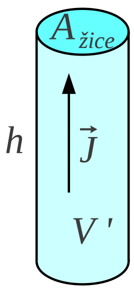
3. Osnovni viri sevanja

Večina nalog iz anten in razširjanje valov zahteva obravnavo v treh dimenzijah prostora. Tako skalarne kot vektorske veličine so funkcije časa in vseh treh dimenzij prostora. Ozkopasovne signale $B \ll f$ radia največkrat smemo v izračunih ponazoriti s harmonskim signalom ene same krožne frekvence $\omega = 2\pi f$, kar poenostavi časovne odvode v $\partial / \partial t = j\omega$.

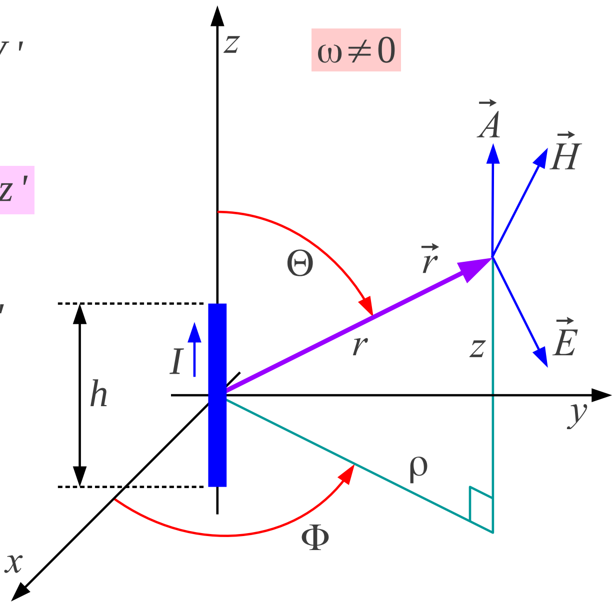
Ko reševanje naloge zahteva dva različna krogelna koordinatna sistema z različnima izhodiščema, je edina smotrna pot preračunavanje preko vmesnih kartezičnih koordinat (x, y, z).



$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{V'} \vec{J}(\vec{r}\,') \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} dV'$$

$$\vec{J}(\vec{r}\,') = \vec{1}_z \frac{I}{A_{žice}} \qquad dV' = A_{žice}\, dz'$$

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{-h/2}^{h/2} \vec{1}_z I \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} dz'$$



Poenostavitve:

(1) $h \ll r \;\rightarrow\; \dfrac{1}{|\vec{r}-\vec{r}\,'|} \approx \dfrac{1}{r}$

(2) $h \ll \lambda = \dfrac{2\pi}{k} \;\rightarrow\; e^{-jk|\vec{r}-\vec{r}\,'|} \approx e^{-jkr}$

$$\vec{A}(\vec{r}) = \vec{1}_z \frac{\mu I h}{4\pi} \frac{e^{-jkr}}{r} = \left(\vec{1}_r \cos\Theta - \vec{1}_\Theta \sin\Theta\right) \frac{\mu I h}{4\pi} \frac{e^{-jkr}}{r}$$

$$\vec{1}_z = \vec{1}_r \cos\Theta - \vec{1}_\Theta \sin\Theta$$

$$\vec{H}(\vec{r}) = \frac{1}{\mu} \operatorname{rot} \vec{A}(\vec{r}) = \vec{1}_\Phi \frac{I h}{4\pi} e^{-jkr} \left(\underbrace{\frac{jk}{r}}_{\text{Sevanje}} + \underbrace{\frac{1}{r^2}}_{\text{Biot-Savart}} \right) \sin\Theta$$

Tokovni element

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\epsilon}\,\mathrm{rot}\,\vec{H} = \frac{I\,h}{4\pi\,j\,\omega\,\epsilon}\,e^{-jkr}\left[\vec{1}_r\left(\frac{jk}{r^2}+\frac{1}{r^3}\right)2\cos\Theta + \vec{1}_\Theta\left(-\frac{k^2}{r}+\frac{jk}{r^2}+\frac{1}{r^3}\right)\sin\Theta\right]$$

Zveznost toka/elektrine $I = j\,\omega\,Q$

Sevanje

$$\vec{E}(\vec{r}) = \frac{Q\,h}{4\pi}\,e^{-jkr}\left[\vec{1}_r\left(\frac{jk}{r^2}+\frac{1}{r^3}\right)2\cos\Theta + \vec{1}_\Theta\left(-\frac{k^2}{r}+\frac{jk}{r^2}+\frac{1}{r^3}\right)\sin\Theta\right]$$

Točkasti statični električni dipol

$$\frac{1}{\omega\,\epsilon} = \frac{1}{\omega\,\sqrt{\mu\,\epsilon}}\sqrt{\frac{\mu}{\epsilon}} = \frac{Z}{k}$$

$$\vec{S}(\vec{r}) = \frac{1}{2}\vec{E}(\vec{r})\times\vec{H}(\vec{r})^* = \frac{|I|^2 h^2 Z}{32\pi^2 k}\left[\vec{1}_r\left(\frac{k^3}{r^2}-\frac{j}{r^5}\right)\sin^2\Theta + \vec{1}_\Theta\left(\frac{j\,k^2}{r^3}+\frac{j}{r^5}\right)2\cos\Theta\sin\Theta\right]$$

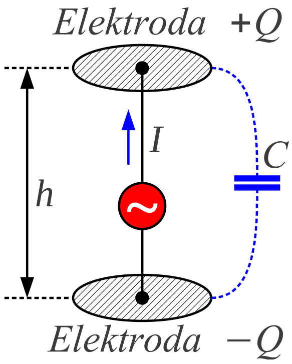
Sevanje

$$P = \oiint_{r\to\infty} \vec{S}(\vec{r})\cdot\vec{1}_r\, r^2 \sin\Theta\, d\Theta\, d\Phi = \frac{|I|^2 h^2 Z k^2}{12\pi}$$

$$R_s = \frac{2P}{|I|^2} = \frac{Z k^2 h^2}{6\pi} = \frac{2\pi Z}{3}\left(\frac{h}{\lambda}\right)^2$$

Dinamični električni dipol

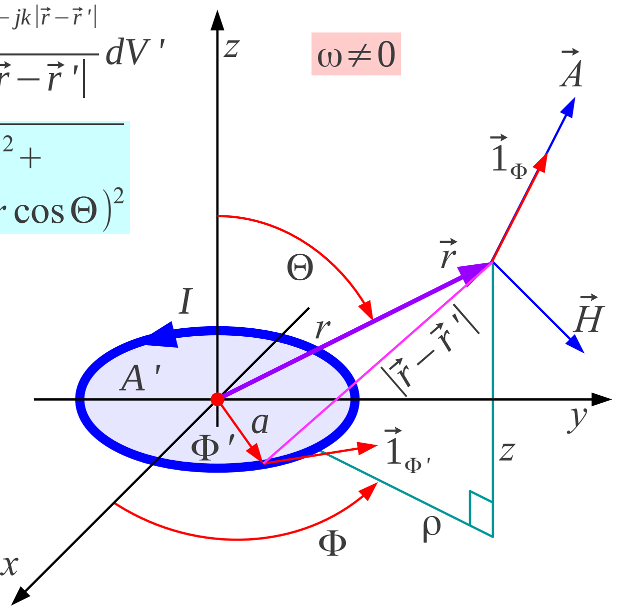
$$h \ll \lambda \;\rightarrow\; R_S \ll \frac{1}{\omega\,C}$$



Tokovna zanka

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{V'} \vec{J}(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dV'$$

$\omega \neq 0$



$$|\vec{r}-\vec{r}\,'| = \sqrt{\begin{matrix}(r\sin\Theta\cos\Phi - a\cos\Phi')^2 + \\ +(r\sin\Theta\sin\Phi - a\sin\Phi')^2 + (r\cos\Theta)^2\end{matrix}}$$

$$\vec{J}(\vec{r}\,') = \vec{1}_{\Phi'}\frac{I}{A_{žice}}$$

$$dV' = A_{žice}\,a\,d\Phi'$$

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_0^{2\pi}\vec{1}_{\Phi'}I\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}a\,d\Phi'$$

$$\vec{1}_{\Phi'} = -\vec{1}_x\sin\Phi' + \vec{1}_y\cos\Phi'$$

$$-\vec{1}_x\sin\Phi + \vec{1}_y\cos\Phi = \vec{1}_\Phi$$

Poenostavitve:

(1) $a \ll r \;\rightarrow\; \dfrac{1}{|\vec{r}-\vec{r}\,'|} \approx \dfrac{1}{r}\left[1 + \dfrac{a}{r}\sin\Theta\cos(\Phi-\Phi')\right]$

(2) $a \ll \lambda \;\rightarrow\; e^{-jk|\vec{r}-\vec{r}\,'|} \approx e^{-jkr}\left[1 + jka\sin\Theta\cos(\Phi-\Phi')\right]$

Površina zanke
$A' = \pi a^2$

$$\vec{A}(\vec{r}) = \vec{1}_\Phi\frac{\mu}{4\pi}I(\pi a^2)e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\Theta = \vec{1}_\Phi\frac{\mu}{4\pi}I\,A'e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\Theta$$

Sevanje

$$\vec{H}(\vec{r})=\frac{1}{\mu}\operatorname{rot}\vec{A}(\vec{r})=\frac{IA'}{4\pi}e^{-jkr}\left[\vec{1}_r\left(\frac{jk}{r^2}+\frac{1}{r^3}\right)2\cos\Theta+\vec{1}_\Theta\left(-\frac{k^2}{r}+\frac{jk}{r^2}+\frac{1}{r^3}\right)\sin\Theta\right]$$

Točkasti statični magnetni dipol

$$\rho(\vec{r}\,')=0 \;\rightarrow\; grad\,V(\vec{r})=0 \;\rightarrow\; \vec{E}(\vec{r})=-j\omega\vec{A}(\vec{r})$$

$$\omega\mu=\omega\sqrt{\mu\epsilon}\sqrt{\frac{\mu}{\epsilon}}=kZ$$

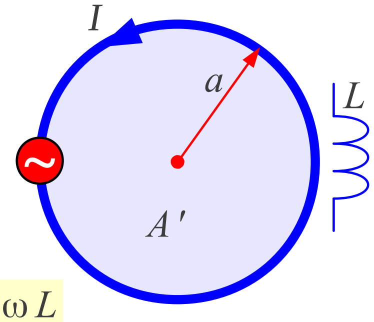
$$\vec{E}(\vec{r})=-\vec{1}_\Phi\frac{j\omega\mu I A'}{4\pi}e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\Theta=\vec{1}_\Phi\frac{Z I A'}{4\pi}e^{-jkr}\left(\frac{k^2}{r}-\frac{jk}{r^2}\right)\sin\Theta$$

$$\vec{S}(\vec{r})=\frac{|I|^2(A')^2 Z}{32\pi}\left[\vec{1}_r\left(\frac{k^4}{r^2}+\frac{jk}{r^5}\right)\sin^2\Theta-\vec{1}_\Theta\left(\frac{jk^3}{r^3}+\frac{jk}{r^5}\right)2\cos\Theta\sin\Theta\right]$$

Sevanje

$$P=\oint\!\!\oint_{r\rightarrow\infty}\vec{S}(\vec{r})\cdot\vec{1}_r r^2\sin\Theta\, d\Theta\, d\Phi=\frac{|I|^2(A')^2 Z k^4}{12\pi}$$

$$R_s=\frac{2P}{|I|^2}=\frac{Z k^4(A')^2}{6\pi}=\frac{8\pi^3 Z}{3}\left(\frac{A'}{\lambda^2}\right)^2$$



Dinamični magnetni dipol

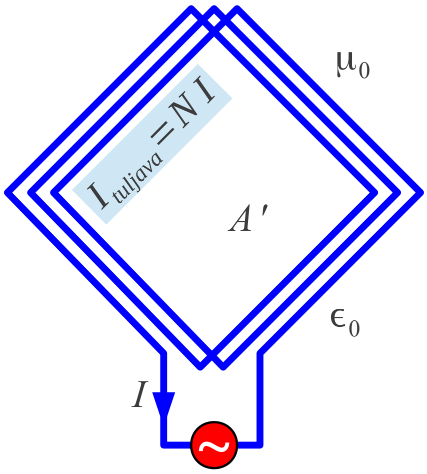
$$\sqrt{A'}\ll\lambda \;\rightarrow\; R_S\ll\omega L$$

$$R_s=\frac{Zk^4(NA')^2}{6\pi}=\frac{8\pi^3 Z}{3}\left(\frac{NA'}{\lambda^2}\right)^2$$

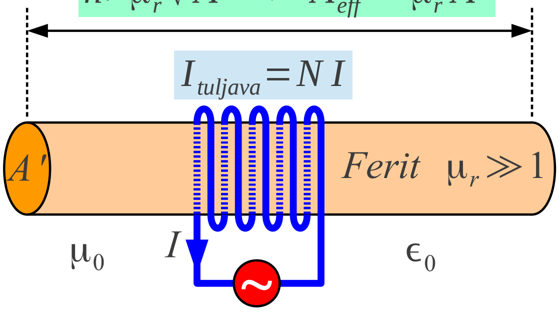
$f\approx 300\mathrm{kHz}$
$A'\approx 1\mathrm{m}^2$
$N\approx 10$

Zrak
$Z_0=\sqrt{\frac{\mu_0}{\epsilon_0}}\approx 377\,\Omega$
$\lambda=c_0/f=1\mathrm{km}$
$R_S\approx 3.1\,\mu\Omega$

Okvirna antena ~1930



$h>\mu_r\sqrt{A'}\ \rightarrow\ A_{eff}'\approx\mu_r A'$



Feritna antena ~1970

$$R_s=\frac{Zk^4(\mu_r NA')^2}{6\pi}=\frac{8\pi^3 Z}{3}\left(\frac{\mu_r NA'}{\lambda^2}\right)^2$$

$f\approx 1\mathrm{MHz}$
$A'\approx 1\mathrm{cm}^2$
$h\approx 20\mathrm{cm}$
$\mu_r\approx 100$
$N\approx 30$

Zrak
$Z_0=\sqrt{\frac{\mu_0}{\epsilon_0}}\approx 377\,\Omega$
$\lambda=c_0/f=300\mathrm{m}$
$R_S\approx 0.35\,\mu\Omega$

Poenostavitve za sevanje

$$k=\omega\sqrt{\mu\epsilon}$$

$$r\gg\frac{1}{k}=\frac{\lambda}{2\pi}\rightarrow\quad\frac{\partial}{\partial r}\approx-jk\qquad\frac{\partial}{\partial\Theta}\approx0\qquad\frac{\partial}{\partial\Phi}\approx0\qquad\nabla\approx\vec{1}_r(-jk)$$

$$\vec{H}=\frac{1}{\mu}\operatorname{rot}\vec{A}=\frac{1}{\mu}\nabla\times\vec{A}\approx-\frac{jk}{\mu}\vec{1}_r\times\vec{A}=-\frac{j\omega}{Z}\vec{1}_r\times\vec{A}=\vec{1}_\Theta\frac{j\omega}{Z}A_\Phi-\vec{1}_\Phi\frac{j\omega}{Z}A_\Theta$$

$$Lorenz\quad j\omega\mu\epsilon V+\operatorname{div}\vec{A}=0\rightarrow\quad V=\frac{j}{\omega\mu\epsilon}\nabla\cdot\vec{A}\approx\frac{j}{\omega\mu\epsilon}(-jk)A_r=\frac{kA_r}{\omega\mu\epsilon}=\frac{A_r}{\sqrt{\mu\epsilon}}$$

$$\vec{E}=-j\omega\vec{A}-\nabla V\approx-j\omega\vec{A}+jk\vec{1}_r\frac{A_r}{\omega\mu\epsilon}=-j\omega\left[\vec{A}-\vec{1}_r\left(\vec{1}_r\cdot\vec{A}\right)\right]=-j\omega\left[\vec{1}_\Theta A_\Theta+\vec{1}_\Phi A_\Phi\right]$$

$$Gauss\quad\frac{\rho}{\epsilon}=0=\nabla\cdot\vec{E}\approx-jk\vec{1}_r\cdot\vec{E}=-jkE_r\qquad0=\nabla\cdot\vec{H}\approx-jk\vec{1}_r\cdot\vec{H}=-jkH_r$$

$$Sevano\quad polje\quad\vec{1}_r\perp\vec{H}\perp\vec{E}\perp\vec{1}_r\qquad E_r=0\qquad H_r=0\qquad\frac{|\vec{E}|}{|\vec{H}|}=Z=\sqrt{\frac{\mu}{\epsilon}}$$

$$Faraday\quad\vec{H}=\frac{j}{\omega\mu}\operatorname{rot}\vec{E}=\frac{j}{\omega\mu}\nabla\times\vec{E}\approx\frac{j}{\omega\mu}(-jk)\vec{1}_r\times\vec{E}=\frac{\vec{1}_r\times\vec{E}}{Z}=-\vec{1}_\Theta\frac{E_\Phi}{Z}+\vec{1}_\Phi\frac{E_\Theta}{Z}$$

$$Poynting\quad\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*\approx\frac{\vec{E}\times\left(\vec{1}_r\times\vec{E}\right)^*}{2Z}=\vec{1}_r\frac{\vec{E}\cdot\vec{E}^*}{2Z}=\vec{1}_r\frac{\left|\vec{E}\right|^2}{2Z}$$

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