Cutoff Wavelengths of Ridged, Circular, and Elliptic Guides

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Abstract—Cutoff wavelengths of TE and TM modes in doubly ridged guides of elliptical cross section have been calculated using an exact method. Closed-form expressions in terms of Mathieu functions have been obtained for the cutoff frequencies and eigenfunctions describing the field distribution for different modes. These solutions are further extended to include the case of doubly and quadruply ridged guides of circular cross section and to calculate the slot coupling of two waveguides with semielliptical or circular cross sections. The dual problem of the coaxial strip guide has also been treated using the same method.

I. Introduction

RIDGES in guides may be introduced to increase the operating frequency bandwidth through lowering the cutoff frequencies of certain modes. Examples are the uses of Π and H rectangular guides in which the cutoff frequency of the TE_{01} mode is lowered by the capacitive loading of the ridges. Similarly, ridged circular guides may be used to stabilize certain field polarizations, as well as to lower the cutoff frequencies of desired modes. Ferrite devices employing ridged circular guides may also have larger bandwidths [1].

II. Formulation of the Problem

We shall consider the elliptical ridged guide structure of Fig. 1, homogeneous in the direction of propagation z, in which thin ridges extend from the walls to the focal points of the elliptical cross section in the plane of the major axis. The configuration of the cross section can be described in terms of an elliptical cylindrical coordinate system μ, ϕ, z, which is related to the Cartesian coordinate system x, y, z by the relations

$$x = \tfrac{1}{2}a \cosh \mu \cos \phi$$

$$y = \tfrac{1}{2}a \sinh \mu \sin \phi.$$

The elliptic boundary of the guide coincides with the surface $\mu = \mu_0$, $-\pi < \phi < \pi$, whereas ridge surfaces are defined by

$$\phi = 0, \qquad 0 \le \mu < \mu_0 \quad \text{and} \quad \phi = \pm\pi, \qquad 0 \le \mu < \mu_0.$$

Ridge tips are located at points $x = \pm a/2$ and $y = 0$. The major and minor axes of the elliptic boundary are given by

$$A = a \cosh \mu_0 \qquad B = a \sinh \mu_0.$$

Electromagnetic wave propagation in such a cylindrical guide can be described in terms of a scalar function ψ satisfying the Helmholtz equation

$$(\nabla^2 + \kappa^2)\psi = 0$$

in the region bounded by S, where S is the contour of the guide cross section, $\kappa^2 = k^2 - \beta^2$, $k^2 = (\omega/c)^2$, and β is the propagation constant in the direction of z. Function ψ satisfies one of the following boundary conditions on S:

$$\partial\psi/\partial n]_S = 0, \qquad \text{for TE modes}$$

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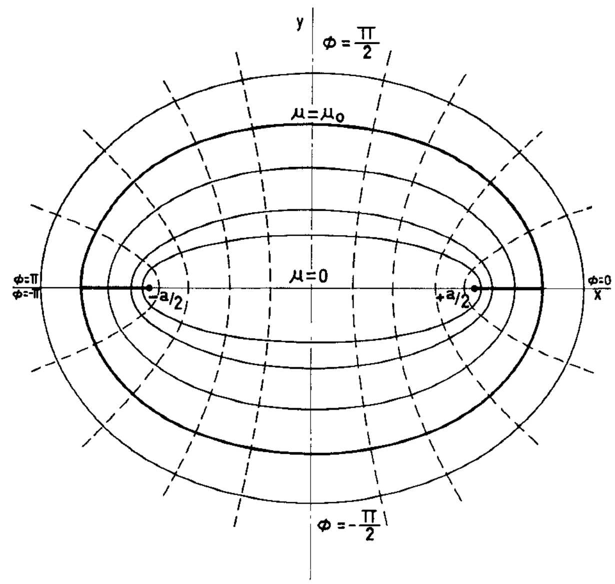

Fig. 1. Cross-section configuration of the elliptic ridged waveguide.

or

$$\psi]_S = 0, \qquad \text{for TM modes.}$$

Function ψ must also be continuous with continuous first derivatives in all points in the region with the exception, maybe, of some points on the boundary where it may have quadratically integrable singularities. In terms of μ and ϕ, the two-dimensional Helmholtz equation can be written:

$$\left[\frac{\partial^2}{\partial\mu^2} + \frac{\partial^2}{\partial\phi^2} + (a\kappa/2)^2(\cosh^2 \mu - \cos^2 \phi)\right]\psi = 0 \qquad (1)$$

and boundary conditions can be formulated in terms of μ and ϕ as follows.

1) For TE modes,

$$\left.\frac{\partial\psi}{\partial\mu}\right]_{\mu=\mu_0} = 0 \qquad -\pi < \phi < \pi \qquad \frac{\partial\psi}{\partial\phi} = 0$$

$$\text{at } \phi = \pm\pi, 0, \qquad 0 \le \mu < \mu_0.$$

2) For TM modes,

$$\psi]_{\mu=\mu_0} = 0 \qquad -\pi < \phi < \pi \qquad \psi = 0$$

$$\text{at } \phi = \pm\pi, 0, \qquad 0 \le \mu < \mu_0.$$

Function ψ must be continuous with continuous derivatives in the internal region:

$$0 \le \mu < \mu_0 \quad 0 < \phi < \pi \quad -\pi < \phi < 0, \qquad (\phi \ne 0, \pi).$$

As (1) separates in elliptic cylinder coordinates, function ψ can be represented as the product

$$\psi = R(\mu)\Phi(\phi).$$

The azimuthal function Φ and the radial function $R(\mu)$ may be determined from the two differential equations

$$\Phi'' + (b^2 - h^2 \cos^2 \phi)\Phi = 0 \tag{2}$$

$$-R'' + (b^2 - h^2 \cosh^2 \mu)R = 0 \tag{3}$$

where $h = a\kappa/2$ and b is the separation constant.

Equation (2) is the Mathieu equation, while (3) is the modified Mathieu equation. The proper azimuthal solutions are those having periods of π and 2π, and therefore Φ is the Mathieu function of either odd or even types denoted by $So_n(h, \phi)$ and $Se_n(h, \phi)$, respectively. The value of the separation constant is completely defined by this choice and b will take the values of bo_n or be_n, where these quantities are the eigenvalues corresponding to odd or even periodic Mathieu functions. Two types of radial functions correspond to each value of h and b. They may be denoted by $Jo_n(h, \mu)$ and $No_n(h, \mu)$ or $Je_n(h, \mu)$ and $Ne_n(h, \mu)$, and they correspond to Bessel and Neumann functions in circular cylindrical coordinates. For details and properties of these functions, the reader may be referred to [2].

III. Transverse Electric Modes

Two types of wave functions with different symmetries about the plane $y = 0$ that satisfy homogeneous Neumann's conditions on the walls and ridges can be distinguished as follows.

1) Even modes for which ψ is symmetrical about the plane $y = 0$ or $\phi = 0$ may be formed by the combinations

$$\psi = Se_n(h, \phi)Je_n(h, \mu), \qquad n = 0, 1, 2, \cdots \tag{4}$$

as we have for all n, $Je_n'(h, 0) = 0$, i.e.,

$$\left.\frac{\partial \psi}{\partial y}\right]_{y=0} = 0.$$

The cutoff wavelength λ_c must be determined from the boundary condition

$$Je_n'(h, \mu_0) = 0, \qquad h = a\pi/\lambda_c.$$

These modes are characterized by the absence of the μ (or the x) component of the electric field on the plane of the ridges so they can also exist in hollow elliptic guides (without ridges), as the presence of ridges does not disturb such fields. These modes were studied in connection with guides of elliptic cross section [3], [4] and will not be treated here further.

2) Odd modes for which ψ has zero value on the plane of symmetry $y = 0$ (i.e., on the plane of the ridges). These modes could have been constructed in the same manner using, instead of Je_n, another regular radial function of an even type having zero value at $\mu = 0$. However, such a function does not exist, as any combination of radial functions containing Ne_n will have a discontinuity in either the slope or value at the origin. Nevertheless, such modes can be constructed as follows:

$$\psi = \begin{cases} Se_n(h, \phi)Re_n(h, \mu), & 0 < \phi < \pi \\ -Se_n(h, \phi)Re_n(h, \mu), & -\pi < \phi < 0 \end{cases} \tag{5}$$

where the radial part Re_n is a linear combination of Je_n and Ne_n having the property

$$Re_n(h, 0) = 0$$

i.e., it may be taken that

$$Re_n = Je_n(h, \mu) + KNe_n(h, \mu), \qquad K \text{ is a constant.}$$

The value of K is determined from the above condition as

$$K = -\frac{Je_n(h, 0)}{Ne_n(h, 0)}.$$

The eigenfrequency h_n may be determined from

$$Re_n'(h_n, \mu_0) = 0. \tag{5a}$$

Now examining (5) we find that it satisfies all the requirements of the boundary value problem. Thus even azimuthal functions satisfy Neumann's boundary conditions on the ridges; the type of symmetry chosen in (5) and the choice of K guarantee the continuity of function ψ and its first derivatives on the slot plane ($|x| \le a/2$, $y = 0$), and finally (5a) satisfies boundary conditions on the elliptic boundary. The wave function suffers discontinuity at $\phi = 0$, π for $\mu > 0$ as expected, due to currents flowing on the ridge conducting surfaces.

Second derivatives may be discontinuous, however, across the slot plane, but this does not violate any physical requirements. Such a solution can be looked upon as two regular solutions in the separate halves of the waveguide section matched together at the common boundary in value and slope, a technique frequently used in irregular waveguide problems. The discontinuity in the second derivative can be attributed to the irregular shape of the boundary giving rise to strong singularities in the field at the sharp tips of the ridges.

For this type of mode the electric field at the plane of the slot is tangential to the slot, and it acquires infinite values at the points of the ridges. These modes clearly do not exist in hollow guides with elliptical cross section and therefore are specific for ridged guides, hence they will be referred to later as ridged guide modes.

In the following sections, TE ridged guide modes will be denoted by H_{nm}^r, where n and m characterize mode type and order, respectively. Regular modes of type 1) will be denoted by H_{nm}^e and are identical with H_{cnm} modes in the theory of hollow elliptic guides. In ridged guides, both H_{nm}^r and H_{nm}^e modes are derived necessarily from even Mathieu functions, and hence the c can be omitted.

IV. Transverse Magnetic Modes

Following the same line we can construct elementary wave functions for TM modes. Thus the proper azimuthal functions are Mathieu odd functions So_n, which are required to satisfy homogeneous Dirichlet's conditions on the ridge surfaces $\phi = 0$, π, $-\pi$. Odd solutions are constructed using Jo_n and elementary wave functions of the form

$$\psi = So_n(h, \phi)Jo_n(h, \mu), \qquad n = 1, 2, 3, \cdots \tag{6}$$

as for all n we have $Jo_n(h, 0) = 0$ and $\psi = 0$ at $y = 0$. While even modes have to be constructed in a way similiar to (5) and will have the form

$$\psi = \begin{cases} So_n(h, \phi)Ro_n(h, \mu), & 0 < \phi < \pi \\ So_n(h, -\phi)Ro_n(h, \mu), & -\pi < \phi < 0 \end{cases} \tag{7}$$

where

$$Ro_n = Jo_n(h, \mu) + LNo_n(h, \mu)$$

and L has to be adjusted to give

$$Ro_n'(h, 0) = Jo_n'(h, 0) + LNo_n'(h, 0) = 0.$$

Solutions of the form (7) satisfy boundary conditions on the ridges, continuity conditions for the function ψ, and its derivatives on the slot, and are even with respect to the plane $y=0$ or $\phi=0$. The electric field of modes (6) is perpendicular to the slot plane, while it is purely tangential for (7). It can be easily seen that waves of the type (6) can exist in hollow guides, while (7) are the ridged guide modes of the TM type. Eigenvalues of h are derived in both cases from the boundary condition on the guide walls, i.e.,

$$Ro_n(h_n, \mu_0) = 0. \tag{7a}$$

V. Numerical Results

The cutoff wavelengths of the two lowest order modes of each of the H^r and E^r sets were computed using (5a) and (7a). Mathieu radial functions were evaluated using their Bessel function expansions [2]. The ratios of the cutoff wavelengths to the major axis λ_c/A were plotted against the ratio of slot width to major axis a/A (ellipse eccentricity) and are shown on Fig. 2.

It must be noted that the ratio of the minor to major axes, which is given by

$$B/A = \tanh \mu_0$$

is very near to unity when μ_0 is sufficiently large; in fact, for $\mu_0 \geq 2$, B/A will lie between 0.95 and 1.0 and $a/A \leq 0.3$. In other words, for values of the ratio of slot width to major axis less than about 0.3, the guide may be considered as being almost circular. This property is important, as it allows the extension of the above results to the case of the circular ridged guide—a configuration that is much more convenient for use in practice than the elliptic one.

As $a/A \rightarrow 0$, the ratios λ_c/A for nearly all modes tend to their values for the corresponding modes in the semicircular guide. Mode H_{01}^r is an important exception; it exhibits anomolous behavior for small values of a/A as its cutoff wavelength tends to infinity. The behavior of the H_{01}^r mode can be explained in two ways. This mode can be viewed as a circular H_{11} mode polarized in the plane of the slot, whose field configuration is modified by the presence of ridges as illustrated in Fig. 3. Electric field concentration in the space between ridge points leads to an equivalent capacitive loading to the guide, which tends to lower its cutoff frequency—an effect analogous to the effect of ridges in rectangular guides. Alternatively, the H_{01}^r mode can be looked upon as a TEM mode between the ridge planes, whose field configuration is perturbed by the bounding walls. As the radius of the shielding cylinder becomes large relative to the slot width, the field will tend to be purely TEM and the cutoff frequency reduced to zero.

Higher H_{0m}^r modes with $m=2, 3, \cdots$ do not behave similarly, and an investigation of (5a) when $n=0$ shows that they tend to $H_{0(m-1)}$ circular guide modes as the eccentricity tends to zero.

For values of eccentricity less than about 0.55, the dominant mode in elliptic ridged guide is the H_{01}^r mode. The bandwidth of the dominant mode propagation increases with decreasing eccentricities and is about an octave for $a/A=0.1$.

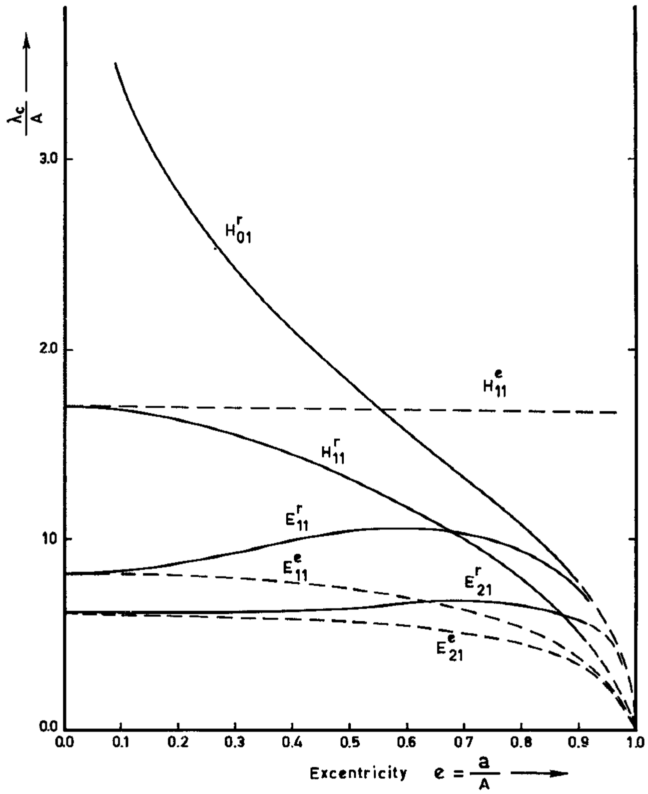

Fig. 2. Cutoff characteristics of H_{01}^r, H_{11}^r, E_{11}^r, and E_{21}^r modes. Characteristics of the conjugate regular modes are shown for comparison.

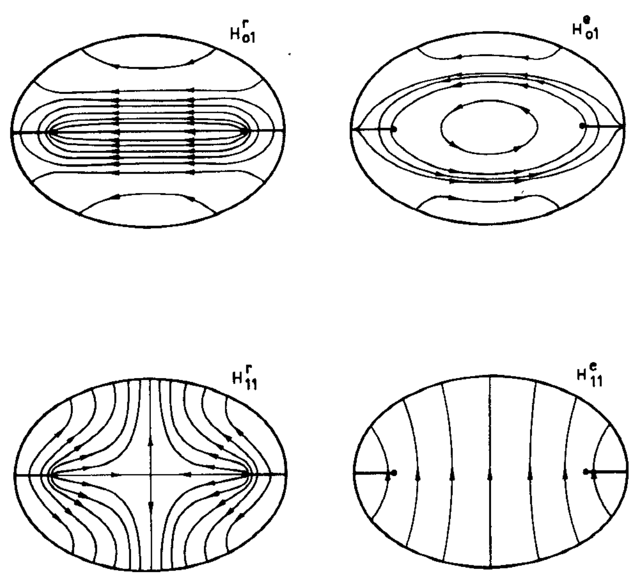

Fig. 3. Electric field configurations for the H_{01}^r and H_{11}^r modes. Field configurations of the conjugate modes are also given for comparison.

Viewing the elliptic guide as a perturbed circular guide it is easily seen that the elongation in the direction of the major axis reduces the cutoff wavelengths of the circular H_{11} mode if it is polarized in this direction, while it has no effect on this mode if it is polarized in the perpendicular direction. The presence of ridges leads to the inverse effect. For small eccentricities the effect of ridges is strong while the elongation is small, thus the cutoff wavelength of the $H_{01}{}^r$ mode is larger than that for the $H_{11}{}^e$, which is dominant in the hollow guide. For large eccentricities the elongation is large, while ridges are less effective because of their smaller relative dimensions; therefore, the $H_{11}{}^e$ mode will dominate. For a value of eccentricity of about 0.55, both effects will compensate each other and these two modes will degenerate. Following the same reasoning we can conclude that the $H_{01}{}^r$ mode will be dominant in the ridged circular guide for all values of slot width to diameter ratios, as there is no elongation in any direction.

It is also remarkable that the H^r modes, except the zero-order one, have cutoff wavelengths lower than those for their conjugate H^e modes. On the contrary, E^r modes have cutoff wavelengths that are higher than those for E^e modes.

VI. Quadruply Ridged Circular Guide

Limiting the discussion to small eccentricities when the elliptic guide can be considered approximately circular, it can be noted that H modes of the type (5) with n restricted to the values 0, 2, 4, $\cdots$ have no electric field component in the plane $\phi=\pi/2$, and therefore are possible modes also for the quadruply ridged guide of Fig. 4. The dominant mode of such a guide is the previously discussed $H_{01}{}^r$ mode with two possible polarizations: either in the plane $\phi=0$ or $\phi=\pi/2$, or any linear combination of them.

This guide may find many applications in broad-band ferrite devices, electron devices with transverse interaction, etc. As there are two polarizations possible perpendicular to each other for the $H_{01}{}^r$ mode, circularly or elliptically polarized modes are evidently among the possible modes of this guide.

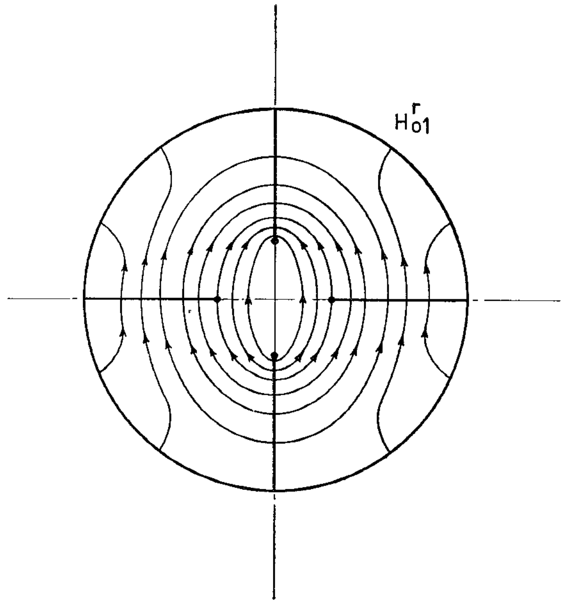

Fig. 4. Electric field configuration of the lowest H mode in quadruply ridged guide ($H_{01}{}^r$).

VII. Coupling Between Semicircular or Semielliptic Guides

Ridged guide modes discussed in Sections III and IV can be used to solve the problem of coupling of semielliptic or semicircular guides through slots. It has been shown that with each regular guide mode of type (4) or (6), there is an associated ridged guide mode with opposite symmetry as given by (5) and (7). For small h we have

$$K = -\frac{Je_n(h, 0)}{Ne_n(h, 0)} \xrightarrow[h\to 0]{} 0$$

and

$$L = -\frac{Jo_n'(h, 0)}{No_n'(h, 0)} \xrightarrow[h\to 0]{} 0$$

as $|Ne_n(h, 0)|$ and $|No_n'(h, 0)|$ tend to infinity as h goes to zero, while $Je_n(h, 0)$ and $Jo_n'(h, 0)$ remain finite. This means that ridged guide modes (with the exception of $H_{01}{}^r$) smoothly degenerate into the corresponding regular guide modes as the slot width reduces to zero. In other words, this means that for small but finite values of h, fields of both types will coincide in magnitude almost everywhere in the guide section except in the vicinity of the slot where the field remains disturbed by the tips.

We can therefore superimpose two H modes of the type

$$\psi^e = Se_n(h_n{}^e, \phi)Je_n(h_n{}^e, \mu)$$

and

$$\psi^r = \pm\, Se_n(h_n{}^r, \phi)Re_n(h_n{}^r, \mu)$$

or E modes

$$\psi^e = So_n(h_n{}^e, \phi)Jo_n(h_n{}^e, \mu)$$

and

$$\psi^r = So_n(h_n{}^r, \pm\,\phi)Ro_n(h_n{}^r, \mu)$$

where h^e, ψ^e and h^r, ψ^r refer to the eigenvalues of h and the eigenfunctions for regular and ridged guide modes, respectively. The resulting fields will vanish in almost all points of one of the semielliptic (or semicircular) regions while having finite values in the other. This clearly is the case when the excitation is applied to one of the halves of the ridged guide.

If we suppose that the semielliptic guide defined by $0<\phi<\pi$ is excited, then the proper TE elementary wave function including the longitudinal phase factor will be

$$\psi = \begin{cases} Se_n(h_n{}^e, \phi)Je_n(h_n{}^e, \mu)e^{-j\beta_n{}^e z} \\ \quad + Se_n(h_n{}^r, \phi)Re_n(h_n{}^r, \mu)e^{-j\beta_n{}^r z}, & \text{for } 0<\phi<\pi \\ Se_n(h_n{}^e, \phi)Je_n(h_n{}^e, \mu)e^{-j\beta_n{}^e z} \\ \quad - Se_n(h_n{}^r, \phi)Re_n(h_n{}^r, \mu)e^{-j\beta_n{}^r z}, & \text{for } -\pi<\phi<0 \end{cases}$$

or for TM modes,

$$\psi = \begin{cases} So_n(h_n{}^e, \phi)Jo_n(h_n{}^e, \mu)e^{-j\beta_n{}^e z} \\ \quad + So_n(h_n{}^r, \phi)Ro_n(h_n{}^r, \mu)e^{-j\beta_n{}^r z}, & \text{for } 0<\phi<\pi \\ So_n(h_n{}^e, \phi)Jo_n(h_n{}^e, \mu)e^{-j\beta_n{}^e z} \\ \quad + So_n(h_n{}^r, -\phi)Ro_n(h_n{}^r, \mu)e^{-j\beta_n{}^r z}, & \text{for } -\pi<\phi<0 \end{cases}$$

where $\beta_n{}^e$ and $\beta_n{}^r$ are the propagation constants for the regular and ridged guide modes, respectively. For narrow slots,

$$h_n^e \simeq h_n^r = h_n \qquad \beta_n^e \simeq \beta_n^r = \beta_n.$$

The field distribution in the section $z=0$ will be given approximately by (for $\mu \neq 0$)

$$\psi \cong \begin{cases} 2Se_n(h_n, \phi)Je_n(h_n, \mu)e^{-j\beta_n z}, & 0 < \phi < \pi \\ 0, & -\pi < \phi < 0 \end{cases}$$

for TE modes, or

$$\psi \cong \begin{cases} 2So_n(h_n, \phi)Jo_n(h_n, \mu)e^{-j\beta_n z}, & 0 < \phi < \pi \\ 0, & -\pi < \phi < 0 \end{cases}$$

for TM modes., For other sections $z \neq 0$, as the propagation constants are not exactly equal; the modes will propagate with slightly different phase velocities and the field power will be transferred from one half to the other periodically as given by

$$\psi \cong \begin{cases} 2Se_n(h_n, \phi)Je_n(h_n, \mu) \cos \delta z e^{-j\beta_n z}, & 0 < \phi < \pi \\ 2Se_n(h_n, \phi)Je_n(h_n, \mu) \sin \delta z e^{-j\beta_n z}, & -\pi < \phi < 0 \end{cases}$$

where $2\delta = \beta_n^e - \beta_n^r$, $\beta_n = (\beta_n^e + \beta_n^r)/2$. Similar expressions can be written for TM fields.

Such an effect is well known from the theory of coupled modes [5] and is utilized in directional couplers and related devices. However, in the present analysis, weak coupling is not assumed and for an elliptic guide the difference 2δ, which determines the length of total power exchange, can be calculated for any slot width, while the slot width should be restricted (for other reasons) to values less than or equal to 0.3 that of the diameter for circular guides.

Simple expressions can be derived for the practically important case of weak coupling when the slot width and therefore h are small. Consider for example the H_{11} circular guide mode, which is dominant in semicircular guides; the characteristic equations for symmetric and antisymmetric modes in a ridged guide of small eccentricity can be written approximately as

$$J_1'(h^e \cosh \mu_0) = 0, \qquad \text{i.e., } h^e \cosh \mu_0 = 1.84$$

and

$$J_1'(h^r \cosh \mu_0) + \frac{\pi}{8} h^{r^2} N_1'(h^r \cosh \mu_0) = 0$$

where J_1 and N_1 are Bessel and Neumann functions of the first order, respectively. In writing the above simplified expressions we are making use of the known asymptotic expansions of radial Mathieu functions for small h:

$$Je_1(h, \mu) \backsim \sqrt{\frac{\pi}{2}} J_1(h \cosh \mu) + 0(h^2)$$

$$Ne_1(h, \mu) \backsim \sqrt{\frac{\pi}{2}} N_1(h \cosh \mu) + 0(h^2), \qquad \mu \neq 0$$

$$Je_1(h, 0) \backsim \frac{h}{2} \sqrt{\frac{\pi}{2}}$$

$$Ne_1(h, 0) \backsim -\frac{2}{h} \sqrt{\frac{2}{\pi}}.$$

Writing $h^r = (1+\Delta)h^e$, expanding Bessel functions in powers of Δ, and keeping first-order terms, we obtain the following asymptotic formula for Δ:

$$\Delta = -\frac{\lambda_c^r - \lambda_c^e}{\lambda_c^r} \simeq 1.048(a/A)^2. \tag{8}$$

Comparison with the results obtained numerically from the exact characteristic equation showed that (8) gives an error of less than 3 percent for eccentricities up to 0.3.

VIII. The Coaxial Elliptic Guide with Inner Strip Conductor

The complementary problem of the coaxial waveguide with an elliptic-shielding cylinder and inner strip conductor having edges at the focal points of the ellipse can be treated using the same approach of Sections III and IV for the ridged guide. This problem is a special case of the more general problem of the coaxial transmission line with elliptic conductors [6], [7], when the eccentricity of the inner conductor is unity. Without many details, the results can be summarized as follows.

In addition to the dominant TEM mode with an infinite cutoff wavelength, the higher TE and TM modes compose two sets: 1) regular or hollow guide modes H_{nm}^e and E_{nm}^e, whose generating functions are given by (4) and (6), and 2) coaxial strip guide modes H_{nm}^s and E_{nm}^s, which have wave functions of the form a) for H_{nm}^s modes:

$$\psi = Ro_n(h, \mu)So_n(h, \phi), \qquad n = 1, 2, \cdots \tag{9}$$

where

$$Ro_n(h, \mu) = Jo_n(h, \mu) - \frac{Jo_n'(h, 0)}{No_n'(h, 0)} No_n(h, \mu).$$

The function satisfies the boundary conditions on the inner strip conductor as

$$Ro_n'(h, 0) = 0$$

and it is odd as So_n is an odd function with respect to the line $\phi = 0$. Cutoff wavelengths may be determined from the characteristic equation

$$Ro_n'(h, \mu_0) = 0. \tag{9a}$$

b) For E_{nm}^s modes,

$$\psi = Re_n(h, \mu)Se_n(h, \phi), \qquad n = 0, 1, 2, \cdots \tag{10}$$

where

$$Re_n(h, \mu) = Je_n(h, \mu) - \frac{Je_n(h, 0)}{Ne_n(h, 0)} Ne_n(h, \mu).$$

Function ψ must have zero values on the elliptic boundary; therefore,

$$Re_n(h, \mu_0) = 0. \tag{10a}$$

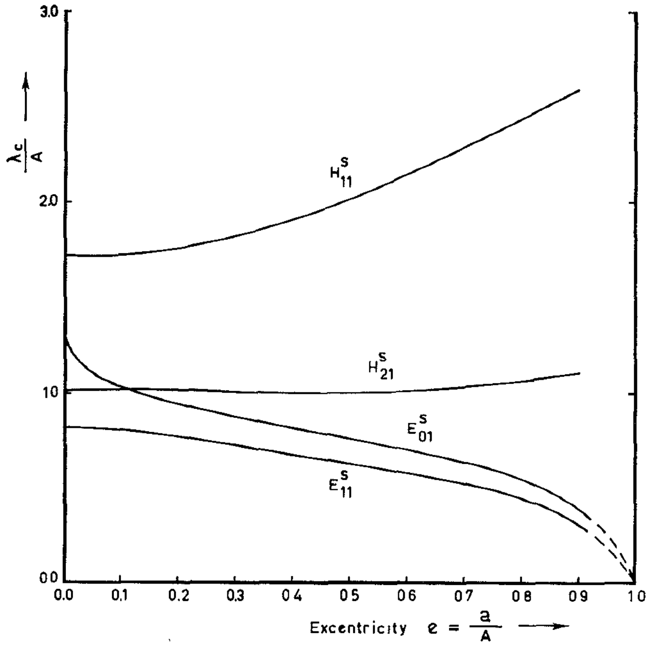

Fig. 5. Cutoff characteristics of the H_{11}^s, H_{21}^s, E_{01}^s, and E_{11}^s modes in the coaxial elliptic waveguide with strip inner conductor.

Equation (10a) determines the cutoff wavelengths of these modes. Modes H_{nm}^s and E_{nm}^s degenerate into their corresponding circular hollow guide modes as the eccentricity (ratio of strip width to major axis) tends to zero.

The dependence of the cutoff wavelengths on the eccentricity was computed for the lowest two of each of the H^s and E^s mode sets and is illustrated by the curves of Fig. 5.

For eccentricities less than or equal to 0.3, the guide is very nearly circular and can be regarded as a coaxial transmission line with strip inner conductor.

IX. Conclusions and Remarks

Needless to say, (5a), (7a), (9a), and (10a) determine the allowed values of h, or the cutoff wavelengths are exact for the case of elliptic guide. Therefore, using a suitable computational method these quantities can be determined to any required degree of accuracy. The transition to circular ridged or strip guides, however, introduces a certain amount of error in the determination of h, which is expected to be of relative magnitude of the order of $(a/A)^2$. If more accurate results are required, corrections have to be made using well-known perturbation methods and regarding the circular guide to be an elliptical one with slightly deformed boundaries.

Ridged and strip guides under study are idealized structures with infinitely thin ridges and strips. Practical guides of course use ridges and strips of finite thickness, and therefore may have cutoff wavelengths different from those computed. However, it is believed that the simplified theory discussed will give values that are at least of the correct order of magnitude and may be helpful in the design. Accurate values can be obtained either by introducing corrections or by direct measurement, if necessary.

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