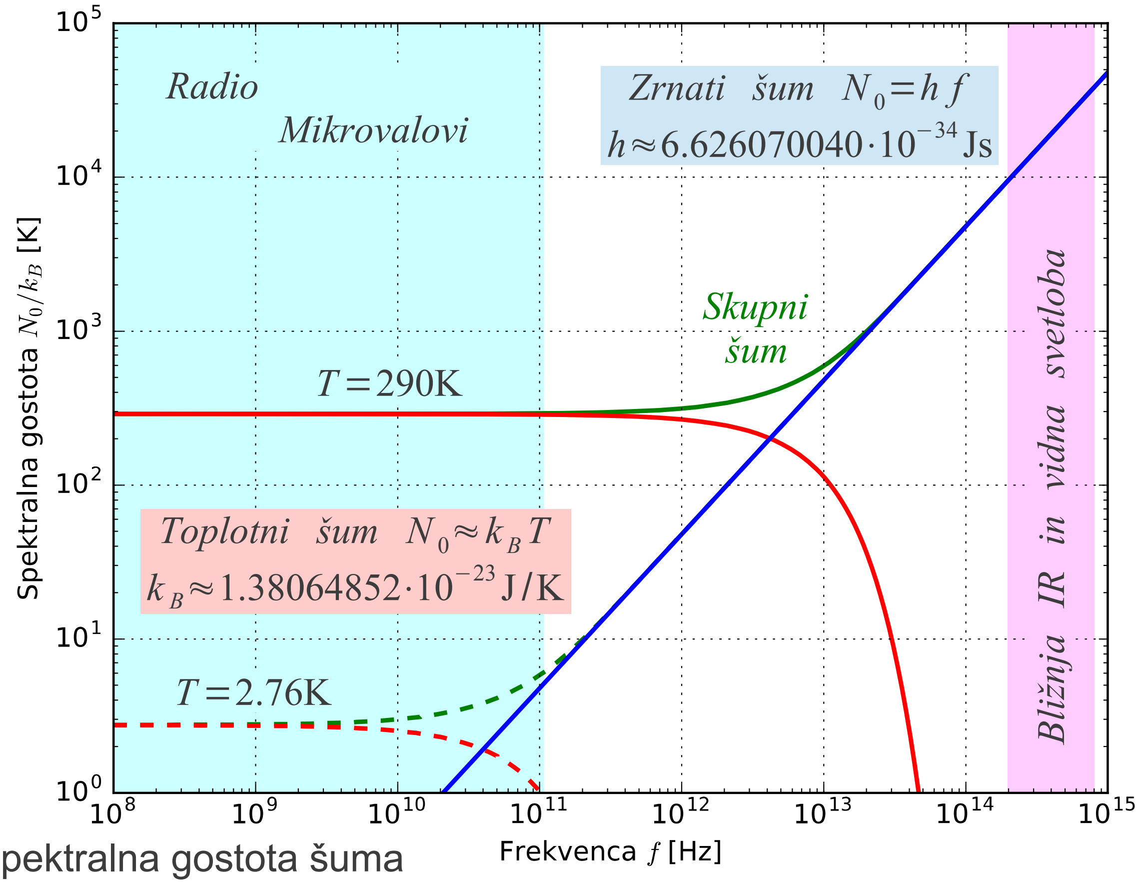
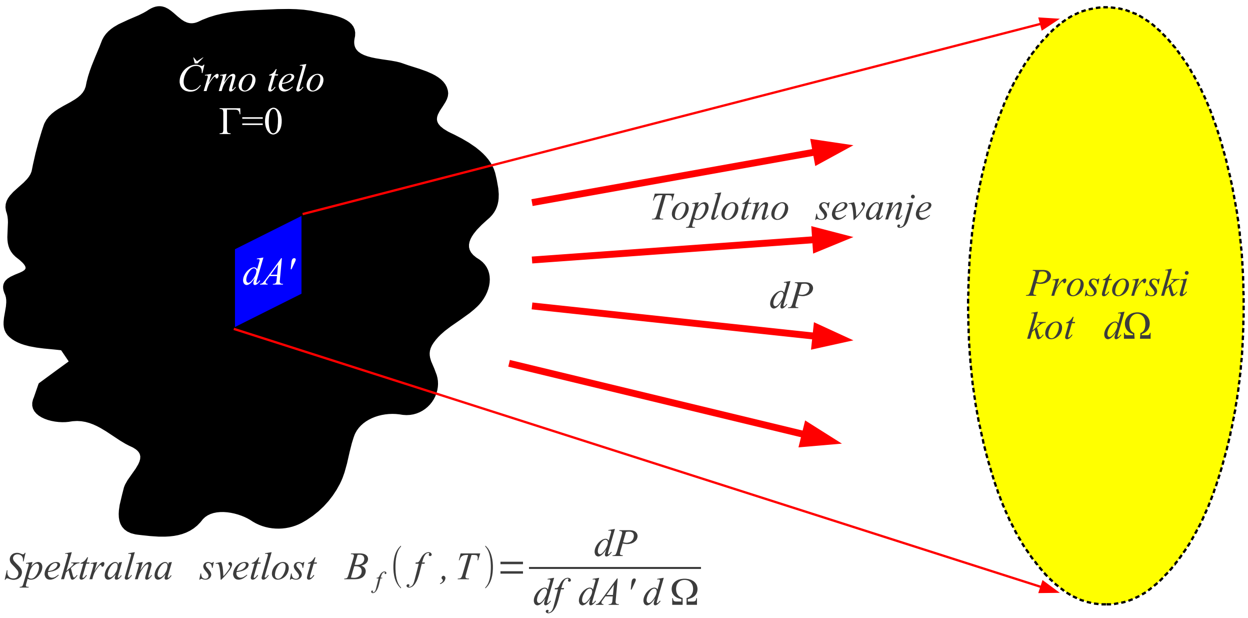
13. Toplotni šum

Večina nalog iz anten in razširjanje valov zahteva obravnavo v treh dimenzijah prostora. Tako skalarne kot tudi vektorske veličine so funkcije časa in vseh treh dimenzij prostora. Ozkopasovne signale $B \ll f$ radia največkrat smemo v izračunih ponazoriti s harmonskim signalom ene same krožne frekvence $\omega = 2\pi f$, kar poenostavi časovne odvode v $\partial/\partial t = j\omega$.

$$P_N = \Delta f\, N_0 \qquad \Delta f \equiv \textit{pasovna širina} \qquad N_0 \equiv \textit{spektralna gostota šuma}$$



Spektralna gostota šuma



Spektralna svetlost $B_f(f,T)=\frac{dP}{df\,dA'\,d\Omega}$

Planckov zakon $B_f(f,T)=\frac{2hf^3}{c_0^2}\cdot\frac{1}{e^{\frac{hf}{k_B T}}-1}$

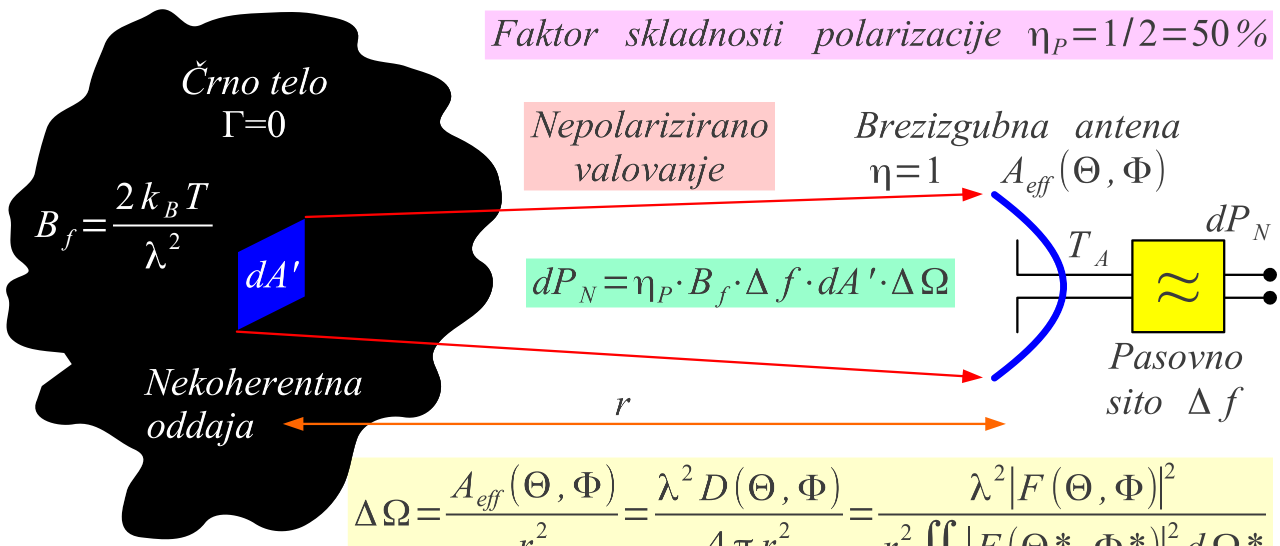
Prazen prostor ϵ_0,μ_0

$c_0=299792458\,\mathrm{m/s}\approx 3\cdot 10^8\,\mathrm{m/s}$

$hf \ll k_B T$ → Rayleigh−Jeansov približek $B_f(f,T)\approx\frac{2k_B T f^2}{c_0^2}=\frac{2k_B T}{\lambda^2}$

Toplotno sevanje črnega telesa

Faktor skladnosti polarizacije $\eta_P = 1/2 = 50\%$



$$dP_N = \eta_P \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta\Omega$$

$$B_f = \frac{2 k_B T}{\lambda^2}$$

$$\Delta\Omega = \frac{A_{eff}(\Theta,\Phi)}{r^2} = \frac{\lambda^2 D(\Theta,\Phi)}{4\pi r^2} = \frac{\lambda^2 |F(\Theta,\Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*,\Phi^*)|^2 d\Omega^*}$$

$$P_N = \iint_{A'} \frac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta\Omega$$

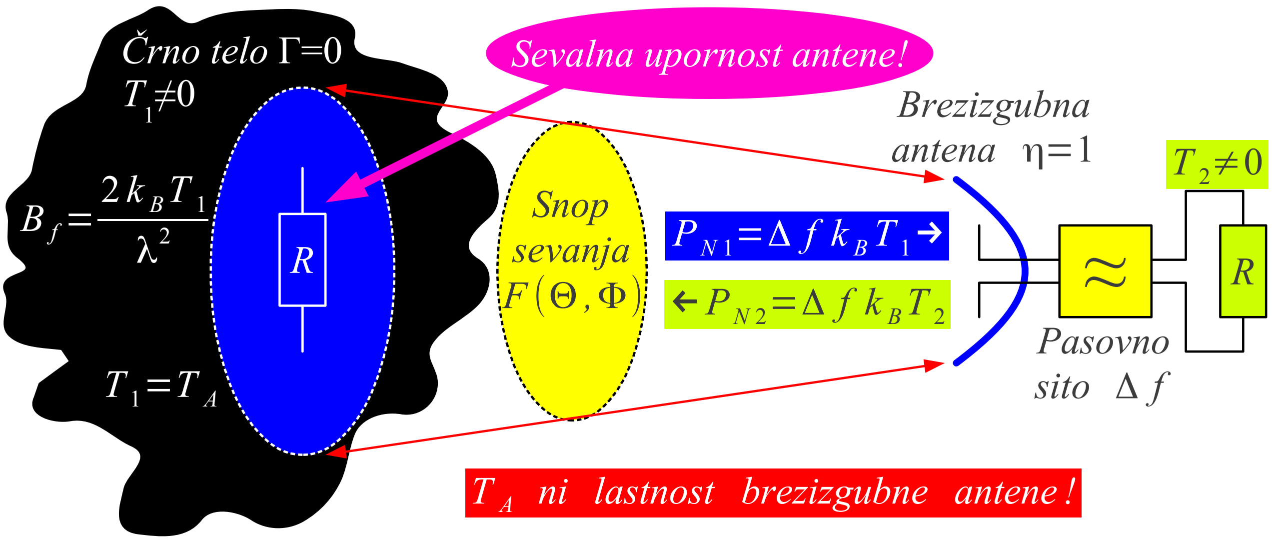
$$dA' = r^2 d\Omega$$

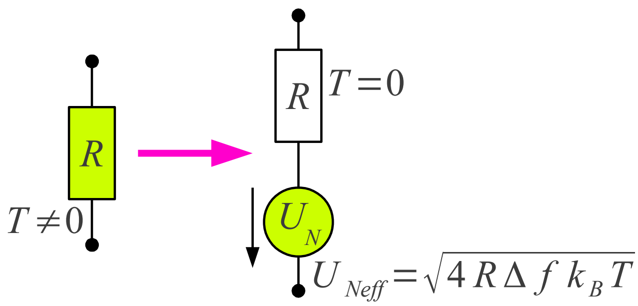
$$P_N = \Delta f\, k_B \frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

Sprejeta moč toplotnega šuma

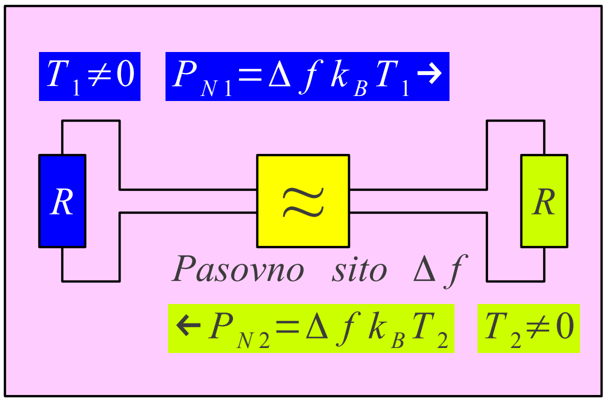
$$P_N = \Delta f\, N_0 = \Delta f\, k_B T_A$$

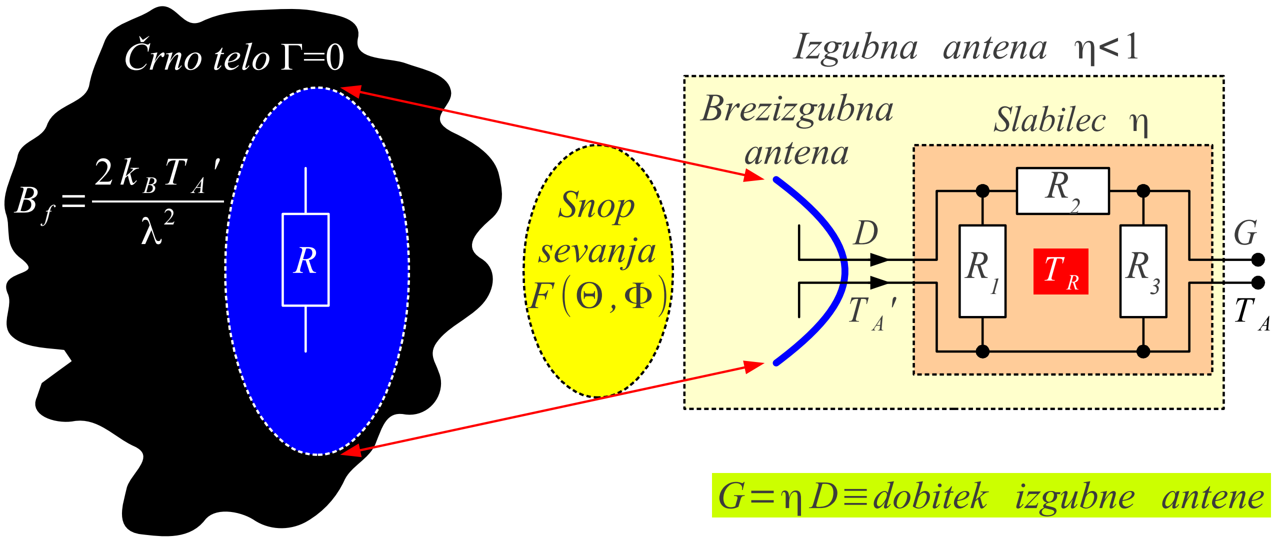
$$T_A = \frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$





Toplotno ravnovesje





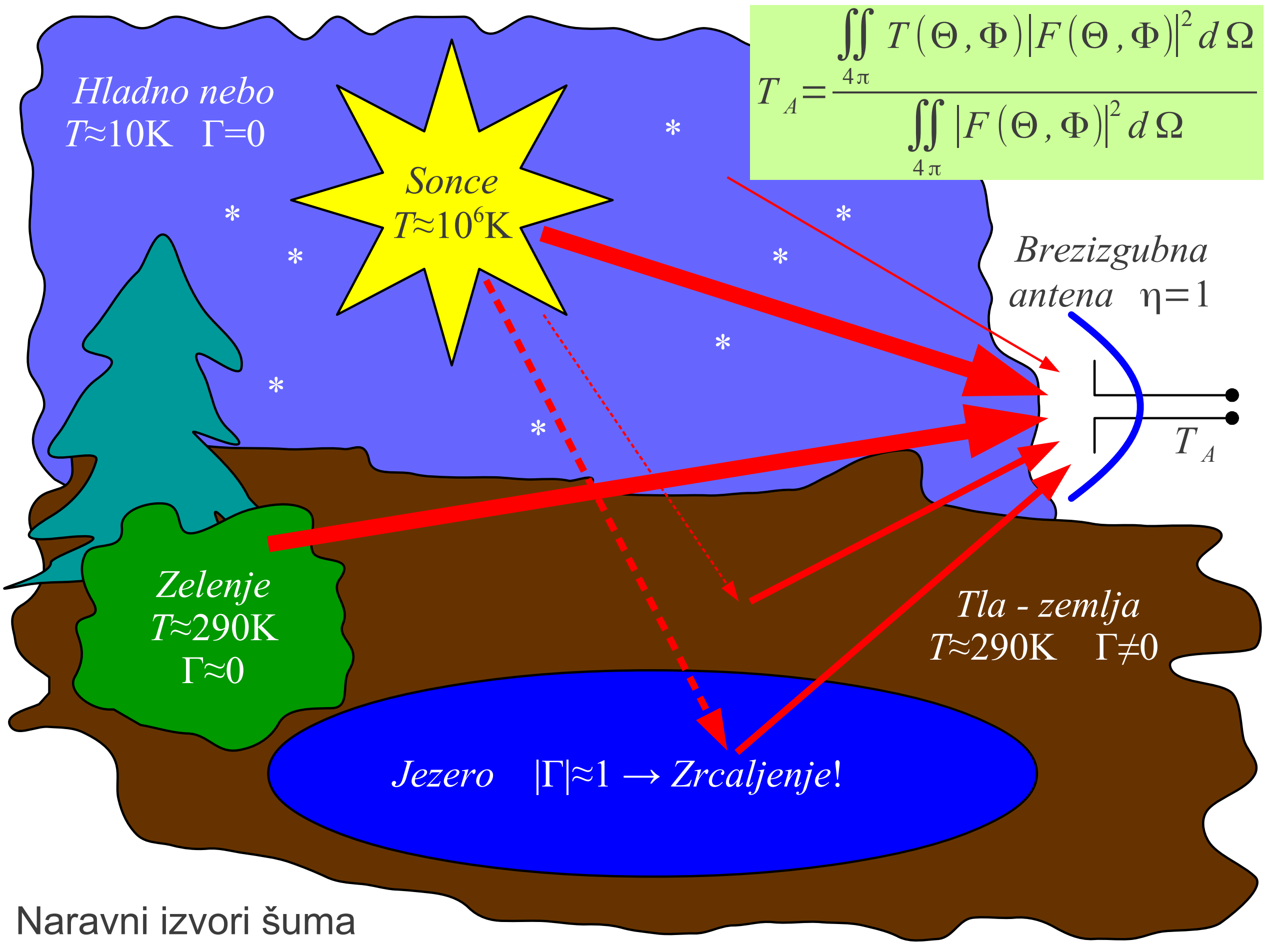
$G = \eta D \equiv$ *dobitek izgubne antene*

$T_A = \eta T_A' + (1-\eta) T_R \equiv$ *šumna temperatura izgubne antene*

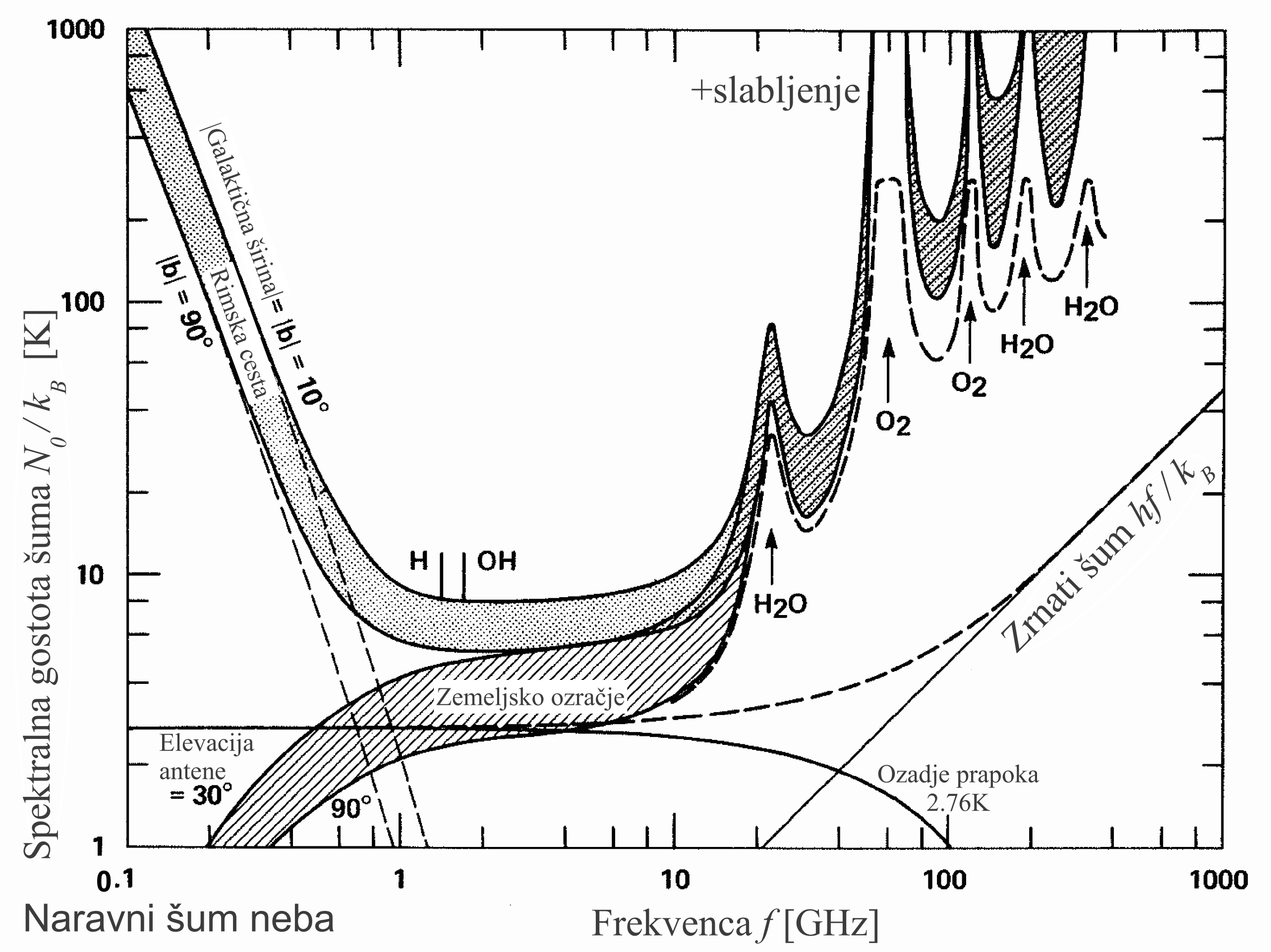
$T_R \approx 290\text{K} \equiv$ *temperatura slabilca*

$$T_A = \eta \left[\frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega} \right] + (1-\eta) T_R$$

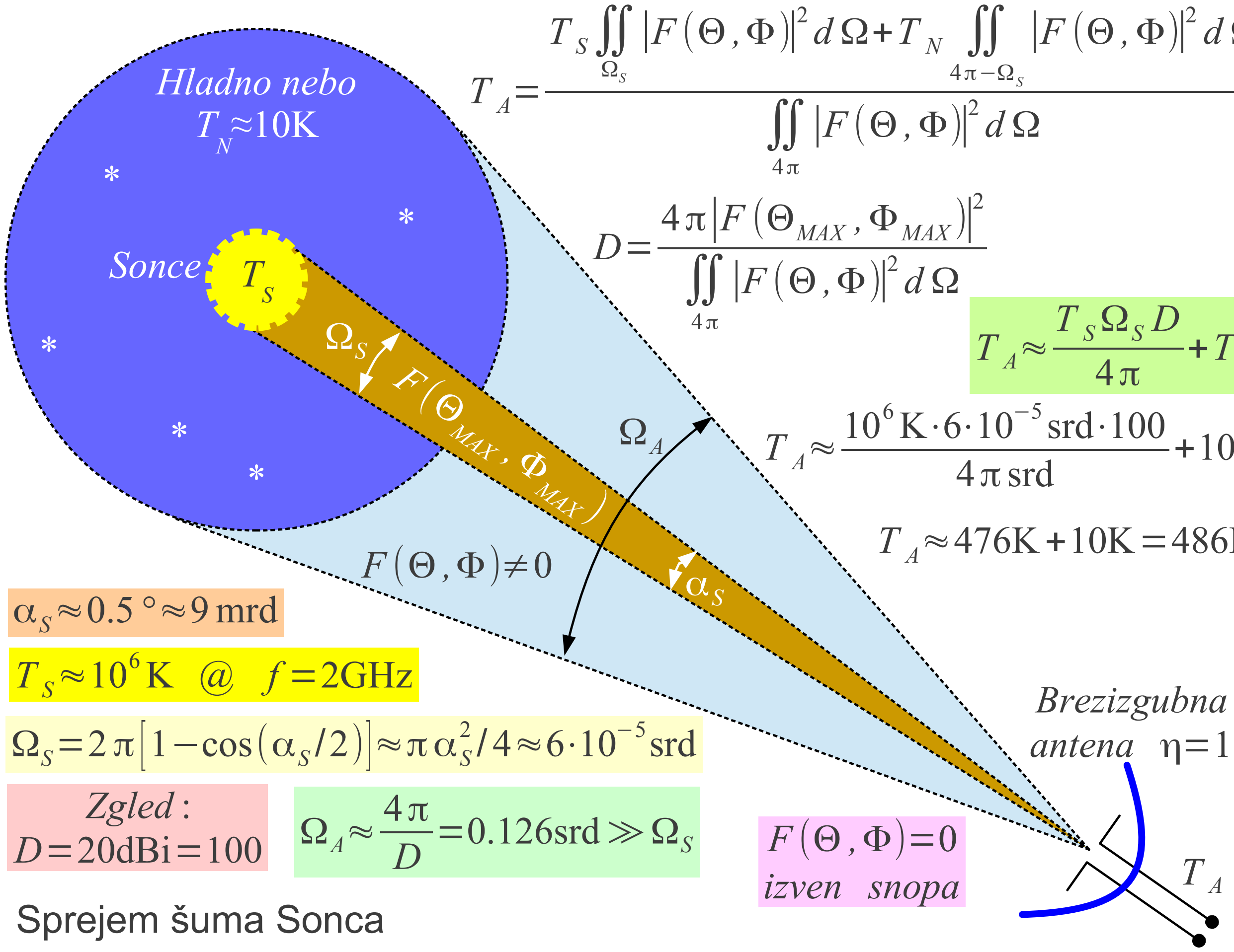
Dobitek in šumna temperatura izgubne antene



Naravni izvori šuma



1000
100
10
1
0.1
1
10
100
1000
+slabljenje
|Galaktična širina|= |b| = 10°
|b| = 90°
Rimska cesta
H
OH
H2O
O2
O2
H2O
H2O
Zrnati šum hf / k_B
Zemeljsko ozračje
Elevacija antene = 30°
90°
Ozadje prapoka 2.76K
Spektralna gostota šuma N_0 / k_B [K]
Frekvenca f [GHz]
Naravni šum neba



$$T_A = \frac{T_S \iint_{\Omega_S} |F(\Theta,\Phi)|^2 d\Omega + T_N \iint_{4\pi-\Omega_S} |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$D = \frac{4\pi |F(\Theta_{MAX},\Phi_{MAX})|^2}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$T_A \approx \frac{T_S \Omega_S D}{4\pi} + T_N$$

$$T_A \approx \frac{10^6\text{K}\cdot 6\cdot 10^{-5}\text{srd}\cdot 100}{4\pi\,\text{srd}} + 10\text{K}$$

$$T_A \approx 476\text{K} + 10\text{K} = 486\text{K}$$

$\alpha_S \approx 0.5° \approx 9\,\text{mrd}$

$T_S \approx 10^6\text{K}$ @ $f = 2\text{GHz}$

$\Omega_S = 2\pi[1-\cos(\alpha_S/2)] \approx \pi\alpha_S^2/4 \approx 6\cdot 10^{-5}\,\text{srd}$

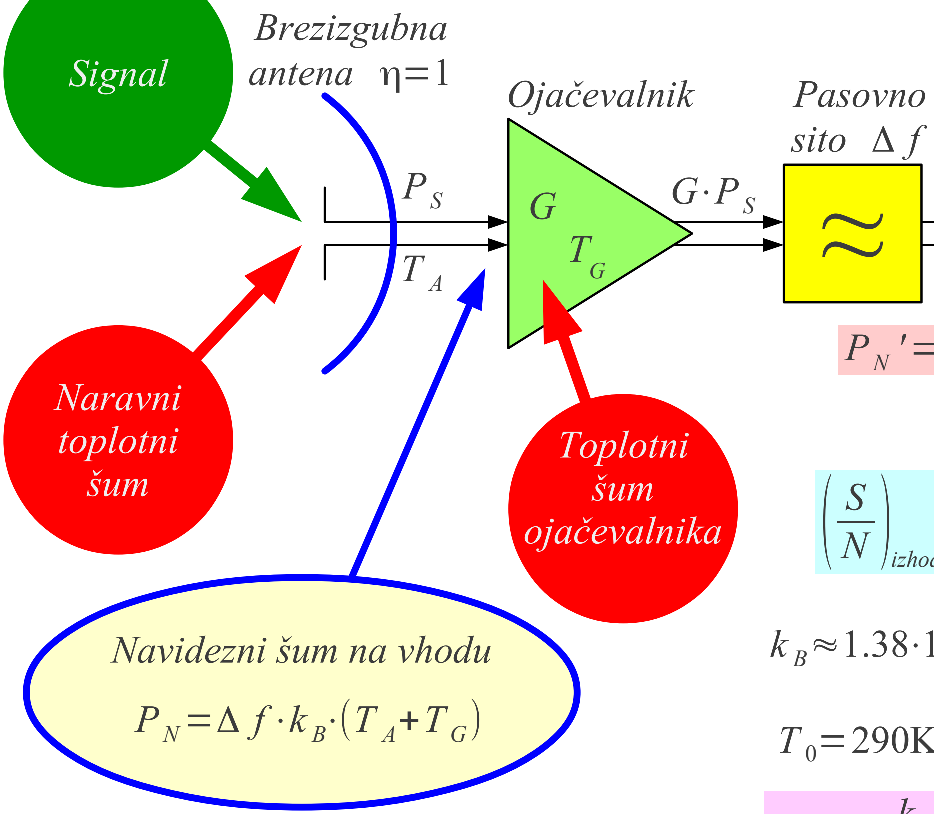
Zgled : $D = 20\text{dBi} = 100$

$$\Omega_A \approx \frac{4\pi}{D} = 0.126\,\text{srd} \gg \Omega_S$$

$F(\Theta,\Phi) = 0$ *izven snopa*

Sprejem šuma Sonca

$T_S \equiv$ *temperatura ojačevalnika preračunana na vhod!*



$$P_S' = G \cdot P_S$$

$$\left(\frac{S}{N}\right)_{izhod} = \frac{P_S'}{P_N'}$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot (T_A + T_G)$$

$$P_N = \Delta f \cdot k_B \cdot (T_A + T_G)$$

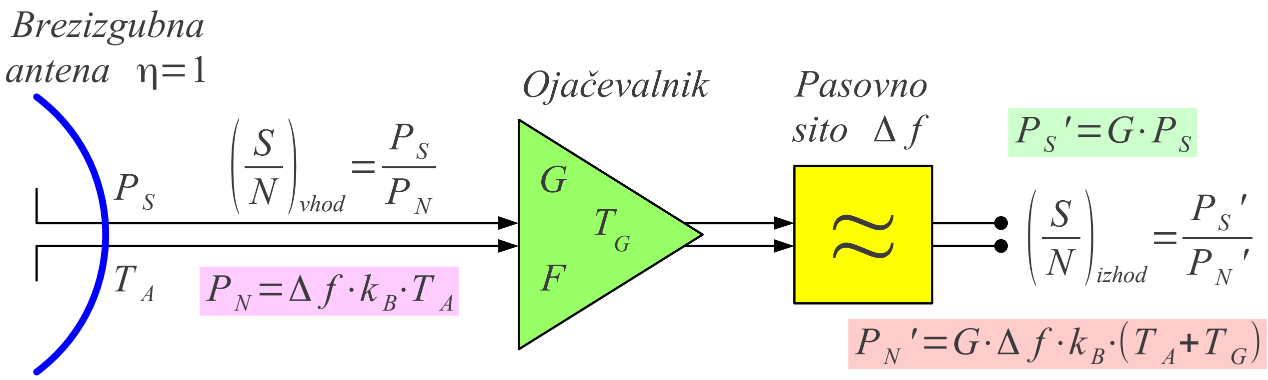
$$\left(\frac{S}{N}\right)_{izhod} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_G)}$$

$$k_B \approx 1.38 \cdot 10^{-23}\,\mathrm{J/K}$$

$$T_0 = 290\,\mathrm{K} \approx 17\,^\circ\mathrm{C}$$

$$10 \log_{10} \frac{k_B T_0}{1\,\mathrm{mJ}} \approx -174\,\mathrm{dBm/Hz}$$

Razmerje signal/šum sprejemnika



Nesmiselna definicija šumnega števila:

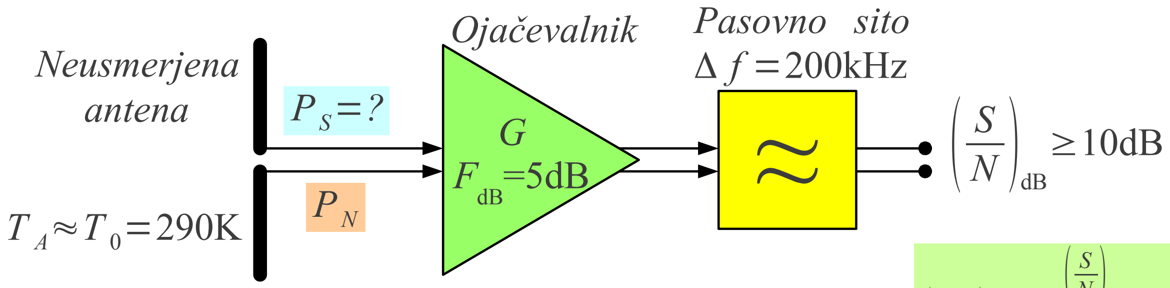
$$F=\frac{\left(\frac{S}{N}\right)_{vhod}}{\left(\frac{S}{N}\right)_{izhod}}=\frac{\frac{P_S}{\Delta f\, k_B T_A}}{\frac{G P_S}{G \Delta f\, k_B (T_A+T_G)}}=\frac{T_A+T_G}{T_A}=1+\frac{T_G}{T_A}$$

Lastnost ojačevalnika ne more biti funkcija T_A!

Smiselna definicija $F=1+\frac{T_G}{T_0}$ @ $T_0=290\mathrm{K}\approx 17\,^\circ\mathrm{C}$ $\leftrightarrow$ $T_G=T_0(F-1)$

Logaritemske enote $F_{\mathrm{dB}}=10\log_{10}F=10\log_{10}\left(1+\frac{T_G}{T_0}\right)$ $\leftrightarrow$ $T_G=T_0\left(10^{\frac{F_{\mathrm{dB}}}{10}}-1\right)$

Šumno število ojačevalnika



$$T_G = T_0 \cdot \left(10^{\frac{F_{dB}}{10}} - 1\right) = 290\text{K} \cdot (3.162 - 1) = 627\text{K}$$

$$\left(\frac{S}{N}\right) = 10^{\frac{\left(\frac{S}{N}\right)_{dB}}{10}} \geq 10$$

$$P_N = \Delta f \cdot k_B \cdot (T_A + T_G) = 200\text{kHz} \cdot 1.38 \cdot 10^{-23}\,\text{J/K} \cdot (290\text{K} + 627\text{K}) = 2.53 \cdot 10^{-15}\,\text{W}$$

$$P_S = P_N \cdot \left(\frac{S}{N}\right) = P_N \cdot 10 = 2.53 \cdot 10^{-14}\,\text{W}$$

$$P_{S\,dBm} = 10 \log_{10} \frac{P_S}{1\text{mW}} = -106\text{dBm}$$

Poenostavljen izračun izključno v primeru $T_A \approx T_0 = 290\text{K}$

$$P_{S\,dBm} \approx (S/N)_{dB} + (\Delta f)_{dB\cdot Hz} + (k_B T_0)_{dBm/Hz} + F_{dB}$$

$$(k_B T_0)_{dBm/Hz} = -174\text{dBm/Hz}$$

$$(\Delta f)_{dB\cdot Hz} = 10 \log_{10}\left(\frac{\Delta f}{1\text{Hz}}\right) = 53\text{dB}\cdot\text{Hz}$$

$$P_{S\,dBm} \approx 10\text{dB} + 53\text{dB}\cdot\text{Hz} - 174\text{dBm/Hz} + 5\text{dB} = -106\text{dBm}$$

Občutljivost GSM telefona

Dva različna sprejemnika #1 in #2:

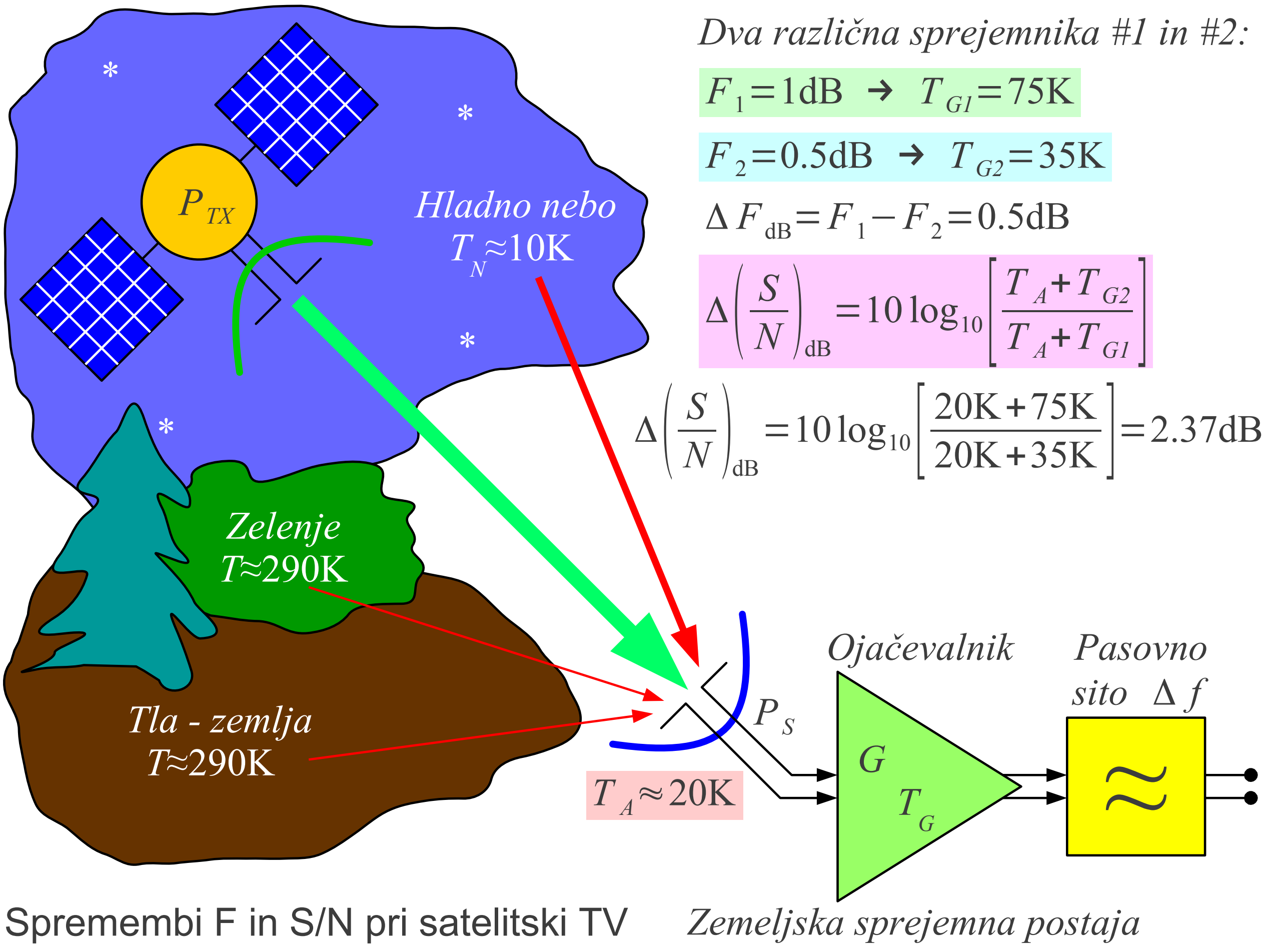
$F_1 = 1\text{dB} \rightarrow T_{G1} = 75\text{K}$

$F_2 = 0.5\text{dB} \rightarrow T_{G2} = 35\text{K}$

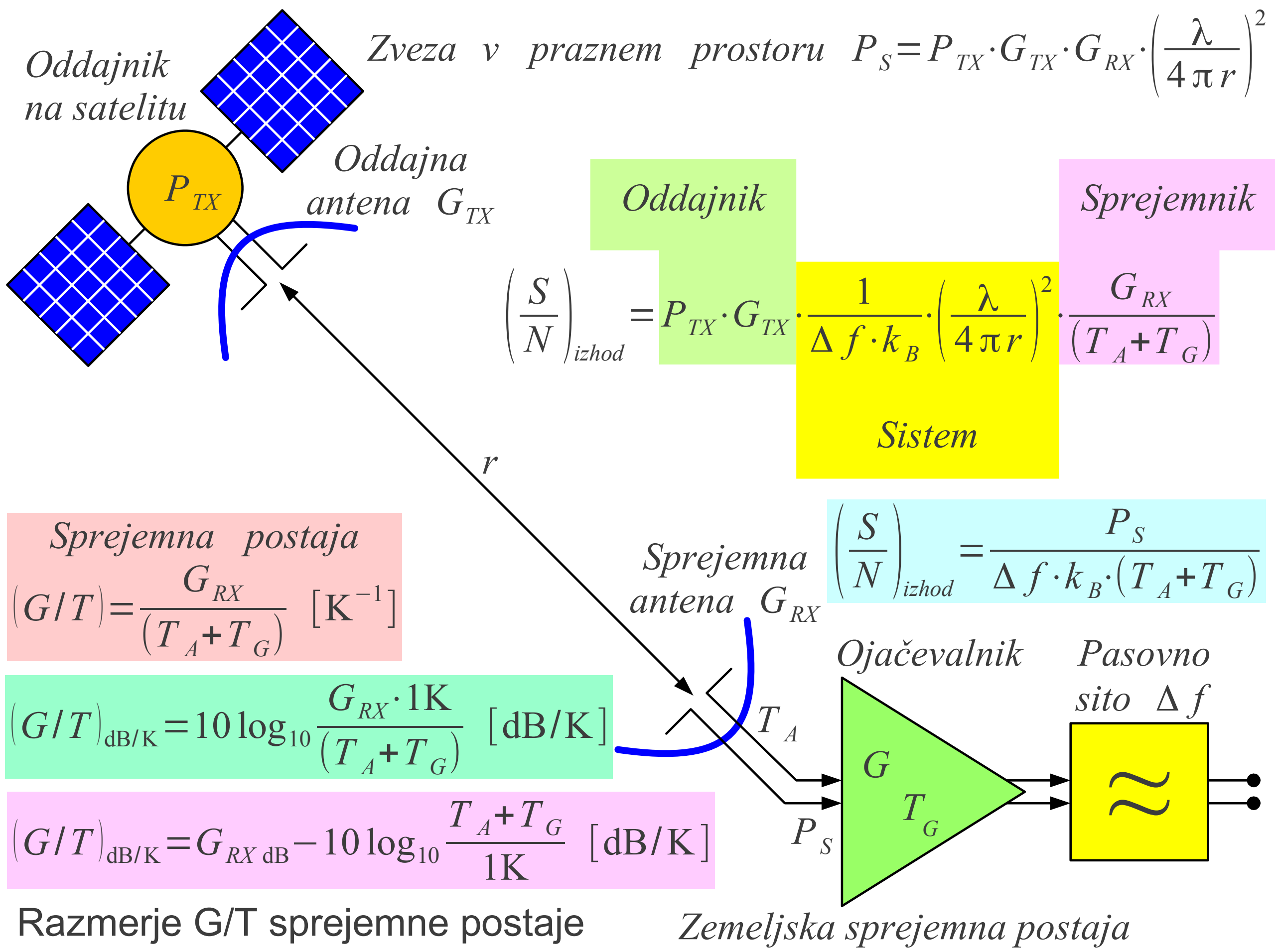
$\Delta F_{\text{dB}} = F_1 - F_2 = 0.5\text{dB}$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{T_A + T_{G2}}{T_A + T_{G1}}\right]$$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{20\text{K}+75\text{K}}{20\text{K}+35\text{K}}\right] = 2.37\text{dB}$$



Spremembi F in S/N pri satelitski TV



Razmerje G/T sprejemne postaje

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