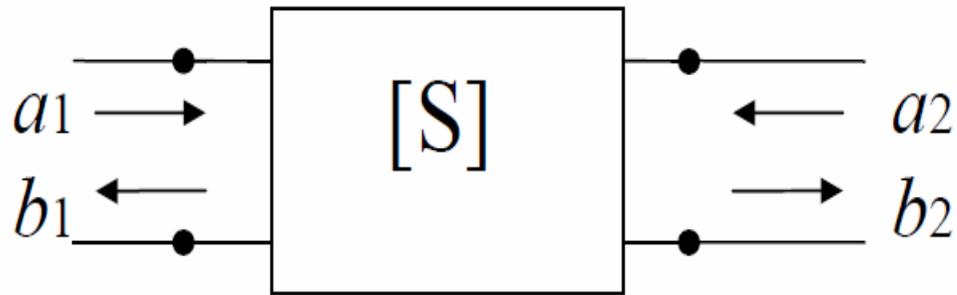
$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = \frac{\text{odbiti val na vhodu}}{\text{vpadni val na vhodu}} \quad \text{pri prilagojenem izhodu}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} = \frac{\text{odbiti (prenešeni) val na izhodu}}{\text{vpadni val na vhodu}} \quad \text{pri prilagojenem izhodu}$$

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} = \frac{\text{odbiti val na izhodu}}{\text{vpadni val na izhodu}} \quad \text{pri prilagojenem vhodu}$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} = \frac{\text{odbiti (prenešeni) val na vhodu}}{\text{vpadni val na izhodu}} \quad \text{pri prilagojenem vhodu}$$

# 4-polno vezje, graf in matrične enačbe



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$b_1 = a_1 S_{11} + a_2 S_{12}$$

$$b_2 = a_1 S_{21} + a_2 S_{22}$$

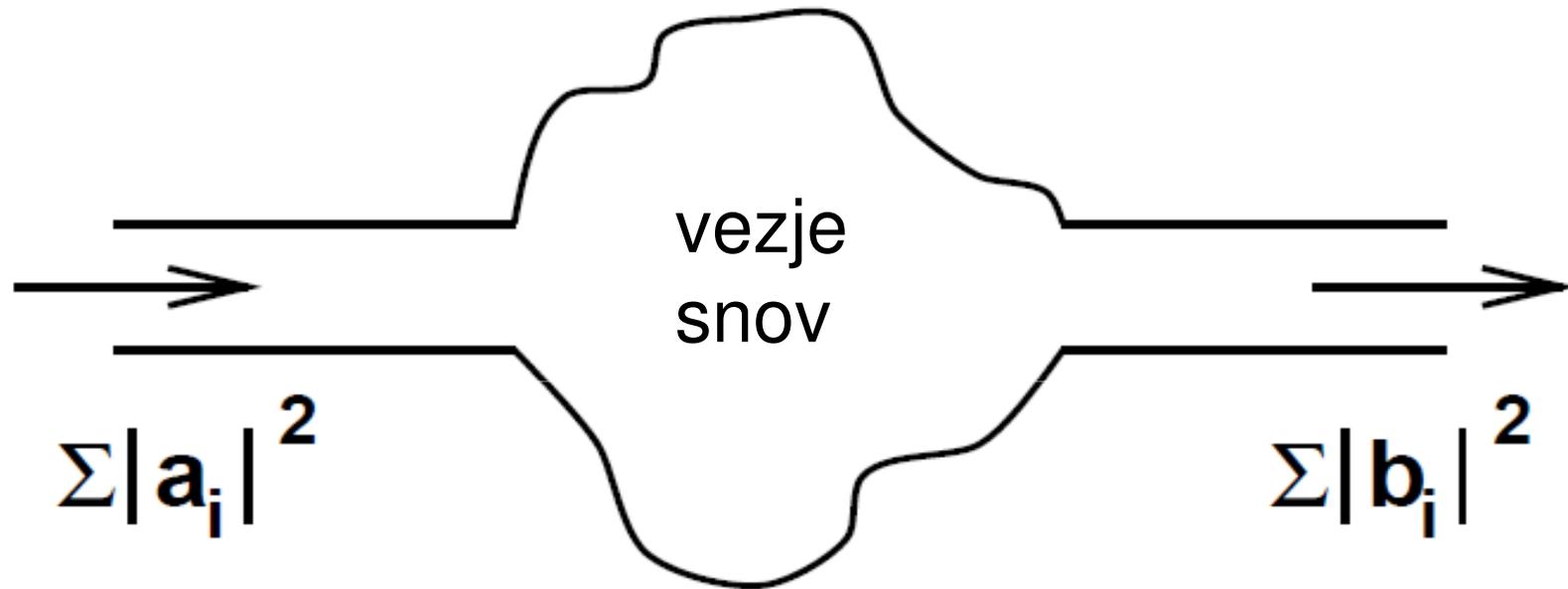
Odbojno slabljenje v dB:

$$-20 \log \left| \frac{b_1}{a_1} \right| = -20 \log |S_{11}|$$

Prehodno slabljenje v dB:

$$-20 \log \left| \frac{b_2}{a_1} \right| = -20 \log |S_{21}|$$

# Vezje brez izgub



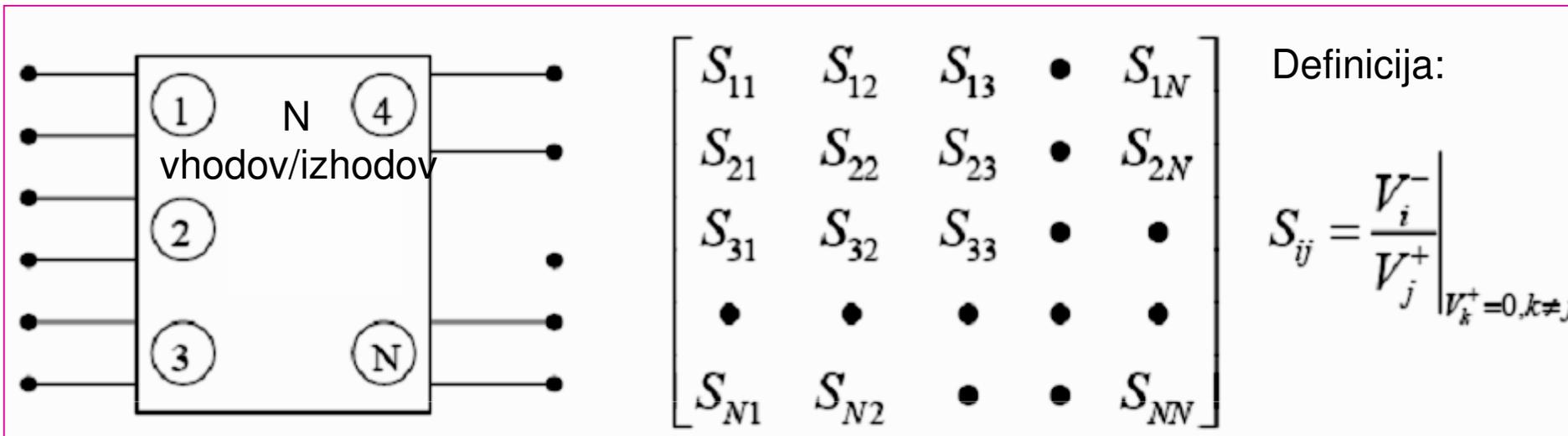
$$\sum_{i=1}^n |a_i|^2 = \sum_{i=1}^n |b_i|^2$$

Vsota moči dotekajočih valov v vezje je enaka vsoti moči odtekajočih valov iz vezja.

# Temeljne lastnosti vezij in matrice [S]

2N-polno vezje:

Matrika [S] 2N-polnega vezja:

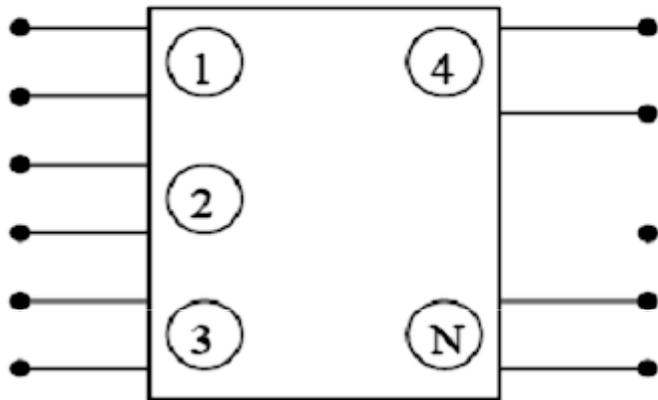


Temeljne lastnosti vezja in lastnosti matrice:

1. Notranja prilagojenost:  $S_{ii} = 0;$
2. Recipročnost:  $[S] = [S]^T, \text{ ali } S_{ij} = S_{ji}$
3. Brezizgubnost (unitarnost):  $([S]^*)^T [S] = [1],$
4. Brezizgubnost in recipročnost:  $[S]^* = [S]^{-1}$

# Splošne lastnosti matrike [S] N-polnega vezja

N-polno vezje:



Matrika S N-polnega vezja:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & \bullet & S_{1N} \\ S_{21} & S_{22} & S_{23} & \bullet & S_{2N} \\ S_{31} & S_{32} & S_{33} & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ S_{N1} & S_{N2} & \bullet & \bullet & S_{NN} \end{bmatrix}$$

Definicija:

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0, k \neq j}$$

Lastnosti vezja in matrike:

1. Notranja prilagojenost
2. Recipročnost
3. Brezizgubnost

$$S_{ii} = 0$$

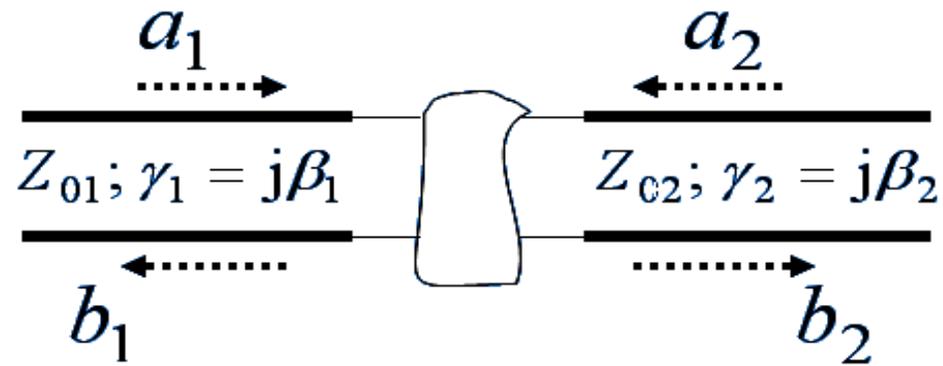
$$S_{ij} = S_{ji}$$

$$S S^{*T} = 1$$

unitarnost

# 4-polno recipročno vezje brez izgub

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



Linearno recipročno vezje

Dvopolno vezje brez izgub

Štiripolno vezje brez izgub

$$S_{12} = S_{21}$$

$$S = \Gamma = e^{j\phi} \rightarrow S \cdot S^* = 1$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{12}^* & S_{22}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{S} \cdot \mathbf{S}^* = \mathbf{1}$$

$\mathbf{S}$  unitarna matrika

$$S_{11}S_{11}^* + S_{12}S_{12}^* = 1 \rightarrow |S_{11}|^2 + |S_{12}|^2 = 1$$

$$S_{11}S_{12}^* + S_{12}S_{22}^* = 0 \rightarrow e^{j(\phi_{11}-\phi_{12})} + e^{j(\phi_{12}-\phi_{22})} = 0$$

$$S_{12}S_{11}^* + S_{22}S_{12}^* = 0$$

$$S_{12}S_{12}^* + S_{22}S_{22}^* = 1 \rightarrow |S_{12}|^2 + |S_{22}|^2 = 1$$

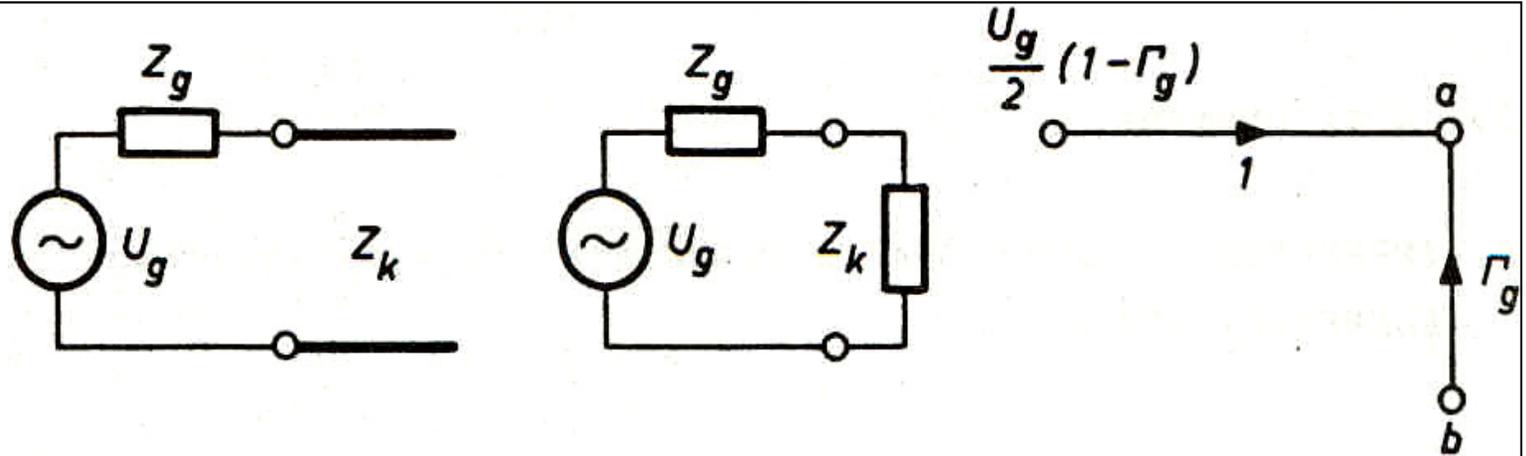
$$|S_{11}| = |S_{22}| = \sqrt{1 - |S_{12}|^2}$$

$$|S_{11}| = |S_{22}| = 0 \rightarrow |S_{12}| = 1$$

$$|S_{11}| = |S_{22}| \neq 0 \rightarrow \phi_{11} + \phi_{22} - 2\phi_{12} = \pi$$

# Smerni graf generatorja in bremena

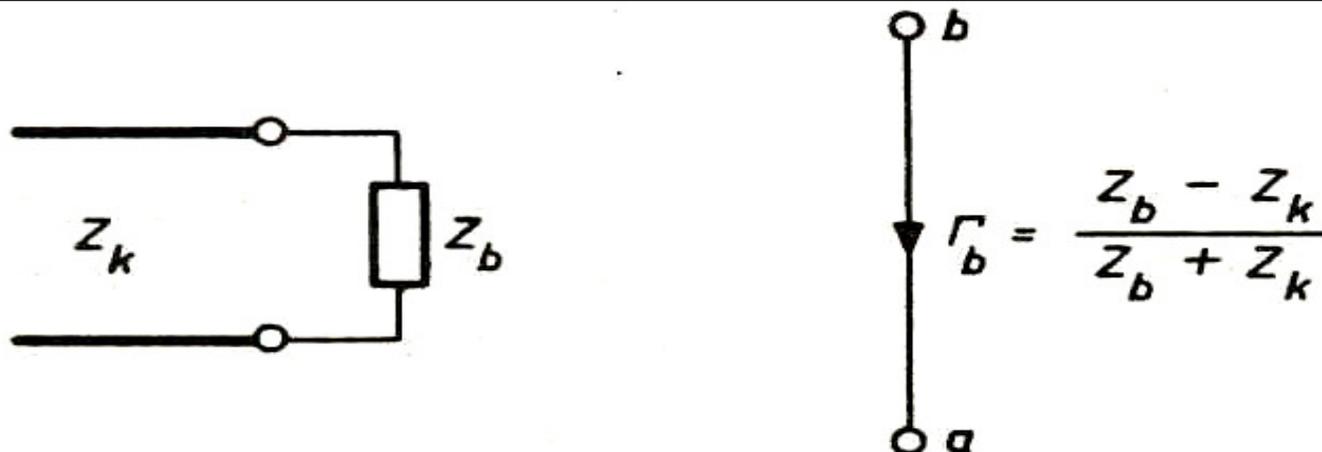
Graf generatorja



$$a = U_g \frac{Z_k}{Z_g + Z_k} = \frac{U_g}{2} (1 - \Gamma_g)$$

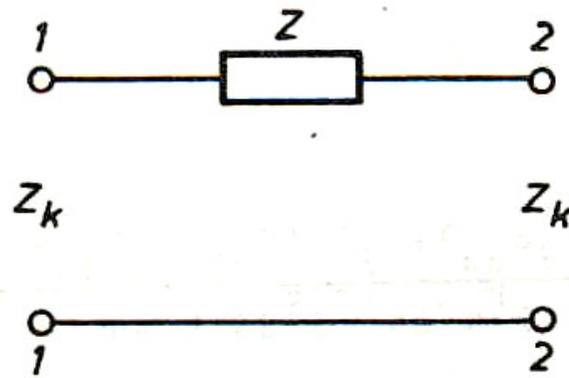
$$a = \Gamma_g b = \frac{Z_g - Z_k}{Z_g + Z_k} b$$

Graf bremena



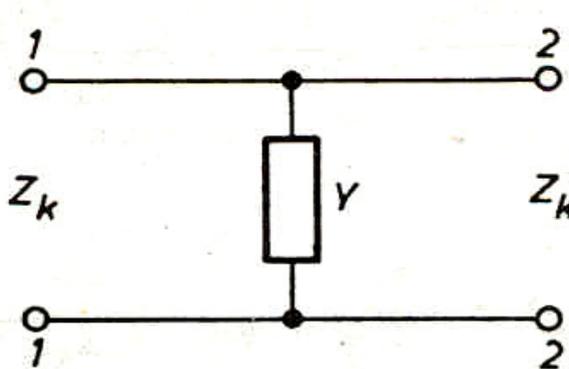
# Graf zaporedne impedance in vzporedne admittance

Zaporedna  
impedanca na  
liniji  
karakteristične  
impedance  $Z_k$



$$S_{11} = \frac{(Z + Z_k) - Z_k}{(Z + Z_k) + Z_k} = \frac{Z}{Z + 2Z_k}$$

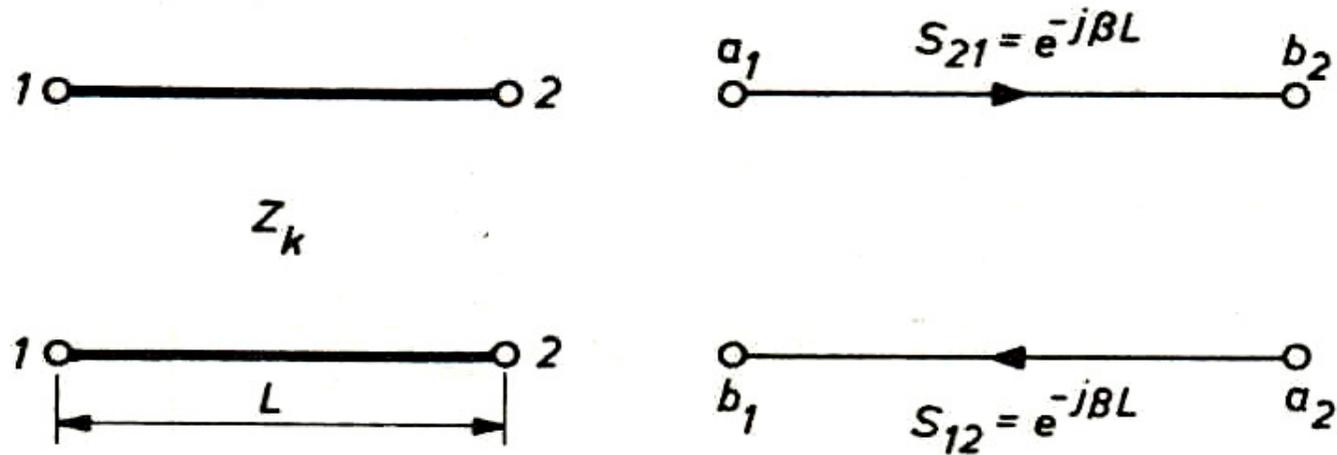
Vzporedna  
admitanca na  
liniji  
karakteristične  
impedance  $Z_k$



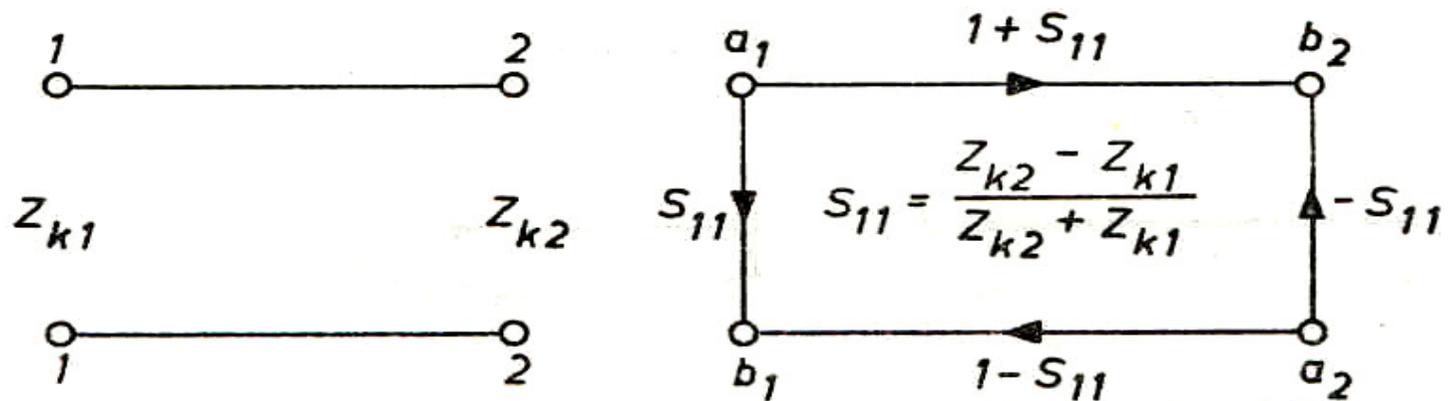
$$S_{11} = \frac{Y_k - (Y + Y_k)}{Y_k + (Y + Y_k)} = -\frac{Y}{Y + 2Y_k}$$

# Graf odseka linije in preskoka karakt. impedance

Odsek  
homogene  
linije

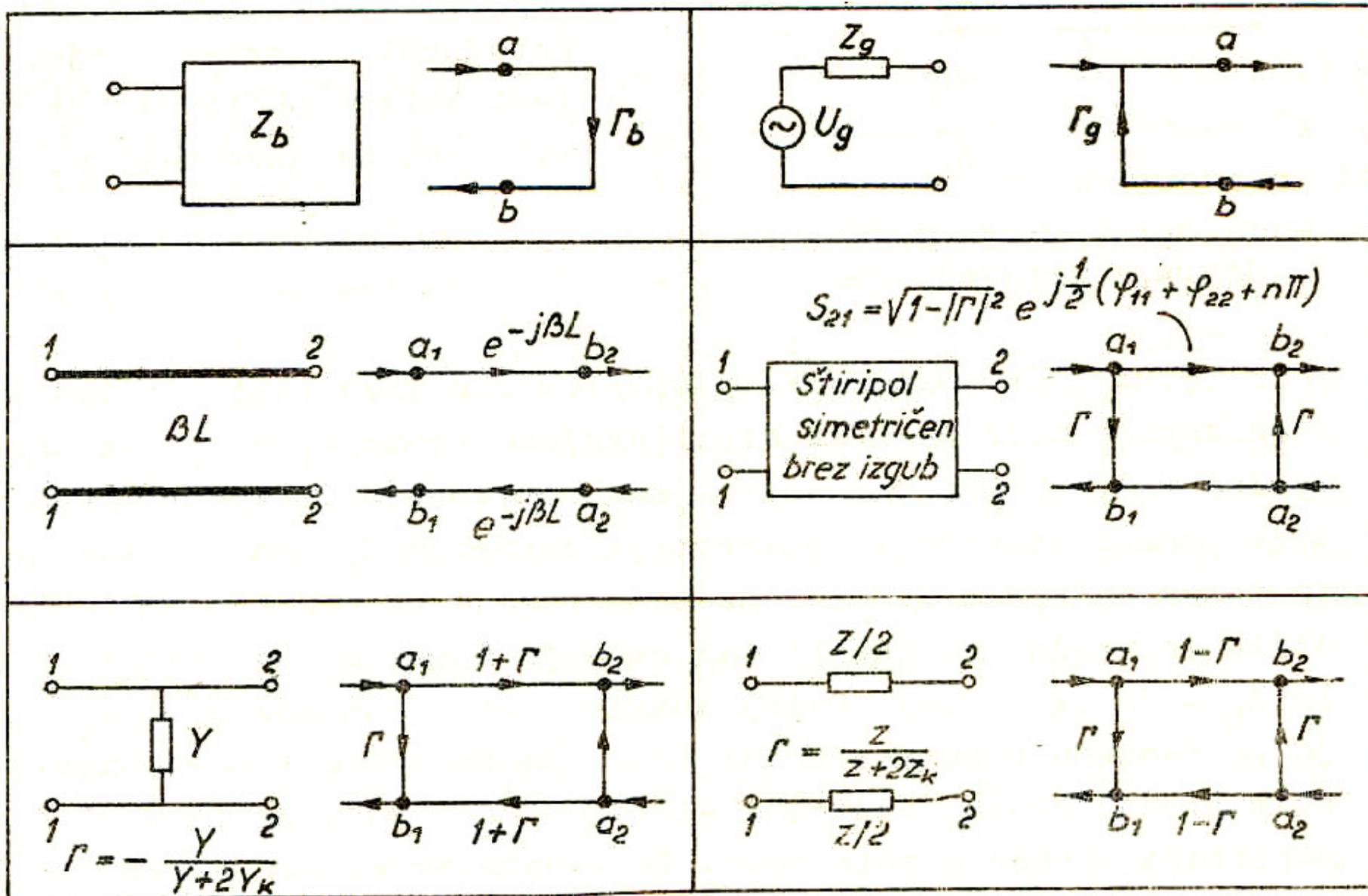


Preskok  
karakteristične  
impedance

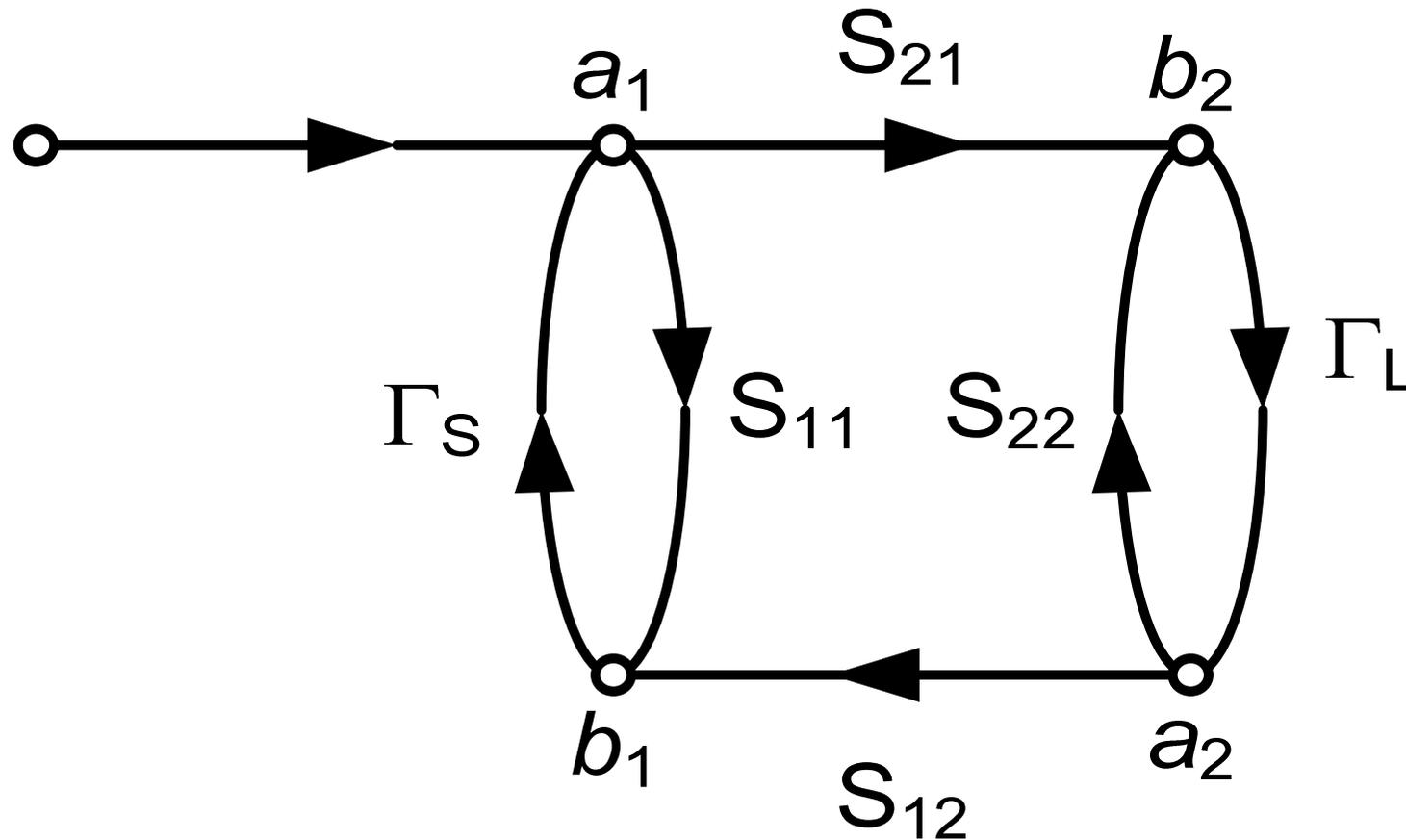


Napetost in tok prehajata zvezno.

# Smerni grafi osnovnih gradnikov, pregled



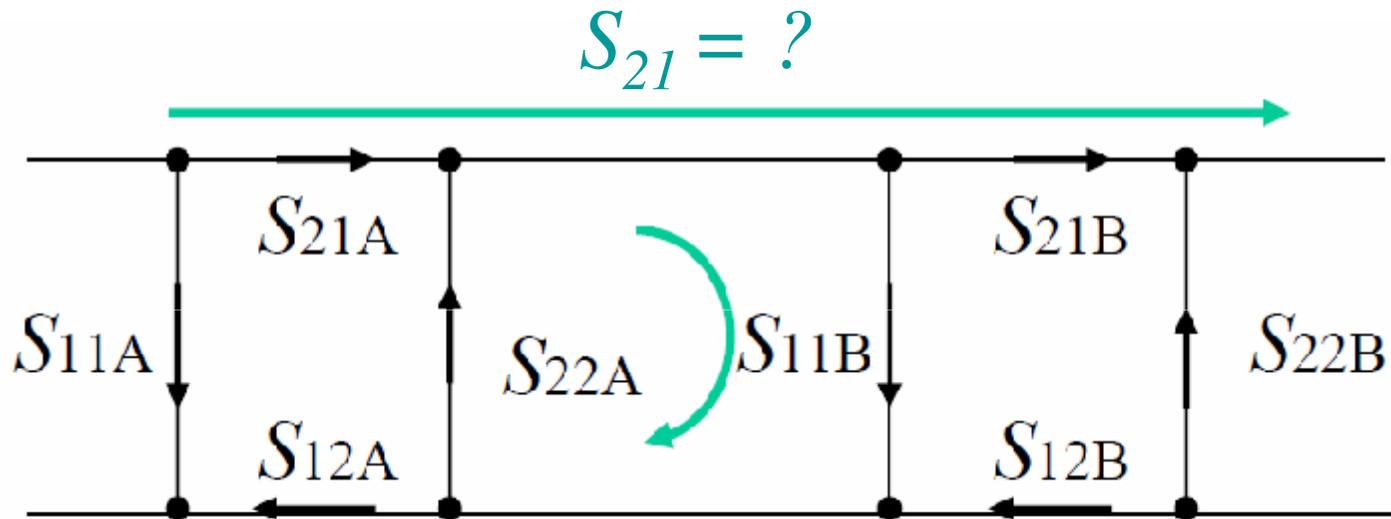
# Smerni graf generatorja, vezja in bremena



Na stiku generatorja z vezjem se pojavlja zanka s koeficientom  $\Gamma_s S_{11}$ . Na stiku vezja in bremena se pojavlja druga zanka s koeficientom  $S_{22} \Gamma_L$ .

# Kaskadna vezava čveropolo

Čveropola A in B, vezana v verigo, sestavljata na stiku odbojno zanko, ki povzroča povratni sklop.



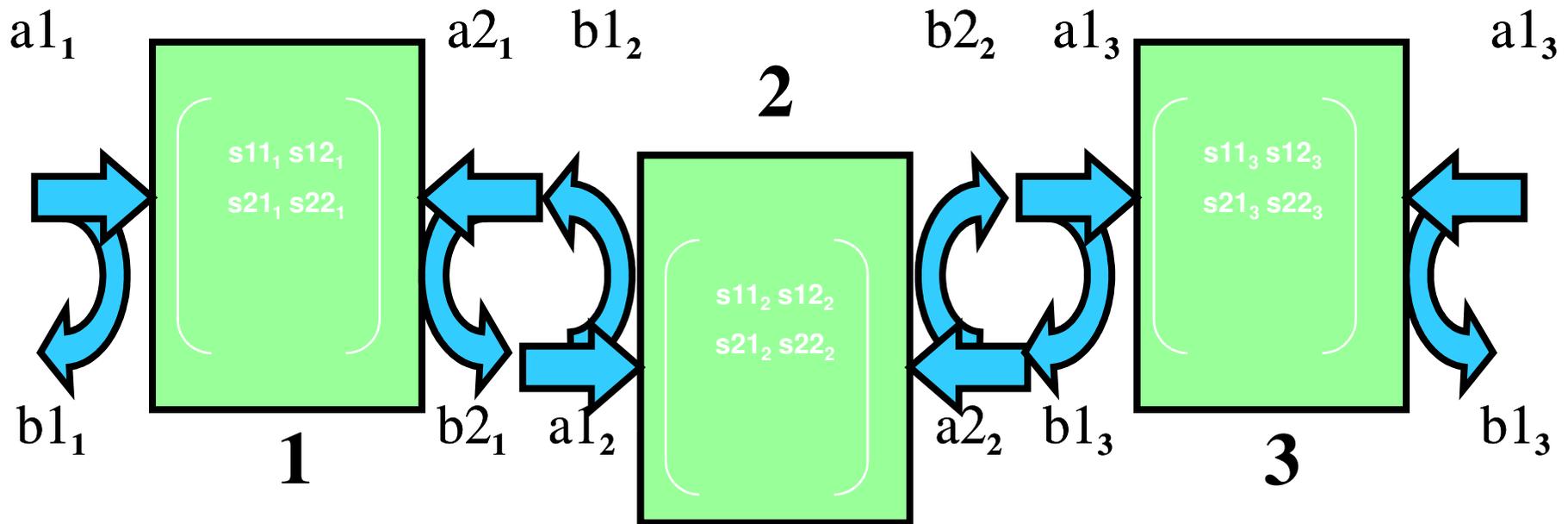
Povratni sklop:  
odvisnost  $1/(1-k)$

$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}$$

Koeficient povratnega sklopa:

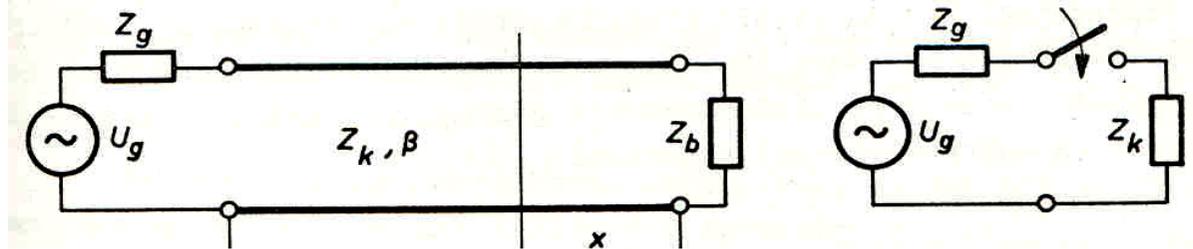
$$k = S_{22}^A S_{11}^B$$

# Povratne zanke na verigi četveropolov



Slika prikazuje tipični valovni pojav, ki nastaja na kaskadni vezavi četveropolov. Vezje obravnavamo z valovno matriko  $[S]$ . Za predstavitev problemov in izpeljavo formul uporabljamo smerne grafe zaradi preglednosti in predstavljalivosti valovnih pojavov.

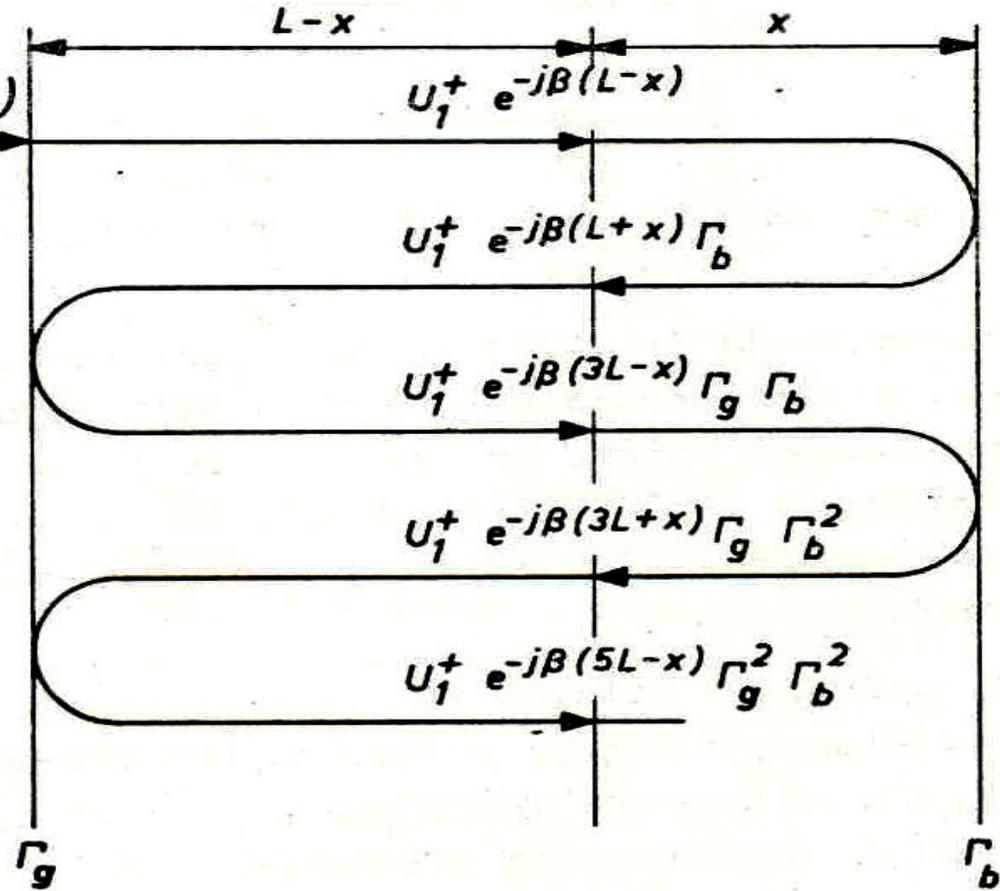
# Valovni pojavi na liniji, valovna obravnava



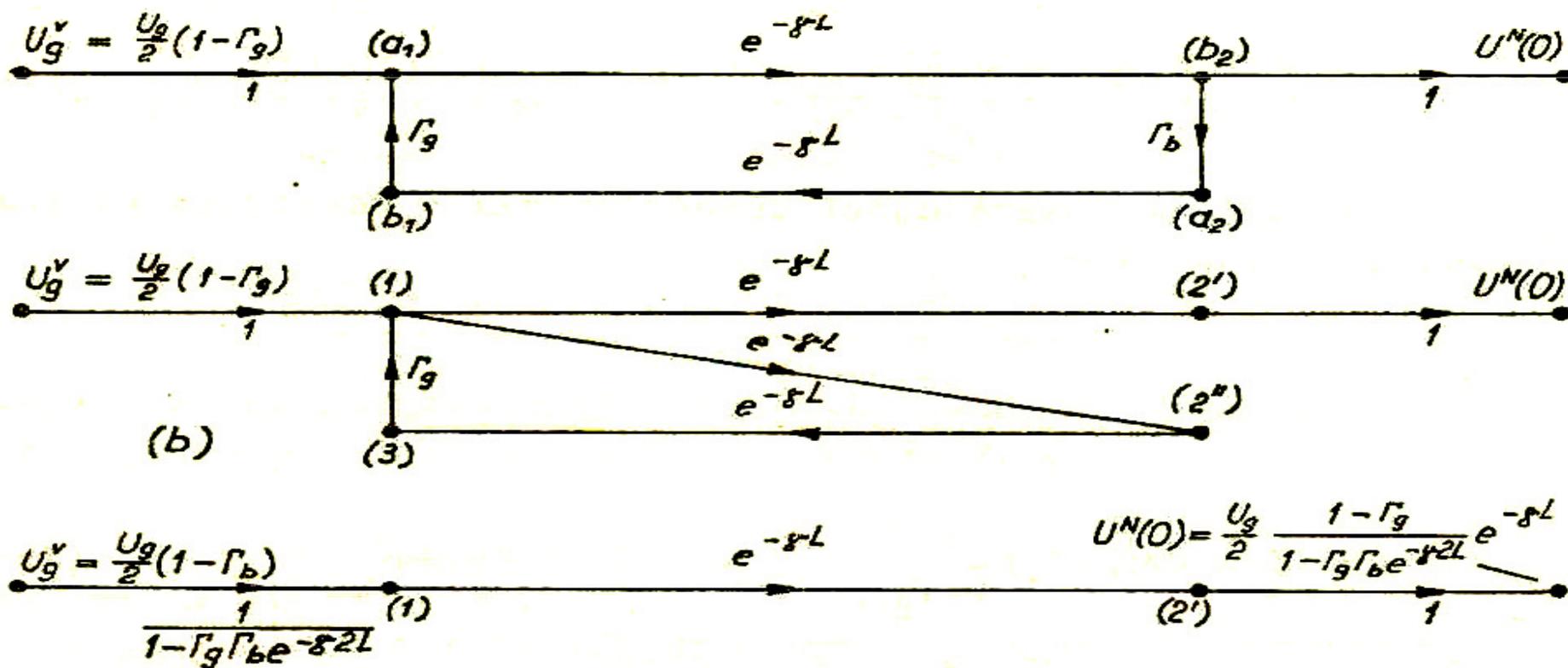
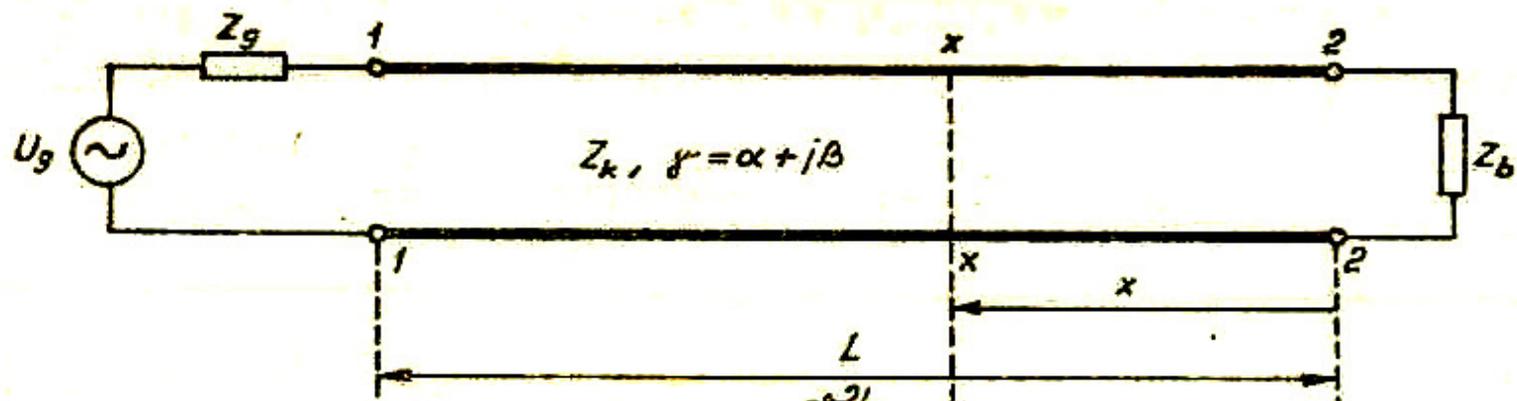
$$U_1^+ = \frac{U_g}{2} (1 - \Gamma_g)$$

$$U^+(x) = U_1^+ e^{-j\beta(L-x)} (1 + \Gamma_g \Gamma_b e^{-j\beta 2L} + \Gamma_g^2 \Gamma_b^2 e^{-j\beta 4L} + \dots)$$

$$U^+(x) = U_1^+ \frac{e^{-j\beta(L-x)}}{1 - \Gamma_g \Gamma_b e^{-j\beta 2L}}$$

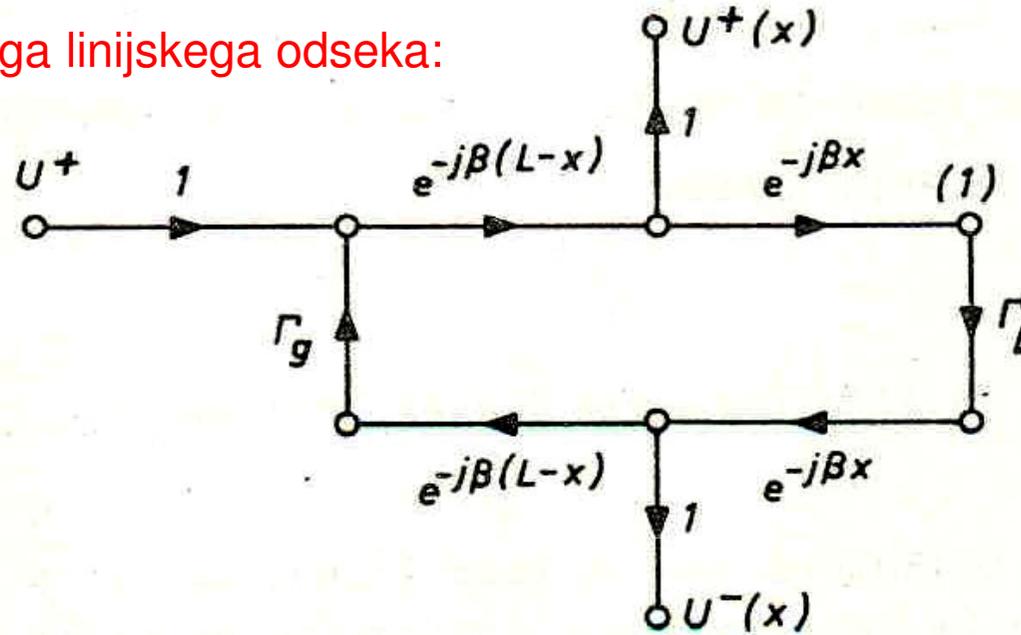


# Smerni graf napredujočega vala na liniji



# Valovni pojavi na liniji, obravnava z grafom

Graf obremenjenega linijskega odseka:



Napredujoči val:

$$U^+(x) = \frac{U_g}{2} (1 - \Gamma_g) \frac{e^{-j\beta(L-x)}}{1 - \Gamma_g \Gamma_b e^{-j\beta 2L}}$$

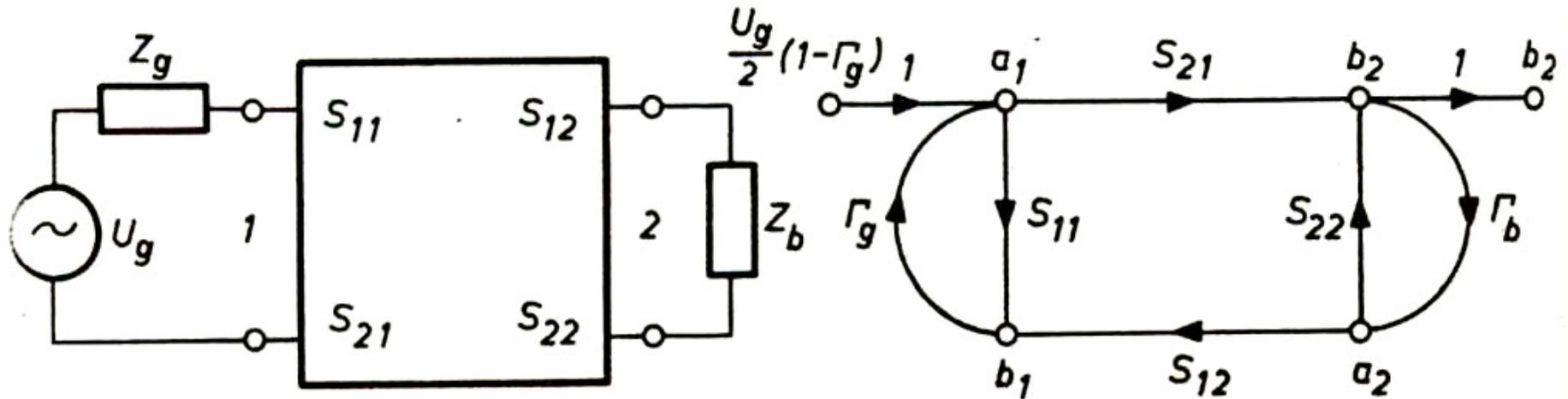
Odbiti val:

$$U^-(x) = \frac{U_g}{2} (1 - \Gamma_g) \frac{e^{-j\beta(L+x)}}{1 - \Gamma_g \Gamma_b e^{-j\beta 2L} \Gamma_b}$$

Stojni val:

$$U(x) = U^+(x) + U^-(x) = \frac{U_g}{2} (1 - \Gamma_g) \frac{e^{-j\beta(L-x)}}{1 - \Gamma_g \Gamma_b e^{-j\beta 2L}} (1 + \Gamma_b e^{-j\beta 2x})$$

# Slabljenje štiripolnega vezja

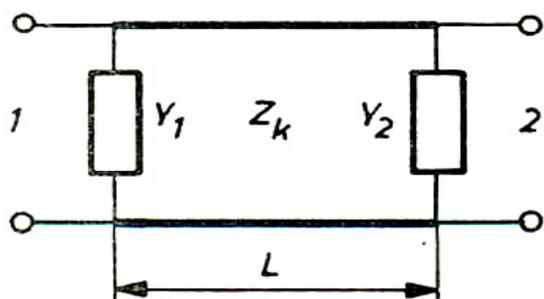


$$b_2 = \frac{U_g}{2} (1 - \Gamma_g) \frac{S_{21}}{(1 - \Gamma_g S_{11})(1 - \Gamma_b S_{22}) - \Gamma_g \Gamma_b S_{21} S_{12}}$$

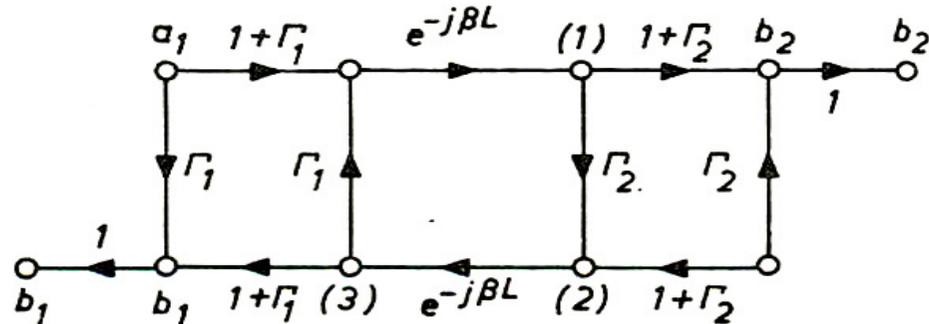
Slabljenje  $L = 20 \log |b_2/a_1|$  je enako  $20 \log |S_{21}|$  v primeru, ko sta vhod in izhod vezja prilagojena ( $\Gamma_g = \Gamma_b = 0$ )

# Vzporedni admitanci na linijskem odseku

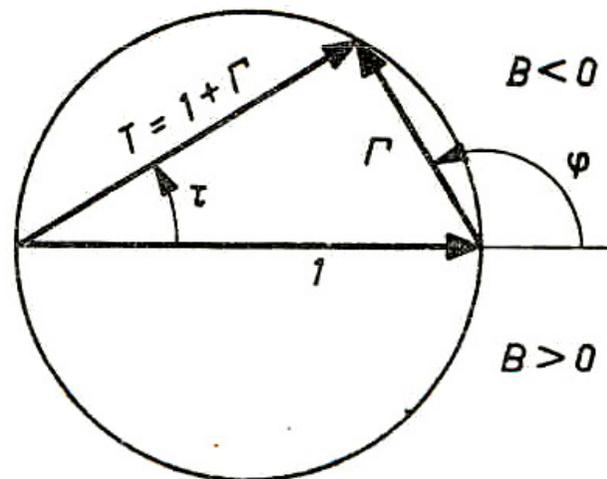
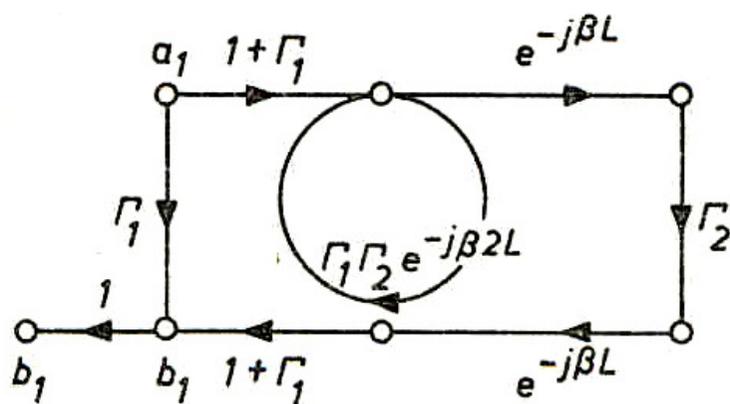
Vezje:



Smerni graf:



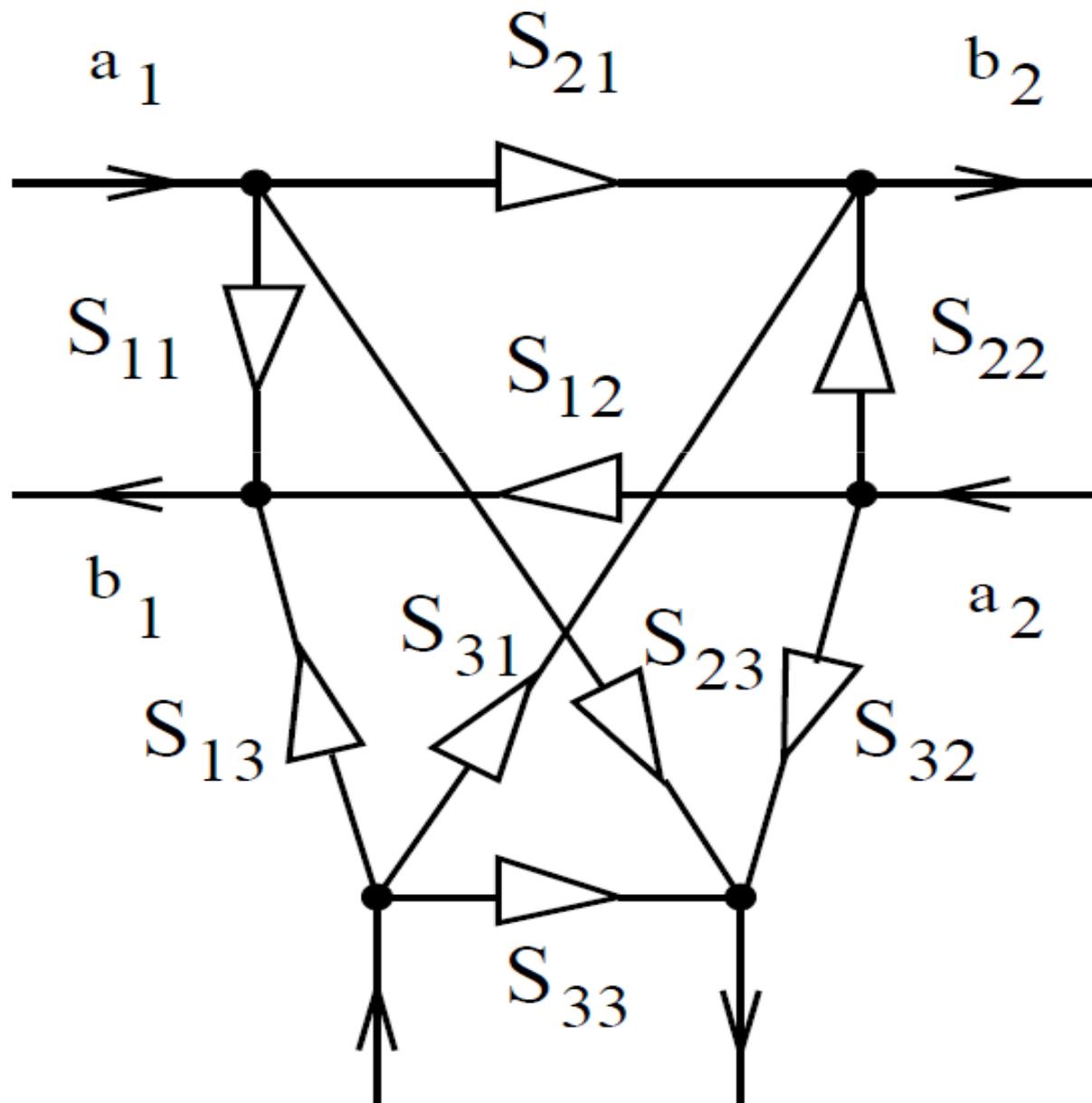
Reduciran smerni graf:



$$S_{11} = \frac{b_1}{a_1} = \Gamma_1 + \frac{(1 + \Gamma_1)^2 \Gamma_2 e^{-j\beta 2L}}{1 - \Gamma_1 \Gamma_2 e^{-j\beta 2L}}$$

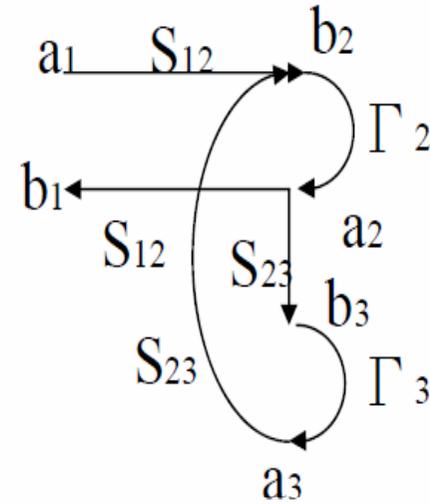
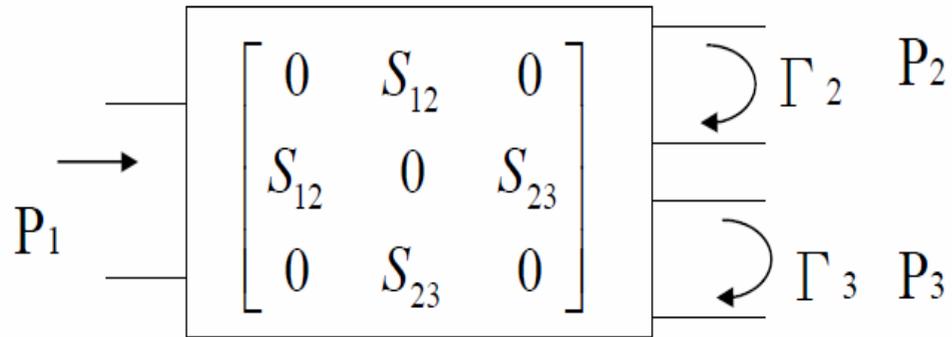
$$S_{21} = \frac{b_2}{a_1} = \frac{(1 + \Gamma_1)(1 + \Gamma_2) e^{-j\beta L}}{1 - \Gamma_1 \Gamma_2 e^{-j\beta 2L}}$$

# Smerni graf šestpolnega vezja



# Primer 6-polnega vezja

6-polno recipročno notranje prilagojeno vezje



$$b_1 = a_1 \frac{S_{12}^2 \Gamma_2}{1 - \Gamma_2 \Gamma_3 S_{23}^2}, b_2 = a_1 \frac{S_{12}}{1 - \Gamma_2 \Gamma_3 S_{23}^2}, b_3 = a_1 \frac{S_{12} \Gamma_2 S_{23}}{1 - \Gamma_2 \Gamma_3 S_{23}^2}$$

$$\frac{P_2}{P_1} = \frac{|b_2|^2 - |a_2|^2}{|a_1|^2 - |b_1|^2} = \frac{|b_2|^2 (1 - |\Gamma_2|^2)}{|a_1|^2 (1 - |\Gamma_{in}|^2)} = \frac{|S_{12}|^2 (1 - |\Gamma_2|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 \left(1 - \frac{|S_{12}^2 \Gamma_2|^2}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2}\right)} = \frac{|S_{12}|^2 (1 - |\Gamma_2|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 - |S_{12}^2 \Gamma_2|^2}$$

$$\frac{P_3}{P_1} = \frac{|b_3|^2 - |a_3|^2}{|a_1|^2 - |b_1|^2} = \frac{|b_3|^2 (1 - |\Gamma_3|^2)}{|a_1|^2 (1 - |\Gamma_{in}|^2)} = \frac{|S_{12}|^2 |\Gamma_2|^2 |S_{23}|^2 (1 - |\Gamma_3|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 \left(1 - \frac{|S_{12}^2 \Gamma_2|^2}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2}\right)} = \frac{|S_{12}|^2 |\Gamma_2|^2 |S_{23}|^2 (1 - |\Gamma_3|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 - |S_{12}^2 \Gamma_2|^2}$$

# Obravnava smernih grafov

## 1. Elementi grafov

- vozli
- veje
- zanke

## 2. Pravila dekompozicije (redukcije) grafov

- adicija
- multiplikacija
- pravilo povratne zanke
- distribucija

## 3. Masonovo pravilo nedotikajočih se zank

# Smerni graf in njegovi elementi

Sistem linearnih enačb:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + c_1 = x_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + c_2 = x_2$$

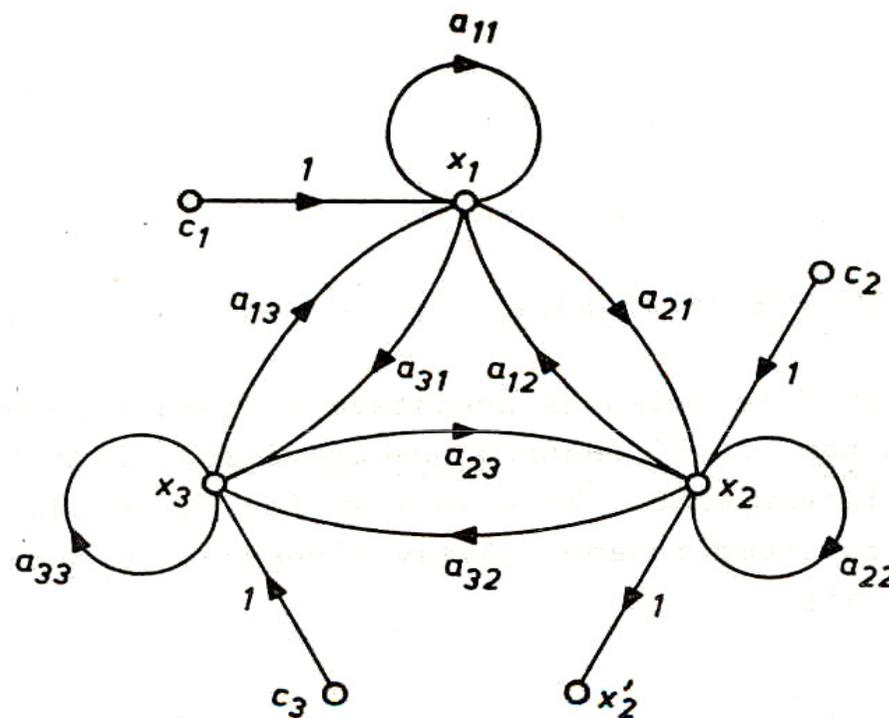
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + c_3 = x_3$$

$x_1, x_2, x_3 \dots$  spremenljivke

$a_{11}, a_{12}$  do  $a_{33} \dots$  koeficienti

$c_1, c_2, c_3 \dots$  konstante

Smerni graf



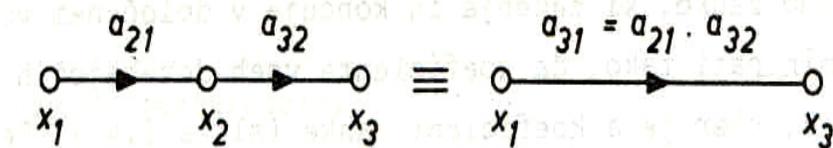
$x_1, x_2, x_3 \dots$  vozli smernega grafa

$a_{11}, a_{12}$  do  $a_{33} \dots$  koeficienti vej

$c_1, c_2, c_3 \dots$  viri smernega grafa

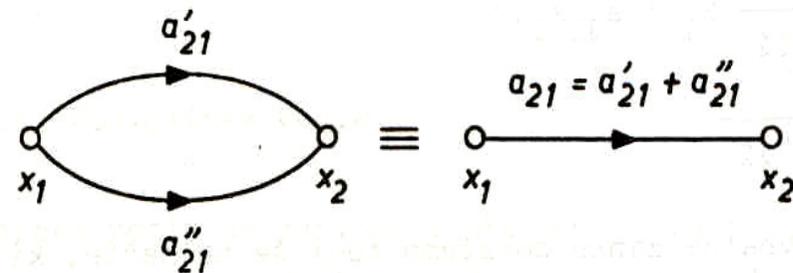
# Pravila redukcije smernega grafa

## 1. Multiplikacija



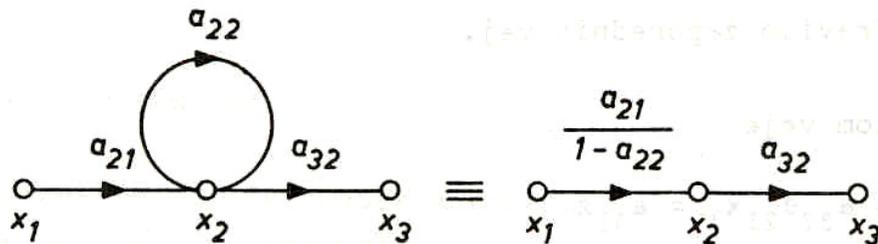
Koeficienti zaporednih vej se množijo.

## 2. Adicija



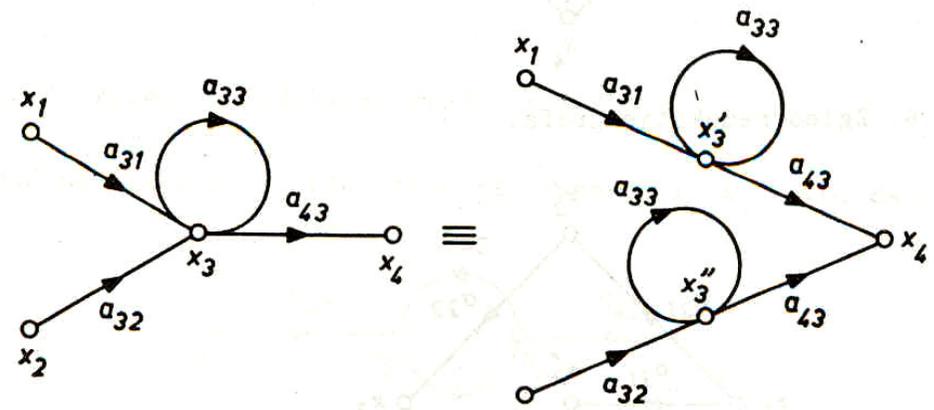
Koeficienti vzporednih vej se seštevajo.

## 3. Pravilo povratne zanke



Povratno zanko eliminiramo tako, da koeficient predhodne veje množimo z  $1/(1 - a_{22})$ , ki je vsota geometrijske vrste.

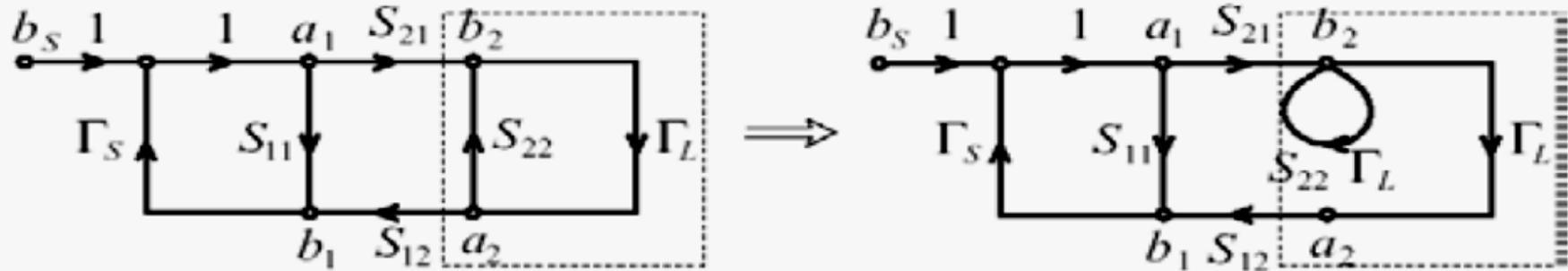
## 4. Distribucija



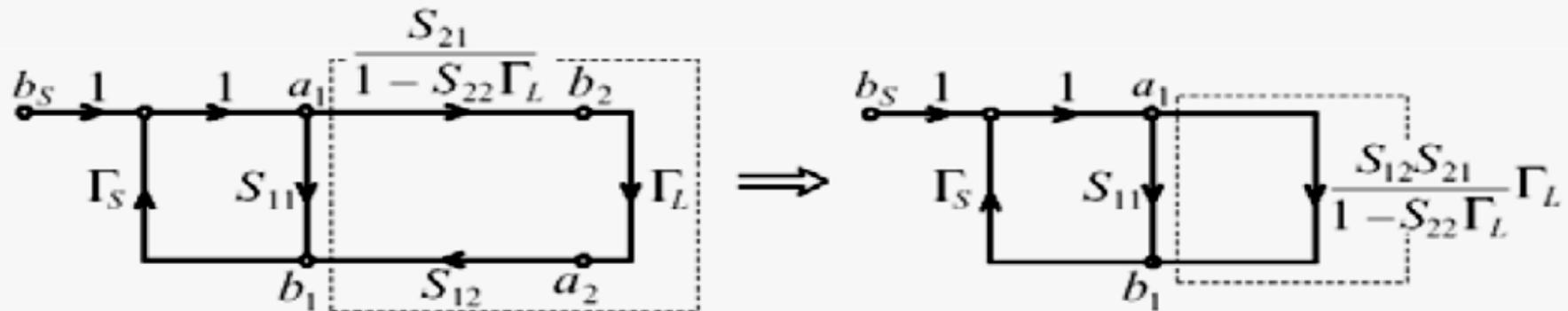
Vozel s povratno zanko dveh ali več dotekajočih vej podvojimo.

# Primer postopne redukcije grafa, 1/2

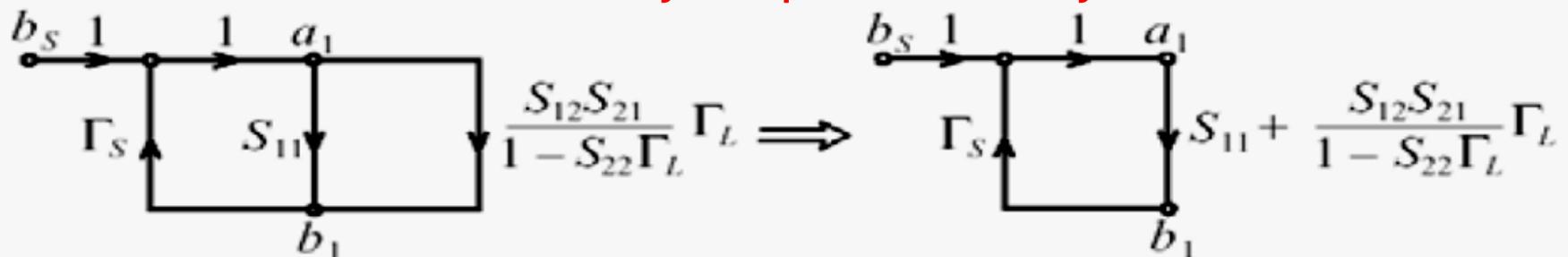
Korak 1: Podvojitv vozla  $a_2$ :



Korak 2: Odprava zanke in multiplikacija vej:

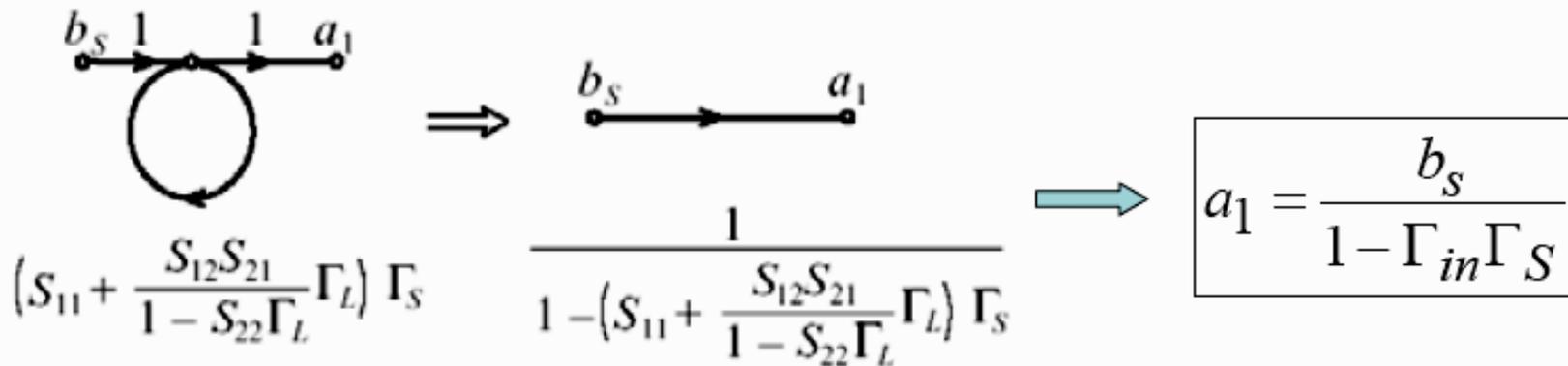
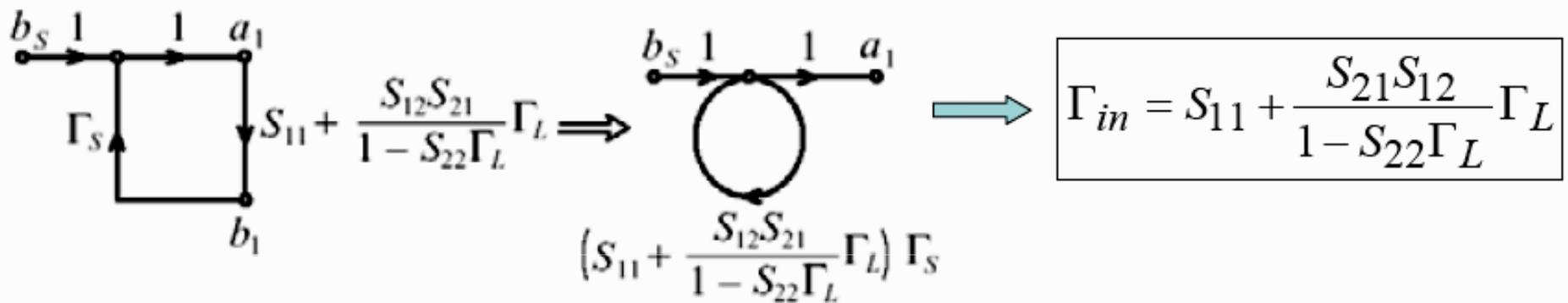


Korak 3: Adicija vzporednih vej:

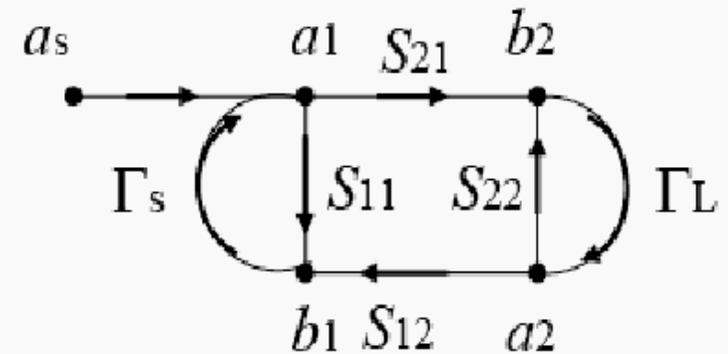
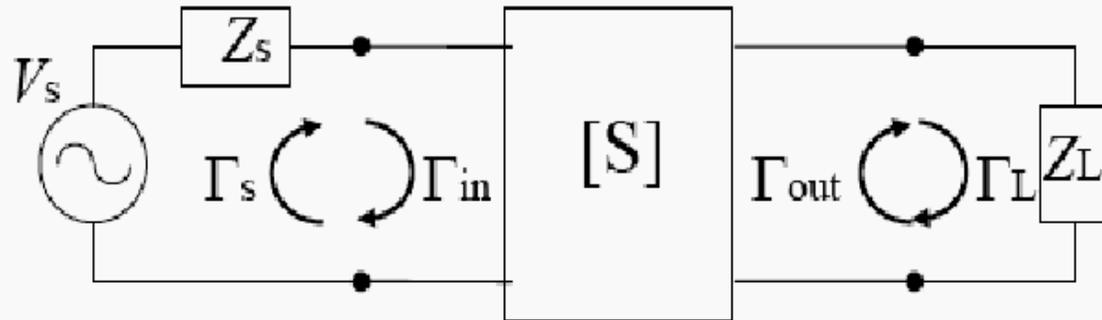


# Primer postopne redukcije grafa, 2/2

Korak 4: Podvojitv a<sub>1</sub> in določitev a<sub>1</sub> ter b<sub>1</sub> (vhodna odbojnost):



# Vhodna in izhodna odbojnost, metoda redukcije

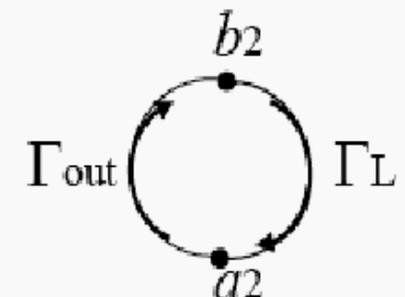
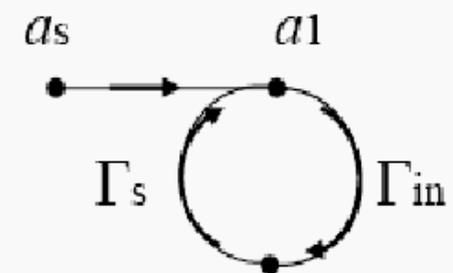


$$b_1 = a_1 S_{11} + a_1 S_{21} \Gamma_L S_{12} (1 + S_{22} \Gamma_L + \dots) = a_1 S_{11} + a_1 \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$\rightarrow \Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$b_2 = a_2 S_{22} + a_2 S_{12} \Gamma_S S_{21} (1 + S_{11} \Gamma_S + \dots) = a_2 S_{22} + a_2 \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$$



# Masonovo pravilo nedotikajočih se zank

$$T = \frac{T_1 [1 - \Sigma L(1)^{(1)} + \Sigma L(2)^{(1)} - \dots] + T_2 [1 - \Sigma L(1)^{(2)} + \dots] + \dots}{1 - \Sigma L(1) + \Sigma L(2) - \Sigma L(3) + \dots}$$

kjer pomenijo:

$T_1, T_2 \dots$  koeficienti poti  $T_m$ , ki povezujejo vozle grafa

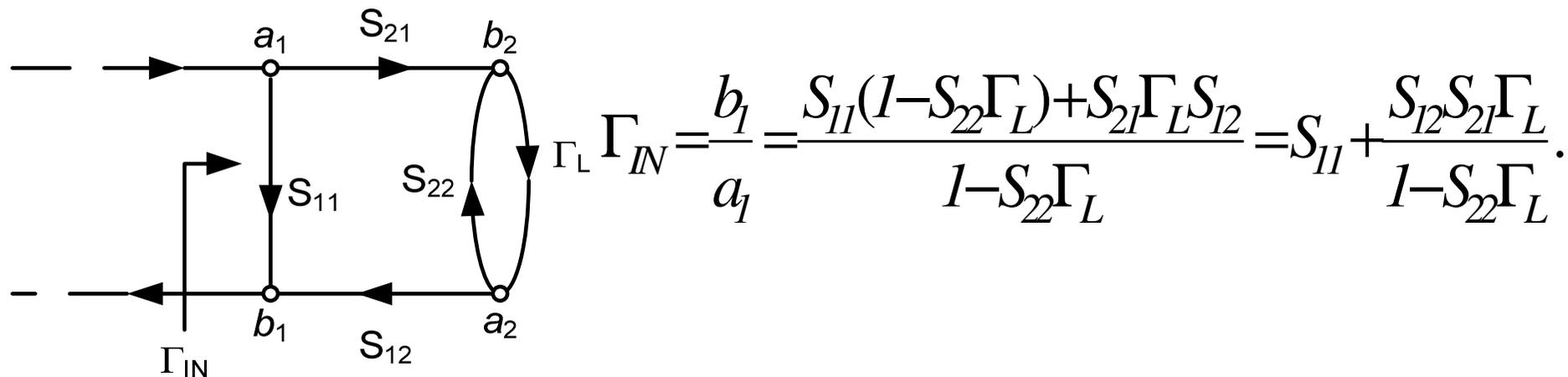
$L(1) \dots$  koeficient zanke prvega reda (produkt koeficientov vej, ki izhajajo iz vozla in se vanj vračajo)

$L(2) \dots$  koeficient zanke drugega reda (produkt koeficientov dveh nedotikajočih se zank prvega reda)

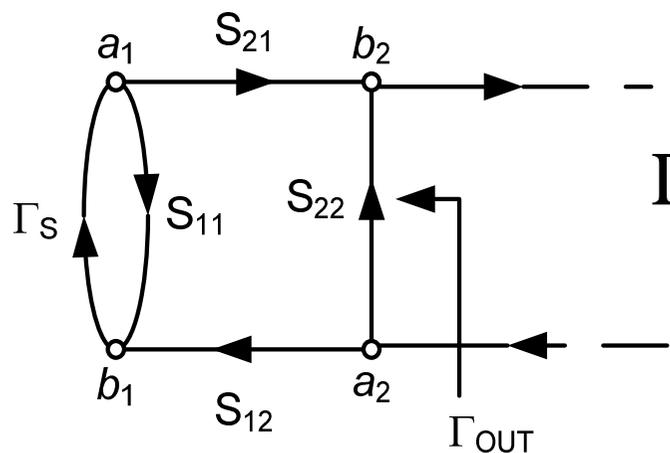
$L(3) \dots$  koeficient zanke tretjega reda (produkt koeficientov treh nedotikajočih se zank drugega reda)

$L(n)^{(T_m)}$  koeficient zanke  $n$ -tega reda, ki se ne dotika poti  $T_m$

# Vhodna in izhodna odbojnost 4-polnega vezja, Masonovo pravilo

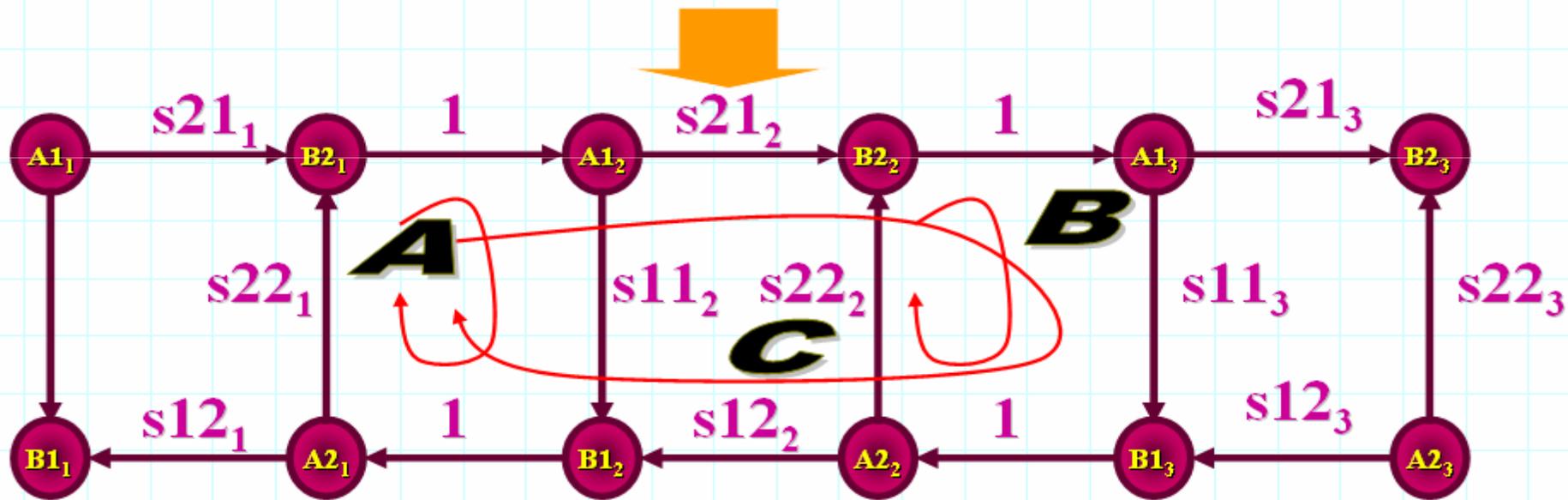
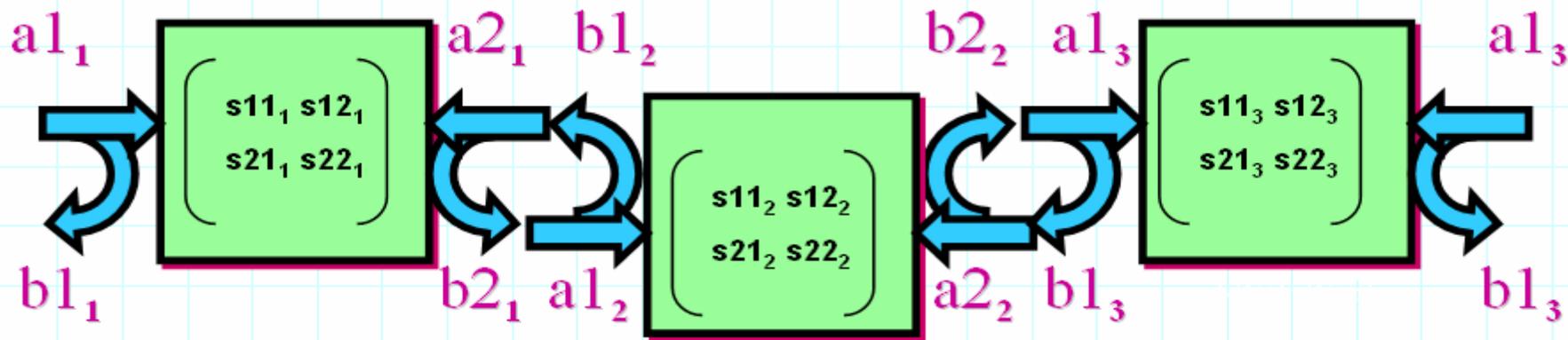


$$\Gamma_{IN} = \frac{b_1}{a_1} = \frac{S_{11}(1 - S_{22}\Gamma_L) + S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$



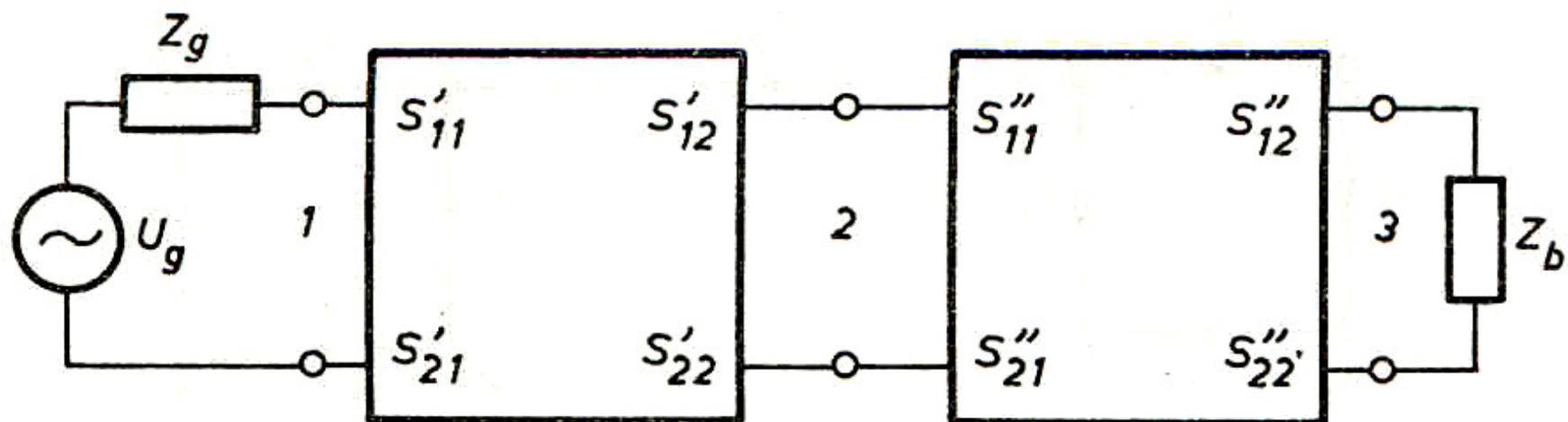
$$\Gamma_{OUT} = \frac{b_2}{a_2} = \frac{S_{22}(1 - S_{11}\Gamma_S) + S_{21}\Gamma_S S_{12}}{1 - S_{11}\Gamma_S} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

# Kaskadna vezava po Masonovem pravilu

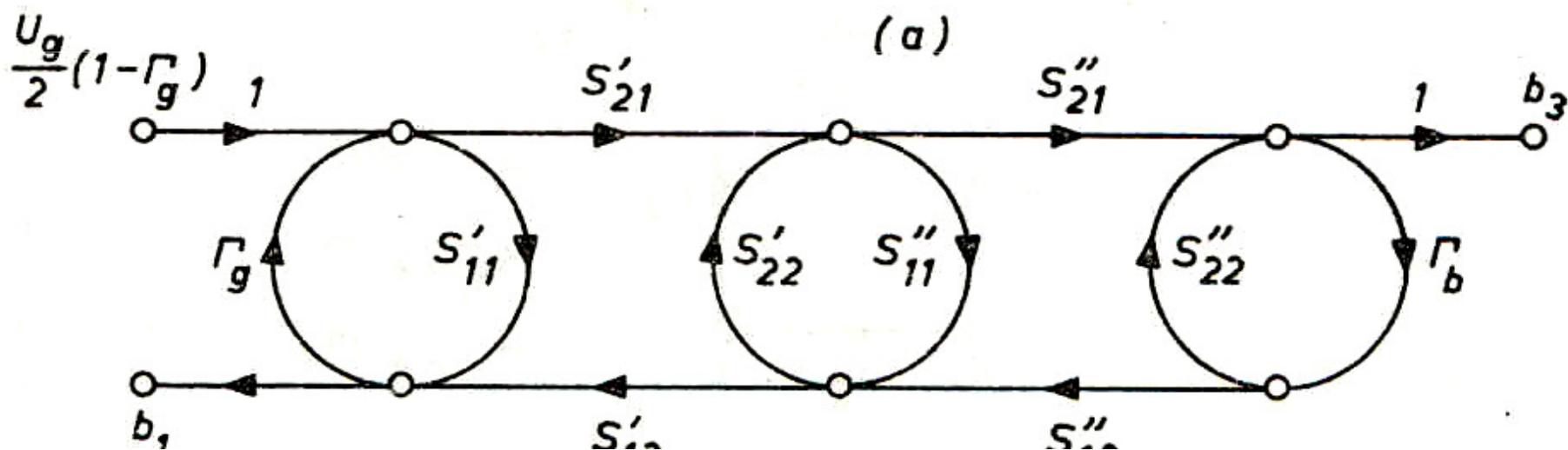


$$\frac{b_6}{a_1} = \frac{s_{21_1} \cdot s_{21_2} \cdot s_{21_3}}{1 - \left( \underbrace{s_{22_2} \cdot s_{11_1}}_B + \underbrace{s_{22_3} \cdot s_{11_2}}_C + \underbrace{s_{11_3} \cdot s_{22_1} \cdot s_{12_2} \cdot s_{21_2}}_A \right) + \underbrace{s_{22_1} \cdot s_{11_2} \cdot s_{22_2} \cdot s_{11_3}}_B}$$

# Kaskada 4- polnih vezij 1/2



(a)



# Kaskada 4- polnih vezij 2/2

Masonovo pravilo nedotikajočih se zank:

$$b_3 = \frac{U_g}{2} (1 - \Gamma_g) \frac{T_1}{1 - \Sigma L^{(1)} + \Sigma L^{(2)} - \Sigma L^{(3)}} \quad T_1 = S_{21}' S_{21}''$$

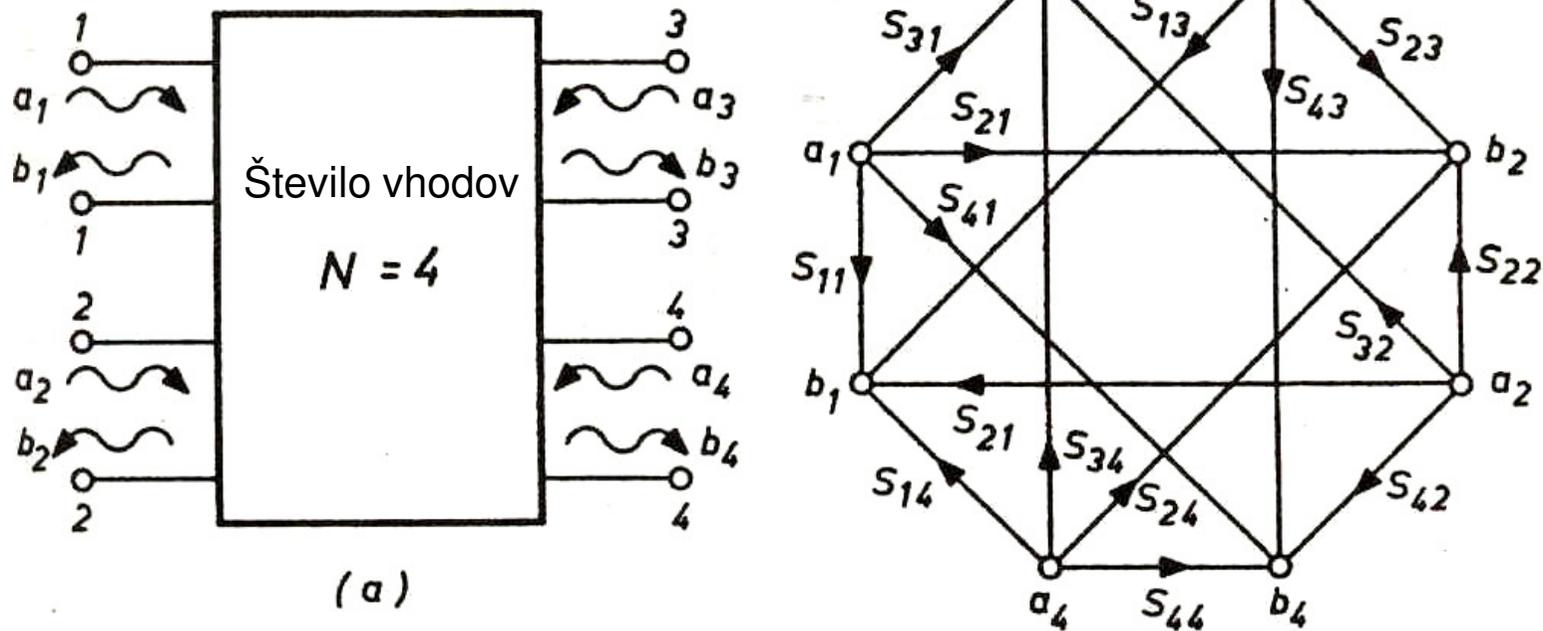
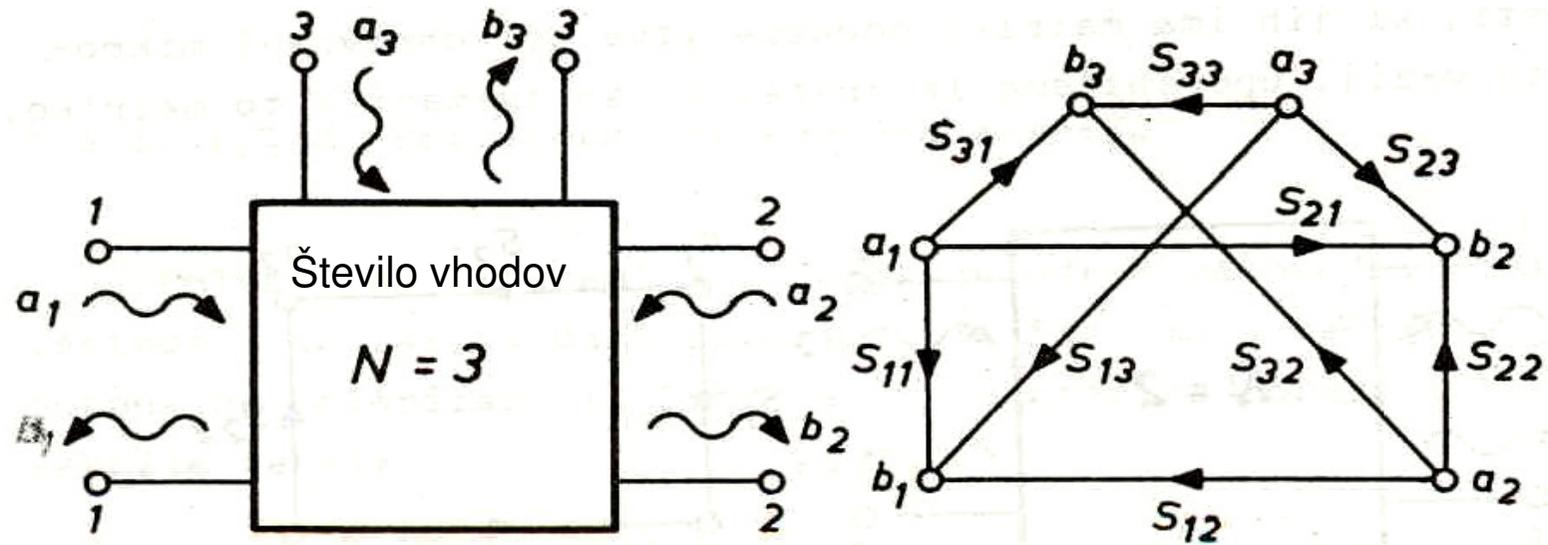
kjer so koeficienti zank:

$$\begin{aligned} \Sigma L^{(1)} &= \Gamma_g S_{11}' + S_{22}'' S_{11}''' + S_{22}'' \Gamma_b + \Gamma_g S_{21}' S_{11}''' S_{12}' + S_{22}'' S_{21}''' \Gamma_b S_{12}'' \\ &+ \Gamma_g S_{21}' S_{21}''' \Gamma_b S_{12}'' S_{12}', \end{aligned}$$

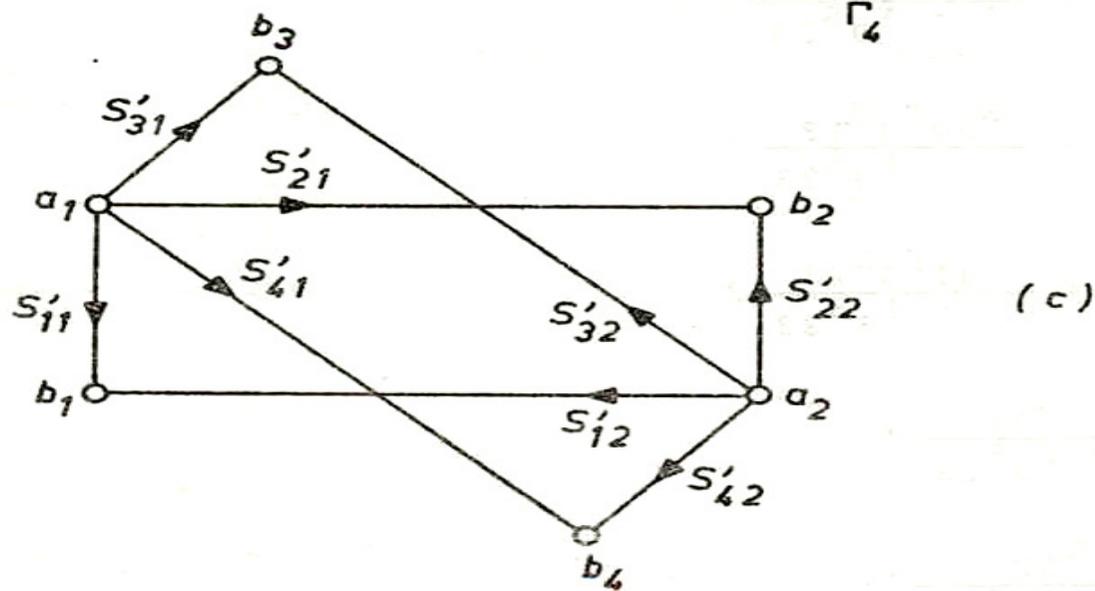
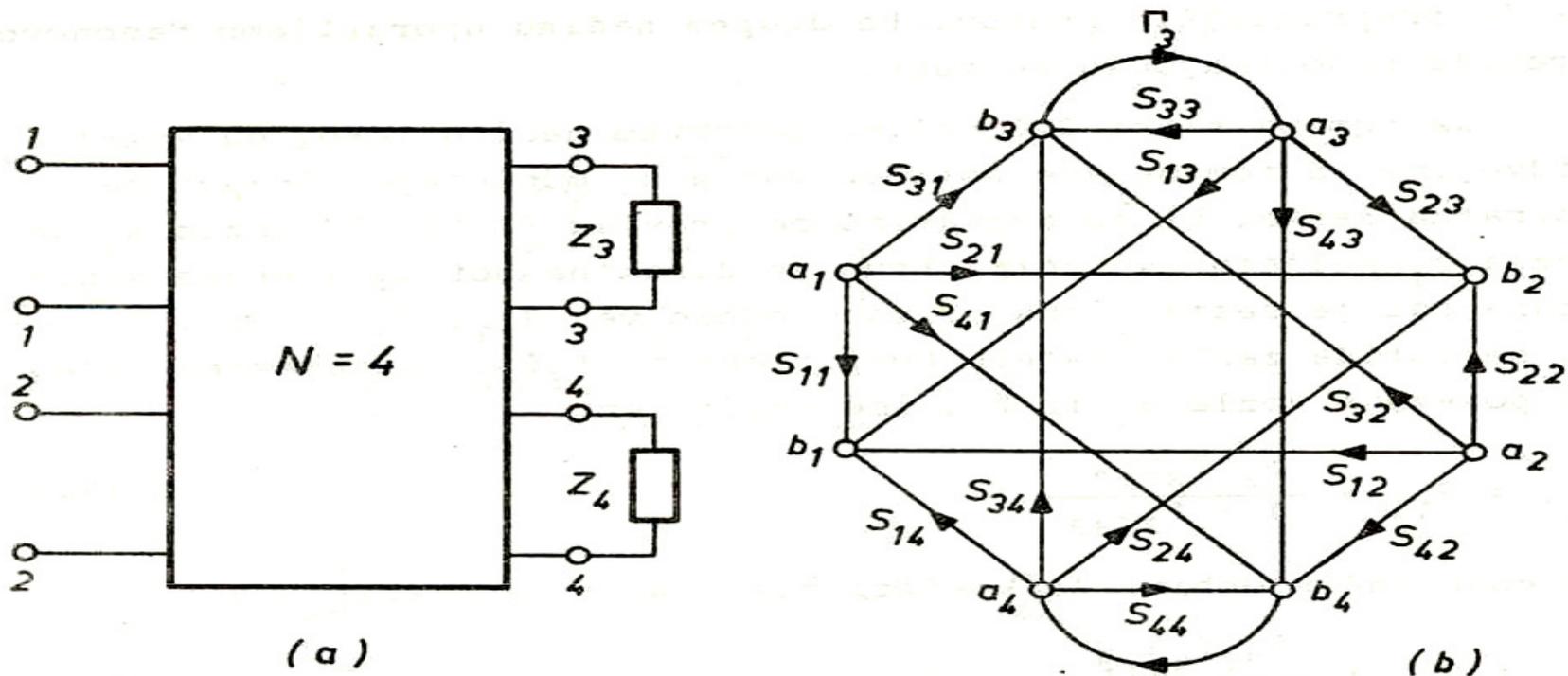
$$\begin{aligned} \Sigma L^{(2)} &= \Gamma_g S_{11}' S_{22}'' S_{11}''' + S_{22}'' S_{11}''' S_{22}'' \Gamma_b + \Gamma_g S_{11}' S_{22}'' \Gamma_b + \\ &+ \Gamma_g S_{21}' S_{11}''' S_{12}' S_{22}'' \Gamma_b + \Gamma_g S_{11}' S_{22}'' S_{21}''' \Gamma_b S_{12}'', \end{aligned}$$

$$\Sigma L^{(3)} = \Gamma_g S_{11}' S_{22}'' S_{11}''' S_{22}'' \Gamma_b.$$

# Smerni graf 6- in 8-polnega vezja



# Redukcija smernega grafa 8- polnega vezja



# Koeficient $S'_{11}$ reduciranega grafa 8-polnega vezja

Koeficienti direktnih poti  $T_1$  ( $a_1 - b_1$ ):

$$\begin{aligned} T_1 &= S_{11}, \\ T_2 &= S_{31} \Gamma_3 S_{13}, \\ T_3 &= S_{41} \Gamma_4 S_{14}, \\ T_4 &= S_{31} \Gamma_3 S_{43} \Gamma_4 S_{14}, \\ T_5 &= S_{41} \Gamma_4 S_{34} \Gamma_3 S_{13}. \end{aligned}$$

Koeficienti zank direktnih poti  $T_2$  in  $T_3$ :

$$\begin{aligned} L_2^{(1)} &= \Gamma_4 S_{44}, \\ L_3^{(1)} &= \Gamma_3 S_{33}. \end{aligned}$$

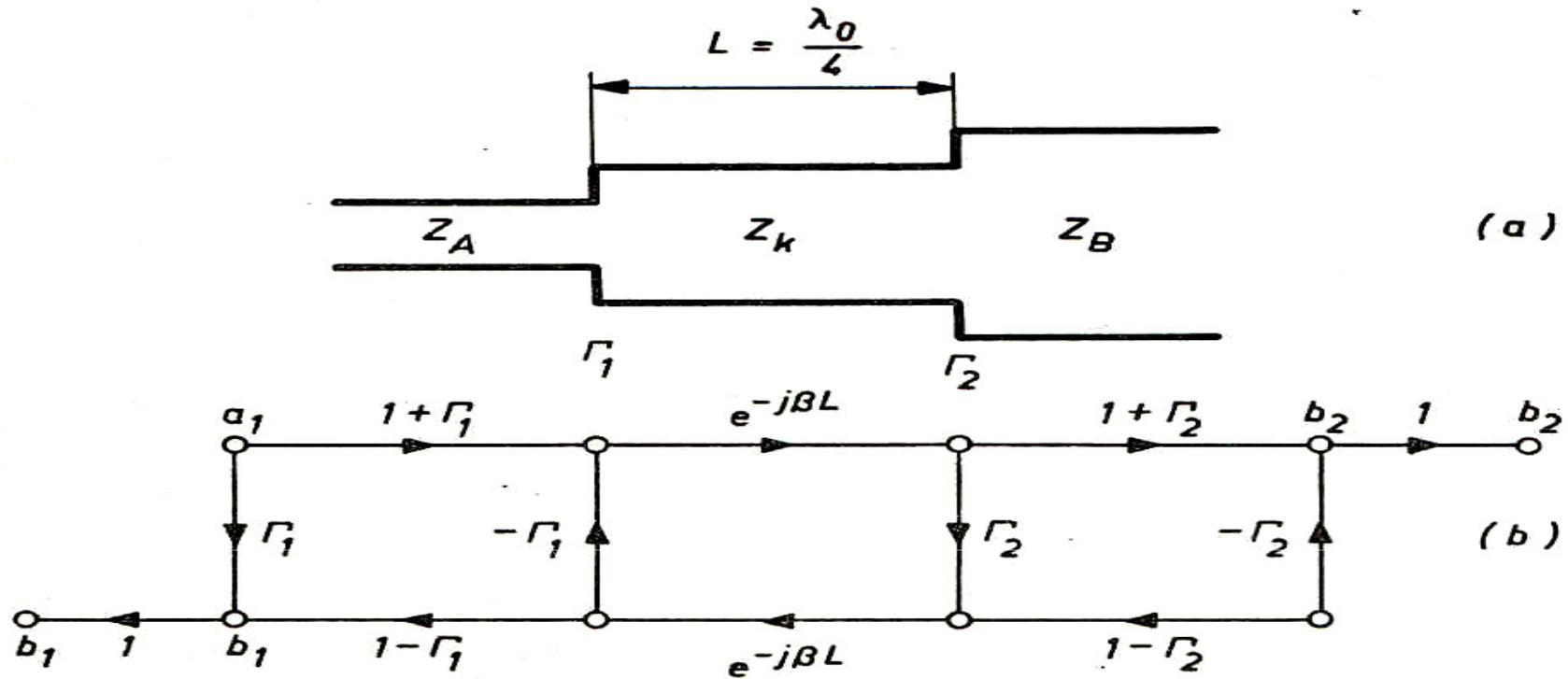
Koeficienti zank poti  $T_2$  in  $T_3$ :

$$\begin{aligned} L_1^{(1)} &= \Gamma_3 S_{33} + \Gamma_4 S_{44} + \Gamma_3 S_{43} \Gamma_4 S_{34}, \\ L_1^{(2)} &= \Gamma_3 S_{33} \Gamma_4 S_{44}. \end{aligned}$$

Koeficient  $S'_{11}$ :

$$S'_{11} = S_{11} + \frac{S_{31} \Gamma_3 S_{13} (1 - \Gamma_4 S_{44}) + S_{41} \Gamma_4 S_{14} (1 - \Gamma_3 S_{33}) + \Gamma_3 \Gamma_4 (S_{31} S_{43} S_{14} + S_{41} S_{34} S_{13})}{1 - \Gamma_3 S_{33} - \Gamma_4 S_{44} - \Gamma_3 S_{43} \Gamma_4 S_{34} + \Gamma_3 S_{33} \Gamma_4 S_{44}}.$$

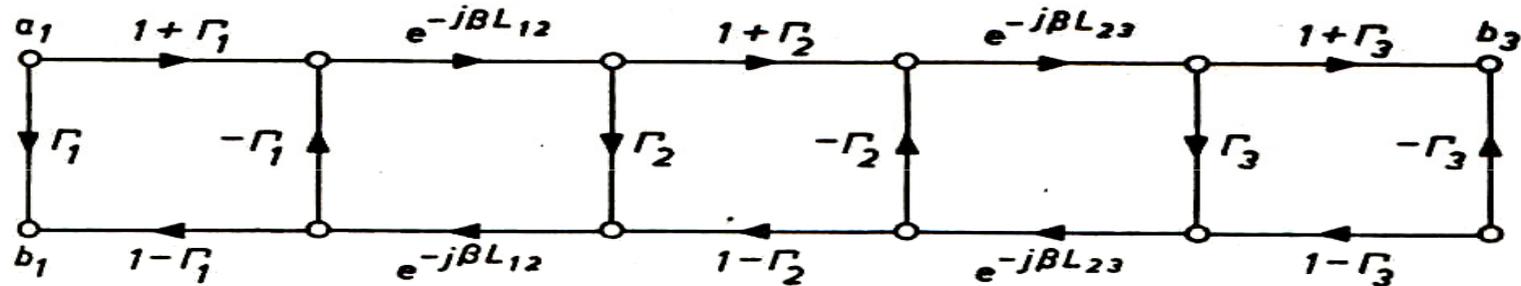
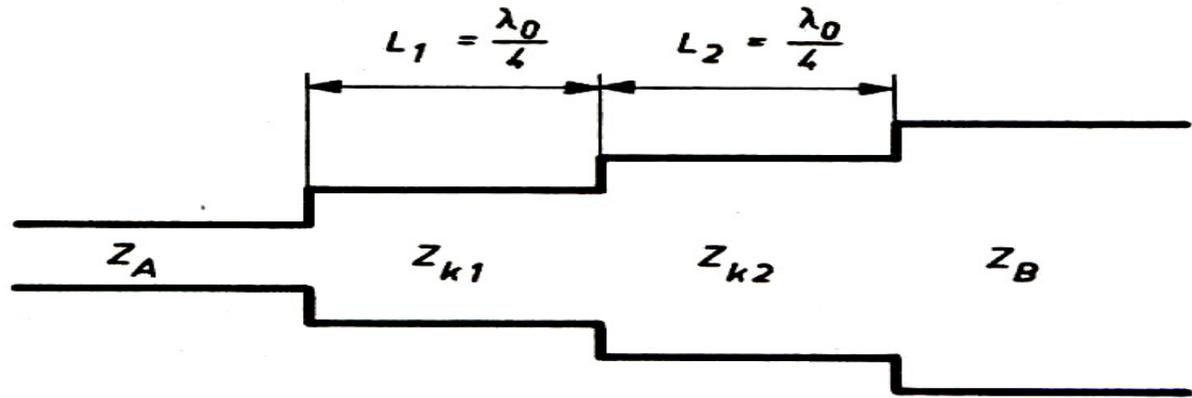
# Enostopenjski impedančni $\lambda/4$ transformator



$$\Gamma_{vh} = \frac{b_1}{a_1} = \Gamma_1 + \frac{(1 - \Gamma_1^2)\Gamma_2 e^{-j\beta 2L}}{1 + \Gamma_1\Gamma_2 e^{-j\beta 2L}} = \frac{\Gamma_1 + \Gamma_2 e^{-j\beta 2L}}{1 + \Gamma_1\Gamma_2 e^{-j\beta 2L}}$$

$$\Gamma_1 = \Gamma_2 \quad \text{ali} \quad \frac{Z_k/Z_A - 1}{Z_k/Z_A + 1} = \frac{Z_B/Z_k - 1}{Z_B/Z_k + 1} \quad \text{ali} \quad Z_k^2 = Z_A Z_B$$

# Dvostopenjski impedančni $\lambda/4$ transformator



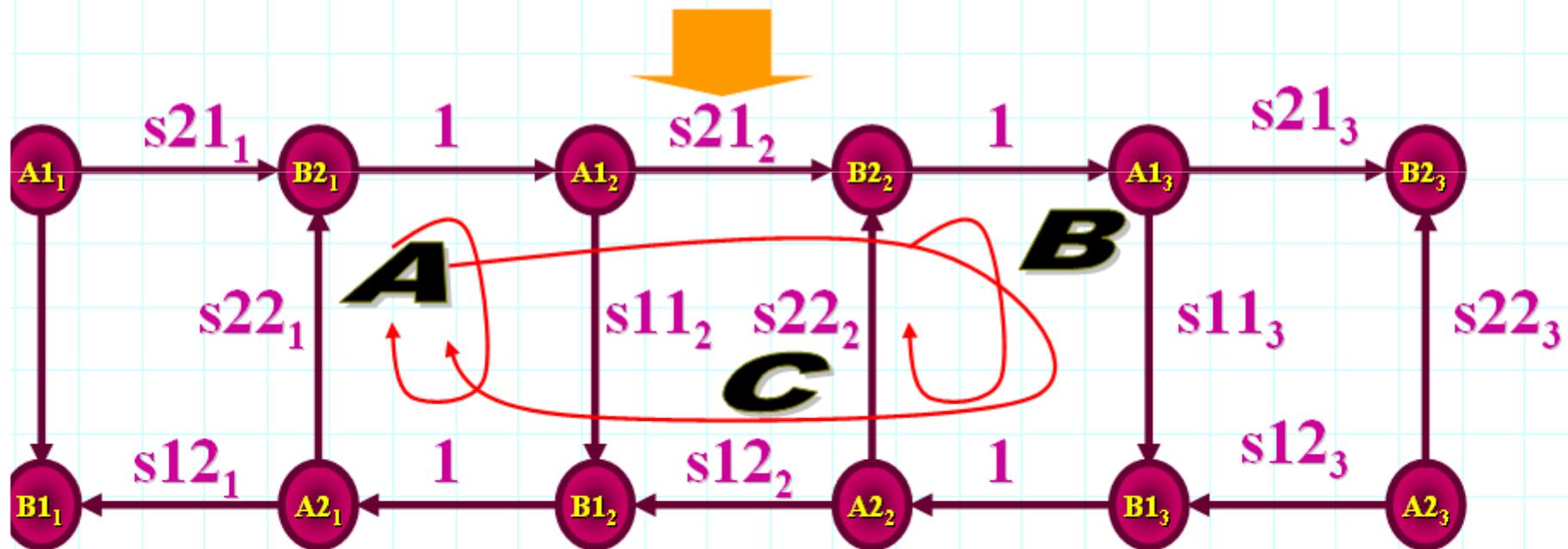
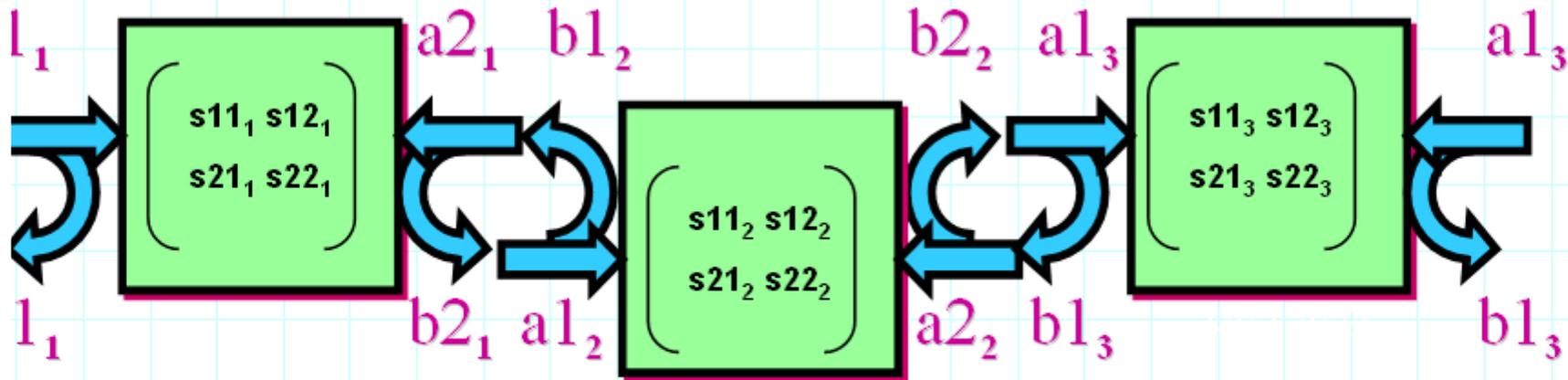
$$\Gamma_{vh} \doteq \Gamma_1 + \frac{(1-\Gamma_1^2)\Gamma_2 e^{-j\beta 2L_{12}} (1+\Gamma_2\Gamma_3 e^{-j\beta 2L_{23}}) + (1-\Gamma_1^2)(1-\Gamma_2^2)\Gamma_3 e^{-j\beta 2(L_{12}+L_{23})}}{1+\Gamma_1\Gamma_2 e^{-j\beta 2L_{12}} + \Gamma_2\Gamma_3 e^{-j\beta 2L_{23}} + \Gamma_1\Gamma_3 (1-\Gamma_2^2) e^{-j\beta 2(L_{12}+L_{23})}}$$

$$\Gamma_{vh} \doteq \Gamma_1 + \frac{\Gamma_2 e^{-j\beta 2L_{12}} + \Gamma_3 e^{-j\beta 2(L_{12}+L_{23})}}{1+\Gamma_1\Gamma_2 e^{-j\beta 2L_{12}} + \Gamma_2\Gamma_3 e^{-j\beta 2L_{23}} + \Gamma_1\Gamma_3 e^{-j\beta 2(L_{12}+L_{23})}}$$

# Sklep

- Grafi signalnega toka so praktična metoda za ocenjevanje valovnih pojavov v mikrovalovni praksi.
- Metodo redukcije grafov uporabljamo pri reševanju preprostejših vezij. Masonovo metodo uporabljamo pri reševanju kompleksnih vezij, grafi katerih imajo zanke višjega reda.
- Smerni grafi dajejo jasnejšo predstavo valovnih pojavov in omogočajo upoštevanje in vrednotenje zanemaritev.
- Alternativa obravnave mikrovalovnih vezij s smernimi grafi je računalniška analiza, ki temelji na S-parametrih.
- Smerni grafi so idealen pripomoček za analizo merilnih napak.

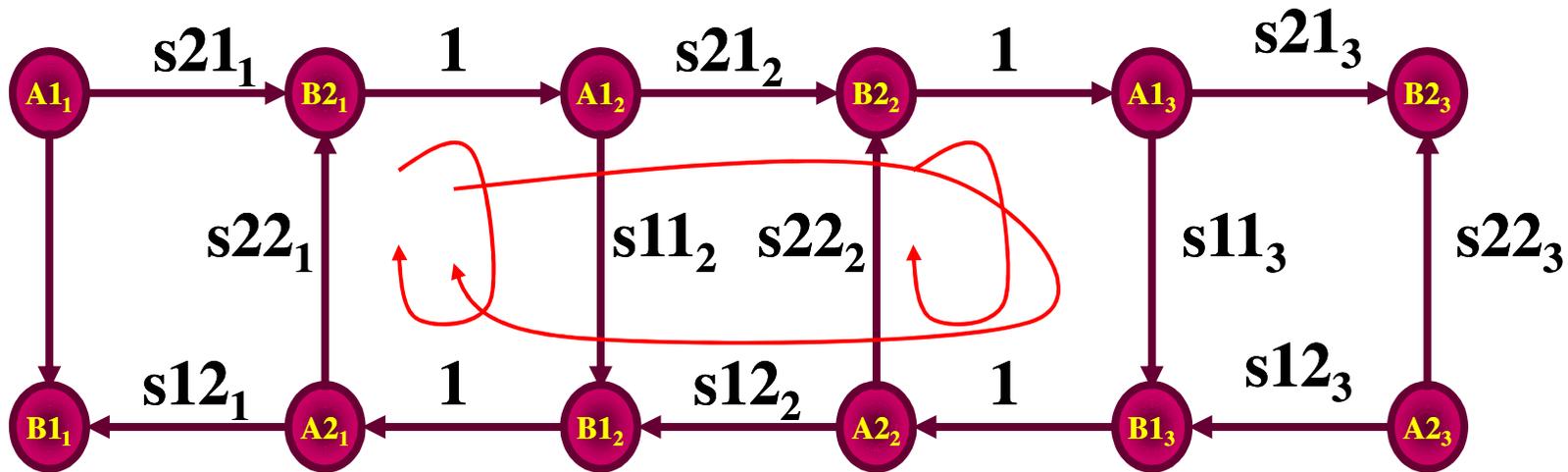
# cutting the signal flow graph



We map output a to input b and visa versa.

Next we define all the loops

Loop "A" and "B" do not touch each other



$$\frac{b_6}{a_1} = \frac{s_{21_1} \cdot s_{21_2} \cdot s_{21_3}}{1 - (s_{22_2} \cdot s_{11_1} + s_{22_3} \cdot s_{11_2} + s_{11_3} \cdot s_{22_1} \cdot s_{12_2} \cdot s_{21_2}) + s_{22_1} \cdot s_{11_2} \cdot s_{22_2} \cdot s_{11_3}}$$

# Variables' descriptions (1)

$P_1, P_2,$  (and so on)

= paths connecting the dependent and independent variables whose transfer function  $T$  is to be determined.

A path is defined as a set of consecutive, codirectional branches along which no node is encountered more than once as we move in the graph from the independent to the dependent node.

$\Sigma L(1)$

= the sum of all first-order loops. A first-order loop is defined as the product of the branches encountered in a round trip as we move from a node in the direction of the arrows back to that original node.

# Variables' descriptions (2)

$\Sigma L(2)$

= the sum of all second-order loops. A second-order loop is defined as the product of any two nontouching first-order loops.

$\Sigma L(3)$

= the sum of all third-order loops. A third-order loop is defined as the product of any three nontouching first-order loops.

$\Sigma L(4)$ ,  $\Sigma L(5)$ , and so on represent fourth-, fifth-, and higher order loops.

# Variables' descriptions (3)

$$\Sigma L(1)^{(P)}$$

= the sum of all first-order loops that do not touch the path  $P$  between the independent and dependent variables.

$$\Sigma L(2)^{(P)}$$

= the sum of all second-order loops that do not touch the path  $P$  between the independent and dependent variables.

$\Sigma L(3)^{(P)}$ ,  $\Sigma L(4)^{(P)}$  and so on represent third-, fourth-, and higher order loops that do not touch the path  $P$ .

## Mason's Rule ~ Non-Touching Loop Rule

$$T = \frac{\sum_k T_k \left(1 + \sum_{mk} (-1)^{mk} L(mk) \Big|^{(k)}\right)}{\left(1 + \sum_{mk} (-1)^{mk} L(mk)\right)}$$

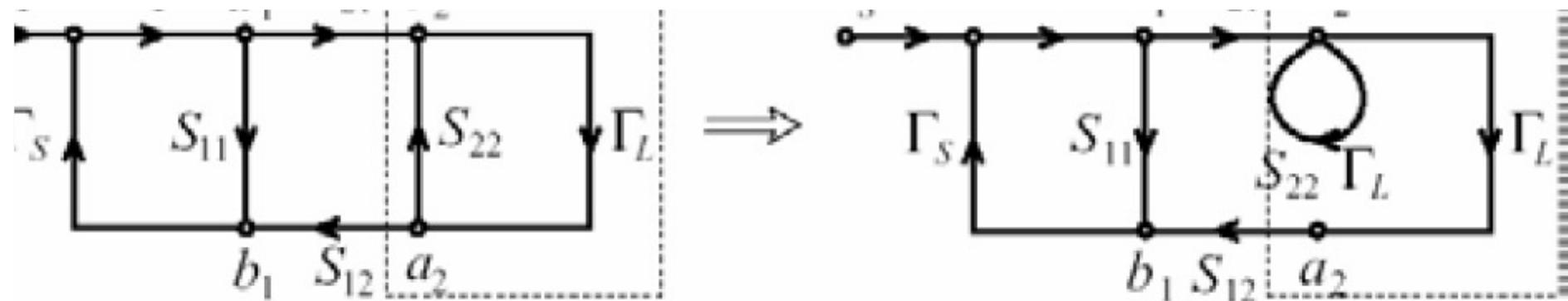
$T$  is the transfer function (often called gain)

$T_k$  is the transfer function of the  $k^{\text{th}}$  forward path

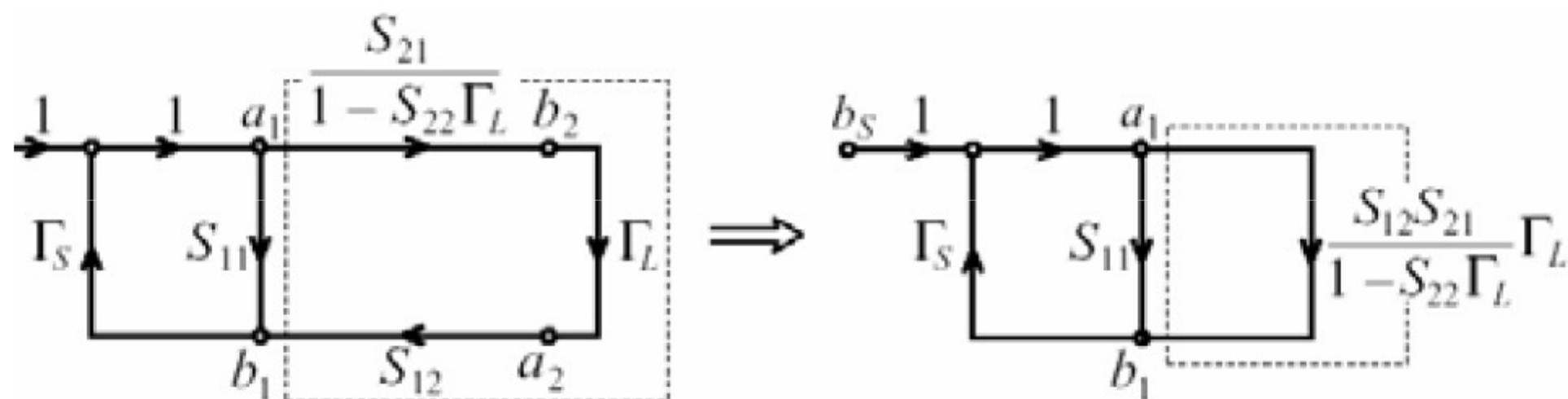
$L(mk)$  is the product of non touching loop gains on path  $k$  and loop  $mk$  at a time.

$L(mk) \Big|^{(k)}$  is the product of non touching loop gains on path  $k$  and loop  $mk$  at a time but not touching path  $k$ .

$k=1$  means all individual loops



Step 1



Step 2

