Wave impedance

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The **wave impedance** of an electromagnetic wave is the ratio of the transverse components of the electric and magnetic fields (the transverse components being those at right angles to the direction of propagation). For a transverse-electric-magnetic (TEM) plane wave traveling through a homogeneous medium, the wave impedance is everywhere equal to the intrinsic impedance of the medium. In particular, for a plane wave travelling through empty space, the wave impedance is equal to the impedance of free space. The symbol *Z* is used to represent it and it is expressed in units of ohms. The symbol η (eta) may be used instead of *Z* for wave impedance to avoid confusion with electrical impedance.

The wave impedance is given by

$$Z = \frac{E_0^-(x)}{H_0^-(x)}$$

where $E_0^-(x)$ is the electric field and $H_0^-(x)$ is the magnetic field, in phasor representation.

In terms of the parameters of an electromagnetic wave and the medium it travels through, the wave impedance is given by

$$Z = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

where μ is the magnetic permeability, ε is the electric permittivity and σ is the electrical conductivity of the material the wave is travelling through. In the equation, *j* is the imaginary unit, and ω is the angular frequency of the wave. In the case of a dielectric (where the conductivity is zero), the equation reduces to

$$Z = \sqrt{\frac{\mu}{\varepsilon}}.$$

As usual for any electrical impedance, the ratio is defined only for the frequency domain and never in the time domain.

Contents

Wave impedance in free space

In free space the wave impedance of plane waves is:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

and:

$$c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 299,792,458$$ m/s (by current SI definition of metre)

hence, to the same accuracy as the current definition of c_0, the value in ohms is:

$$Z_0 = \mu_0 c_0 = 4\pi \times 10^{-7} H/m \times 299,792,458 m/s = 376.730313\Omega$$

Wave impedance in an unbounded dielectric

In a isotropic, homogeneous dielectric with negligible magnetic properties, i.e. $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m and $\varepsilon = \varepsilon_r \times 8.854 \times 10^{-12}$ F/m. So, the value of wave impedance in a perfect dielectric is

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \frac{Z_0}{\sqrt{\varepsilon_r}} \approx \frac{377}{\sqrt{\varepsilon_r}}\,\Omega.$$

In a perfect dielectric, the wave impedance can be found by dividing Z_0 by the square root of the dielectric constant.

Wave impedance in a waveguide

For any waveguide in the form of a hollow metal tube, (such as rectangular guide, circular guide, or double-ridge guide), the wave impedance of a travelling wave is dependent on the frequency f, but is the same throughout the guide. For transverse electric (TE) modes of propagation the wave impedance is:

$$Z = \frac{Z_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \qquad \text{(TE modes)},$$

where f_c is the cut-off frequency of the mode, and for transverse magnetic (TM) modes of propagation the wave impedance is:

$$Z = Z_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad \text{(TM modes)}$$

Above the cut-off ($f > f_c$), the impedance is real (resistive) and the wave carries energy. Below cut-off the impedance is imaginary (reactive) and the wave is evanescent. These expressions neglect the effect of resistive loss in the walls of the waveguide. For a waveguide entirely filled with a homogeneous dielectric medium, similar expressions apply, but with the wave impedance of the medium replacing Z_0. The presence of the dielectric also modifies the cut-off frequency f_c.

For a waveguide or transmission line containing more than one type of dielectric medium (such as microstrip), the wave impedance will in general vary over the cross-section of the line.

References

This article incorporates public domain material from the General Services Administration document "Federal Standard 1037C" (http://www.its.bldrdoc.gov/fs-1037/fs-1037c.htm) (in support of MIL-STD-188).

External links

- Standing Wave Diagram (http://swdiagram.tumblr.com/) Application for drawing Standing Wave Diagrams, specifying the wave impedance whenever the wave changes mediums.

See also

- Characteristic impedance
- Impedance (disambiguation)

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