14. Votlinski rezonator

Ravninski val nam kot gradnik omogoča rešiti marsikatero nalogo. Kaj se zgodi, ko žarek vpada iz prve brezizgubne snovi (μ_1, ϵ_1) pod pravim kotom na drugo brezizigubno snov (μ_2, ϵ_2) ? Nalogo rešijo trije ravninski valovi: vpadni val in odbiti val v prvi snovi ter lomljeni val v drugi snovi:

Fabry-Perot

Vpad $\vec{k}_V = \vec{1}_z k$

$$\vec{E}_V(z) = \vec{1}_x \frac{C}{2} e^{-jkz}$$

$$\vec{H}_V(z) = \vec{1}_y \frac{C}{2Z} e^{-jkz}$$

Odboj $\vec{k}_O = -\vec{1}_z k$

$$\vec{E}_O(z) = \vec{1}_x \frac{C}{2} e^{jkz}$$

$$\vec{H}_O(z) = -\vec{1}_y \frac{C}{2Z} e^{jkz}$$

$$\vec{E}(z) = \vec{E}_V(z) + \vec{E}_O(z) = \vec{1}_x \frac{C}{2} e^{-jkz} + \vec{1}_x \frac{C}{2} e^{jkz}$$

$$\vec{E}(z) = \vec{1}_x C \cos(kz)$$

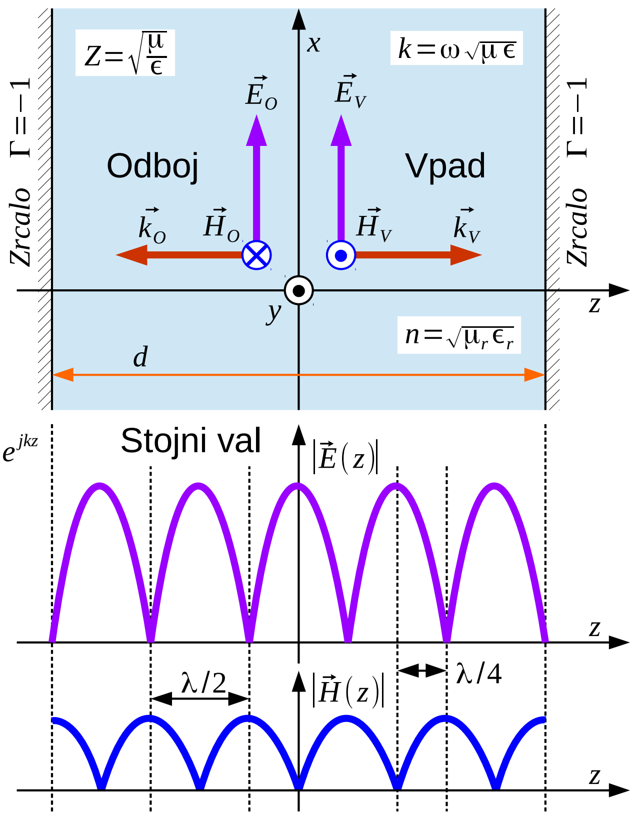
$$\vec{H}(z) = \vec{H}_V(z) + \vec{H}_O(z) = \vec{1}_y \frac{C}{2} Z e^{-jkz} - \vec{1}_y \frac{C}{2} Z e^{jkz}$$

$$\vec{H}(z) = -\vec{1}_y \frac{jC}{Z} \sin(kz)$$

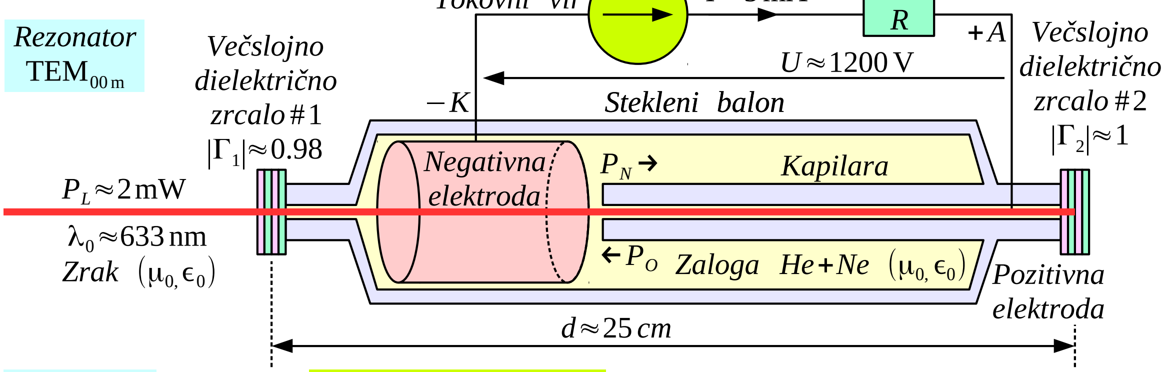
$$\vec{S}(z) = \frac{1}{2} \vec{E} \times \vec{H}^* = \vec{1}_z \frac{j|C|^2}{2Z} \cos(kz) \sin(kz)$$

$kd = m\pi \qquad \mathrm{TEM}_{00\,m} \qquad m = 1,2,3,4,5,6\ldots$

$$f_{00m} = k_{00m} \frac{v}{2\pi} = m \frac{c_0}{2d\sqrt{\mu_r \epsilon_r}} = m \frac{c_0}{2nd}$$



Helij-neonski laser



$$f_{00m}=m\frac{c_0}{2d}$$

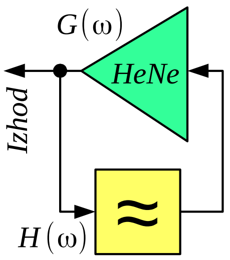
$$m=\frac{2df}{c_0}=\frac{2d}{\lambda_0}\approx 7.9\cdot 10^5$$

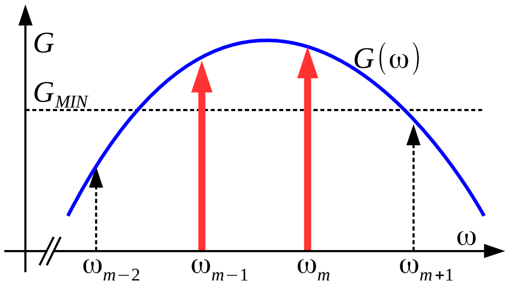
$$W\approx(P_N+P_O)\frac{d}{c_0}\approx\frac{2P_O d}{c_0}$$

$$P=P_L\approx P_O\left(1-|\Gamma_1|^2\right)$$

$$Q=\omega\frac{W}{P}\approx\omega\frac{2d}{c_0\left(1-|\Gamma_1|^2\right)}=\frac{2k_0 d}{1-|\Gamma_1|^2}$$

$$Q\approx\frac{4\pi d}{\lambda_0\left(1-|\Gamma_1|^2\right)}\approx 1.24\cdot 10^8$$





2D stojni val

$$k_x^2+k_y^2=k^2$$

$$\vec{E}(\vec{r})=\vec{1}_z C\sin(k_x x)\sin(k_y y)$$

$$\vec{E}(\vec{r})=-\vec{1}_z\frac{C}{4}\left(e^{jk_x x}-e^{-jk_x x}\right)\left(e^{jk_y y}-e^{-jk_y y}\right)$$

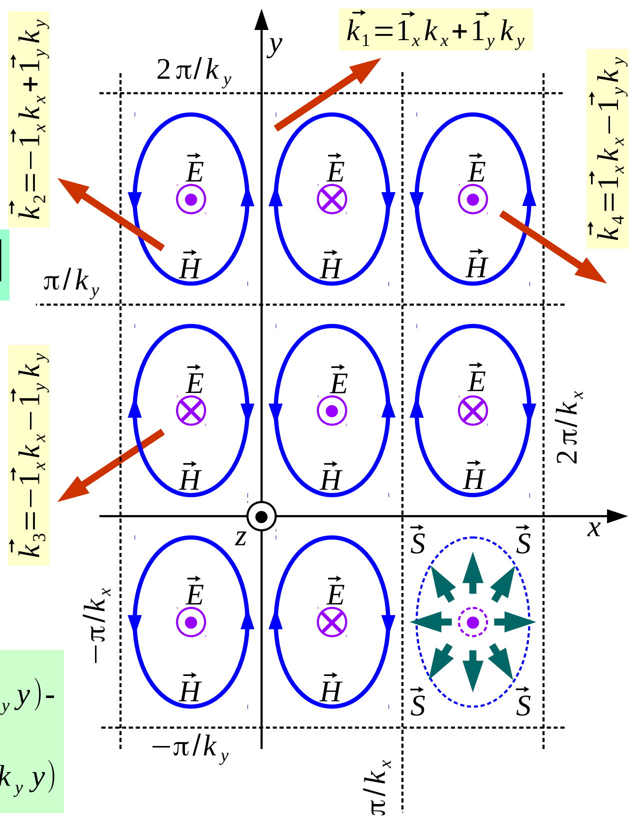
$$\vec{E}(\vec{r})=\vec{1}_z\frac{C}{4}\left[-e^{-j\vec{k}_1\cdot\vec{r}}+e^{-j\vec{k}_2\cdot\vec{r}}-e^{-j\vec{k}_3\cdot\vec{r}}+e^{-j\vec{k}_4\cdot\vec{r}}\right]$$

$$\vec{H}(\vec{r})=\frac{j}{\omega\mu}\operatorname{rot}\vec{E}(\vec{r})=\frac{j}{kZ}\operatorname{rot}\vec{E}(\vec{r})$$

$$\vec{H}(\vec{r})=\vec{1}_x\frac{jCk_y}{kZ}\sin(k_x x)\cos(k_y y)-$$
$$-\vec{1}_y\frac{jCk_x}{kZ}\cos(k_x x)\sin(k_y y)$$

$$\vec{S}(\vec{r})=\frac{1}{2}\vec{E}(\vec{r})\times\vec{H}(\vec{r})^*$$

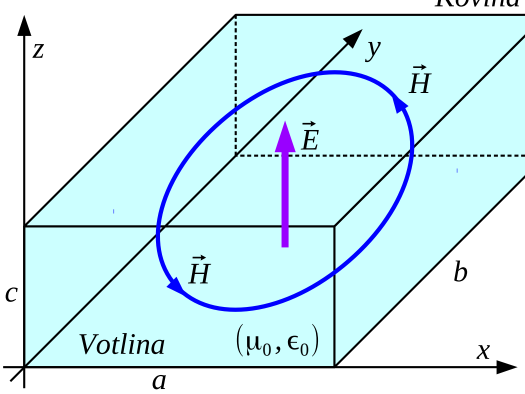
$$\vec{S}(\vec{r})=-\vec{1}_x\frac{j|C|^2 k_x}{2kZ}\sin(k_x x)\cos(k_x x)\sin^2(k_y y)-$$
$$-\vec{1}_y\frac{j|C|^2 k_y}{2kZ}\sin^2(k_x x)\sin(k_y y)\cos(k_y y)$$

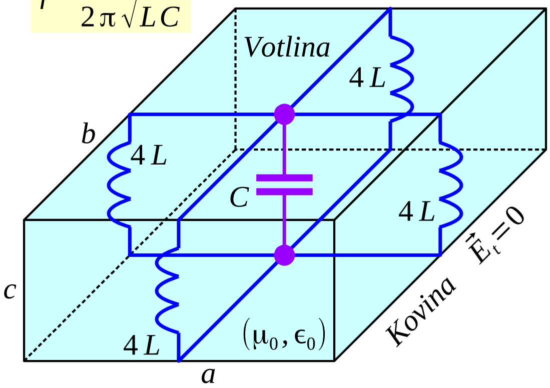


Pravokotna votlina

$$\vec{E}(\vec{r})=\vec{1}_z C\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)$$

$$f=\frac{1}{2\pi\sqrt{LC}}$$

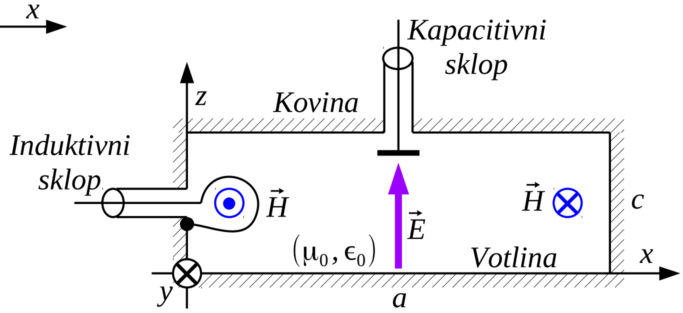




$$k_0^2=k_x^2+k_y^2=\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2$$

$$f_{110}=\frac{\omega}{2\pi}=\frac{k_0c_0}{2\pi}=\frac{c_0}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2}$$

$$f_{110}=\frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$$



Valjni Laplace

Valjne koordinate $\vec{E}(\rho,\varphi,z)$

$\vec{1}_\rho\neq konst.\quad \vec{1}_\varphi\neq konst.\quad \vec{1}_z=konst.$

$$0=\Delta\vec{E}+k^2\vec{E}=\Delta_t\vec{E}_t+\vec{1}_z\Delta E_z+k^2\vec{E}$$

$$\operatorname{grad} E_z=\vec{1}_\rho\frac{\partial E_z}{\partial\rho}+\vec{1}_\varphi\frac{1}{\rho}\frac{\partial E_z}{\partial\varphi}+\vec{1}_z\frac{\partial E_z}{\partial z}$$

$$\Delta E_z=\operatorname{div}(\operatorname{grad} E_z)=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\varphi^2}+\frac{\partial^2 E_z}{\partial z^2}$$

$$\Delta E_z(\rho,\varphi,z)+k^2E_z(\rho,\varphi,z)=0$$

$$E_z(\rho,\varphi,z)=R(\rho)F(\varphi)Z(z)$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\varphi^2}+\frac{\partial^2 E_z}{\partial z^2}+k^2E_z=0$$

$$\frac{1}{\rho R}\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right)+\frac{1}{\rho^2}\underbrace{\frac{1}{F}\frac{d^2F}{d\varphi^2}}_{-m^2}+\underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{-\beta^2}+k^2=0$$

$$\frac{1}{\rho R}\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right)-\frac{m^2}{\rho^2}+k^2-\beta^2=0$$

$$k_\rho^2=k^2-\beta^2$$

$$\frac{1}{\rho R}\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right)-\frac{m^2}{\rho^2}+k_\rho^2=0$$

$$\frac{1}{F}\frac{d^2F}{d\varphi^2}=-m^2\ \rightarrow\ F(\varphi)=C_1\cos(m\varphi)+C_2\sin(m\varphi)$$

$R(\rho)=J_m(k_\rho\rho)$ *brez singularnosti* $(\rho=0)$

$$\frac{1}{Z}\frac{d^2Z}{dz^2}=-\beta^2\ \rightarrow\ Z(z)=C_3\cos(\beta z)+C_4\sin(\beta z)$$

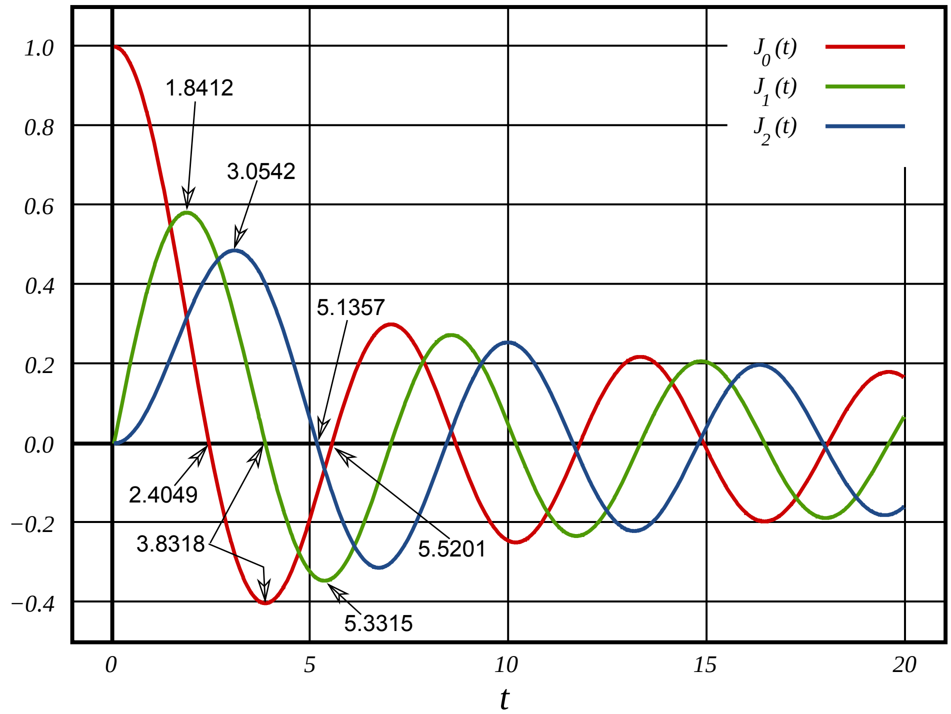
$$J_m(t)=\sum_{n=0}^{\infty}\frac{(-1)^n(t/2)^{2n+m}}{n!(n+m)!}$$

$$E_z(\rho,\varphi,z)=J_m(k_\rho\rho)\left[C_1\cos(m\varphi)+C_2\sin(m\varphi)\right]\left[C_3\cos(\beta z)+C_4\sin(\beta z)\right]$$

$$J_m(t)=\sum_{n=0}^{\infty}\frac{(-1)^n (t/2)^{2n+m}}{n!(n+m)!}$$

Besselove funkcije

$$J_m(t\gg 1)\approx\sqrt{2/(\pi t)}\cos(t-\pi/4-m\pi/2)$$



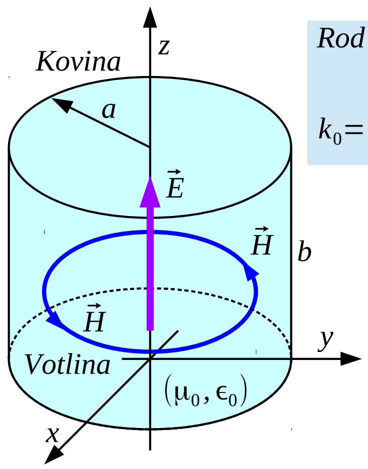
Valjni rezonator

$$\frac{\partial}{\partial\varphi}=0 \rightarrow m=0$$

$$\frac{\partial}{\partial z}=0 \rightarrow \beta=0$$

$$\vec{E}(\vec{r})=\vec{1}_z C J_0(k_0\rho)$$

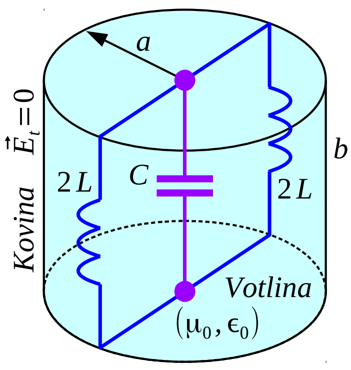
$$\vec{H}(\vec{r})=\frac{j}{k_0 Z_0}\operatorname{rot}\vec{E}(\vec{r})=\vec{1}_\varphi\frac{-jC}{Z_0}J_0{}'(k_0\rho)$$



Rod TM_{100}

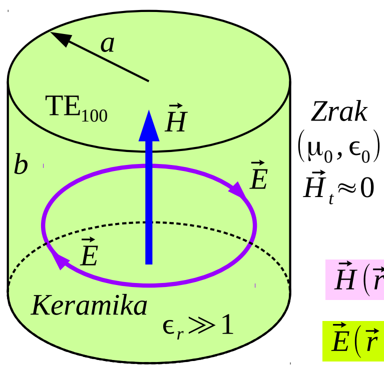
$$k_0=\frac{2.4049}{a}$$

$$f_{100}=\frac{k_0 c_0}{2\pi}=\frac{2.4049 c_0}{2\pi a}\approx\frac{114.75\,\mathrm{MHz.m}}{a}$$



$$f=\frac{1}{2\pi\sqrt{LC}}$$

Dielektrični rezonator



TiO_2 $\epsilon_r\approx 100$

$$f_{100}\approx\frac{2.4049 c_0}{2\pi a\sqrt{\epsilon_r}}$$

$$\vec{H}(\vec{r})=\vec{1}_z C J_0(k\rho)$$

$$\vec{E}(\vec{r})=\vec{1}_\varphi jCZ J_0{}'(k\rho)$$

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