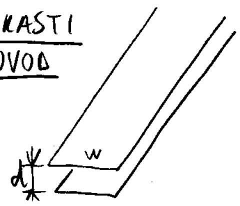
Elektrodinamika #1 8/10/2012

TRAKASTI DVOVOD



$$C/\ell = \varepsilon_0 \frac{w}{d}$$

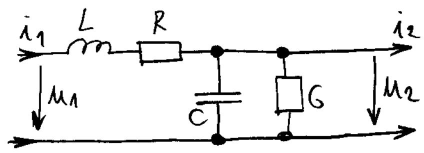
$$L/\ell = \mu_0 \frac{d}{w}$$

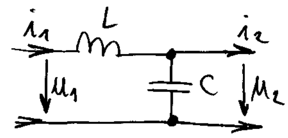
KOAKSIALNI KABEL

$$C/\ell = \frac{2\pi\varepsilon_0}{\ln b/a}$$

$$L/\ell = \frac{\mu_0}{2\pi} \ln b/a$$

NADOMESTNO VEZJE ⟶ **BREZ IZGUB**





$$\Delta u = u_2 - u_1 = -L\frac{di_1}{dt} - Ri_1$$

$$\Delta i = i_2 - i_1 = -C\frac{du_2}{dt} - Gu_2$$

$$\frac{\partial u(z,t)}{\partial z} = -L/\ell \frac{\partial i(z,t)}{\partial t} - R/\ell\, i(z,t)$$

$$\frac{\partial i(z,t)}{\partial z} = -C/\ell \frac{\partial u(z,t)}{\partial t} - G/\ell\, u(z,t)$$

$$\Delta u = u_2 - u_1 = -L\frac{di_1}{dt}$$

$$\Delta i = i_2 - i_1 = -C\frac{di_1}{dt}$$

$$\frac{\partial u(z,t)}{\partial z} = -L/\ell \frac{\partial i(z,t)}{\partial t} \quad /\frac{\partial}{\partial z}$$

$$\frac{\partial i(z,t)}{\partial z} = -C/\ell \frac{\partial u(z,t)}{\partial t} \quad /\frac{\partial}{\partial t}$$

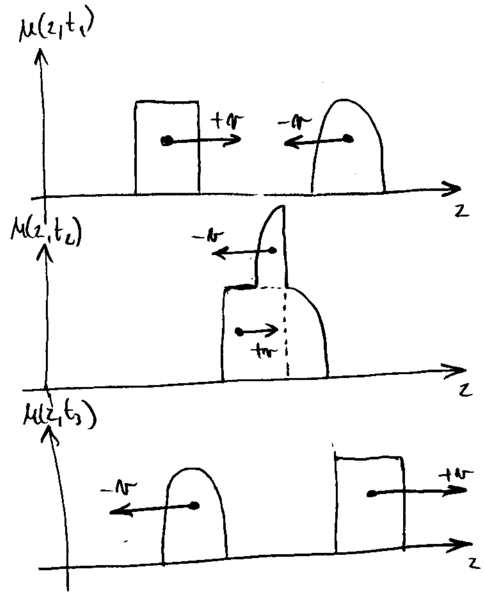
$$\frac{\partial^2 u(z,t)}{\partial z^2} = L/\ell \cdot C/\ell \frac{\partial^2 u(z,t)}{\partial t^2}$$

$$u(z,t) = f(x)\;;\; x = t \pm \frac{z}{v}$$

$$\frac{\partial^2 u(z,t)}{\partial z^2} = f''(x)\frac{1}{v^2} \qquad \frac{\partial^2 u(z,t)}{\partial t^2} = f''(x) \longrightarrow \underline{v = 1/\sqrt{L/\ell \cdot C/\ell}}$$

$$u(z,t) = C_1 f_1\left(t - \frac{z}{v}\right) + C_2 f_2\left(t + \frac{z}{v}\right)$$

NAPREDUJOČI VAL ODBITI VAL



TRAKASTI DVOVOD

$$v = 1/\sqrt{\mu_0 \frac{d}{w}\varepsilon_0\frac{w}{d}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \underline{c_0 \approx 3\cdot 10^8\,\mathrm{m/s}}$$

KOAKSIALNI KABEL

$$v = 1/\sqrt{\frac{\mu_0}{2\pi}\ln b/a \,\frac{2\pi\varepsilon_0}{\ln b/a}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \underline{c_0 \approx 3\cdot 10^8\,\mathrm{m/s}}$$

$$\frac{\partial}{\partial z} u\left(t \pm \frac{z}{v}\right) = -L/\ell \frac{\partial}{\partial t} i\left(t \pm \frac{z}{v}\right)$$

$$\pm\frac{1}{v} u'\left(t \pm \frac{z}{v}\right) = -L/\ell\, i'\left(t \pm \frac{z}{v}\right)$$

$$\underline{\frac{u'}{i'} = \frac{u}{i} = \mp v L/\ell = \mp\sqrt{\frac{L/\ell}{C/\ell}} = \mp Z_k}$$

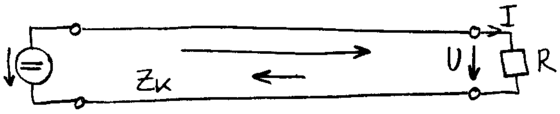
TRAKASTI DVOVOD $\quad Z_k = \sqrt{\dfrac{\mu_0 \frac{d}{w}}{\varepsilon_0\frac{w}{d}}} = \underline{\dfrac{d}{w}\sqrt{\dfrac{\mu_0}{\varepsilon_0}}}$

KOAKSIALNI KABEL $\quad Z_k = \sqrt{\dfrac{\frac{\mu_0}{2\pi}\ln b/a}{2\pi\varepsilon_0/\ln b/a}} = \underline{\dfrac{1}{2\pi}\sqrt{\dfrac{\mu_0}{\varepsilon_0}}\ln b/a}$

PROSTOR: $\dfrac{|E|}{|H|} = \underline{Z_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 120\pi\,\Omega = 377\,\Omega}$

Elektrodinamika #2 15/10/2012

STATIKA



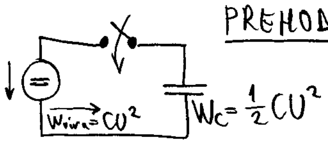
$$U = U_N + U_0$$

$$I = I_N + I_0 = \frac{U_N}{Z_k} - \frac{U_0}{Z_k} = \frac{U}{R} = \frac{U_N + U_0}{R} \quad / \cdot \frac{1}{U_N}$$

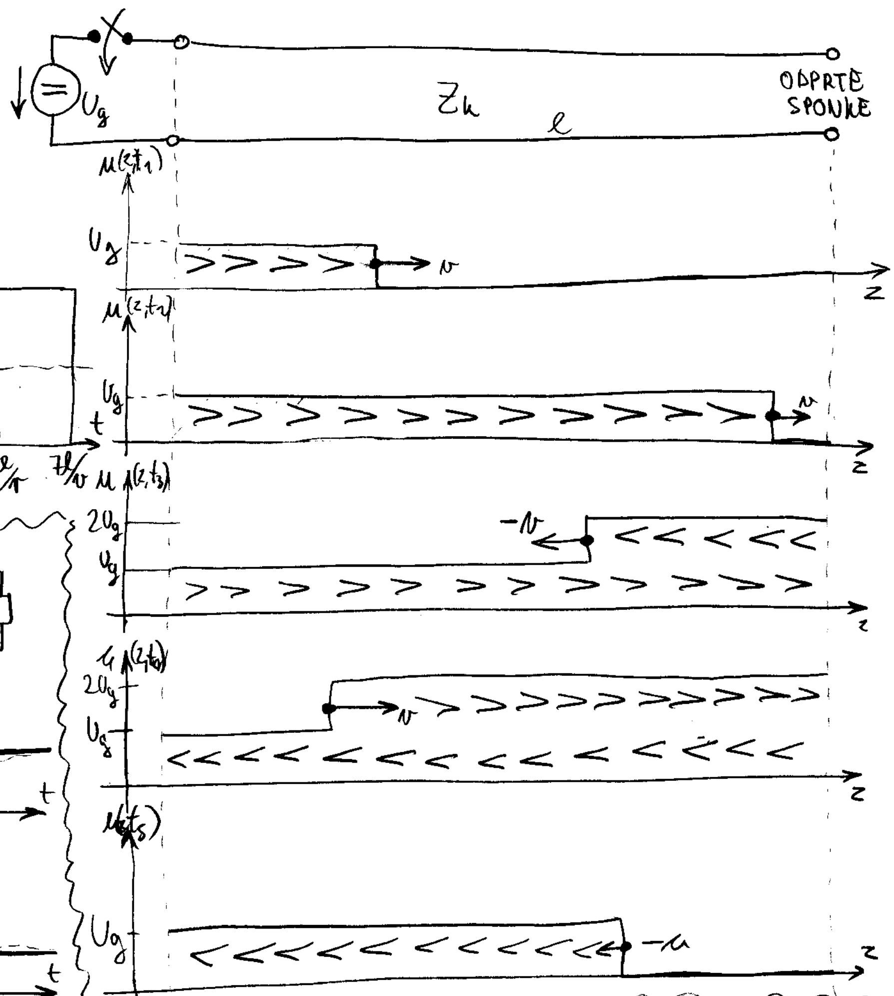
ODBOJNOST $\Gamma = \frac{U_0}{U_N} \longrightarrow \frac{1}{Z_k} - \frac{\Gamma}{Z_k} = \frac{1+\Gamma}{R} \longrightarrow \boxed{\Gamma = \frac{R - Z_k}{R + Z_k}}$ PASIVNO BREME

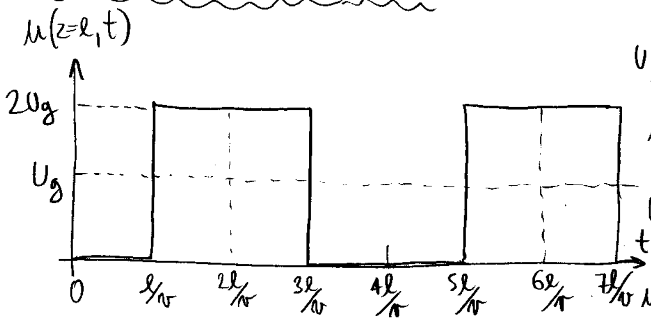
OS : $\Gamma = +1$, KS: $\Gamma = -1$; $Z_k = R \to \Gamma = 0$; $Z_k > R \to \Gamma < 0$; $Z_k < R \to \Gamma > 0$; $|\Gamma| \leqslant 1$

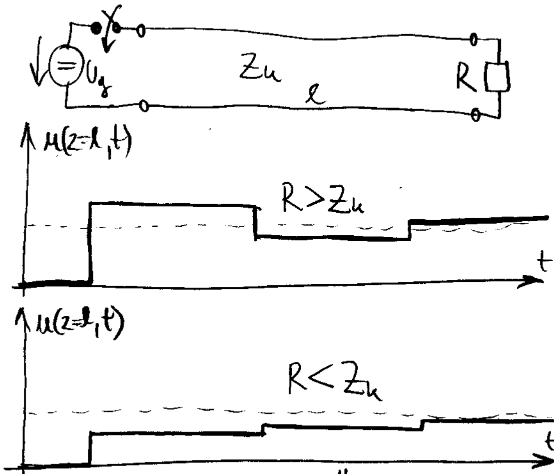
PREHODNI POJAV

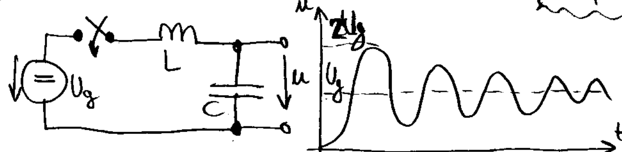


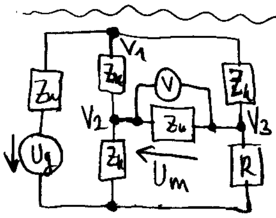
KAM GRE POLOVICA ENERGIJE ?











① $\frac{V_1 - U_g}{Z_k} + \frac{V_1 - V_2}{Z_k} + \frac{V_1 - V_3}{Z_k} = 0$

② $\frac{V_2 - V_1}{Z_k} + \frac{V_2 - V_3}{Z_k} + \frac{V_2}{Z_k} = 0$

③ $\frac{V_3 - V_2}{Z_k} + \frac{V_3 - V_1}{Z_k} + \frac{V_3}{R} = 0$

① $3V_1 = U_g + V_2 + V_3 = 9V_2 - 3V_3 \longrightarrow 8V_2 = U_g + 4V_3$

② $3V_2 = V_1 + V_3 \longrightarrow V_1 = 3V_2 - V_3$ MOSTIČ za Γ

③ $\left(2 + \frac{Z_k}{R}\right)V_3 = V_1 + V_2 = 4V_2 - V_3 \longrightarrow \left(3 + \frac{Z_k}{R}\right)V_3 = 4V_2$

$\left(6 + 2\frac{Z_k}{R}\right)V_3 = U_g + 4V_3 \longrightarrow V_3 = \frac{U_g}{2\left(1 + \frac{Z_k}{R}\right)}$ $U_m = V_3 - V_2$

$V_2 = \frac{U_g}{8} + \frac{V_3}{2} = \frac{U_g}{8} \frac{3 + \frac{Z_k}{R}}{\left(1 + \frac{Z_k}{R}\right)}$ $\boxed{U_m = \frac{U_g}{8} \frac{1 - \frac{Z_k}{R}}{1 + \frac{Z_k}{R}} = \frac{U_g}{8}\Gamma}$

FREKVENČNI PROSTOR

$\frac{\omega}{v} = \beta \equiv$ fazna konstanta

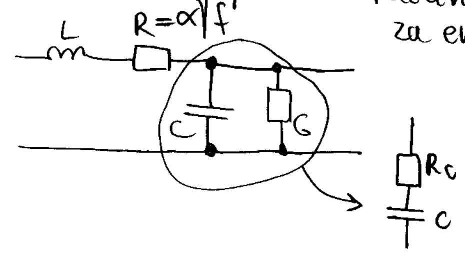
$$u(t,z) = \mathrm{Re}\left[U_0 e^{j\omega\left(t \pm \frac{z}{v}\right)}\right] = \mathrm{Re}\left[U_0 e^{j(\omega t \pm \beta z)}\right]$$

$$\frac{\partial}{\partial t} = j\omega \ ; \ \frac{\partial}{\partial z} = \pm j\beta \qquad \beta = \omega\sqrt{\mu_0 \varepsilon_0}$$

$$\beta\lambda = 2\pi \longrightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} = \frac{c_0}{f}$$

TEŽAVE Z IZGUBAMI

račun možen za eno frekvenco



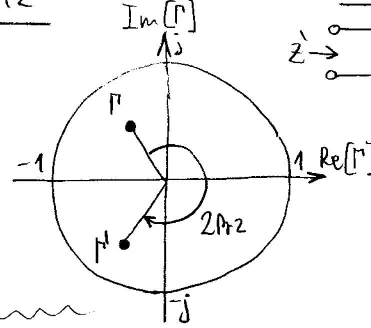
Elektrodinamika #3 22/10/2012

$$u(t,z)=\mathrm{Re}\left[U_N e^{j(\omega t-\beta z)}+U_O e^{j(\omega t+\beta z)}\right]$$

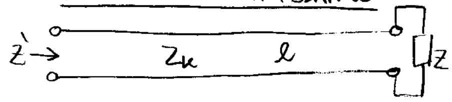
$$\Gamma=\frac{Z-Z_k}{Z+Z_k}=\frac{Y_k-Y}{Y_k+Y}$$

BREZIZGUBNI VOD

$|\Gamma|\leq 1$ pasivno breme



PRESLIKAVA IMPEDANCE

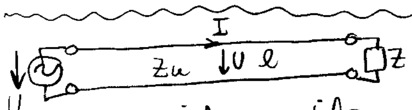


① $\Gamma=\dfrac{Z-Z_k}{Z+Z_k}$

② $\Gamma'=\Gamma e^{-j2\beta l}$

③ $Z'=Z_k\dfrac{1+\Gamma'}{1-\Gamma'}$

$$Z'=Z_k\frac{1+\Gamma'}{1-\Gamma'}=Z_k\frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}=Z_k\frac{Z+Z_k+(Z-Z_k)e^{-j2\beta l}}{Z+Z_k-(Z-Z_k)e^{-j2\beta l}}=Z_k\frac{Z(e^{j\beta l}+e^{-j\beta l})+Z_k(e^{j\beta l}-e^{-j\beta l})}{Z(e^{j\beta l}-e^{-j\beta l})+Z_k(e^{j\beta l}+e^{-j\beta l})}=\underline{Z_k\frac{Z\cos\beta l+jZ_k\sin\beta l}{jZ\sin\beta l+Z_k\cos\beta l}}$$



$$U=U_N e^{-j\beta z}+U_O e^{j\beta z}$$

$$I=\frac{U_N}{Z_k}e^{-j\beta z}-\frac{U_O}{Z_k}e^{j\beta z}$$

MOČ

$$P=\frac{1}{2}UI^*=\frac{1}{2}\left(U_N e^{-j\beta z}+U_O e^{+j\beta z}\right)\left(\frac{U_N^*}{Z_k}e^{+j\beta z}-\frac{U_O^*}{Z_k}e^{-j\beta z}\right)$$

$$P=\frac{|U_N|^2}{2Z_k}-\frac{|U_O|^2}{2Z_k}+\frac{U_O U_N^*}{2Z_k}e^{+j2\beta z}-\frac{U_N U_O^*}{2Z_k}e^{-j2\beta z}$$

$U_O U_N^*=|U_O U_N^*|e^{j\varphi}$

$U_N U_O^*=|U_O U_N^*|e^{-j\varphi}$

$$P=\underbrace{\frac{|U_N|^2}{2Z_k}}_{\text{NAPREDUJOČA MOČ}}-\underbrace{\frac{|U_O|^2}{2Z_k}}_{\text{ODBITA MOČ}}+\underbrace{j\frac{|U_O U_N^*|}{Z_k}\sin(2\beta z+\varphi)}_{\text{JALOVA MOČ}}$$

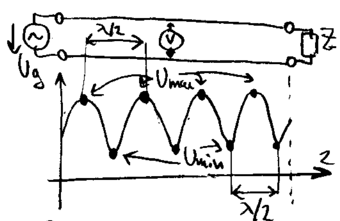
JALOVA MOČ → ENERGIJA STOJNEGA VALA

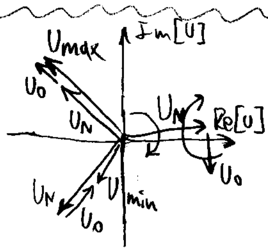
$\mathrm{Re}[P]=P_N-P_O \qquad P_O=|\Gamma|^2 P_N$

$$\underline{\mathrm{Re}[P]=P_N(1-|\Gamma|^2)}$$

STOJNI VAL (BREZIZGUBNI)

$$\rho=\frac{U_{max}}{U_{min}}$$





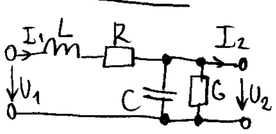
$$U_{max}=|U_N|+|U_O|=|U_N|(1+|\Gamma|)$$

$$U_{min}=|U_N|-|U_O|=|U_N|(1-|\Gamma|)$$

$$\rho=\frac{1+|\Gamma|}{1-|\Gamma|}$$

RAZMERJE STOJNEGA VALA
STANDING-WAVE RATIO
VALOVITOST

IZGUBNI VOD



$U_1-U_2=(j\omega L+R)I_1 \qquad \dfrac{\partial U}{\partial z}=-(j\omega L/l+R/l)I \;\Big/\dfrac{\partial}{\partial z}$

$I_1-I_2=(j\omega C+G)U_2 \qquad \dfrac{\partial I}{\partial z}=-(j\omega C/l+G/l)U$

$U=Ae^{\pm jkz}\longrightarrow -k^2=(j\omega L+R)(j\omega C+G)$

$$k=\sqrt{-(j\omega L/l+R/l)(j\omega C/l+G/l)}=\underset{[\mathrm{rd/m}]}{\beta}-j\underset{[\mathrm{Np/m}]}{\alpha}$$

$$u(z,t)=\mathrm{Re}\left[U_N e^{-\alpha z}e^{j(\omega t-\beta z)}+U_O e^{\alpha z}e^{j(\omega t+\beta z)}\right]$$

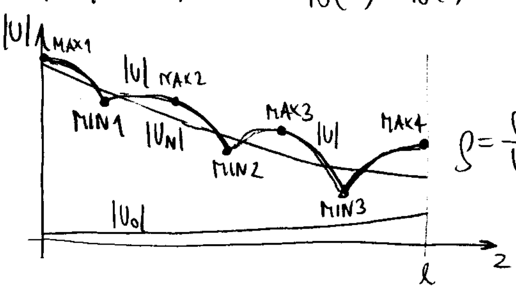
$$Z_k=\frac{U_N}{I_N};\; U_N=A_N e^{-jkz};\; I_N=\frac{-jk}{-(j\omega L/l+R/l)}U_N\longrightarrow Z_k=\frac{j\omega L/l+R/l}{jk}=\sqrt{\frac{j\omega L/l+R/l}{j\omega C/l+G/l}}\approx\sqrt{\frac{L/l}{C/l}}$$

ZA MAJHNE IZGUBE!

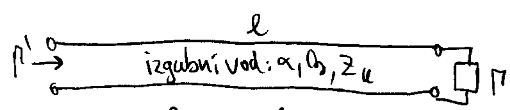
$$|U_N|=|A_N|e^{-\alpha z}\longrightarrow P_N(z)=P_N(0)e^{-2\alpha z}$$

$$|U_O|=|A_O|e^{+\alpha z}\longrightarrow P_O(z)=P_O(0)e^{+2\alpha z}$$

$$\Gamma(z)=\frac{U_O}{U_N}=\frac{e^{\alpha z}e^{j\beta z}}{e^{-\alpha z}e^{-j\beta z}}\Gamma(0)=\underline{e^{2\alpha z}e^{j2\beta z}\Gamma(0)}$$

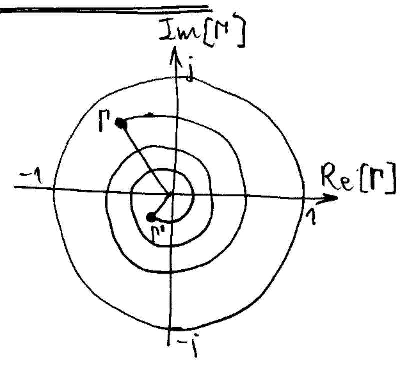


$$\rho=\frac{U_{max}}{U_{min}}=\text{NEDEFINIRAN}$$



$$\underline{\Gamma'=e^{-2\alpha l}e^{-j2\beta l}\Gamma}$$

$$\underline{|\Gamma'|=|\Gamma|e^{-2\alpha l}}$$



LOG. ENOTE

PRILAGOJENOST
RETURN LOSS

$$\Gamma_{dB}=20\log_{10}|\Gamma|$$

$$\Gamma_{Np}=\ln|\Gamma|=\ln\frac{U_O}{U_N}$$

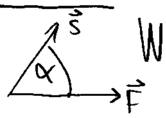
$$\Gamma'_{dB}=\Gamma_{dB}-\frac{20}{\ln 10}(2\alpha l)$$

$$\Gamma'_{dB}=\Gamma_{dB}-2a_{dB}$$

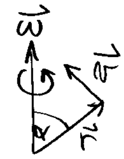
$$a_{dB}=10\log_{10}\frac{P(0)}{P(l)}=20\log_{10}\frac{U(0)}{U(l)}=\frac{20}{\ln 10}a_{Np} \qquad a_{dB}/l=\frac{20}{\ln 10}\alpha$$

Elektrodinamika #4 29/10/2012

FIZIKA



$$W = |\vec{F}||\vec{s}|\cos\varphi = \vec{F}\cdot\vec{s}$$

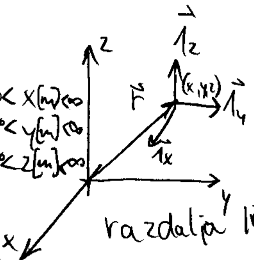


$$|\vec{v}| = |\vec{r}||\vec{\omega}|\sin\alpha$$

$$\vec{v} \perp \vec{r},\ \vec{v} \perp \vec{\omega}$$

$$\vec{v} = \vec{\omega}\times\vec{r}$$

KARTEZIČNI KS



$-\infty < x[m] < \infty$

$-\infty < y[m] < \infty$

$-\infty < z[m] < \infty$

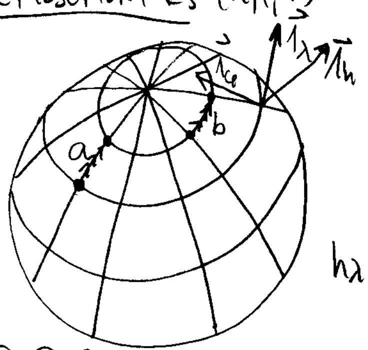
— 3D (x, y, z)

— pravokotni $\vec{1}_x \cdot \vec{1}_y = 0$

— desnoročni $\vec{1}_x \times \vec{1}_y = \vec{1}_z$

razdalja $|\vec{r}_2 - \vec{r}_1| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

ZEMLJEPISNI KS (λ, φ, h)



$$a = h_\varphi \Delta\varphi$$

$0 \leq \lambda[°] \leq 360°$

$-90° \leq \varphi[°] \leq +90°$

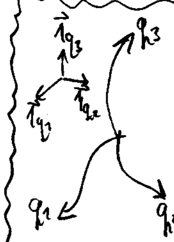
$6378000\,m < h[m] < \infty$

$$h_\varphi = \frac{40000\,km}{360°} = 111\,km/°$$

$$b = h_\lambda \Delta\lambda$$

$$h_\lambda = \frac{40000\,km}{360°}\cos\varphi = 111\,km/° \cdot \cos\varphi$$

SPLOŠNI KRIVOČRTNI KS (q_1, q_2, q_3)



$$dl_1 = \sqrt{dx^2 + dy^2 + dz^2} = h_1 dq_1$$

$$dx = \frac{\partial x}{\partial q_1}dq_1 \quad dy = \frac{\partial y}{\partial q_1}dq_1 \quad dz = \frac{\partial z}{\partial q_1}dq_1$$

$$dl_1 = \sqrt{\left(\frac{\partial x}{\partial q_1}\right)^2 + \left(\frac{\partial y}{\partial q_1}\right)^2 + \left(\frac{\partial z}{\partial q_1}\right)^2}\, dq_1$$

$$h_i = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2}$$

$x = x(q_1, q_2, q_3)$

$y = y(q_1, q_2, q_3)$

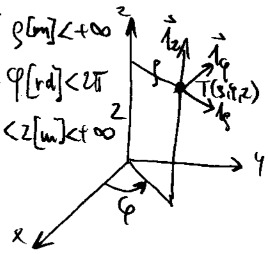
$z = z(q_1, q_2, q_3)$

VALJNI KS (ρ, φ, z)

$0 \leq \rho[m] < +\infty$

$0 \leq \varphi[rd] < 2\pi$

$-\infty < z[m] < +\infty$



$\rho = \sqrt{x^2 + y^2}$

$\varphi = \mathrm{arctg}\, y/x$ (kvadrant!)

$z = z$

$x = \rho\cos\varphi \qquad h_\rho = 1$

$y = \rho\sin\varphi \qquad h_\varphi = \rho$

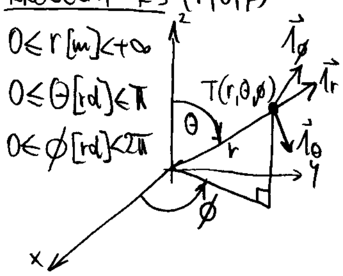
$z = z \qquad h_z = 1$

KROGELNI KS (r, θ, ϕ)

$0 \leq r[m] < +\infty$

$0 \leq \theta[rd] \leq \pi$

$0 \leq \phi[rd] < 2\pi$



$r = \sqrt{x^2 + y^2 + z^2}$

$\theta = \arccos z/r$

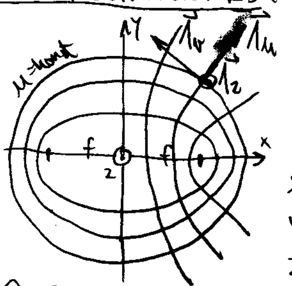
$\phi = \mathrm{arc\,tg}\, y/x$ (kvadrant!)

$x = r\sin\theta\cos\phi \qquad h_r = 1$

$y = r\sin\theta\sin\phi \qquad h_\theta = r$

$z = r\cos\theta \qquad h_\phi = r\sin\theta$

VALJNI-ELIPTIČNI KS (u, v, z)



$0 \leq u[\ ] < +\infty$

$0 \leq v[rd] < 2\pi$

$-\infty < z[m] < +\infty$

podatek $f[m]$

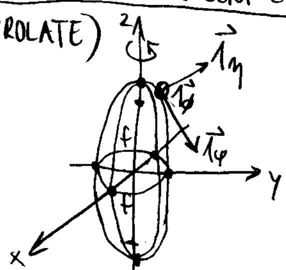
$x = f\,\mathrm{ch}u\cos v \qquad h_z = 1$

$y = f\,\mathrm{sh}u\sin v$

$z = z \qquad h_u = h_v = f\sqrt{\mathrm{sh}^2 u + \sin^2 v}$

PODOLGOVATI KROGELNI-ELIPTIČNI KS (η, ψ, ϕ)

(PROLATE)



$0 \leq \eta[\ ] < +\infty$ podatek $f[m]$

$0 \leq \psi[rd] \leq \pi$

$0 \leq \phi[rd] < 2\pi$

$x = f\,\mathrm{sh}\eta\sin\psi\cos\phi$

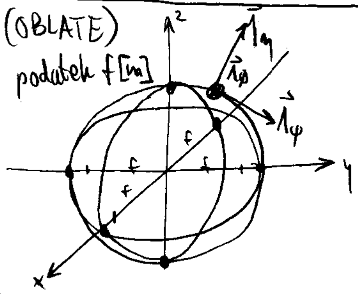
$y = f\,\mathrm{sh}\eta\sin\psi\sin\phi$

$z = f\,\mathrm{ch}\eta\cos\psi$

SPLOŠČENI KROGELNI-ELIPTIČNI KS (η, ψ, ϕ)

(OBLATE)

podatek $f[m]$



$0 \leq \eta[\ ] < +\infty$

$0 \leq \psi[rd] \leq \pi$

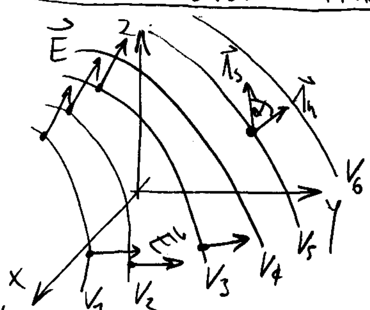
$0 \leq \phi[rd] < 2\pi$

$x = f\,\mathrm{ch}\eta\sin\psi\cos\phi$

$y = f\,\mathrm{ch}\eta\sin\psi\sin\phi$

$z = f\,\mathrm{sh}\eta\cos\psi$

SMERNI ODVOD = GRADIENT $V(x, y, z)$



$$\max\frac{\partial V}{\partial s} = \frac{\partial V}{\partial n}$$

$$\frac{\partial V}{\partial s} = \vec{1}_s\cdot\vec{1}_n\frac{\partial V}{\partial n} = \frac{\partial V}{\partial n}\cos\alpha$$

$$\vec{1}_n\frac{\partial V}{\partial n} = \mathrm{grad}\,V$$

$$(x, y, z) \rightarrow \vec{1}_x\cdot\mathrm{grad}V = \frac{\partial V}{\partial x},\ \vec{1}_y\cdot\mathrm{grad}V = \frac{\partial V}{\partial y},\ \vec{1}_z\cdot\mathrm{grad}V = \frac{\partial V}{\partial z} \rightarrow \underline{\mathrm{grad}V = \vec{1}_x\frac{\partial V}{\partial x} + \vec{1}_y\frac{\partial V}{\partial y} + \vec{1}_z\frac{\partial V}{\partial z}}$$

$$\vec{\nabla} = \vec{1}_x\frac{\partial}{\partial x} + \vec{1}_y\frac{\partial}{\partial y} + \vec{1}_z\frac{\partial}{\partial z} \rightarrow \mathrm{grad}\,V = \vec{\nabla}V$$

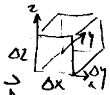
$dl_1 = h_1 dq_1,\ dl_2 = h_2 dq_2,\ dl_3 = h_3 dq_3$

$$(q_1, q_2, q_3) \rightarrow \vec{1}_{q_1}\cdot\mathrm{grad}V = \frac{\partial V}{\partial l_1},\ \vec{1}_{q_2}\cdot\mathrm{grad}V = \frac{\partial V}{\partial l_2},\ \vec{1}_{q_3}\cdot\mathrm{grad}V = \frac{\partial V}{\partial l_3} \rightarrow \underline{\mathrm{grad}V = \vec{1}_{q_1}\frac{1}{h_1}\frac{\partial V}{\partial q_1} + \vec{1}_{q_2}\frac{1}{h_2}\frac{\partial V}{\partial q_2} + \vec{1}_{q_3}\frac{1}{h_3}\frac{\partial V}{\partial q_3}}$$

$$(\rho, \varphi, z) \rightarrow \underline{\mathrm{grad}V = \vec{1}_\rho\frac{\partial V}{\partial \rho} + \vec{1}_\varphi\frac{1}{\rho}\frac{\partial V}{\partial \varphi} + \vec{1}_z\frac{\partial V}{\partial z}}$$

$$(r, \theta, \phi) \rightarrow \underline{\mathrm{grad}V = \vec{1}_r\frac{\partial V}{\partial r} + \vec{1}_\theta\frac{1}{r}\frac{\partial V}{\partial \theta} + \vec{1}_\phi\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}}$$

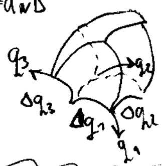
Elektrodinamika 5/11/2012



IZVORNOST

$$\int_V \rho\, dv = Q = \oint_A \vec{D}\cdot d\vec{A} \rightarrow \rho = \lim_{\Delta V\to 0} \frac{\oint \vec{D}\cdot d\vec{A}}{\Delta V} = \mathrm{div}\,\vec{D}$$

$$\rho = \lim_{\Delta V\to 0} \frac{\Delta y\Delta z\Delta D_x + \Delta x\Delta z\Delta D_y + \Delta x\Delta y\Delta D_z}{\Delta x\Delta y\Delta z} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \vec{\nabla}\cdot\vec{D}$$



$$\mathrm{div}\,\vec{D} = \frac{1}{h_1h_2h_3}\left(\frac{\partial}{\partial q_1}(h_2h_3D_1) + \frac{\partial}{\partial q_2}(h_1h_3D_2) + \frac{\partial}{\partial q_3}(h_1h_2D_3)\right)$$

$$\rho = \lim_{\Delta V\to 0} \frac{\Delta(l_2l_3D_1) + \Delta(l_1l_3D_2) + \Delta(l_1l_2D_3)}{\Delta l_1\Delta l_2\Delta l_3} = \frac{\frac{\partial(h_2h_3D_1)}{\partial q_1} + \frac{\partial(h_1h_3D_2)}{\partial q_2} + \frac{\partial(h_1h_2D_3)}{\partial q_3}}{h_1h_2h_3}$$

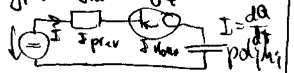
VRTINČENJE

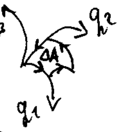
$$\int_A \vec{J}\cdot d\vec{A} = I = \oint_S \vec{H}\cdot d\vec{s} \longrightarrow J_z = \lim_{\Delta A\to 0} \frac{\oint \vec{H}\cdot d\vec{s}}{\Delta A} = \vec{1}_z\cdot \mathrm{rot}\,\vec{H}$$

$$J_z = \lim_{\Delta A\to 0} \frac{\Delta y H_y - \Delta x H_x}{\Delta x\Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \rightarrow \vec{J} = \vec{\nabla}\times\vec{H} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\vec{J} = \mathrm{rot}\,\vec{H}$$

$$\vec{J} = \vec{J}_{prev} + \vec{J}_{konv} + \frac{\partial\vec{D}}{\partial t}$$





$$J_3 = \lim_{\Delta A\to 0} \frac{\Delta l_2 H_2 - \Delta l_1 H_1}{\Delta l_1\Delta l_2} = \frac{1}{h_1h_2}\left(\frac{\partial(h_2H_2)}{\partial q_1} - \frac{\partial(h_1H_1)}{\partial q_2}\right) \qquad \vec{J} = \frac{1}{h_1h_2h_3}\begin{vmatrix} \vec{1}_{q_1}h_1 & \vec{1}_{q_2}h_2 & \vec{1}_{q_3}h_3 \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1H_1 & h_2H_2 & h_3H_3 \end{vmatrix}$$

ME v dif. obliki

1. $\mathrm{rot}\,\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$
2. $\mathrm{rot}\,\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
3. $\mathrm{div}\,\vec{D} = \rho$

Harmonske veličine

$\vec{E}(x,y,z,t) = \vec{E}_0(x,y,z)e^{j\omega t}$

$\frac{\partial}{\partial t} = j\omega$

1. $\mathrm{rot}\,\vec{H} = \vec{J} + j\omega\vec{D}$
2. $\mathrm{rot}\,\vec{E} = -j\omega\vec{B}$
3. $\mathrm{div}\,\vec{D} = \rho$

Snov: ε, μ (skalarni konstanti)

1. $\mathrm{rot}\,\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$
2. $\mathrm{rot}\,\vec{E} = -j\omega\mu\vec{H}$
3. $\mathrm{div}(\varepsilon\vec{E}) = \rho$

EM naloga

izvori $(\rho, \vec{J}) \rightarrow$ polje $(\vec{E}, \vec{H})$

$$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = \mathrm{grad}\left(\frac{\rho}{\varepsilon}\right) + j\omega\mu\vec{J}$$

$$\Delta\vec{H} + \omega^2\mu\varepsilon\vec{H} = -\mathrm{rot}\,\vec{J}$$

Sestavljene operacije

$$\mathrm{rot}(\mathrm{grad}\,V) = \vec{\nabla}\times(\vec{\nabla}V) = 0$$

$$\mathrm{div}(\mathrm{rot}\,\vec{F}) = \vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) = 0$$

$$\mathrm{rot}(\mathrm{rot}\,\vec{F}) = \mathrm{grad}(\mathrm{div}\,\vec{F}) - \Delta\vec{F}$$

$$\mathrm{div}(\mathrm{grad}\,V) = \vec{\nabla}\cdot(\vec{\nabla}V) = \Delta V$$

Potenciali

OE I $\rightarrow \omega = 0$, $\vec{E} = -\mathrm{grad}\,V$, $\Delta V = -\frac{\rho}{\varepsilon}$

OE II $\rightarrow \omega = 0$, $\vec{J} = 0$, $\vec{H} = -\mathrm{grad}\,V_m$, $\Delta V_m = 0$

VEKTORSKI POTENCIAL $\vec{B} = \mathrm{rot}\,\vec{A}$

2. $\mathrm{rot}\,\vec{E} = -j\omega\vec{B} = -j\omega\,\mathrm{rot}\,\vec{A} \rightarrow \mathrm{rot}(\vec{E} + j\omega\vec{A}) = 0 \rightarrow \vec{E} + j\omega\vec{A} = -\mathrm{grad}\,V$, $\underline{\vec{E} = -j\omega\vec{A} - \mathrm{grad}\,V}$

1. $\mathrm{rot}\,\vec{H} = \mathrm{rot}\left(\frac{1}{\mu}\mathrm{rot}\,\vec{A}\right) = \vec{J} + j\omega\varepsilon\vec{E} \rightarrow \mathrm{rot}(\mathrm{rot}\,\vec{A}) = \mathrm{grad}(\mathrm{div}\,\vec{A}) - \Delta\vec{A} = \mu\vec{J} + j\omega\mu\varepsilon(-j\omega\vec{A} - \mathrm{grad}\,V)$

$$\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J} + \mathrm{grad}(\mathrm{div}\,\vec{A} + j\omega\mu\varepsilon V)$$

Columb: $\mathrm{div}\,\vec{A} = 0$ $\quad$ $\underline{\text{Lorentz: } \mathrm{div}\,\vec{A} = -j\omega\mu\varepsilon V}$

$$\boxed{\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J}}$$

3. $\mathrm{div}(\varepsilon\vec{E}) = \rho \rightarrow \frac{\rho}{\varepsilon} = -j\omega\,\mathrm{div}\,\vec{A} - \mathrm{div}(\mathrm{grad}\,V)$

$$\boxed{\Delta V + \omega^2\mu\varepsilon V = -\frac{\rho}{\varepsilon}}$$

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OE I, $\omega=0$ $W_e=\frac{1}{2}\int_v \vec{E}\cdot\vec{D}\,dv$ $\mathrm{div}(\varepsilon V\,\mathrm{grad}V)=\varepsilon\,\mathrm{grad}V\cdot\mathrm{grad}V+\varepsilon V\Delta V=\vec{E}\cdot\vec{D}-\rho V$

$\Delta V=-\frac{\rho}{\varepsilon}$ $\int_v \mathrm{div}(\varepsilon V\,\mathrm{grad}V)\,dv=\oint_A \varepsilon V\,\mathrm{grad}V\cdot\vec{1}_n dA=\int_v \vec{E}\cdot\vec{D}\,dv-\int_v \rho V\,dv \longrightarrow W_e=\frac{1}{2}\int_v \rho V\,dv$

$A\to\infty$ $V(\infty)=0$

OE II, $\omega=0$ $W_m=\frac{1}{2}\int_v \vec{H}\cdot\vec{B}\,dv$ $\mathrm{div}(\vec{H}\times\vec{A})=\vec{\nabla}\cdot(\vec{H}\times\vec{A})=\vec{A}\cdot\mathrm{rot}\vec{H}-\vec{H}\cdot\mathrm{rot}\vec{A}=\vec{J}\cdot\vec{A}-\vec{H}\cdot\vec{B}$

$\int_v \mathrm{div}(\vec{H}\times\vec{A})\,dv=\oint_A(\vec{H}\times\vec{A})\cdot\vec{1}_n dA=0=\int_v \vec{J}\cdot\vec{A}\,dv-\int_v \vec{H}\cdot\vec{B}\,dv \longrightarrow W_m=\frac{1}{2}\int_v \vec{J}\cdot\vec{A}\,dv$

$A\to\infty$ $\vec{A}(\infty)=0$

POYNTING $W=\frac{1}{2}\int_v \vec{E}\cdot\vec{D}\,dv+\frac{1}{2}\int_v \vec{H}\cdot\vec{B}\,dv$, $\frac{\partial\vec{D}}{\partial t}=\mathrm{rot}\vec{H}-\vec{J}$, $\frac{\partial\vec{B}}{\partial t}=-\mathrm{rot}\vec{E}$, $P=\int_v \vec{J}\cdot\vec{E}\,dv$

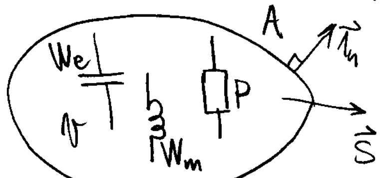
$\frac{dW}{dt}=\frac{1}{2}\int_v\left[2\vec{E}\cdot(\mathrm{rot}\vec{H}-\vec{J})-2\vec{H}\cdot\mathrm{rot}\vec{E}\right]dv=-\int_v \vec{J}\cdot\vec{E}\,dv+\int_v\left[\vec{E}\cdot\mathrm{rot}\vec{H}-\vec{H}\cdot\mathrm{rot}\vec{E}\right]dv=$

$=-\int_v \vec{J}\cdot\vec{E}\,dv+\int_v \mathrm{div}(\vec{H}\times\vec{E})\,dv=-P-\oint_A(\vec{E}\times\vec{H})\cdot\vec{1}_n dA \longrightarrow \underline{\oint_A(\vec{E}\times\vec{H})\cdot\vec{1}_n dA=\oint_A \vec{S}\cdot\vec{1}_n dA=-P-\frac{dW}{dt}}$

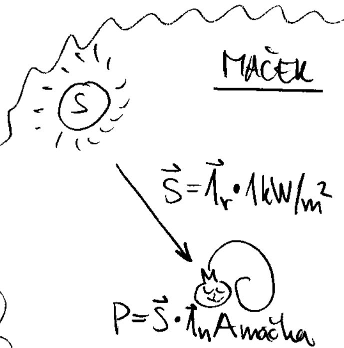
$\underline{\vec{S}=\vec{E}\times\vec{H}}$ $[\mathrm{W/m^2}]$

$\vec{E}\,[\mathrm{V/m}]$

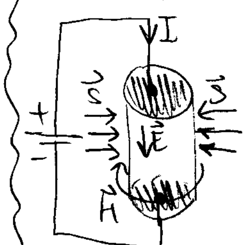
$\vec{H}\,[\mathrm{A/m}]$



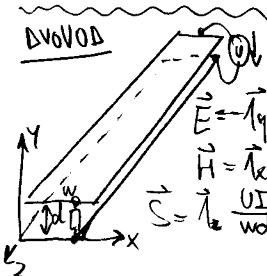
MAČEK



VALJASTI UPOR



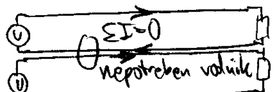
DVOVOD



$\vec{E}=-\vec{1}_y\frac{U}{d}$

$\vec{H}=\vec{1}_x\frac{I}{w}$

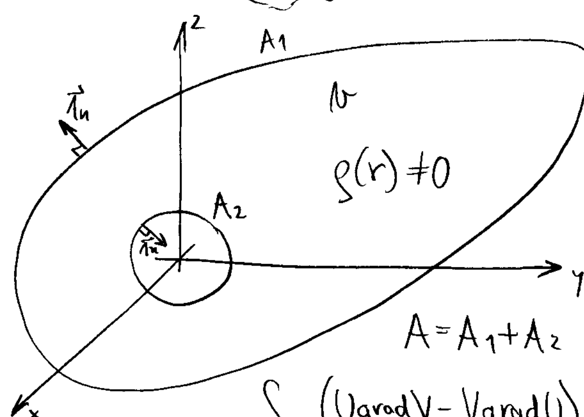
$\vec{S}=\vec{1}_z\frac{UI}{wd}$



DVA DVOVODA

$\mathrm{div}(U\,\mathrm{grad}V-V\,\mathrm{grad}U)=U\Delta V-V\Delta U$ GREEN

$\oint_A(U\,\mathrm{grad}V-V\,\mathrm{grad}U)\cdot\vec{1}_n dA=\int_v(U\Delta V-V\Delta U)\,dv$



$A=A_1+A_2$

$\omega=0 \to U=\frac{1}{r}$; $\mathrm{grad}U=-\vec{1}_r\frac{1}{r^2}$; $\Delta U=0$

$\Delta V=-\frac{\rho}{\varepsilon}$; $\int_v(U\Delta V-V\Delta U)\,dv=-\int_v\frac{\rho}{\varepsilon r}dv$

$\int_{A_1}(U\,\mathrm{grad}V-V\,\mathrm{grad}U)\cdot\vec{1}_n dA=\underline{0}$

$\omega\neq 0 \to U=\frac{e^{-jkr}}{r}$; $\mathrm{grad}U=-\vec{1}_r\left(\frac{1}{r}+jk\right)U$; $\Delta U=-k^2U$

$\Delta V=-\frac{\rho}{\varepsilon}-k^2V$; $\int_v(U\Delta V-V\Delta U)\,dv=-\int_v\frac{\rho}{\varepsilon}\frac{e^{-jkr}}{r}dv$

$\int_{A_2}(U\,\mathrm{grad}V-V\,\mathrm{grad}U)\cdot\vec{1}_n dA=-\underline{4\pi V(r=0)}$

$\omega=0 \to V(r=0)=\frac{1}{4\pi\varepsilon}\int_v\frac{\rho}{r}dv$; $\omega\neq 0 \to V(r=0)=\frac{1}{4\pi\varepsilon}\int_v\rho\frac{e^{-jkr}}{r}dv$

POTENCIAL V POLJUBNI TOČKI

$$V(\vec{r})=\frac{1}{4\pi\varepsilon}\int_{v'}\rho(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dv'$$

$$V(\vec{r}\,')=\frac{1}{4\pi\varepsilon}\sum_i Q_i\frac{e^{-jk|\vec{r}-\vec{r}_i|}}{|\vec{r}-\vec{r}_i|}$$

VEKTORSKI POTENCIAL $\Delta\vec{A}+k^2\vec{A}=-\mu\vec{J}$

$\Delta\vec{A}=\vec{1}_x A_x+\vec{1}_y A_y+\vec{1}_z A_z$ (x,y,z)

$\Delta A_x+k^2A_x=-\mu J_x$ .....

$$A_x(\vec{r})=\frac{\mu}{4\pi}\int_{v'}J_x(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dv'$$

$$\vec{A}(\vec{r})=\frac{\mu}{4\pi}\int_{v'}\vec{J}(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dv'$$

3D

Poljuben KS

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PONOVITEV ME, $\frac{\partial}{\partial t}=j\omega$

① $\mathrm{rot}\vec{H}=\vec{J}+j\omega\varepsilon\vec{E}$ $\xrightarrow{\mathrm{div}\,+③}$ $\mathrm{div}(\mathrm{rot}\vec{H})=0=\mathrm{div}\vec{J}+j\omega\,\mathrm{div}(\varepsilon\vec{E})$

② $\mathrm{rot}\vec{E}=-j\omega\mu\vec{H}$

③ $\mathrm{div}(\varepsilon\vec{E})=\rho$

$$\boxed{0=\mathrm{div}\vec{J}+j\omega\rho}$$

Zveznost (kontinuiteta) toka

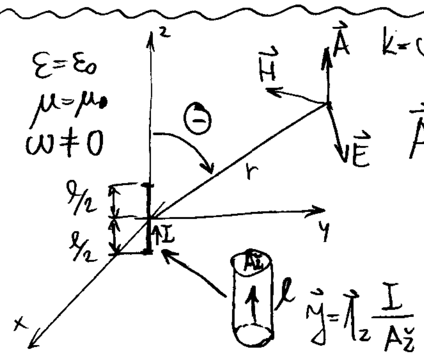
POTENCIALI

$\vec{B}=\mathrm{rot}\vec{A}=\mathrm{rot}\vec{V}_m$

$V=-j\omega\vec{A}-\mathrm{grad}V$

Lorentz-ova izbira $\longrightarrow$ $\Delta\vec{A}+\omega^2\mu\varepsilon\vec{A}=-\mu\vec{J}$ $\qquad \vec{A}(\vec{r})=\frac{\mu}{4\pi}\int_{v'}\vec{J}(\vec{r}')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}dv'$

$\Delta V+\omega^2\mu\varepsilon V=-\frac{\rho}{\varepsilon}$ $\qquad V(\vec{r})=\frac{1}{4\pi\varepsilon}\int_{v'}\rho(\vec{r}')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}dv'=\frac{1}{4\pi\varepsilon}\sum_i Q_i\frac{e^{-jk|\vec{r}-\vec{r}_i|}}{|\vec{r}-\vec{r}_i|}$



$\vec{J}=\vec{1}_z\frac{I}{A_z}$

$$\vec{A}=\frac{\mu}{4\pi}\int_{v'}\vec{1}_z\frac{I}{A_z}\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}A_z\,dz\approx\vec{1}_z\frac{\mu I\ell}{4\pi}\frac{e^{-jkr}}{r}$$

KRATKA ŽICA

① $\ell\ll r$ $\nearrow \frac{1}{|\vec{r}-\vec{r}'|}\approx\frac{1}{r}$

② $\ell\ll\frac{2\pi}{k}\rightarrow e^{-jk|\vec{r}-\vec{r}'|}\approx e^{-jkr}$

$(x,y,z)\rightarrow(r,\theta,\phi)$: $\vec{1}_z=\vec{1}_r\cos\theta-\vec{1}_\theta\sin\theta$

$$\underline{\underline{\vec{A}=(\vec{1}_r\cos\theta-\vec{1}_\theta\sin\theta)\frac{\mu I\ell}{4\pi}\frac{e^{-jkr}}{r}}}$$

$\vec{H}=\frac{1}{\mu}\mathrm{rot}\vec{A}$

$$\vec{H}=\frac{1}{\mu}\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & \vec{1}_\theta r & \vec{1}_\phi r\sin\theta\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ \frac{\mu I\ell}{4\pi}\frac{e^{-jkr}}{r}\cos\theta & -r\frac{\mu I\ell}{4\pi}\frac{e^{-jkr}}{r}\sin\theta & 0\end{vmatrix}=\vec{1}_\phi\frac{I\ell}{4\pi}e^{-jkr}\Big(\underbrace{\frac{jk}{r}}_{\text{SEVANJE}}+\underbrace{\frac{1}{r^2}}_{\text{BIOT-SAVART}}\Big)\sin\theta$$

Zveznost toka $(+Q)$ $I=\frac{dQ}{dt}=j\omega Q$ $\;(-Q)$

$\vec{E}=-j\omega\vec{A}-\mathrm{grad}V$

② ME @ $\vec{J}=0$

$$\vec{E}=\frac{1}{j\omega\varepsilon}\mathrm{rot}\vec{H}=\frac{1}{j\omega\varepsilon}\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & \vec{1}_\theta r & \vec{1}_\phi r\sin\theta\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & r0 & r\sin\theta\frac{I\ell}{4\pi}e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta\end{vmatrix}$$

$$\vec{E}=\frac{Q\ell}{4\pi\varepsilon}e^{-jkr}\Big[\vec{1}_r\Big(\frac{jk}{r^2}+\underbrace{\frac{1}{r^3}}_{\text{TOČKASTI STAT. DIPOL}}\Big)2\cos\theta+\vec{1}_\theta\Big(\underbrace{-\frac{k^2}{r}}_{\text{SEVANJE}}+\frac{jk}{r^2}+\underbrace{\frac{1}{r^3}}_{\text{TOČKASTI STAT. DIPOL}}\Big)\sin\theta\Big]$$

$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*$

PRIMERJAVA

	$f=50\,\mathrm{Hz}$	$f=900\,\mathrm{MHz}$	$\lambda=0{,}5\,\mu m$
k	10^{-6} rd/m	19 rd/m	$1{,}3\cdot10^{7}$ rd/m
k^2	10^{-12} rd²/m²	380 rd²/m²	$1{,}7\cdot10^{14}$ rd/m
$r=\frac{1}{k}$	1000 km	5,3 cm	80 nm

$k=\omega\sqrt{\mu\varepsilon}$ $\qquad Z_0=\sqrt{\frac{\mu}{\varepsilon}}$ $\qquad \frac{1}{\omega\varepsilon}=\frac{Z_0}{k}$

$$\vec{S}=\frac{1}{2}\frac{I\ell}{j\omega4\pi\varepsilon}e^{-jkr}\left[\vec{1}_r\left(\frac{jk}{r^2}+\frac{1}{r^3}\right)2\cos\theta+\vec{1}_\theta\left(-\frac{k^2}{r}+\frac{jk}{r^2}+\frac{1}{r^3}\right)\sin\theta\right]\times$$

$$\times\vec{1}_\phi\frac{I^*\ell}{4\pi}e^{+jkr}\left[-\frac{jk}{r}+\frac{1}{r^2}\right]\sin\theta$$

$$\vec{S}=\frac{|I|^2\ell^2Z_0}{32\pi^2}\left[\vec{1}_r\left(\underbrace{\frac{k^2}{r^2}}_{\text{DELOVNA SEVANA MOČ}}-\frac{j}{kr^5}\right)\sin^2\theta+\vec{1}_\theta\left(\frac{jk}{r^3}+\frac{j}{kr^5}\right)2\cos\theta\sin\theta\right]$$

JALOVA MOČ

$$P=\oint_{r\to\infty}\vec{S}\cdot\vec{1}_r r^2\sin\theta\,d\theta\,d\phi$$

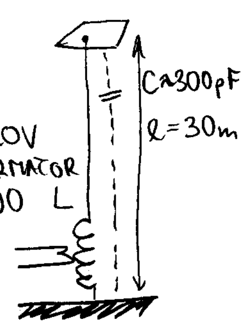
$$P=\frac{|I|^2\ell^2Z_0}{32\pi^2}\int_0^{\pi}\int_0^{2\pi}\left(\frac{k^2}{r^2}-\frac{j}{kr^5}\right)\sin^2\theta\sin\theta\,d\theta=\frac{|I|^2\ell^2Z_0k^2}{16\pi}\int_{-1}^{+1}(1-u^2)du=\frac{|I|^2\ell^2Z_0k^2}{12\pi}=\frac{1}{2}|I|^2R_s$$

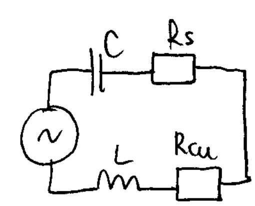
$$\underline{R_s=\frac{\ell^2Z_0k^2}{6\pi}=\frac{2}{3}\pi Z_0\left(\frac{\ell}{\lambda}\right)^2}$$

$f=30\,\mathrm{kHz}\rightarrow\lambda=10\,\mathrm{km}$

$\underline{R_s\approx7{,}2\,\mathrm{m}\Omega}$

TESLOV TRANSFORMATOR





$R_{cu}=\frac{\omega L}{Q}=\frac{1}{\omega CQ}\approx\underline{60\,\Omega}$

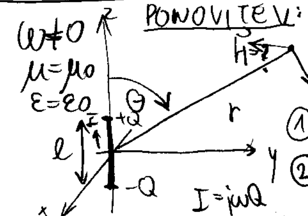
$Q\approx300$

$\eta=\frac{R_s}{R_s+R_{cu}}\approx\underline{0{,}012\,\%}$

Elektrodinamika #8 26/11/2012

PONOVITEV:

$\omega \neq 0$, $\mu=\mu_0$, $\varepsilon=\varepsilon_0$



① $\ell \ll r$

② $\ell \ll \frac{1}{k}$

$$\vec{A}=\left(\vec{1}_r\cos\theta-\vec{1}_\theta\sin\theta\right)\frac{\mu I\ell}{4\pi}\frac{e^{-jkr}}{r}$$

$$\vec{H}=\vec{1}_\phi\frac{I\ell}{4\pi}e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta$$

$$\vec{E}=\frac{Q\ell}{4\pi\varepsilon}e^{-jkr}\left[\vec{1}_r\left(\frac{jk}{r^2}+\frac{1}{r^3}\right)2\cos\theta+\vec{1}_\theta\left(-\frac{k^2}{r}+\frac{jk}{r^2}+\frac{1}{r^3}\right)\sin\theta\right]$$

$$R_S=\frac{2\pi Z_0}{3}\left(\frac{\ell}{\lambda}\right)^2$$

POENOSTAVITVE ZA SEVANJE

$r \gg \frac{1}{k} \longrightarrow \frac{\partial}{\partial r}\approx -jk$, $\frac{\partial}{\partial\theta}\to 0$, $\frac{\partial}{\partial\phi}\to 0$

$r \gg \frac{\lambda}{2\pi}$

$$\operatorname{rot}\vec{A}=\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ -jk & 0 & 0\\ A_r & rA_\theta & r\sin\theta A_\phi\end{vmatrix}=\vec{\nabla}\times\vec{A}=\vec{1}_\theta jkA_\phi-\vec{1}_\phi jkA_\theta$$

$\vec{\nabla}=\vec{1}_r(-jk)$ $\omega\mu=kZ_0$

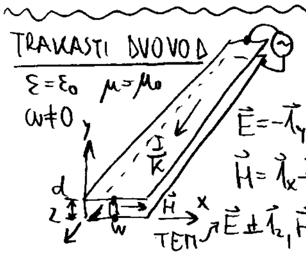
$$\vec{H}\approx\vec{1}_\phi\frac{jk}{4\pi}I\ell\frac{e^{-jkr}}{r}\sin\theta$$

$$\vec{E}=\frac{1}{j\omega\varepsilon}\vec{\nabla}\times\left(\frac{1}{\mu}\vec{\nabla}\times\vec{A}\right)\approx-j\omega\left(\vec{A}-\vec{1}_r(\vec{1}_r\cdot\vec{A})\right)=\vec{1}_\theta\frac{jkZ_0}{4\pi}I\ell\frac{e^{-jkr}}{r}\sin\theta$$

LASTNOSTI SEVANJA $\overset{\text{TEM}}{}$ $\vec{H}\perp\vec{1}_r$; $\vec{E}\perp\vec{1}_r$; $\vec{E}\perp\vec{H}$; $|\vec{E}|=Z_0|\vec{H}|$; $\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\vec{1}_r\frac{|\vec{E}|^2}{2Z_0}$ REALEN = DELOVNA MOČ

TRAKASTI DVOVOD $\frac{U}{I}=Z_K=\frac{d}{w}Z_0$

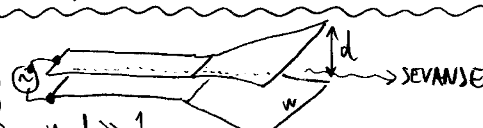
$\varepsilon=\varepsilon_0$ $\mu=\mu_0$ $\omega\neq 0$



$$\vec{E}=-\vec{1}_y\frac{U}{d}=-\vec{1}_y\frac{U_0}{d}e^{-j\beta z}$$

$$\vec{H}=\vec{1}_x\frac{I}{w}=\vec{1}_x\frac{I_0}{w}e^{-j\beta z}$$

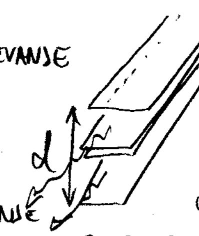
TEM $\vec{E}\perp\vec{1}_z$, $\vec{H}\perp\vec{1}_z$, $\vec{E}\perp\vec{H}$, $|\vec{E}|=Z_0|\vec{H}|$



$w, d \gg \frac{1}{k}$

NI ODBOJA NA OS! $\Gamma\to 0$!

(OS = ODPRTE SPONKE)

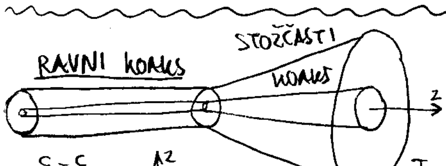


$\Gamma\to 0$

NI ODBOJA NA OS

ZAPOREDNE VEZAVE

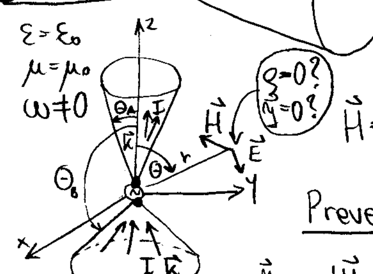
$d \gg \frac{1}{k}=\frac{\lambda}{2\pi}$



Ugibam rešitev: $\vec{E}=\vec{1}_\theta\frac{C}{r\sin\theta}e^{-jkr}$

Preverim ③ ME: $\rho=\operatorname{div}(\varepsilon\vec{E})=\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(r\sin\theta\,\varepsilon\frac{C}{r\sin\theta}e^{-jkr}\right)=\underline{\underline{0}}$ ✓

$\varepsilon=\varepsilon_0$, $\mu=\mu_0$, $\omega\neq 0$



Izračunam $\vec{H}$ iz ② ME:

$\frac{k}{\omega\mu}=\frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu}=\frac{1}{Z_0}$

$$\vec{H}=\frac{j}{\omega\mu}\operatorname{rot}\vec{E}=\frac{j}{\omega\mu}\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & r\frac{C}{r\sin\theta}e^{-jkr} & r\sin\theta\cdot 0\end{vmatrix}=\underline{\underline{\vec{1}_\phi\frac{C/Z_0}{r\sin\theta}e^{-jkr}}}$$

TEM: $\vec{E}\perp\vec{1}_r$; $\vec{H}\perp\vec{1}_r$; $\vec{E}\perp\vec{H}$, $|\vec{E}|=Z_0|\vec{H}|$

Preverim ① ME:

$\frac{k}{Z_0}=\frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{\frac{\mu}{\varepsilon}}}=\omega\varepsilon$

$$\vec{J}=\operatorname{rot}\vec{H}-j\omega\varepsilon\vec{E}=\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & r0 & r\sin\theta\frac{C/Z_0}{r\sin\theta}e^{-jkr}\end{vmatrix}-j\omega\varepsilon\vec{1}_\theta\frac{C}{r\sin\theta}e^{-jkr}=\vec{1}_\theta\frac{j\omega\varepsilon C}{r\sin\theta}e^{-jkr}-\vec{1}_\theta\frac{j\omega\varepsilon C}{r\sin\theta}e^{-jkr}=\underline{\underline{0}}$$ ✓

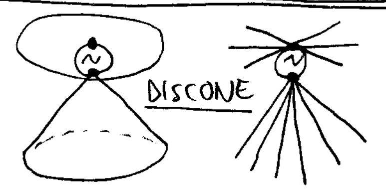
Izračunam tok I: $d\vec{s}=\vec{1}_\phi r\sin\theta d\phi$

$$I=\oint_s\vec{H}\cdot d\vec{s}=\int_0^{2\pi}\vec{1}_\phi\frac{C/Z_0}{r\sin\theta}e^{-jkr}\cdot\vec{1}_\phi r\sin\theta d\phi=\underline{\underline{\frac{2\pi C}{Z_0}e^{-jkr}}}$$

$$Z_K=\frac{U}{I}=\underline{\underline{\frac{Z_0}{2\pi}\ln\left(\frac{\operatorname{tg}(\theta_B/2)}{\operatorname{tg}(\theta_A/2)}\right)\approx 60\,\Omega\ln\left(\frac{\operatorname{tg}(\theta_B/2)}{\operatorname{tg}(\theta_A/2)}\right)}}$$

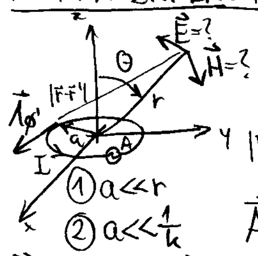
Izračunam napetost U: $d\vec{s}=\vec{1}_\theta r d\theta$

$$U=\int_A^B\vec{E}\cdot d\vec{s}=\int_{\theta_A}^{\theta_B}\vec{1}_\theta\frac{C}{r\sin\theta}e^{-jkr}\cdot\vec{1}_\theta r d\theta=Ce^{-jkr}\int_{\theta_A}^{\theta_B}\frac{d\theta}{\sin\theta}=\underline{\underline{Ce^{-jkr}\ln\left(\frac{\operatorname{tg}(\theta_B/2)}{\operatorname{tg}(\theta_A/2)}\right)}}$$



MAJHNA ZANKA (xy)

$$\vec{A}(\vec{r})=\frac{\mu}{4\pi}\int_{v'}\vec{J}(\vec{r}')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}dv'=\frac{\mu}{4\pi}\int_0^{2\pi}\vec{1}_{\phi'}\frac{I}{A_z}\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}A_z a\,d\phi'=\frac{\mu I a}{4\pi}\int_0^{2\pi}\vec{1}_{\phi'}\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}d\phi'$$



$$|\vec{r}-\vec{r}'|=\sqrt{(r\sin\theta\cos\phi-a\cos\phi')^2+(r\sin\theta\sin\phi-a\sin\phi')^2+(r\cos\theta)^2}=\sqrt{\underline{r^2}+\underline{a^2}-2ar\sin\theta\cos(\phi-\phi')}$$

$|\vec{r}-\vec{r}'|\approx r-a\sin\theta\cos(\phi-\phi')$; $\frac{1}{|\vec{r}-\vec{r}'|}\approx\frac{1}{r}\left(1+\frac{a}{r}\sin\theta\cos(\phi-\phi')\right)$; $e^{-jk|\vec{r}-\vec{r}'|}\approx e^{-jkr}\left(1+jka\sin\theta\cos(\phi-\phi')\right)$

① $a\ll r$

② $a\ll\frac{1}{k}$

$$\vec{A}(\vec{r})\approx\frac{\mu I a}{4\pi}\frac{e^{-jkr}}{r}\underbrace{\int_0^{2\pi}\left(-\vec{1}_x\sin\phi'+\vec{1}_y\cos\phi'\right)\left(1+\frac{a}{r}\sin\theta\cos(\phi-\phi')\right)\left(1+jka\sin\theta\cos(\phi-\phi')\right)d\phi'}_{\vec{1}_\phi\pi a\sin\theta}$$

$\vec{1}_{\phi'}=-\vec{1}_x\sin\phi'+\vec{1}_y\cos\phi'$

$Q=0\to\operatorname{grad}V=0$

$\omega\mu=\omega\sqrt{\mu\varepsilon}\sqrt{\frac{\mu}{\varepsilon}}=kZ_0$

$$\vec{A}(\vec{r})\approx\vec{1}_\phi\frac{\mu}{4\pi}I\pi a^2\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\sin\theta$$

$$\vec{E}=-j\omega\vec{A}-\operatorname{grad}V=\underline{\underline{\vec{1}_\phi\frac{-jkZ_0}{4\pi}IAe^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta}}$$

$\underline{\underline{\vec{H}=\frac{j}{\omega\mu}\operatorname{rot}\vec{E}}}$

Elektrodinamika #9   3.12.2012

VALOVNA ENAČBA BREZ IZVOROV

$\omega^2\mu\varepsilon = k^2$

$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0 \quad (x,y,z) \longrightarrow \Delta\vec{E} = \vec{1}_x\Delta E_x + \vec{1}_y\Delta E_y + \vec{1}_z\Delta E_z$

$\Delta\vec{H} + \omega^2\mu\varepsilon\vec{H} = 0 \qquad \Delta E_x = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}$

$\Delta\vec{E} = \mathrm{grad}(\mathrm{div}\vec{E}) - \mathrm{rot}(\mathrm{rot}\vec{E})$

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0$$

$$E_x(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{-k_x^2} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{-k_y^2} + \underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{-k_z^2} + k^2 = 0$$

$$k_z^2 > 0 \longrightarrow Z(z) = \overbrace{C_1\cos k_z z + C_2\sin k_z z}^{\text{STOJNI VAL}} = \overbrace{C_3 e^{jk_z z}}^{\text{ODBITI}} + \overbrace{C_4 e^{-jk_z z}}^{\text{NAPREDUJOČI}}$$

$$k_z^2 < 0 \longrightarrow Z(z) = C_5\,\mathrm{ch}|k_z|z + C_6\,\mathrm{sh}|k_z|z = C_7 e^{|k_z|z} + \underbrace{C_8 e^{-|k_z|z}}_{\text{exp. usihanje}}$$

Zgledi: preveri $\mathrm{div}(\varepsilon\vec{E}) = 0$
izračunaj $\vec{H} = \frac{j}{\omega\mu}\mathrm{rot}\vec{E}$
preveri $\vec{J} = \mathrm{rot}\vec{H} - j\omega\varepsilon\vec{E} = 0$
izračunaj $\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*$

VALOVNI VEKTOR

za potujoči val $\vec{k} = \vec{1}_x k_x + \vec{1}_y k_y + \vec{1}_z k_z$

$$\left.\begin{aligned} E_x(x,y,z) &= E_{x0}\,e^{-jk_x x}e^{-jk_y y}e^{-jk_z z} \\ E_y(x,y,z) &= E_{y0}\,e^{-jk_x x}e^{-jk_y y}e^{-jk_z z} \\ E_z(x,y,z) &= E_{z0}\,e^{-jk_x x}e^{-jk_y y}e^{-jk_z z} \end{aligned}\right\} \vec{E}(x,y,z) = \vec{E}_0\,e^{-j\vec{k}\cdot\vec{r}}$$

$|\vec{k}| = k = \omega^2\mu\varepsilon$

$\vec{k} = \vec{1}_k\,\omega^2\mu\varepsilon = \vec{1}_k k$

$$\mathrm{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = -jk_x E_x - jk_y E_y - jk_z E_z = -j\vec{k}\cdot\vec{E} \longrightarrow \text{FIZIKALNA REŠITEV } \rho=0 \rightarrow \vec{k} \perp \vec{E}$$

$$\mathrm{rot}\vec{E} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\vec{k}\times\vec{E} \longrightarrow \vec{H} = \frac{\vec{k}\times\vec{E}}{\omega\mu} = \frac{\vec{1}_k\times\vec{E}}{Z_0} \longrightarrow \vec{E}\perp\vec{H};\ \vec{H}\perp\vec{k};\ |\vec{H}| = \frac{|\vec{E}|}{Z_0}$$

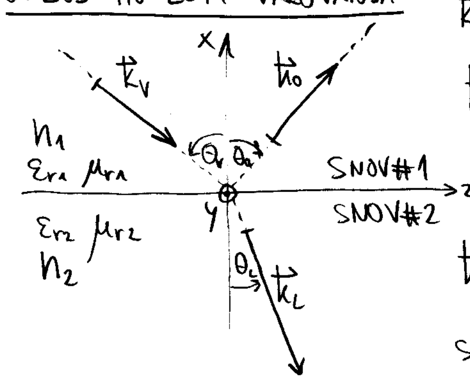
$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{E}_{eff}\times\vec{H}_{eff}^* = \vec{1}_s\frac{|\vec{E}|^2}{2Z_0} = \vec{1}_s\frac{|H|^2 Z_0}{2};\quad \vec{1}_s = \vec{1}_k$$

SNOV $\varepsilon_r \neq 1,\ \mu_r \neq 1$

Lomni količnik $n = \sqrt{\mu_r\varepsilon_r}$

$$\vec{k} = \vec{1}_k k = \vec{1}_k\,\omega\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0} = \vec{1}_k\frac{\omega}{c_0}\sqrt{\mu_r\varepsilon_r} = \vec{1}_k\frac{\omega}{c_0}n \qquad c = \frac{c_0}{n}$$

ODBOJ IN LOM VALOVANJA



$$\left.\begin{aligned} \vec{k}_v &= \vec{1}_x k_{vx} + \vec{1}_z\beta \\ \vec{k}_o &= \vec{1}_x k_{ox} + \vec{1}_z\beta \\ \vec{k}_L &= \vec{1}_x k_{Lx} + \vec{1}_z\beta \end{aligned}\right\}$$

isti pojav hkrati na meji snovi — isti $\beta = k_z$

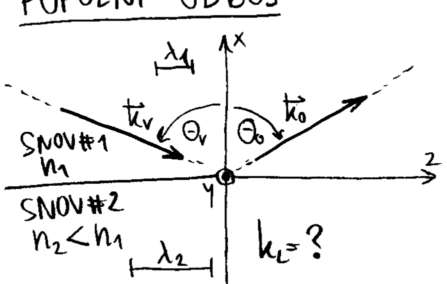
$k_{vx}^2 + \beta^2 = k_{ox}^2 + \beta^2 = k_1^2 = n_1\omega\sqrt{\mu_0\varepsilon_0}$

$$\underbrace{\sin\theta_v = \frac{\beta}{k_1} \qquad \sin\theta_o = \frac{\beta}{k_1}}_{\underline{\underline{\theta_v = \theta_o}}}$$

$k_{Lx}^2 + \beta^2 = k_2^2 = n_2\omega\sqrt{\mu_0\varepsilon_0}$

$$\sin\theta_L = \frac{\beta}{k_2} \longrightarrow \underline{\underline{n_2\sin\theta_L = n_1\sin\theta_v}} \quad \text{Snell-ov lomni zakon}$$

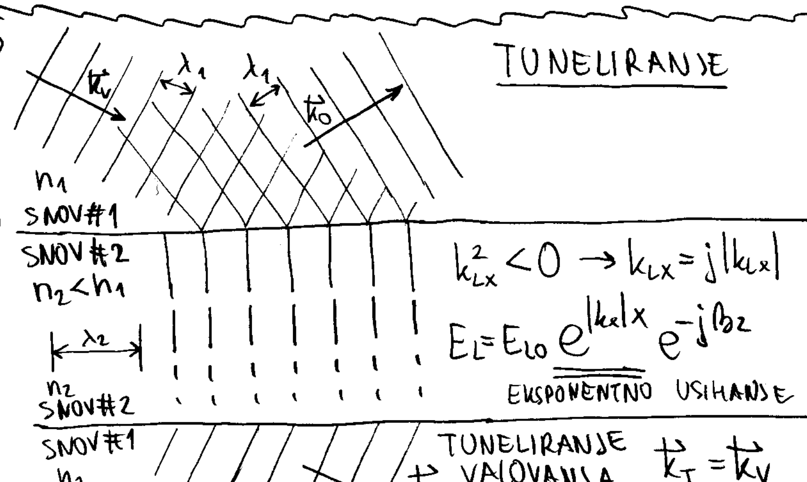
POPOLNI ODBOJ



$$\theta_L = \arcsin\left(\frac{n_1}{n_2}\sin\theta_v\right)$$

$$\frac{n_1}{n_2}\sin\theta_v > 1\ ?$$

TUNELIRANJE



$k_{Lx}^2 < 0 \rightarrow k_{Lx} = j|k_{Lx}|$

$$E_L = E_{L0}\,\underbrace{e^{|k_{Lx}|x}}_{\text{EKSPONENTNO USIHANJE}}\,e^{-j\beta z}$$

TUNELIRANJE VALOVANJA $\vec{k}_T = \vec{k}_v$

Elektrodinamika #10   10.12.2012

Ponovitev $\Delta\vec{E}+k^2\vec{E}=0 \longrightarrow \underbrace{\vec{E}=\vec{E}_0 e^{-j\vec{k}\cdot\vec{r}}}_{\text{POTUJOČI VAL}}$, $\vec{k}\perp\vec{E}$, $\vec{E}\perp\vec{H}$, $\vec{H}\perp\vec{k}$, $\frac{|\vec{E}|}{|\vec{H}|}=Z_0=\sqrt{\frac{\mu}{\varepsilon}}$

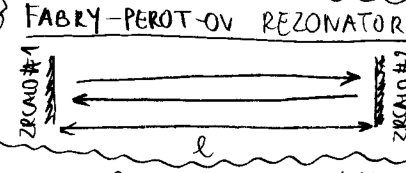
1D STOJNI VAL

$\frac{E}{H}$ nestalno razmerje, kvadratura

$$\underbrace{\vec{E}=\vec{1}_x C\cos kz}_{\text{STOJNI VAL}}=\vec{1}_x\frac{C}{2}\left(\overset{\text{ODBITI VAL}}{e^{jkz}}+\overset{\text{NAPREDUJOČI VAL}}{e^{-jkz}}\right)\qquad \vec{H}=\frac{j}{\omega\mu}\operatorname{rot}\vec{E}=\frac{j}{\omega\mu}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ C\cos kz & 0 & 0\end{vmatrix}=\vec{1}_y\frac{-jkC}{\omega\mu}\sin kz=\underline{\vec{1}_y\frac{-j}{Z_0}C\sin kz}$$

$$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\underbrace{\vec{1}_z\frac{j|C|^2}{2Z_0}\cos kz\sin kz}_{\text{JALOVA MOČ}}$$

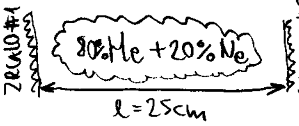
FABRY-PEROT-OV REZONATOR



$kl=m\pi$ $f=\frac{c_0}{2l}m$, $m=1,2,3,4\ldots$

$k=\frac{2\pi f}{c_0}$ TEM_{00m} PRIMERJAVA S STRUNO!

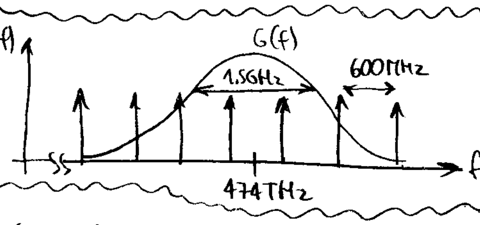
HeNe laser



$f\sim 474\text{THz}$ $c_0=3\cdot10^8\text{m/s}$

Doppler $\pm1{,}5\cdot10^{-6}$ (toplotno gibanje $\sim450\text{m/s}$)

$\Delta f=\frac{c_0}{2l}=\underline{600\text{MHz}}$

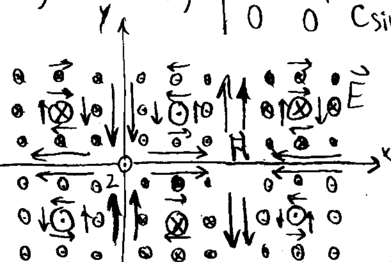


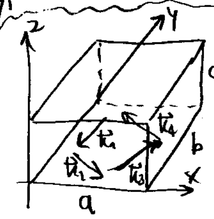
2D STOJNI VAL

$$\vec{E}=\vec{1}_z C\sin k_x x\sin k_y y=\vec{1}_z\frac{C}{-4}\left(e^{jk_x x}-e^{-jk_x x}\right)\left(e^{jk_y y}-e^{-jk_y y}\right)=\vec{1}_z\frac{C}{-4}\left[\underset{\vec{k}_1}{e^{j(k_x x+k_y y)}}-\underset{\vec{k}_2}{e^{j(-k_x x+k_y y)}}-\underset{\vec{k}_4}{e^{j(k_x x-k_y y)}}+\underset{\vec{k}_3}{e^{j(-k_x x-k_y y)}}\right]$$

$$\vec{H}=\frac{j}{\omega\mu}\operatorname{rot}\vec{E}=\frac{j}{\omega\mu}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & 0 & C\sin k_x x\sin k_y y\end{vmatrix}=\underline{\vec{1}_x\frac{jk_y}{\omega\mu}C\sin k_x x\cos k_y y-\vec{1}_y\frac{jk_x}{\omega\mu}C\cos k_x x\sin k_y y}$$

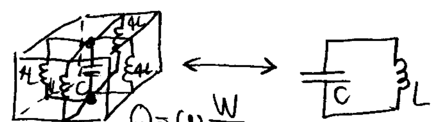
kvadratura $\vec{E},\vec{H}$ $\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=$ jalov!





VOTLINSKI REZONATOR

$TM_{110}: f=\frac{c_0}{2}\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2}$



$Q=\omega\frac{W}{P}$ $Q=\frac{\omega L}{R_L}$

$TM_{mn0}\rightarrow f_{mn}=\frac{c_0}{2}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$ $Q\uparrow\Rightarrow m\uparrow, n\uparrow$ = LASER visokih

3D STOJNI VAL

$f_{lmn}=\frac{c_0}{2}\sqrt{\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2+\left(\frac{n\pi}{c}\right)^2}$

8 valovnih vektorjev

PREPREČEVANJE VIŠJIH RODOV

$c<\frac{a}{2},\frac{b}{2}$

DIELEKTRIK

$\sqrt{\varepsilon_r}=n$ $f_{lmn}=\frac{c_0}{2n}\sqrt{\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2+\left(\frac{n\pi}{c}\right)^2}=\frac{c_0}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2+\left(\frac{n\pi}{c}\right)^2}$

1D STOJNI (Y) + 1D POTUJOČI (z)

$$\vec{E}=\vec{1}_x C\sin k_y y\, e^{-j\beta z}\qquad \vec{H}=\frac{j}{\omega\mu}\operatorname{rot}\vec{E}=\frac{j}{\omega\mu}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ C\sin k_y y e^{-j\beta z} & 0 & 0\end{vmatrix}=\underbrace{\vec{1}_y\frac{\beta}{\omega\mu}C\sin k_y y e^{-j\beta z}}_{\text{V FAZI}\rightarrow\mathrm{Re}[\vec{S}]}-\underbrace{\vec{1}_z\frac{jk_y}{\omega\mu}C\cos k_y y e^{-j\beta z}}_{\text{V KVADRATURI}\rightarrow\mathrm{Im}[\vec{S}]}$$

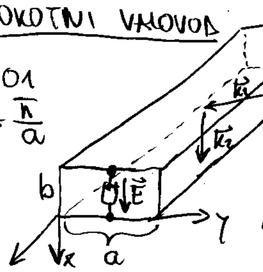
$k_y^2+\beta^2=k^2=\omega^2\mu\varepsilon$

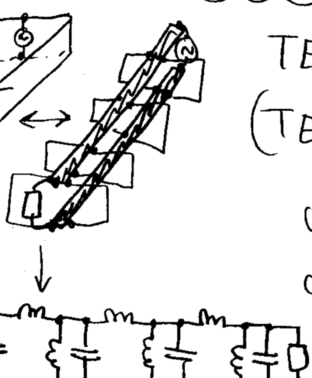
$$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\frac{1}{2}\vec{1}_x C\sin k_y y e^{-j\beta z}\times\left[\vec{1}_y\frac{\beta}{\omega\mu}C^*\sin k_y y e^{+j\beta z}+\vec{1}_z\frac{jk_y}{\omega\mu}C^*\cos k_y y e^{+j\beta z}\right]=\vec{1}_z\frac{\beta|C|^2}{2\omega\mu}\sin^2 k_y y-\vec{1}_y\frac{jk_y|C|^2}{\omega\mu}\sin k_y y\cos k_y y$$

PRAVOKOTNI VALOVOD

TE_{01}

$k_y=\frac{\pi}{a}$





VISOKOPREPUSTNO SITO

$TE_{01}\rightarrow\beta=\sqrt{\omega^2\mu\varepsilon-\left(\frac{\pi}{a}\right)^2}$

$\left(TE_{0m}\rightarrow\beta=\sqrt{\omega^2\mu\varepsilon-\left(\frac{m\pi}{a}\right)^2}\right)$

$\omega^2\mu\varepsilon>\left(\frac{\pi}{a}\right)^2\rightarrow\beta$ = realen (potujoči val) $\rightarrow \vec{E}=\vec{1}_x C\sin k_y y e^{-j\beta z}$

$\omega^2\mu\varepsilon<\left(\frac{\pi}{a}\right)^2\rightarrow\beta$ = imaginaren (exponentno usihanje) $\rightarrow \vec{E}=\vec{1}_x C\sin k_y y e^{-|\beta|z}$

$\omega_k^2\mu\varepsilon=\left(\frac{\pi}{a}\right)^2$

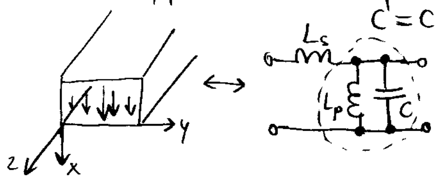
$$\underline{\underline{\omega_k=\frac{\pi c_0}{a}\rightarrow f_k=\frac{c_0}{2a}}}$$

Elektrodinamika #11 17.12.2012

PRAVOKOTNI VALOVOD

$\vec{E} = \vec{1}_x C \sin k_y y \, e^{-j\beta z}$

$k_y^2 + \beta_z^2 = k^2 = \omega^2 \mu\varepsilon \rightarrow \beta_z = \sqrt{\omega^2\mu\varepsilon - k_y^2} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$



$C' = C - \frac{1}{\omega^2 L_p}$

$Z_u = \sqrt{\frac{L_s}{C'}} = \sqrt{\frac{L_s}{C - \frac{1}{\omega^2 L_p}}}$

$\omega_k = \frac{1}{\sqrt{\mu\varepsilon}}\left(\frac{m\pi}{a}\right) = m\frac{\pi c_0}{a}$

$\beta_z \equiv$ realen

$\omega > \frac{1}{\sqrt{CL_p}} \rightarrow Z_u \equiv$ realen = prepust $\longleftarrow \omega > \omega_k$

$\omega < \frac{1}{\sqrt{CL_p}} \rightarrow Z_u \equiv$ imaginaren = zapora $\longleftarrow \omega < \omega_k$

$\beta \equiv$ imaginaren

FAZNA HITROST

$\vec{E} = Re\left[\vec{1}_x C \sin k_y y \, e^{j(\omega t - \beta z)}\right]$

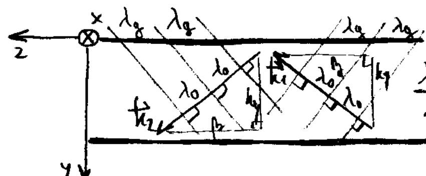
$\varphi = \omega t - \beta z = konst \Big/ \frac{d}{dt}$

$\omega - \beta\frac{dz}{dt} = 0 \rightarrow v_f = \frac{dz}{dt} = \frac{\omega}{\beta}$

$v_f = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - k_y^2}} = \frac{c_0}{\sqrt{1 - \frac{k_y^2}{\omega^2\mu\varepsilon}}} = \frac{c_0}{\sqrt{1 - \left(\frac{\omega_k}{\omega}\right)^2}} \geq c_0\,!$

VALOVODNA VALOVNA DOLŽINA

$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - k_y^2}} = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\omega_k}{\omega}\right)^2}} \geq \lambda_0\,!$



$\frac{\lambda_g}{\lambda_0} = \frac{k}{\beta}$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = Re\left[\vec{1}_x \sin k_y y\left(C_1 e^{j\varphi_1} + C_2 e^{j\varphi_2}\right)\right] \qquad \Delta\varphi = \varphi_1 - \varphi_2$

$\vec{E} = \vec{1}_x \sin k_y y \; Re\left[e^{j\frac{\varphi_1+\varphi_2}{2}}\left(C_1 e^{j\frac{\varphi_1-\varphi_2}{2}} + C_2 e^{j\frac{\varphi_2-\varphi_1}{2}}\right)\right]$

SKUPINSKA HITROST

Vsota $\vec{E}$ dveh frekvenc ω_1, ω_2

$\varphi_1 = \omega_1 t - \beta_1 z$

$\varphi_2 = \omega_2 t - \beta_2 z$

$\vec{E}_1 = Re\left[\vec{1}_x C_1 \sin k_y y \, e^{j(\omega_1 t - \beta_1 z)}\right] = Re\left[\vec{1}_x C_1 \sin k_y y \, e^{j\varphi_1}\right]$

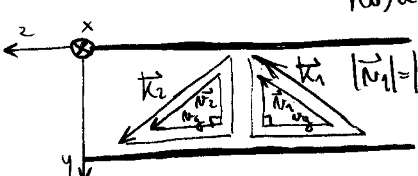
$\vec{E}_2 = Re\left[\vec{1}_x C_2 \sin k_y y \, e^{j(\omega_2 t - \beta_2 z)}\right] = Re\left[\vec{1}_x C_2 \sin k_y y \, e^{j\varphi_2}\right]$

amplituda $|\vec{E}| = \sin k_y y \left|C_1 e^{j\Delta\varphi} + C_2 e^{-j\Delta\varphi}\right|$

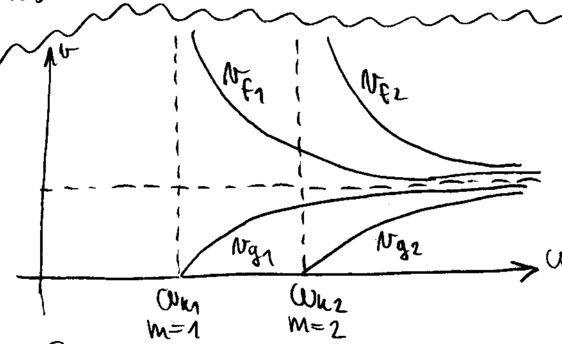
ovojnico sledim $\Delta\varphi = konst = \Delta\omega t - \Delta\beta z \Big/ \frac{d}{dt}$

$\frac{dz}{dt} = v_g = \frac{\Delta\omega}{\Delta\beta}\Big|_{\Delta\omega \to 0} = \frac{d\omega}{d\beta}$ $\qquad \boxed{v_f \geq c_0 \geq v_g}$

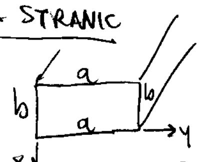
$\beta = \sqrt{\omega^2\mu\varepsilon - k_y^2} \rightarrow \frac{d\beta}{d\omega} = \frac{1}{2\sqrt{\omega^2\mu\varepsilon - k_y^2}} 2\omega\mu\varepsilon = \frac{1}{c_0\sqrt{1 - \left(\frac{\omega_k}{\omega}\right)^2}} \longrightarrow v_g = c_0\sqrt{1 - \left(\frac{\omega_k}{\omega}\right)^2} \leq c_0$



$\frac{v_g}{c_0} = \frac{\beta}{k}$



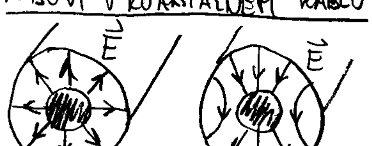
IZBIRA STRANIC



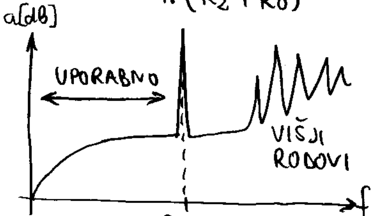
$b \leq \frac{a}{2} \longleftrightarrow \omega_{kTE10} \geq \omega_{kTE02}$

$\omega_{kTE02} = 2\omega_{kTE01}$

RODOVI V KOAKSIALNEM KABLU



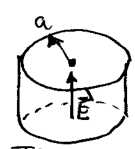
$f_{kTE11} \approx \frac{c_0}{\pi(R_z + R_0)}$



VOTLINSKI REZONATOR (ρ, φ, z)

$\Delta\vec{E} + k^2\vec{E} = 0 \qquad \vec{1}_z \cdot \Delta\vec{E} = \Delta E_z$

$\Delta E_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\varphi^2} + \frac{\partial^2 E_z}{\partial z^2}$



TM100

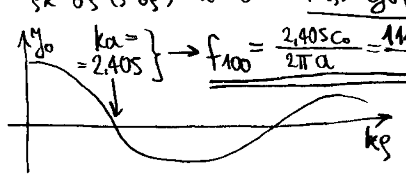
$\vec{E} = \vec{1}_z E_z$

$E_z(\rho, \varphi, z) = R(\rho)F(\varphi)Z(z)$

$\frac{1}{\rho R}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial R}{\partial\rho}\right) + \frac{1}{\rho^2}\underbrace{\frac{1}{F}\frac{\partial^2 F}{\partial\varphi^2}}_{-m^2} + \underbrace{\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}}_{\pm\ell^2} + k^2 = 0$

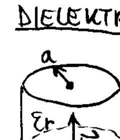
Zgled: $\frac{\partial}{\partial\varphi} = \frac{\partial}{\partial z} = 0$

$\frac{1}{\rho R}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial R}{\partial\rho}\right) + k^2 = 0 \rightarrow R(\rho) = C J_0(k\rho)$



$\left.\begin{matrix} ka = \\ = 2{,}405 \end{matrix}\right\} \rightarrow f_{100} = \frac{2{,}405\,c_0}{2\pi a} = \frac{114{,}8\,\mathrm{MHz \cdot m}}{a}$

DIELEKTRIČNI REZONATOR



TE100

$\varepsilon_r \gg 1 \rightarrow \vec{H}(\rho = a) \approx 0$

$\vec{H} = \vec{1}_z C J_0(k\rho)$

$\Delta\vec{H} + k^2\vec{H} = 0$

$f_{100} \approx \frac{2{,}405\,c_0}{2\pi a\sqrt{\varepsilon_r}}$

$f_{100} \approx \frac{114.8\,\mathrm{MHz \cdot m}}{a\sqrt{\varepsilon_r}}$

Elektrodinamika #12 7/1/2013

VALOVANJE V IZGUBNI SNOVI

$$\vec{E} = \vec{C} e^{-j\vec{k}\cdot\vec{r}}$$

$$\vec{E} = \vec{1}_x C e^{-jkz}$$

$$\vec{E} = \vec{1}_x C e^{-j\beta z} e^{-\alpha z}$$

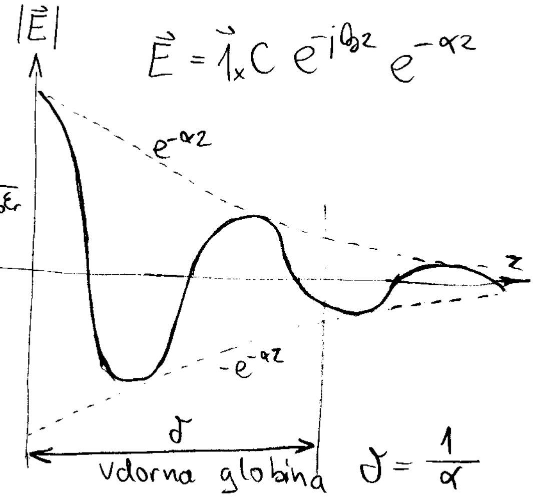
① ME: $\mathrm{rot}\vec{H} = \vec{J} + j\omega\varepsilon\vec{E} = (\gamma + j\omega\varepsilon_0\varepsilon_r)\vec{E} = j\omega\varepsilon_0 \underbrace{\left(\varepsilon_r - \frac{j\gamma}{\omega\varepsilon_0}\right)}_{\varepsilon_r'}\vec{E}$

$\vec{J} = \gamma\vec{E}$ $\quad \varepsilon = \varepsilon_0\varepsilon_r$

$$\Delta\vec{E} + k^2\vec{E} = 0 \longrightarrow k = \omega\sqrt{\mu\varepsilon_0\left(\varepsilon_r - \frac{j\gamma}{\omega\varepsilon_0}\right)} = \beta - j\alpha$$

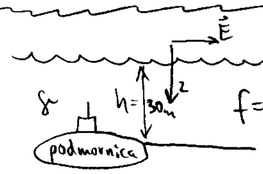
baker $\gamma = 56\cdot10^6\,S/m$ $\varepsilon_r = 1$	morska voda $\gamma = 5\,S/m$ $\varepsilon_r = 80$	SiO_2 steklo $\varepsilon_r = 3{,}9$
$f_m \approx 10^{18}\,Hz$	$f_m = 1{,}13\cdot10^9\,Hz$	$f_m = 3\cdot10^{-10}\,Hz$
dober prevodnik $\alpha = \beta$	vmesni primer $0 < \alpha < \beta$	dober dielektrik $\alpha \to 0$

$f_m = \frac{\gamma}{2\pi\varepsilon_0\varepsilon_r}$



$\delta = \frac{1}{\alpha}$

Dober prevodnik $\frac{\gamma}{\omega\varepsilon_0} \gg \varepsilon_r$ $\quad k = \omega\sqrt{\mu\left(-\frac{j\gamma}{\omega}\right)} = \sqrt{-j}\sqrt{\omega\mu\gamma} = \beta - j\alpha \longrightarrow \alpha = \beta = \sqrt{\frac{\omega\mu\gamma}{2}}$



$f = 10\,kHz \ll f_m \longrightarrow \alpha = 0{,}44\,Np/m$

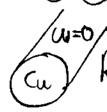
$$a[dB] = \frac{20}{\ln 10}\alpha h = 115\,dB$$

Kovina $\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\gamma}} = \frac{1}{\beta}$

f	δ_{Cu} ($\gamma_{Cu} = 56\cdot10^6\,S/m$, $\mu_{Cu} = 1$)	δ_{Fe} ($\gamma_{Fe} = \frac{1}{10}\gamma_{Cu}$, $\mu_{Fe} = 1000$)
1Hz	67mm	6,7mm
100Hz	6,7mm	0,67mm
10kHz	0,67mm	67μm
1MHz	67μm	6.7μm
100MHz	6,7μm	0,67μm
10GHz	0,67μm	67nm

$$\vec{K} = \int_0^\infty \vec{J}\,dz = \gamma\int_0^\infty \vec{E}_0 e^{-j\beta z} e^{-\alpha z}\,dz = \frac{\gamma\vec{E}_0}{(\alpha + j\beta)}$$

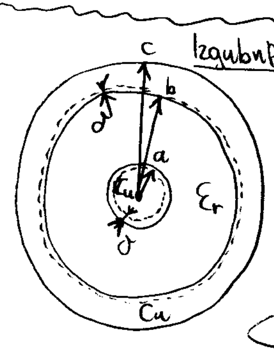
$$\vec{E}_0 = Z_p\vec{K} \longrightarrow Z_p = R_p + jX_p = \sqrt{\frac{\omega\mu}{2\gamma}} + j\sqrt{\frac{\omega\mu}{2\gamma}}$$

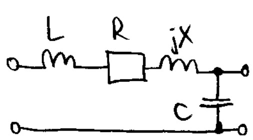


$R = \frac{l}{\gamma\pi r^2}$

$R \approx \frac{l}{2\pi r}R_p = \frac{l}{2\pi r}\sqrt{\frac{\omega\mu}{2\gamma}}$

Izgube koaksialnega kabla





$\omega L \gg X$

$L/l = \frac{\mu_0}{2\pi}\ln\frac{b}{a}$

$C/l = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}$

$$Z_K = \sqrt{\frac{L/l + \frac{R/l}{j\omega} + \frac{X/l}{\omega}}{C/l}} \approx \sqrt{\frac{L/l}{C/l}} = \frac{Z_0}{2\pi\sqrt{\varepsilon_r}}\ln\frac{b}{a}$$

$$k = \beta - j\alpha = \sqrt{\omega\left(L/l + \frac{R/l}{j\omega} + \frac{X/l}{\omega}\right)\omega C/l} \approx \omega\sqrt{\left(L/l + \frac{R/l}{j\omega}\right)C/l} =$$

$$= \omega\sqrt{L/l\, C/l}\sqrt{1 - \frac{jR/l}{\omega C/l}} \approx \omega\sqrt{L/l\, C/l}\left(1 - \frac{jR/l}{2\omega C/l}\right)$$

$\longrightarrow \beta \approx \omega\sqrt{L/l\, C/l}$

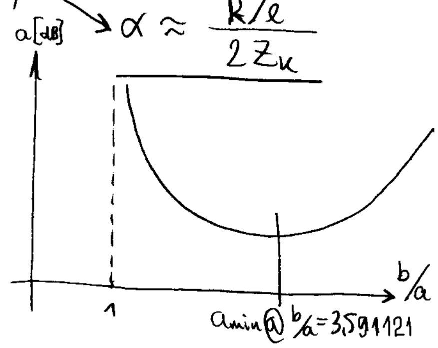
$\longrightarrow \alpha \approx \frac{R/l}{2Z_K}$

$$a[dB]/l = \frac{20}{\ln 10}\alpha = \frac{10}{\ln 10}\frac{R/l}{Z_K} = \frac{10}{\ln 10}\frac{\sqrt{\frac{\omega\mu}{2\gamma}}}{Z_0}\sqrt{\varepsilon_r}\frac{1}{b}\frac{(b/a + 1)}{\ln b/a}$$

$$R/l = R_\check{z}/l + R_o/l = \frac{R_p}{2\pi a} + \frac{R_p}{2\pi b} = \frac{R_p}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$R_p = \sqrt{\frac{\omega\mu}{2\gamma}} = 5{,}94\,m\Omega$

$f = 500\,MHz,\ \gamma = 56\cdot10^6\,S/m$



$\left.\begin{array}{l} a = 1\,mm \\ b = 3{,}5\,mm \\ \varepsilon_r = 2 \end{array}\right\}$ $Z_K = 53{,}2\,\Omega$

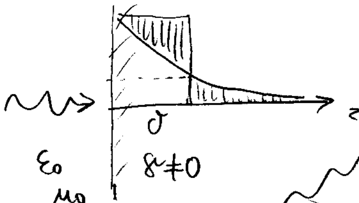
$$a[dB]/l = \frac{10}{\ln 10}\frac{5{,}94\,m\Omega}{377\,\Omega}\sqrt{2}\frac{1}{3{,}5\cdot10^{-3}\,m}\frac{3{,}5+1}{\ln 3{,}5} = 0{,}0993\,dB/m = 99{,}3\,dB/km$$

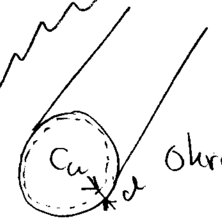
Elektrodinamika #13 14/1/2013

Ponovitev: DOBER PREVODNIK $\gamma \gg \omega\varepsilon$ (kovine $f <$ rentgen)

$\delta = \sqrt{\frac{2}{\omega\mu\gamma}}$

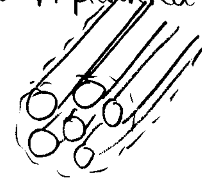
$R_p = R_\square = \sqrt{\frac{\omega\mu}{2\gamma}}$

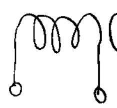




$R = \rho \frac{l}{A} = \frac{l}{2\pi r \delta \gamma} = \frac{l}{2\pi r} R_p$

Okrogel vodnik $\rightarrow$ VF pletenica

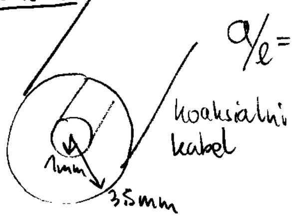




$Q = \frac{\omega L}{R} < 100 \longrightarrow$

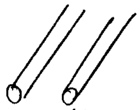
Ponovitev:

$\alpha/l = \frac{10}{\ln 10} \frac{R/l}{Z_k}$



$\alpha/l = \frac{10}{\ln 10} \frac{\sqrt{\varepsilon_r} R_p}{Z_0} \frac{1}{b} \left(\frac{b/a+1}{\ln b/a}\right)$ min @ $b/a = 3{,}591121 \approx 3{,}6$

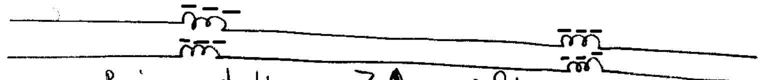
$\alpha/l \approx 0{,}1 dB/m$ @ 500MHz



SIMETRIČNI DVOVOD

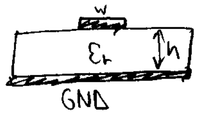
$Z_k \approx (2x \ldots 6x) Z_{koaks}$

$R/l \approx 2 \times (R/l)_{koaks}$



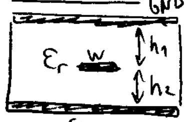
Pupinove tuljave $Z_k\uparrow$, $\Delta f\downarrow$

MIKROSTRIP



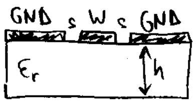
MIKROTRAKASTI

STRIPLINE

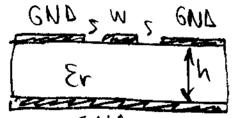


TRAKASTI

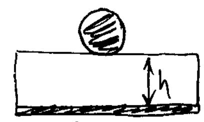
KOPLANARNI



KOPLANARNI Z MASO



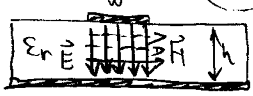
ŽICA NA TV



MIKROSTRIP BREZ STRESANJA $L/l = \mu_0 \frac{h}{w}$

zelo grob približek TEM

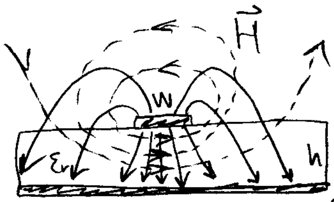
$C/l = \varepsilon_0 \varepsilon_r \frac{w}{h}$

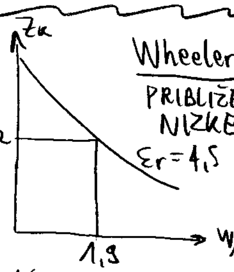


$Z_k \approx \frac{Z_0}{\sqrt{\varepsilon_r}} \frac{h}{w}$

$\varepsilon_r = 4{,}5$, $h = 1{,}6mm$, $Z_k = 50\Omega \longrightarrow \underline{\underline{w = 6mm?}}$

MIKROSTRIP S STRESANJEM





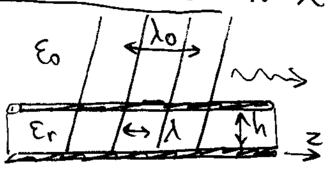
Wheeler TEM

PRIBLIŽEK ZA NIZKE f

$\varepsilon_r = 4{,}5$ $h \ll \lambda$

$\varepsilon_r = 4{,}5$, $h = 1{,}6mm$, $Z_k = 50\Omega \rightarrow \underline{\underline{w = 3mm!}}$

HIBRIDNI RODOVI $h \sim \lambda$



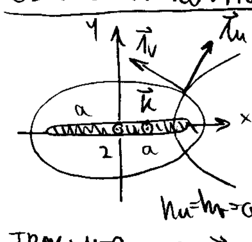
$E_z \neq 0$ TOČNA FIZIKALNA SLIKA

$H_z \neq 0$

$U \equiv$ nedefiniran

$Z_k \equiv$ nedefiniran

OSAMLJEN KOVINSKI TRAK za



(u, v, z)

$x = a \operatorname{ch} u \cos v$

$y = a \operatorname{sh} u \sin v$

$z = z$

$h_u = h_v = a\sqrt{\operatorname{sh}^2 u + \sin^2 v}$

Isti induciron $\vec{E} \rightarrow \vec{A} = \vec{1}_z A_z(u)$

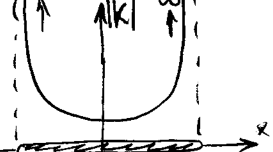
$\vec{H} = \frac{1}{\mu} \operatorname{rot} \vec{A} = \frac{1}{\mu h_u h_v} \begin{vmatrix} h_u \vec{1}_u & h_v \vec{1}_v & \vec{1}_z \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial z} \\ h_u 0 & h_v 0 & A_z \end{vmatrix}$

$\vec{H} = \vec{1}_v \frac{-1}{\mu h_u} \frac{\partial A_z(u)}{\partial u} = \vec{1}_v \frac{f(u)}{h_u}$

TRAK: $u = 0$ $\rightarrow \vec{H} = \vec{1}_v \frac{C}{a \sin v} = \vec{1}_v \frac{C}{\sqrt{a^2 - x^2}}$

$\cos v = \frac{x}{a}$ $\sin v = \sqrt{1 - \left(\frac{x}{a}\right)^2}$

$\vec{K} = \vec{1}_z 2|\vec{H}| = \vec{1}_z \frac{2C}{\sqrt{a^2 - x^2}}$

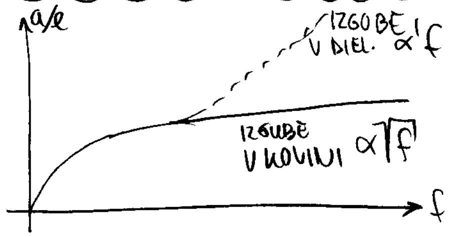


IZRIV TOKA NA ROB TRAKU!

IZGUBE MIKROTRAKASTEGA VODA

1. IZRIV TOKA NA POVRŠINO $\rightarrow \delta$!
2. IZRIV TOKA NA ROB TRAKU!
3. HRAPAVOST BAKRA ZA LEPLJENJE!

$(\alpha/l)_{mikrostrip} > 10 \times (\alpha/l)_{koaks}$ (isti dielektrik, izmere, f)



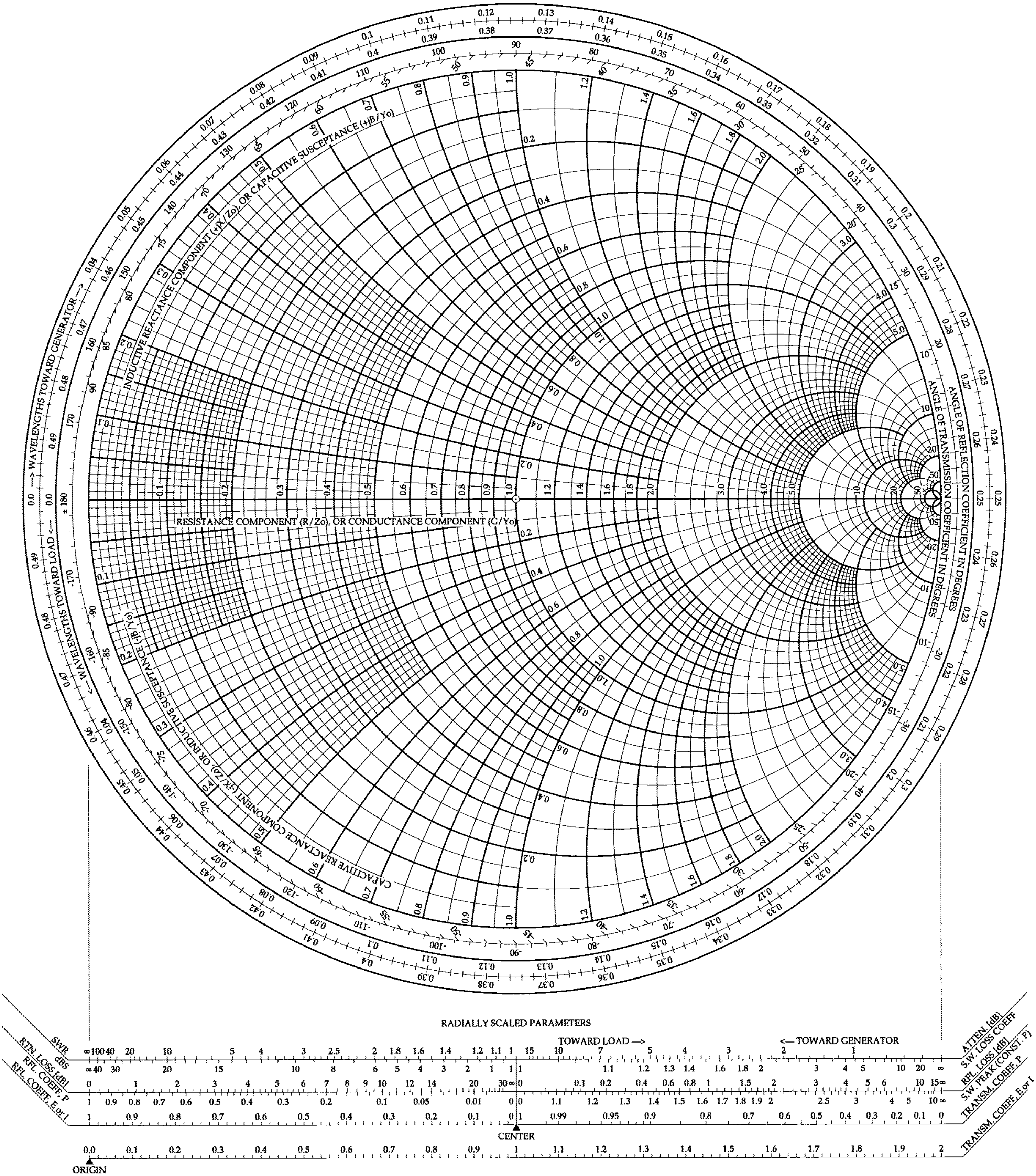
IZGUBE V BAKRU / DIELEKTRIKU

KOVINA: $R_p = \sqrt{\frac{\omega\mu}{2\gamma}} \propto \sqrt{f}$

DIELEKTRIK Z UMAZANIJO:

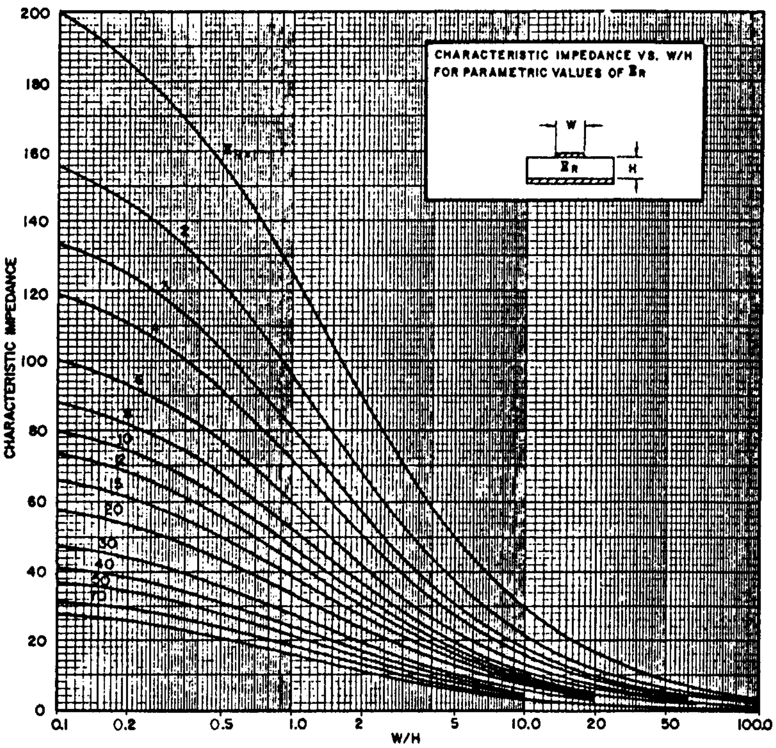
$\rightarrow \propto f$

Smith-ov diagram: impedanca/admitanca v merilu odbojnosti



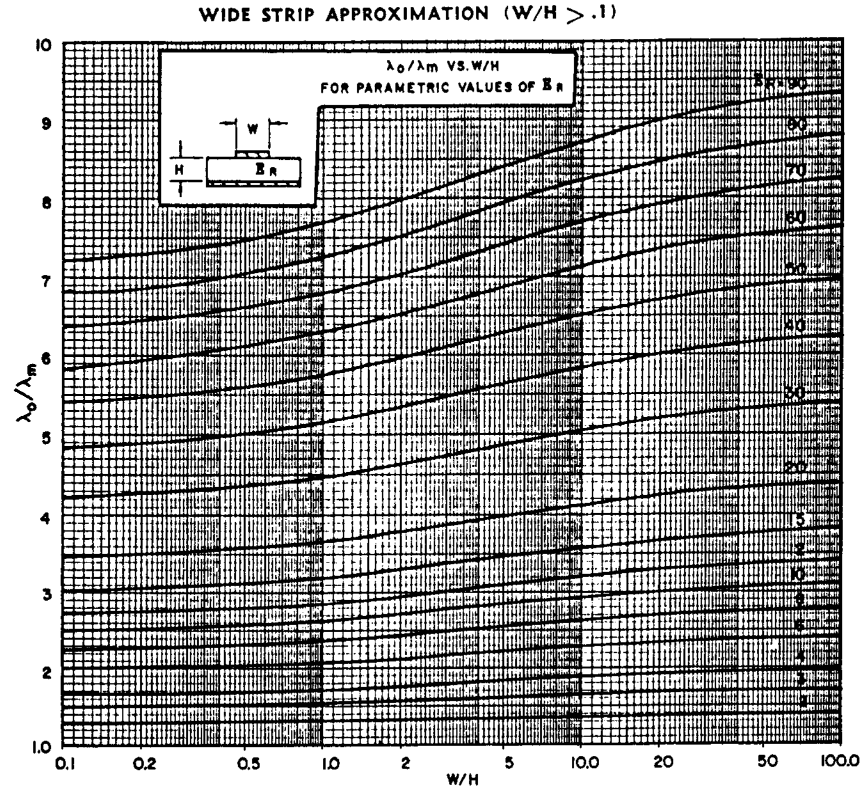
MICROSTRIP CHARACTERISTIC IMPEDANCE CALCULATED FROM WORK OF WHEELER

WIDE STRIP APPROXIMATION (W/H > .1)



RATIO OF FREE SPACE WAVELENGTH (λ_o) TO MICROSTRIP WAVELENGTH (λ_m) CALCULATED FROM WORK OF WHEELER

WIDE STRIP APPROXIMATION (W/H > .1)



Karakteristična impedanca Zk in navidezni lomni količnik n mikrotrakastega voda