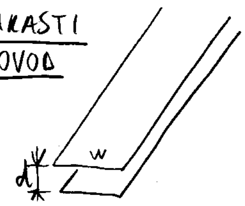
Elektrodinamika #1 8/10/2012

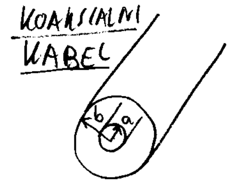
TRAKASTI DVOVOD



$$C/\ell = \varepsilon_0 \frac{w}{d}$$

$$L/\ell = \mu_0 \frac{d}{w}$$

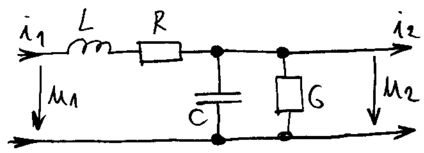
KOAKSIALNI KABEL

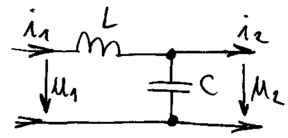


$$C/\ell = \frac{2\pi\varepsilon_0}{\ln b/a}$$

$$L/\ell = \frac{\mu_0}{2\pi} \ln b/a$$

NADOMESTNO VEZJE $\longrightarrow$ BREZ IZGUB





$$\Delta u = u_2 - u_1 = -L\frac{di_1}{dt} - Ri_1$$

$$\Delta i = i_2 - i_1 = -C\frac{du_2}{dt} - Gu_2$$

$$\frac{\partial u(z,t)}{\partial z} = -L/\ell \frac{\partial i(z,t)}{\partial t} - R/\ell\, i(z,t)$$

$$\frac{\partial i(z,t)}{\partial z} = -C/\ell \frac{\partial u(z,t)}{\partial t} - G/\ell\, u(z,t$$

$$\Delta u = u_2 - u_1 = -L\frac{di_1}{dt}$$

$$\Delta i = i_2 - i_1 = -C\frac{di_1}{dt}$$

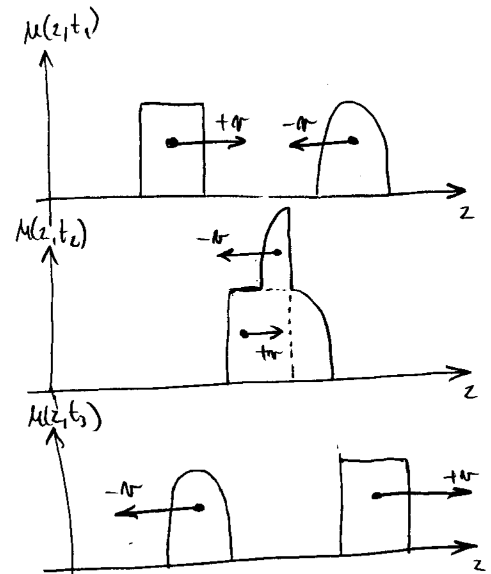
$$\frac{\partial u(z,t)}{\partial z} = -L/\ell \frac{\partial i(z,t)}{\partial t} \quad /\frac{\partial}{\partial z}$$

$$\frac{\partial i(z,t)}{\partial z} = -C/\ell \frac{\partial u(z,t)}{\partial t} \quad /\frac{\partial}{\partial t}$$

$$\frac{\partial^2 u(z,t)}{\partial z^2} = L/\ell \cdot C/\ell \frac{\partial^2 u(z,t)}{\partial t^2}$$

$$u(z,t) = f(x)\;;\; x = t \pm \frac{z}{v}$$

$$\frac{\partial^2 u(z,t)}{\partial z^2} = f''(x)\frac{1}{v^2} \qquad \frac{\partial^2 u(z,t)}{\partial t^2} = f''(x) \longrightarrow v = 1/\sqrt{L/\ell \cdot C/\ell}$$



$$u(z,t) = C_1 f_1\left(t - \frac{z}{v}\right) + C_2 f_2\left(t + \frac{z}{v}\right)$$

NAPREDUJOČI VAL   ODBITI VAL

TRAKASTI DVOVOD

$$v = 1/\sqrt{\mu_0 \frac{d}{w}\varepsilon_0\frac{w}{d}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c_0 \approx 3\cdot 10^8\,\mathrm{m/s}$$

KOAKSIALNI KABEL

$$v = 1/\sqrt{\frac{\mu_0}{2\pi}\ln b/a \,\frac{2\pi\varepsilon_0}{\ln b/a}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c_0 \approx 3\cdot 10^8\,\mathrm{m/s}$$

$$\frac{\partial}{\partial z} u\left(t \pm \frac{z}{v}\right) = -L/\ell \frac{\partial}{\partial t} i\left(t \pm \frac{z}{v}\right)$$

$$\pm\frac{1}{v} u'\left(t \pm \frac{z}{v}\right) = -L/\ell\, i'\left(t \pm \frac{z}{v}\right)$$

$$\frac{u'}{i'} = \frac{u}{i} = \mp v L/\ell = \mp\sqrt{\frac{L/\ell}{C/\ell}} = \mp Z_k$$

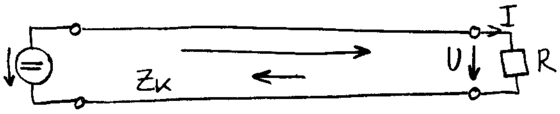
TRAKASTI DVOVOD $Z_k = \sqrt{\dfrac{\mu_0 \frac{d}{w}}{\varepsilon_0 \frac{w}{d}}} = \dfrac{d}{w}\sqrt{\dfrac{\mu_0}{\varepsilon_0}}$

KOAKSIALNI KABEL $Z_k = \sqrt{\dfrac{\frac{\mu_0}{2\pi}\ln b/a}{2\pi\varepsilon_0/\ln b/a}} = \dfrac{1}{2\pi}\sqrt{\dfrac{\mu_0}{\varepsilon_0}}\ln b/a$

PROSTOR: $\dfrac{|E|}{|H|} = Z_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 120\pi\,\Omega = 377\,\Omega$

Elektrodinamika #2 15/10/2012

STATIKA



$U = U_N + U_0$

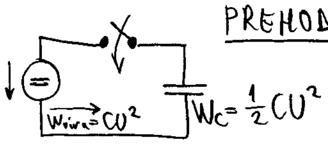
$I = I_N + I_0 = \frac{U_N}{Z_k} - \frac{U_0}{Z_k} = \frac{U}{R} = \frac{U_N + U_0}{R} \quad / \cdot \frac{1}{U_N}$

ODBOJNOST $\Gamma = \frac{U_0}{U_N} \longrightarrow \frac{1}{Z_k} - \frac{\Gamma}{Z_k} = \frac{1+\Gamma}{R} \longrightarrow \boxed{\Gamma = \frac{R - Z_k}{R + Z_k}}$

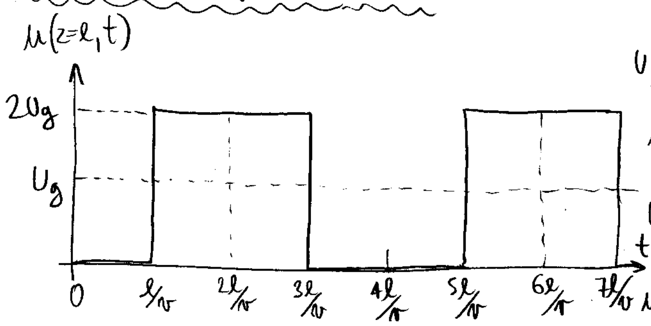
PASIVNO BREME

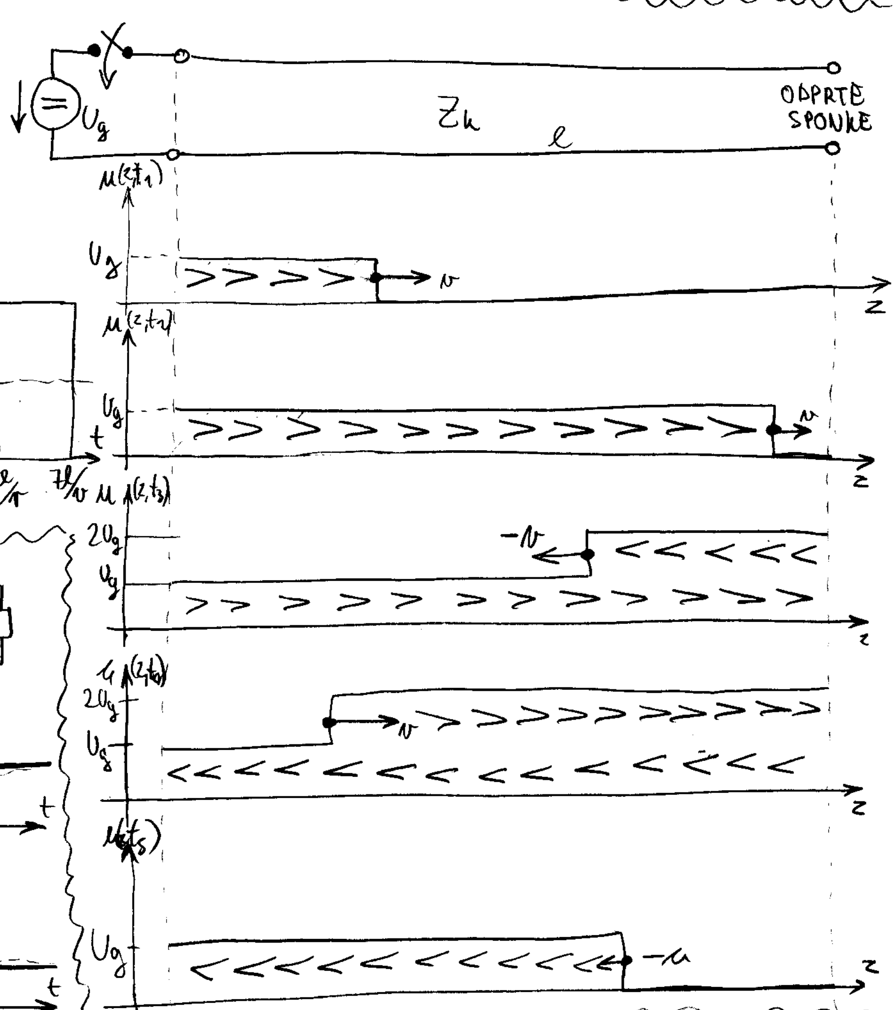
OS: $\Gamma = +1$, KS: $\Gamma = -1$; $Z_k = R \to \Gamma = 0$; $Z_k > R \to \Gamma < 0$; $Z_k < R \to \Gamma > 0$; $|\Gamma| \leq 1$

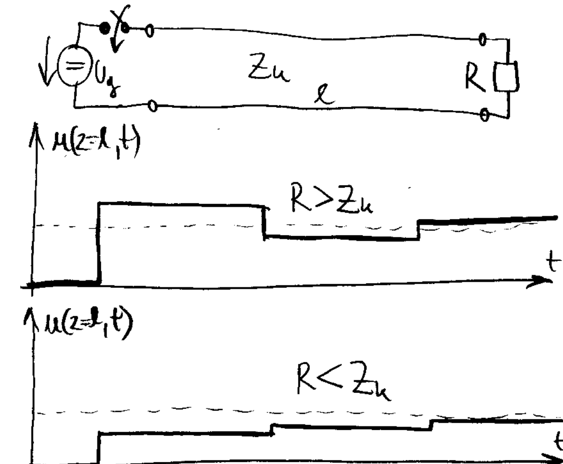
PREHODNI POJAV

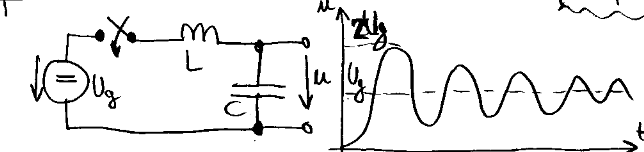


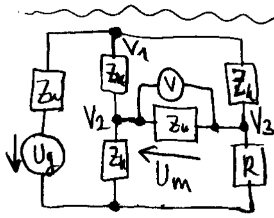
KAM GRE POLOVICA ENERGIJE?











(1) $\frac{V_1 - U_g}{Z_k} + \frac{V_1 - V_2}{Z_k} + \frac{V_1 - V_3}{Z_k} = 0$

(2) $\frac{V_2 - V_1}{Z_k} + \frac{V_2 - V_3}{Z_k} + \frac{V_2}{Z_k} = 0$

(3) $\frac{V_3 - V_2}{Z_k} + \frac{V_3 - V_1}{Z_k} + \frac{V_3}{R} = 0$

(1) $3V_1 = U_g + V_2 + V_3 = 9V_2 - 3V_3 \longrightarrow 8V_2 = U_g + 4V_3$

(2) $3V_2 = V_1 + V_3 \longrightarrow V_1 = 3V_2 - V_3$

MOSTIČ za Γ

(3) $\left(2 + \frac{Z_k}{R}\right) V_3 = V_1 + V_2 = 4V_2 - V_3 \longrightarrow \left(3 + \frac{Z_k}{R}\right) V_3 = 4V_2$

$\left(6 + 2\frac{Z_k}{R}\right) V_3 = U_g + 4V_3 \longrightarrow V_3 = \frac{U_g}{2\left(1 + \frac{Z_k}{R}\right)}$ $\quad U_m = V_3 - V_2$

$V_2 = \frac{U_g}{8} + \frac{V_3}{2} = \frac{U_g}{8} \frac{3 + \frac{Z_k}{R}}{\left(1 + \frac{Z_k}{R}\right)}$

$\boxed{U_m = \frac{U_g}{8} \frac{1 - \frac{Z_k}{R}}{1 + \frac{Z_k}{R}} = \frac{U_g}{8} \Gamma}$

FREKVENČNI PROSTOR

$\frac{\omega}{v} = \beta \equiv$ fazna konstanta

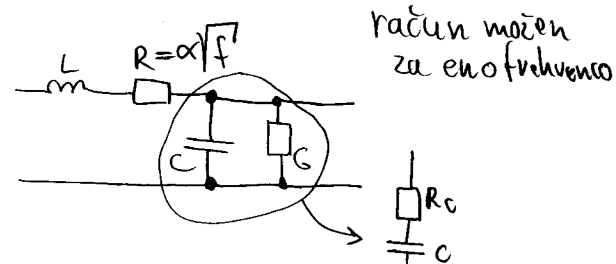
$u(t, z) = \mathrm{Re}\left[U_0 e^{j\omega\left(t \pm \frac{z}{v}\right)}\right] = \mathrm{Re}\left[U_0 e^{j(\omega t \pm \beta z)}\right]$

$\frac{\partial}{\partial t} = j\omega$; $\frac{\partial}{\partial z} = \pm j\beta$ $\qquad \beta = \omega\sqrt{\mu_0 \varepsilon_0}$

$\beta \lambda = 2\pi \longrightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu_0 \varepsilon_0}} = \frac{c_0}{f}$

TEŽAVE Z IZGUBAMI

račun možen za eno frekvenco



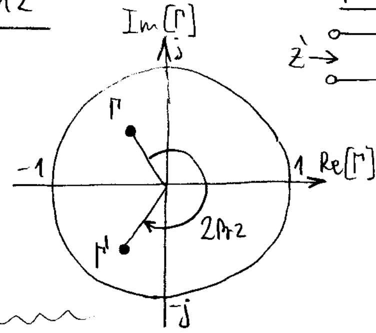
Elektrodinamika #3 22/10/2012

$$u(t,z) = \mathrm{Re}\left[U_N e^{j(\omega t - \beta z)} + U_O e^{j(\omega t + \beta z)}\right]$$

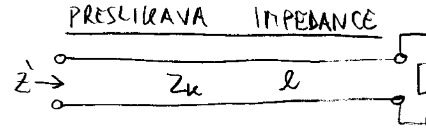
$$\Gamma = \frac{Z - Z_K}{Z + Z_K} = \frac{Y_K - Y}{Y_K + Y}$$

BREZIZGUBNI VOD

$|\Gamma| \leq 1$ pasivno breme



PRESLIKAVA IMPEDANCE

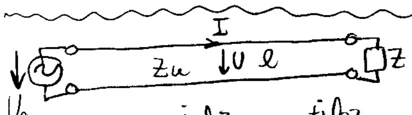


① $\Gamma = \frac{Z - Z_K}{Z + Z_K}$

② $\Gamma' = \Gamma e^{-j2\beta l}$

③ $Z' = Z_K \frac{1+\Gamma'}{1-\Gamma'}$

$$Z' = Z_K \frac{1+\Gamma'}{1-\Gamma'} = Z_K \frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}} = Z_K \frac{Z+Z_K+(Z-Z_K)e^{-j2\beta l}}{Z+Z_K-(Z-Z_K)e^{-j2\beta l}} = Z_K \frac{Z(e^{j\beta l}+e^{-j\beta l})+Z_K(e^{j\beta l}-e^{-j\beta l})}{Z(e^{j\beta l}-e^{-j\beta l})+Z_K(e^{j\beta l}+e^{-j\beta l})} = Z_K \frac{Z\cos\beta l + jZ_K \sin\beta l}{jZ\sin\beta l + Z_K\cos\beta l}$$



$$U = U_N e^{-j\beta z} + U_O e^{j\beta z}$$

$$I = \frac{U_N}{Z_K} e^{-j\beta z} - \frac{U_O}{Z_K} e^{j\beta z}$$

MOČ

$$P = \frac{1}{2} U I^* = \frac{1}{2}\left(U_N e^{-j\beta z} + U_O e^{+j\beta z}\right)\left(\frac{U_N^*}{Z_K} e^{j\beta z} - \frac{U_O^*}{Z_K} e^{-j\beta z}\right)$$

$$P = \frac{|U_N|^2}{2Z_K} - \frac{|U_O|^2}{2Z_K} + \frac{U_O U_N^*}{2Z_K} e^{j2\beta z} - \frac{U_N U_O^*}{2Z_K} e^{-j2\beta z}$$

$U_O U_N^* = |U_O U_N^*| e^{j\varphi}$

$U_N U_O^* = |U_O U_N^*| e^{-j\varphi}$

$$P = \underbrace{\frac{|U_N|^2}{2Z_K}}_{\text{NAPREDUJOČA MOČ}} - \underbrace{\frac{|U_O|^2}{2Z_K}}_{\text{ODBITA MOČ}} + \underbrace{j\frac{|U_O U_N^*|}{Z_K}\sin(2\beta z + \varphi)}_{\text{JALOVA MOČ}}$$

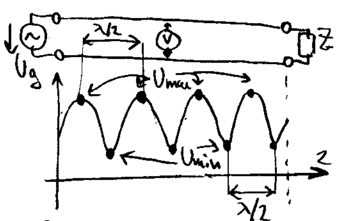
JALOVA MOČ → ENERGIJA STOJNEGA VALA

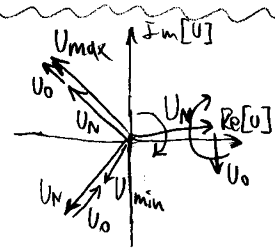
$\mathrm{Re}[P] = P_N - P_O$ $P_O = |\Gamma|^2 P_N$

$$\mathrm{Re}[P] = P_N(1 - |\Gamma|^2)$$

STOJNI VAL (BREZIZGUBNI)

$$\rho = \frac{U_{max}}{U_{min}}$$



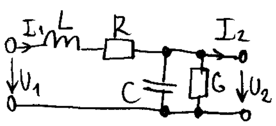


$$U_{max} = |U_N| + |U_O| = |U_N|(1 + |\Gamma|)$$

$$U_{min} = |U_N| - |U_O| = |U_N|(1 - |\Gamma|)$$

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|}$$ RAZMERJE STOJNEGA VALA, STANDING-WAVE RATIO, VALOVITOST

IZGUBNI VOD



$U_1 - U_2 = (j\omega L + R) I_1$ $\frac{\partial U}{\partial z} = -(j\omega L/l + R/l) I$ $/\frac{\partial}{\partial z}$

$I_1 - I_2 = (j\omega C + G) U_2$ $\frac{\partial I}{\partial z} = -(j\omega C/l + G/l) U$

$U = A e^{\pm jkz} \longrightarrow -k^2 = (j\omega L + R)(j\omega C + G)$

$$k = \sqrt{-(j\omega L/l + R/l)(j\omega C/l + G/l)} = \underset{[rd/m]}{\beta} - j\underset{[Np/m]}{\alpha}$$

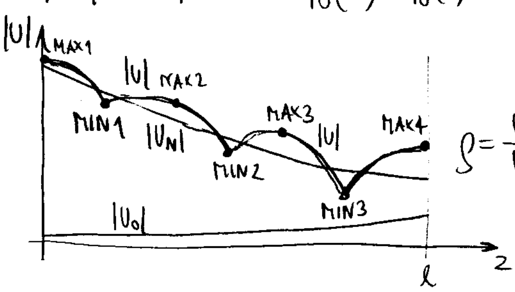
$$u(z,t) = \mathrm{Re}\left[U_N e^{-\alpha z} e^{j(\omega t - \beta z)} + U_O e^{\alpha z} e^{j(\omega t + \beta z)}\right]$$

$$Z_K = \frac{U_N}{I_N};\ U_N = A_N e^{-jkz};\ I_N = \frac{-jk}{-(j\omega L/l + R/l)} U_N \rightarrow Z_K = \frac{j\omega L/l + R/l}{jk} = \sqrt{\frac{j\omega L/l + R/l}{j\omega C/l + G/l}} \approx \sqrt{\frac{L/l}{C/l}}$$ ZA MAJHNE IZGUBE!

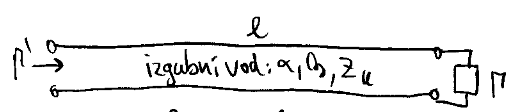
$$|U_N| = |A_N| e^{-\alpha z} \rightarrow P_N(z) = P_N(0) e^{-2\alpha z}$$

$$|U_O| = |A_O| e^{+\alpha z} \rightarrow P_O(z) = P_O(0) e^{+2\alpha z}$$

$$\Gamma(z) = \frac{U_O}{U_N} = \frac{e^{\alpha z} e^{j\beta z}}{e^{-\alpha z} e^{-j\beta z}} \Gamma(0) = e^{2\alpha z} e^{j2\beta z} \Gamma(0)$$

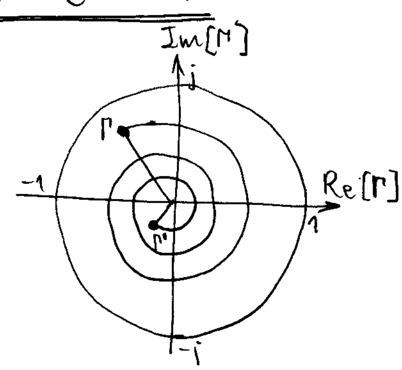


$$\rho = \frac{U_{max}}{U_{min}} = \text{NEDEFINIRAN}$$



$$\Gamma' = e^{-2\alpha l} e^{-j2\beta l} \Gamma$$

$$|\Gamma'| = |\Gamma| e^{-2\alpha l}$$



LOG. ENOTE

PRILAGOJENOST, RETURN LOSS

$$\Gamma_{dB} = 20\log_{10}|\Gamma|$$

$$\Gamma'_{dB} = \Gamma_{dB} - \frac{20}{\ln 10}(2\alpha l)$$

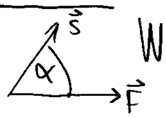
$$\Gamma'_{dB} = \Gamma_{dB} - 2a_{dB}$$

$$\Gamma_{Np} = \ln|\Gamma| = \ln\frac{U_O}{U_N}$$

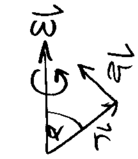
$$a_{dB} = 10\log_{10}\frac{P(0)}{P(l)} = 20\log_{10}\frac{U(0)}{U(l)} = \frac{20}{\ln 10} a_{Np} \qquad a_{dB}/l = \frac{20}{\ln 10}\alpha$$

Elektrodinamika #4 29/10/2012

FIZIKA



$W = |\vec{F}||\vec{s}|\cos\varphi = \vec{F}\cdot\vec{s}$

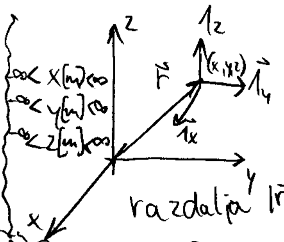


$|\vec{v}| = |\vec{r}||\vec{\omega}|\sin\alpha$

$\vec{v} \perp \vec{r}\ ,\ \vec{v} \perp \vec{\omega}$

$\vec{v} = \vec{\omega} \times \vec{r}$

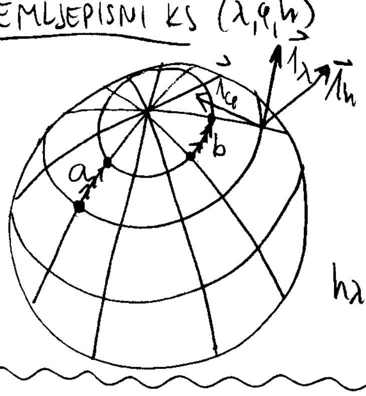
KARTEZIČNI KS



— 3D (x,y,z)
— pravokotni $\vec{1}_x \cdot \vec{1}_y = 0$
— desnoročni $\vec{1}_x \times \vec{1}_y = \vec{1}_z$

razdalja $|\vec{r}_2 - \vec{r}_1| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

ZEMLJEPISNI KS (λ, φ, h)



$a = h_\varphi \Delta\varphi$

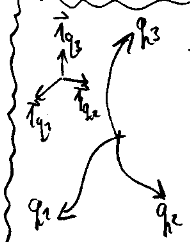
$0 \leq \lambda[°] \leq 360°$
$-90° \leq \varphi[°] \leq +90°$
$6378000\,m < h[m] < \infty$

$h_\varphi = \frac{40000\,km}{360°} = 111\,km/°$

$b = h_\lambda \Delta\lambda$

$h_\lambda = \frac{40000\,km}{360°}\cos\varphi = 111\,km/° \cdot \cos\varphi$

SPLOŠNI KRIVOČRTNI KS (q_1, q_2, q_3)



$dl_1 = \sqrt{dx^2+dy^2+dz^2} = h_1 dq_1$

$dx = \frac{\partial x}{\partial q_1} dq_1 \quad dy = \frac{\partial y}{\partial q_1} dq_1 \quad dz = \frac{\partial z}{\partial q_1} dq_1$

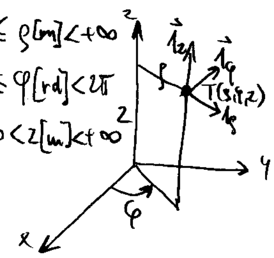
$dl_1 = \sqrt{\left(\frac{\partial x}{\partial q_1}\right)^2 + \left(\frac{\partial y}{\partial q_1}\right)^2 + \left(\frac{\partial z}{\partial q_1}\right)^2}\ dq_1$

$h_i = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2}$

$x = x(q_1,q_2,q_3)$
$y = y(q_1,q_2,q_3)$
$z = z(q_1,q_2,q_3)$

VALJNI KS (ρ, φ, z)

$0 \leq \rho[m] < +\infty$
$0 \leq \varphi[rd] < 2\pi$
$-\infty < z[m] < +\infty$



$\rho = \sqrt{x^2+y^2}$

$\varphi = \mathrm{arctg}\, y/x$ (kvadrant!)

$z = z$

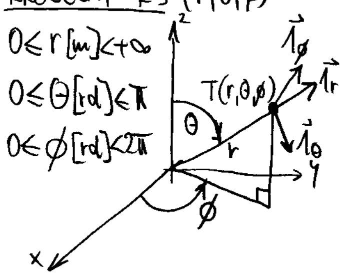
$x = \rho\cos\varphi \qquad h_\rho = 1$

$y = \rho\sin\varphi \qquad h_\varphi = \rho$

$z = z \qquad h_z = 1$

KROGELNI KS (r, θ, ϕ)

$0 \leq r[m] < +\infty$
$0 \leq \theta[rd] \leq \pi$
$0 \leq \phi[rd] < 2\pi$



$r = \sqrt{x^2+y^2+z^2}$

$\theta = \arccos z/r$

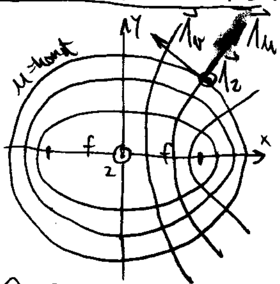
$\phi = \mathrm{arctg}\, y/x$ (kvadrant!)

$x = r\sin\theta\cos\phi \qquad h_r = 1$

$y = r\sin\theta\sin\phi \qquad h_\theta = r$

$z = r\cos\theta \qquad h_\phi = r\sin\theta$

VALJNI-ELIPTIČNI KS (u, v, z)



$0 \leq u[\ ] < +\infty$
$0 \leq v[rd] < 2\pi$
$-\infty < z[m] < +\infty$

podatek $f[m]$

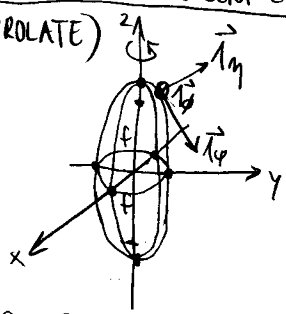
$x = f\,\mathrm{ch}u\cos v \qquad h_z = 1$

$y = f\,\mathrm{sh}u\sin v$

$z = z \qquad h_u = h_v = f\sqrt{\mathrm{sh}^2u + \sin^2 v}$

PODOLGOVATI KROGELNI-ELIPTIČNI KS (η, ψ, ϕ)

(PROLATE)



$0 \leq \eta[\ ] < +\infty$
$0 \leq \psi[rd] \leq \pi$
$0 \leq \phi[rd] < 2\pi$

podatek $f[m]$

$x = f\,\mathrm{sh}\eta\sin\psi\cos\phi$

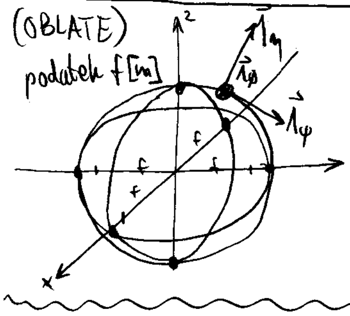
$y = f\,\mathrm{sh}\eta\sin\psi\sin\phi$

$z = f\,\mathrm{ch}\eta\cos\psi$

SPLOŠČENI KROGELNI-ELIPTIČNI KS (η, ψ, ϕ)

(OBLATE)

podatek $f[m]$



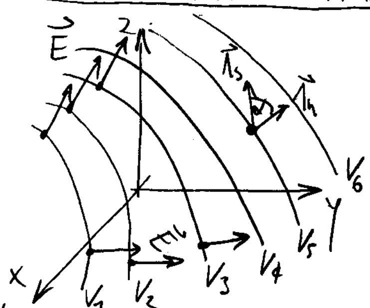
$0 \leq \eta[\ ] < +\infty$
$0 \leq \psi[rd] \leq \pi$
$0 \leq \phi[rd] < 2\pi$

$x = f\,\mathrm{ch}\eta\sin\psi\cos\phi$

$y = f\,\mathrm{ch}\eta\sin\psi\sin\phi$

$z = f\,\mathrm{sh}\eta\cos\psi$

SMERNI ODVOD = GRADIENT $V(x,y,z)$



$\max \frac{\partial V}{\partial s} = \frac{\partial V}{\partial n}$

$\frac{\partial V}{\partial s} = \vec{1}_s \cdot \vec{1}_n \frac{\partial V}{\partial n} = \frac{\partial V}{\partial n}\cos\alpha$

$\vec{1}_n \frac{\partial V}{\partial n} = \mathrm{grad}\, V$

$(x,y,z) \rightarrow \vec{1}_x \cdot \mathrm{grad}V = \frac{\partial V}{\partial x}\ ,\ \vec{1}_y \cdot \mathrm{grad}V = \frac{\partial V}{\partial y}\ ,\ \vec{1}_z \cdot \mathrm{grad}V = \frac{\partial V}{\partial z} \rightarrow \underline{\mathrm{grad}V = \vec{1}_x\frac{\partial V}{\partial x} + \vec{1}_y\frac{\partial V}{\partial y} + \vec{1}_z\frac{\partial V}{\partial z}}$

$\vec{\nabla} = \vec{1}_x\frac{\partial}{\partial x} + \vec{1}_y\frac{\partial}{\partial y} + \vec{1}_z\frac{\partial}{\partial z} \rightarrow \mathrm{grad}\, V = \vec{\nabla} V$

$dl_1 = h_1 dq_1,\ dl_2 = h_2 dq_2,\ dl_3 = h_3 dq_3$

$(q_1,q_2,q_3) \rightarrow \vec{1}_{q_1} \cdot \mathrm{grad}V = \frac{\partial V}{\partial l_1}\ ,\ \vec{1}_{q_2} \cdot \mathrm{grad}V = \frac{\partial V}{\partial l_2}\ ,\ \vec{1}_{q_3} \cdot \mathrm{grad}V = \frac{\partial V}{\partial l_3} \rightarrow \underline{\mathrm{grad}V = \vec{1}_{q_1}\frac{1}{h_1}\frac{\partial V}{\partial q_1} + \vec{1}_{q_2}\frac{1}{h_2}\frac{\partial V}{\partial q_2} + \vec{1}_{q_3}\frac{1}{h_3}\frac{\partial V}{\partial q_3}}$

$(\rho,\varphi,z) \rightarrow \underline{\mathrm{grad}V = \vec{1}_\rho\frac{\partial V}{\partial \rho} + \vec{1}_\varphi\frac{1}{\rho}\frac{\partial V}{\partial \varphi} + \vec{1}_z\frac{\partial V}{\partial z}}$

$(r,\theta,\phi) \rightarrow \underline{\mathrm{grad}V = \vec{1}_r\frac{\partial V}{\partial r} + \vec{1}_\theta\frac{1}{r}\frac{\partial V}{\partial \theta} + \vec{1}_\phi\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}}$

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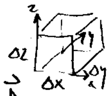
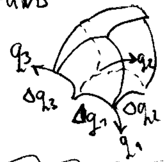
IZVORNOST



$$\int_V \rho\, dv = Q = \oint_A \vec{D}\cdot d\vec{A} \rightarrow \rho = \lim_{\Delta V\to 0} \frac{\oint \vec{D}\cdot d\vec{A}}{\Delta V} = \operatorname{div}\vec{D}$$

$$\rho = \lim_{\Delta V\to 0} \frac{\Delta y \Delta z \Delta D_x + \Delta x \Delta z \Delta D_y + \Delta x \Delta y \Delta D_z}{\Delta x \Delta y \Delta z} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \vec{\nabla}\cdot\vec{D}$$



$$\operatorname{div}\vec{D} = \frac{1}{h_1h_2h_3}\left(\frac{\partial}{\partial q_1}(h_2h_3D_1) + \frac{\partial}{\partial q_2}(h_1h_3D_2) + \frac{\partial}{\partial q_3}(h_1h_2D_3)\right)$$

$$\rho = \lim_{\Delta V\to 0} \frac{\Delta(l_2l_3D_1) + \Delta(l_1l_3D_2) + \Delta(l_1l_2D_3)}{\Delta l_1 \Delta l_2 \Delta l_3} = \frac{\frac{\partial(h_2h_3D_1)}{\partial q_1} + \frac{\partial(h_1h_3D_2)}{\partial q_2} + \frac{\partial(h_1h_2D_3)}{\partial q_3}}{h_1h_2h_3}$$

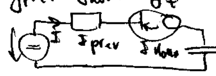
VRTINČENJE

$$\int_A \vec{J}\cdot d\vec{A} = I = \oint_S \vec{H}\cdot d\vec{s} \longrightarrow J_z = \lim_{\Delta A\to 0}\frac{\oint \vec{H}\cdot d\vec{s}}{\Delta A} = \vec{1}_z\cdot \operatorname{rot}\vec{H}$$

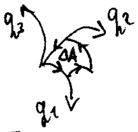
$$J_z = \lim_{\Delta A\to 0}\frac{\Delta y H_y - \Delta x H_x}{\Delta x \Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \rightarrow \vec{J} = \vec{\nabla}\times\vec{H} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\vec{J} = \operatorname{rot}\vec{H}$$

$$\vec{J} = \vec{J}_{prev} + \vec{J}_{konv} + \frac{\partial\vec{D}}{\partial t}$$



$I = \frac{dQ}{dt}$ pol...



$$J_3 = \lim_{\Delta A\to 0}\frac{\Delta l_2 H_2 - \Delta l_1 H_1}{\Delta l_1 \Delta l_2} = \frac{1}{h_1h_2}\left(\frac{\partial(h_2H_2)}{\partial q_1} - \frac{\partial(h_1H_1)}{\partial q_2}\right) \quad \vec{J} = \frac{1}{h_1h_2h_3}\begin{vmatrix} \vec{1}_{q_1}h_1 & \vec{1}_{q_2}h_2 & \vec{1}_{q_3}h_3 \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1H_1 & h_2H_2 & h_3H_3 \end{vmatrix}$$

ME v dif. obliki

1. $\operatorname{rot}\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$
2. $\operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
3. $\operatorname{div}\vec{D} = \rho$

Harmonske veličine

$$\vec{E}(x,y,z,t) = \vec{E}_0(x,y,z)e^{j\omega t}$$

$$\frac{\partial}{\partial t} = j\omega$$

1. $\operatorname{rot}\vec{H} = \vec{J} + j\omega\vec{D}$
2. $\operatorname{rot}\vec{E} = -j\omega\vec{B}$
3. $\operatorname{div}\vec{D} = \rho$

Snov: ε, μ (skalarni konstanti)

1. $\operatorname{rot}\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$
2. $\operatorname{rot}\vec{E} = -j\omega\mu\vec{H}$
3. $\operatorname{div}(\varepsilon\vec{E}) = \rho$

EM naloga

izvori $(\rho, \vec{J}) \rightarrow$ polje $(\vec{E}, \vec{H})$

$$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = \operatorname{grad}\left(\frac{\rho}{\varepsilon}\right) + j\omega\mu\vec{J}$$

$$\Delta\vec{H} + \omega^2\mu\varepsilon\vec{H} = -\operatorname{rot}\vec{J}$$

Sestavljene operacije

$$\operatorname{rot}(\operatorname{grad} V) = \vec{\nabla}\times(\vec{\nabla}V) = 0$$

$$\operatorname{div}(\operatorname{rot}\vec{F}) = \vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) = 0$$

$$\operatorname{rot}(\operatorname{rot}\vec{F}) = \operatorname{grad}(\operatorname{div}\vec{F}) - \Delta\vec{F}$$

$$\operatorname{div}(\operatorname{grad} V) = \vec{\nabla}\cdot(\vec{\nabla}V) = \Delta V$$

Potenciali

OE I $\rightarrow \omega = 0$, $\vec{E} = -\operatorname{grad} V$, $\Delta V = -\frac{\rho}{\varepsilon}$

OE II $\rightarrow \omega = 0$, $\vec{J} = 0$, $\vec{H} = -\operatorname{grad} V_m$, $\Delta V_m = 0$

VEKTORSKI POTENCIAL $\vec{B} = \operatorname{rot}\vec{A}$

2. $\operatorname{rot}\vec{E} = -j\omega\vec{B} = -j\omega\operatorname{rot}\vec{A} \rightarrow \operatorname{rot}(\vec{E} + j\omega\vec{A}) = 0 \rightarrow \vec{E} + j\omega\vec{A} = -\operatorname{grad}V$, $\underline{\vec{E} = -j\omega\vec{A} - \operatorname{grad}V}$

1. $\operatorname{rot}\vec{H} = \operatorname{rot}\left(\frac{1}{\mu}\operatorname{rot}\vec{A}\right) = \vec{J} + j\omega\varepsilon\vec{E} \rightarrow \operatorname{rot}(\operatorname{rot}\vec{A}) = \operatorname{grad}(\operatorname{div}\vec{A}) - \Delta\vec{A} = \mu\vec{J} + j\omega\mu\varepsilon(-j\omega\vec{A} - \operatorname{grad}V)$

$$\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J} + \operatorname{grad}(\operatorname{div}\vec{A} + j\omega\mu\varepsilon V)$$

Columb: $\operatorname{div}\vec{A} = 0$ Lorentz: $\operatorname{div}\vec{A} = -j\omega\mu\varepsilon V$

$$\boxed{\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J}}$$

3. $\operatorname{div}(\varepsilon\vec{E}) = \rho \rightarrow \frac{\rho}{\varepsilon} = -j\omega\operatorname{div}\vec{A} - \operatorname{div}(\operatorname{grad}V)$

$$\boxed{\Delta V + \omega^2\mu\varepsilon V = -\frac{\rho}{\varepsilon}}$$

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OE I, $\omega=0$ $\quad W_e=\frac{1}{2}\int_v \vec{E}\cdot\vec{D}\,dv \qquad \operatorname{div}(\varepsilon V \operatorname{grad} V)=\varepsilon \operatorname{grad}V\cdot\operatorname{grad}V+\varepsilon V\Delta V=\vec{E}\cdot\vec{D}-\rho V$

$$\Delta V=-\frac{\rho}{\varepsilon} \qquad \int_v \operatorname{div}(\varepsilon V \operatorname{grad}V)\,dv=\oint_A \varepsilon V \operatorname{grad}V\cdot\vec{1}_n dA=\int_v \vec{E}\cdot\vec{D}\,dv-\int_v \rho V\,dv \longrightarrow W_e=\frac{1}{2}\int_v \rho V\,dv$$

$A\to\infty \quad V(\infty)=0$

OE II, $\omega=0$ $\quad W_m=\frac{1}{2}\int_v \vec{H}\cdot\vec{B}\,dv \qquad \operatorname{div}(\vec{H}\times\vec{A})=\vec{\nabla}\cdot(\vec{H}\times\vec{A})=\vec{A}\cdot\operatorname{rot}\vec{H}-\vec{H}\cdot\operatorname{rot}\vec{A}=\vec{J}\cdot\vec{A}-\vec{H}\cdot\vec{B}$

$$\int_v \operatorname{div}(\vec{H}\times\vec{A})\,dv=\oint_A (\vec{H}\times\vec{A})\cdot\vec{1}_n dA=0=\int_v \vec{J}\cdot\vec{A}\,dv-\int_v \vec{H}\cdot\vec{B}\,dv \longrightarrow W_m=\frac{1}{2}\int_v \vec{J}\cdot\vec{A}\,dv$$

$A\to\infty \quad \vec{A}(\infty)=0$

POYNTING $\quad W=\frac{1}{2}\int_v \vec{E}\cdot\vec{D}\,dv+\frac{1}{2}\int_v \vec{H}\cdot\vec{B}\,dv, \quad \frac{\partial\vec{D}}{\partial t}=\operatorname{rot}\vec{H}-\vec{J}, \quad \frac{\partial\vec{B}}{\partial t}=-\operatorname{rot}\vec{E}, \quad P=\int_v \vec{J}\cdot\vec{E}\,dv$

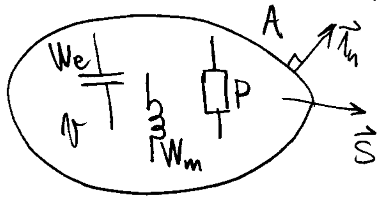
$$\frac{dW}{dt}=\frac{1}{2}\int_v \left[2\vec{E}\cdot(\operatorname{rot}\vec{H}-\vec{J})-2\vec{H}\cdot\operatorname{rot}\vec{E}\right]dv=-\int_v \vec{J}\cdot\vec{E}\,dv+\int_v\left[\vec{E}\cdot\operatorname{rot}\vec{H}-\vec{H}\cdot\operatorname{rot}\vec{E}\right]dv=$$

$$=-\int_v \vec{J}\cdot\vec{E}\,dv+\int_v \operatorname{div}(\vec{H}\times\vec{E})\,dv=-P-\oint_A(\vec{E}\times\vec{H})\cdot\vec{1}_n dA \longrightarrow \oint_A(\vec{E}\times\vec{H})\cdot\vec{1}_n dA=\oint_A \vec{S}\cdot\vec{1}_n dA=-P-\frac{dW}{dt}$$

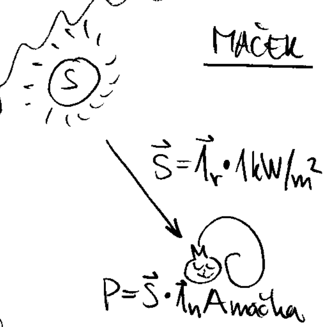
$\vec{S}=\vec{E}\times\vec{H}\ [W/m^2]$

$\vec{E}\,[V/m]$

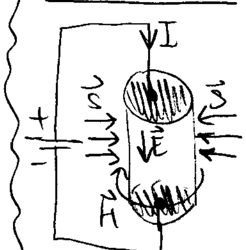
$\vec{H}\,[A/m]$



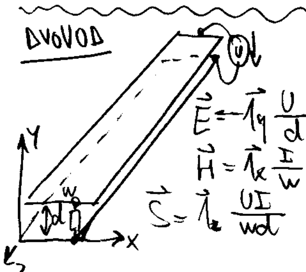
MAČEK



VALJASTI UPOR



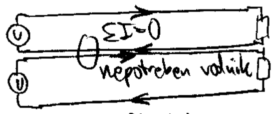
DVOVOD



$\vec{E}=-\vec{1}_y\frac{U}{d}$

$\vec{H}=\vec{1}_x\frac{I}{w}$

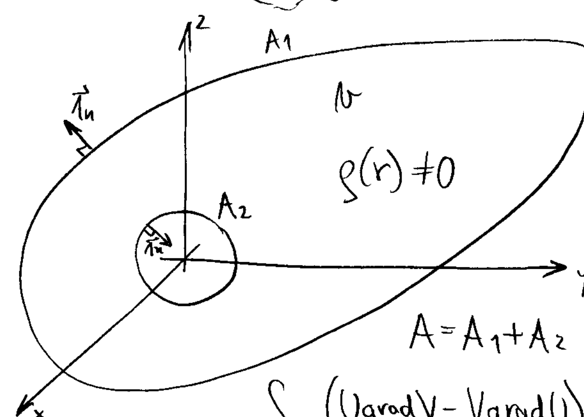
$\vec{S}=\vec{1}_z\frac{UI}{wd}$



DVA DVOVODA

$$\operatorname{div}(U\operatorname{grad}V-V\operatorname{grad}U)=U\Delta V-V\Delta U \quad \text{GREEN}$$

$$\oint_A(U\operatorname{grad}V-V\operatorname{grad}U)\cdot\vec{1}_n dA=\int_v(U\Delta V-V\Delta U)\,dv$$



$A=A_1+A_2$

$$\omega=0 \to U=\frac{1}{r}; \ \operatorname{grad}U=-\vec{1}_r\frac{1}{r^2}; \ \Delta U=0$$

$$\Delta V=-\frac{\rho}{\varepsilon}; \ \int_v(U\Delta V-V\Delta U)\,dv=-\int_v\frac{\rho}{\varepsilon r}\,dv$$

$$\omega\neq 0 \to U=\frac{e^{-jkr}}{r}; \ \operatorname{grad}U=-\vec{1}_r\left(\frac{1}{r}+jk\right)U; \ \Delta U=-k^2U$$

$$\Delta V=-\frac{\rho}{\varepsilon}-k^2V; \ \int_v(U\Delta V-V\Delta U)\,dv=-\int_v\frac{\rho}{\varepsilon}\frac{e^{-jkr}}{r}\,dv$$

$$\int_{A_1}(U\operatorname{grad}V-V\operatorname{grad}U)\cdot\vec{1}_n dA=0$$

$$\int_{A_2}(U\operatorname{grad}V-V\operatorname{grad}U)\cdot\vec{1}_n dA=-4\pi V(r=0)$$

$$\omega=0 \to V(r=0)=\frac{1}{4\pi\varepsilon}\int_v\frac{\rho}{r}\,dv; \qquad \omega\neq 0\to V(r=0)=\frac{1}{4\pi\varepsilon}\int_v\rho\frac{e^{-jkr}}{r}\,dv$$

POTENCIAL V POLJUBNI TOČKI

$$V(\vec{r})=\frac{1}{4\pi\varepsilon}\int_{v'}\rho(\vec{r}')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dv'$$

$$V(\vec{r}')=\frac{1}{4\pi\varepsilon}\sum_i Q_i\frac{e^{-jk|\vec{r}-\vec{r}_i|}}{|\vec{r}-\vec{r}_i|}$$

VEKTORSKI POTENCIAL $\quad \Delta\vec{A}+k^2\vec{A}=-\mu\vec{J}$

$$\Delta\vec{A}=\vec{1}_x A_x+\vec{1}_y A_y+\vec{1}_z A_z \quad (x,y,z)$$

$$\Delta A_x+k^2A_x=-\mu J_x \ \ldots\ldots$$

$$\vec{A}(\vec{r})=\frac{\mu}{4\pi}\int_{v'}\vec{J}(\vec{r}')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dv'$$

$$A_x(\vec{r})=\frac{\mu}{4\pi}\int_{v'}J_x(\vec{r}')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dv'$$

3D

Poljuben KS

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PONOVITEV ME, $\frac{\partial}{\partial t} = j\omega$

① $\mathrm{rot}\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$ $\xrightarrow{\mathrm{div}\ +③}$ $\mathrm{div}(\mathrm{rot}\vec{H}) = 0 = \mathrm{div}\vec{J} + j\omega\,\mathrm{div}(\varepsilon\vec{E})$

② $\mathrm{rot}\vec{E} = -j\omega\mu\vec{H}$

$$\boxed{0 = \mathrm{div}\vec{J} + j\omega\rho}$$

③ $\mathrm{div}(\varepsilon\vec{E}) = \rho$

zveznost (kontinuiteta) toka

POTENCIALI

$\vec{B} = \mathrm{rot}\vec{A} = \mathrm{rot}\vec{V}_m$

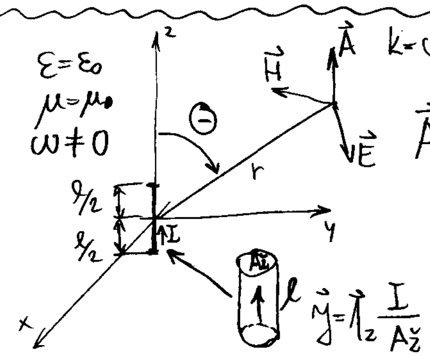
$V = -j\omega\vec{A} - \mathrm{grad}V$

Lorentz-ova izbira $\longrightarrow$

$$\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J} \qquad \vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{v'}\vec{J}(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\,dv'$$

$$\Delta V + \omega^2\mu\varepsilon V = -\frac{\rho}{\varepsilon} \qquad V(\vec{r}) = \frac{1}{4\pi\varepsilon}\int_{v'}\rho(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dv' = \frac{1}{4\pi\varepsilon}\sum_i Q_i\frac{e^{-jk|\vec{r}-\vec{r}_i|}}{|\vec{r}-\vec{r}_i|}$$

$\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $\omega \neq 0$, $k = \omega\sqrt{\mu\varepsilon}$



$$\vec{A} = \frac{\mu}{4\pi}\int_{v'}\vec{1}_z\frac{I}{A_z}\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}A_z\,dz \approx \vec{1}_z\frac{\mu I l}{4\pi}\frac{e^{-jkr}}{r}$$

$\vec{J} = \vec{1}_z\frac{I}{A_z}$

KRATKA ŽICA

① $l \ll r \nearrow \frac{1}{|\vec{r}-\vec{r}\,'|} \approx \frac{1}{r}$

② $l \ll \frac{2\pi}{k} \rightarrow e^{-jk|\vec{r}-\vec{r}\,'|} \approx e^{-jkr}$

$(x,y,z) \rightarrow (r,\theta,\phi)$: $\vec{1}_z = \vec{1}_r\cos\Theta - \vec{1}_\Theta\sin\Theta$

$$\underline{\underline{\vec{A} = (\vec{1}_r\cos\Theta - \vec{1}_\Theta\sin\Theta)\frac{\mu I l}{4\pi}\frac{e^{-jkr}}{r}}}$$

$\vec{H} = \frac{1}{\mu}\mathrm{rot}\vec{A}$

$$\vec{H} = \frac{1}{\mu}\frac{1}{r^2\sin\Theta}\begin{vmatrix}\vec{1}_r & \vec{1}_\Theta r & \vec{1}_\phi r\sin\Theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\Theta} & \frac{\partial}{\partial\phi} \\ \frac{\mu I l}{4\pi}\frac{e^{-jkr}}{r}\cos\Theta & -r\frac{\mu I l}{4\pi}\frac{e^{-jkr}}{r}\sin\Theta & 0\end{vmatrix} = \vec{1}_\phi\frac{Il}{4\pi}e^{-jkr}\Big(\underbrace{\frac{jk}{r}}_{\text{SEVANJE}} + \underbrace{\frac{1}{r^2}}_{\text{BIOT-SAVART}}\Big)\sin\Theta$$

---

zveznost toka: $+Q$, $I = \frac{dQ}{dt} = j\omega Q$, $-Q$

$\vec{E} = -j\omega\vec{A} - \mathrm{grad}V$

② ME @ $\vec{J} = 0$

$$\vec{E} = \frac{1}{j\omega\varepsilon}\mathrm{rot}\vec{H} = \frac{1}{j\omega\varepsilon}\frac{1}{r^2\sin\Theta}\begin{vmatrix}\vec{1}_r & \vec{1}_\Theta r & \vec{1}_\phi r\sin\Theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\Theta} & \frac{\partial}{\partial\phi} \\ 0 & r0 & r\sin\Theta\frac{Il}{4\pi}e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\Theta\end{vmatrix}$$

$$\vec{E} = \frac{Ql}{4\pi\varepsilon}e^{-jkr}\Big[\vec{1}_r\Big(\frac{jk}{r^2} + \underbrace{\frac{1}{r^3}}_{\text{TOČKASTI STAT. DIPOL}}\Big)2\cos\Theta + \vec{1}_\Theta\Big(\underbrace{-\frac{k^2}{r}}_{\text{SEVANJE}} + \frac{jk}{r^2} + \underbrace{\frac{1}{r^3}}_{\text{TOČKASTI STAT. DIPOL}}\Big)\sin\Theta\Big]$$

$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*$

PRIMERJAVA

	$f = 50\,\mathrm{Hz}$	$f = 900\,\mathrm{MHz}$	$\lambda = 0{,}5\,\mu m$
k	10^{-6} rd/m	19 rd/m	$1{,}3\cdot10^{7}$ rd/m
k^2	10^{-12} rd²/m²	380 rd²/m²	$1{,}7\cdot10^{14}$ rd/m
$r = \frac{1}{k}$	1000 km	5,3 cm	80 nm

$k = \omega\sqrt{\mu\varepsilon}$, $Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$, $\frac{1}{\omega\varepsilon} = \frac{Z_0}{k}$

$$\vec{S} = \frac{1}{2}\frac{Il}{j\omega 4\pi\varepsilon}e^{-jkr}\left[\vec{1}_r\left(\frac{jk}{r^2}+\frac{1}{r^3}\right)2\cos\Theta + \vec{1}_\Theta\left(-\frac{k^2}{r}+\frac{jk}{r^2}+\frac{1}{r^3}\right)\sin\Theta\right]\times$$

$$\times\,\vec{1}_\phi\frac{I^*l}{4\pi}e^{+jkr}\left[-\frac{jk}{r}+\frac{1}{r^2}\right]\sin\Theta$$

$$\vec{S} = \frac{|I|^2 l^2 Z_0}{32\pi^2}\Big[\vec{1}_r\Big(\underbrace{\frac{k^2}{r^2}}_{\text{DELOVNA SEVANA MOČ}} - \frac{j}{kr^5}\Big)\sin^2\Theta + \vec{1}_\Theta\Big(\frac{jk}{r^3} + \frac{j}{kr^5}\Big)2\cos\Theta\sin\Theta\Big]$$

JALOVA MOČ

$$P = \oint_{r\to\infty}\vec{S}\cdot\vec{1}_r r^2\sin\Theta\,d\Theta\,d\phi$$

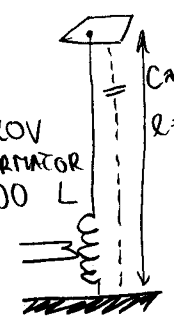
$$P = \frac{|I|^2 l^2 Z_0}{32\pi^2}\int_0^{\pi}\int_0^{2\pi}\left(\frac{k^2}{r^2} - \frac{j}{kr^5}\right)\sin^2\Theta\sin\Theta\,d\Theta = \frac{|I|^2 l^2 Z_0 k^2}{16\pi}\int_{-1}^{+1}(1-u^2)\,du = \frac{|I|^2 l^2 Z_0 k^2}{12\pi} = \frac{1}{2}|I|^2 R_s$$

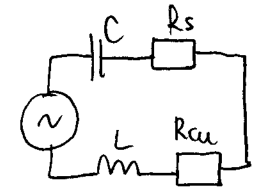
$$\underline{R_s = \frac{l^2 Z_0 k^2}{6\pi} = \frac{2}{3}\pi Z_0\left(\frac{l}{\lambda}\right)^2}$$

$f = 30\,\mathrm{kHz} \rightarrow \lambda = 10\,\mathrm{km}$

$\underline{R_s \approx 7{,}2\,\mathrm{m}\Omega}$

TESLOV TRANSFORMATOR ~ 1900



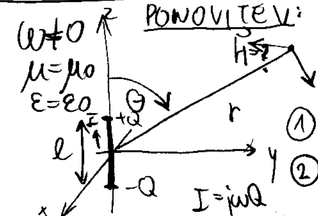


$R_{cu} = \frac{\omega L}{Q} = \frac{1}{\omega CQ} \approx \underline{60\,\Omega}$

$Q \approx 300$

$\eta = \frac{R_s}{R_s + R_{cu}} \approx \underline{0{,}012\%}$

Elektrodinamika #8 26/11/2012



PONOVITEV:

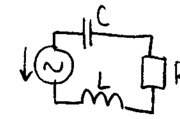
$$\vec{A}=(\vec{1}_r\cos\theta-\vec{1}_\theta\sin\theta)\frac{\mu I\ell}{4\pi}\frac{e^{-jkr}}{r}$$

① $\ell \ll r$

② $\ell \ll \frac{1}{k}$

$$\vec{H}=\vec{1}_\phi\frac{I\ell}{4\pi}e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta$$

$$\vec{E}=\frac{Q\ell}{4\pi\varepsilon}e^{-jkr}\left[\vec{1}_r\left(\frac{jk}{r^2}+\frac{1}{r^3}\right)2\cos\theta+\vec{1}_\theta\left(-\frac{k^2}{r}+\frac{jk}{r^2}+\frac{1}{r^3}\right)\sin\theta\right]$$



$$R_S=\frac{2\pi Z_0}{3}\left(\frac{\ell}{\lambda}\right)^2$$

POENOSTAVITVE ZA SEVANJE

$r \gg \frac{1}{k}$, $r \gg \frac{\lambda}{2\pi}$ $\longrightarrow$ $\frac{\partial}{\partial r}\approx -jk$, $\frac{\partial}{\partial\theta}\to 0$, $\frac{\partial}{\partial\phi}\to 0$

$$\mathrm{rot}\vec{A}=\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ -jk & 0 & 0\\ A_r & rA_\theta & r\sin\theta A_\phi\end{vmatrix}=\vec{\nabla}\times\vec{A}=\vec{1}_\theta jkA_\phi-\vec{1}_\phi jkA_\theta$$

$\vec{\nabla}=\vec{1}_r(-jk)$ $\qquad \omega\mu=kZ_0$

$$\vec{H}\approx\vec{1}_\phi\frac{jk}{4\pi}I\ell\frac{e^{-jkr}}{r}\sin\theta$$

$$\vec{E}=\frac{1}{j\omega\varepsilon}\vec{\nabla}\times\left(\frac{1}{\mu}\vec{\nabla}\times\vec{A}\right)\approx -j\omega\left(\vec{A}-\vec{1}_r(\vec{1}_r\cdot\vec{A})\right)=\vec{1}_\theta\frac{jkZ_0}{4\pi}I\ell\frac{e^{-jkr}}{r}\sin\theta$$

LASTNOSTI SEVANJA

TEM $\vec{H}\perp\vec{1}_r$; $\vec{E}\perp\vec{1}_r$; $\vec{E}\perp\vec{H}$; $|\vec{E}|=Z_0|\vec{H}|$; $\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\vec{1}_r\frac{|E|^2}{2Z_0}$ REALEN = DELOVNA MOČ

TRAKASTI DVOVOD

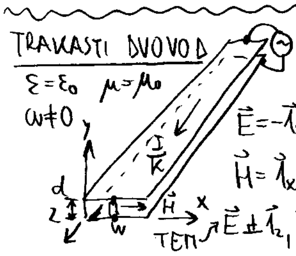
$\varepsilon=\varepsilon_0$ $\mu=\mu_0$

$\omega\neq 0$

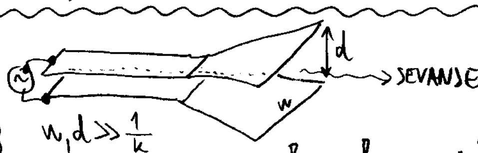
$\frac{U}{I}=Z_K=\frac{d}{w}Z_0$

$\vec{E}=-\vec{1}_y\frac{U}{d}=-\vec{1}_y\frac{U_0}{d}e^{-j\beta z}$

$\vec{H}=\vec{1}_x\frac{I}{w}=\vec{1}_x\frac{I_0}{w}e^{-j\beta z}$



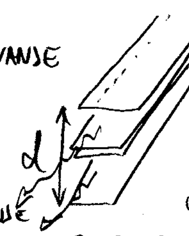
TEM: $\vec{E}\perp\vec{1}_z$, $\vec{H}\perp\vec{1}_z$, $\vec{E}\perp\vec{H}$, $|\vec{E}|=Z_0|\vec{H}|$



$w, d \gg \frac{1}{k}$

NI ODBOJA NA OS! $\Gamma\to 0$!

(OS = ODPRTE SPONKE)

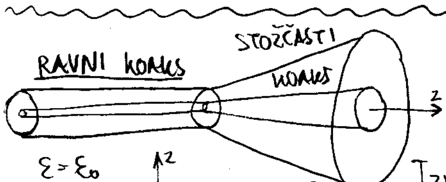


$\Gamma\to 0$

NI ODBOJA NA OS

ZAPOREDNE VEZAVE

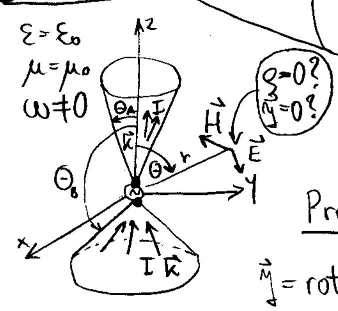
$d \gg \frac{1}{k}=\frac{\lambda}{2\pi}$



$\varepsilon=\varepsilon_0$

$\mu=\mu_0$

$\omega\neq 0$



Ugibam rešitev: $\vec{E}=\vec{1}_\theta\frac{C}{r\sin\theta}e^{-jkr}$

Preverim ③ ME: $\varrho=\mathrm{div}(\varepsilon\vec{E})=\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(r\sin\theta\,\varepsilon\frac{C}{r\sin\theta}e^{-jkr}\right)=\underline{\underline{0}}$ ✓

Izračunam $\vec{H}$ iz ② ME:

$\frac{k}{\omega\mu}=\frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu}=\frac{1}{Z_0}$

$$\vec{H}=\frac{j}{\omega\mu}\mathrm{rot}\vec{E}=\frac{j}{\omega\mu}\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & r\frac{C}{r\sin\theta}e^{-jkr} & r\sin\theta\cdot 0\end{vmatrix}=\underline{\underline{\vec{1}_\phi\frac{C/Z_0}{r\sin\theta}e^{-jkr}}}$$

TEM: $\vec{E}\perp\vec{1}_r$; $\vec{H}\perp\vec{1}_r$

$\vec{E}\perp\vec{H}$, $|\vec{E}|=Z_0|\vec{H}|$

Preverim ① ME:

$\frac{k}{Z_0}=\frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{\frac{\mu}{\varepsilon}}}=\omega\varepsilon$

$$\vec{J}=\mathrm{rot}\vec{H}-j\omega\varepsilon\vec{E}=\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & r0 & r\sin\theta\frac{C/Z_0}{r\sin\theta}e^{-jkr}\end{vmatrix}-j\omega\varepsilon\vec{1}_\theta\frac{C}{r\sin\theta}e^{-jkr}=\vec{1}_\theta\frac{j\omega\varepsilon C}{r\sin\theta}e^{-jkr}-\vec{1}_\theta\frac{j\omega\varepsilon C}{r\sin\theta}e^{-jkr}=\underline{\underline{0}}$$ ✓

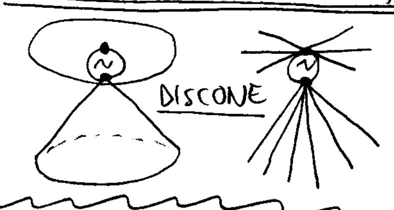
Izračunam tok I: $d\vec{s}=\vec{1}_\phi r\sin\theta d\phi$

$$I=\oint_s\vec{H}\cdot d\vec{s}=\int_0^{2\pi}\vec{1}_\phi\frac{C/Z_0}{r\sin\theta}e^{-jkr}\cdot\vec{1}_\phi r\sin\theta d\phi=\underline{\underline{\frac{2\pi C}{Z_0}e^{-jkr}}}$$

$$Z_K=\frac{U}{I}=\frac{Z_0}{2\pi}\ln\left(\frac{\mathrm{tg}(\theta_B/2)}{\mathrm{tg}(\theta_A/2)}\right)\approx 60\,\Omega\ln\left(\frac{\mathrm{tg}(\theta_B/2)}{\mathrm{tg}(\theta_A/2)}\right)$$

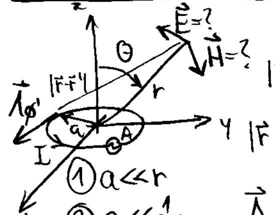
Izračunam napetost U: $d\vec{s}=\vec{1}_\theta r d\theta$

$$U=\int_A^B\vec{E}\cdot d\vec{s}=\int_{\theta_A}^{\theta_B}\vec{1}_\theta\frac{C}{r\sin\theta}e^{-jkr}\cdot\vec{1}_\theta r d\theta=Ce^{-jkr}\int_{\theta_A}^{\theta_B}\frac{d\theta}{\sin\theta}=\underline{\underline{Ce^{-jkr}\ln\left(\frac{\mathrm{tg}(\theta_B/2)}{\mathrm{tg}(\theta_A/2)}\right)}}$$



MAJHNA ZANKA (xy)

$$\vec{A}(\vec{r})=\frac{\mu}{4\pi}\int_{V'}\vec{J}(\vec{r}')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}dV'=\frac{\mu}{4\pi}\int_0^{2\pi}\vec{1}_{\phi'}\frac{I}{A_Z}\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}A_Z a\,d\phi'=\frac{\mu Ia}{4\pi}\int_0^{2\pi}\vec{1}_{\phi'}\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}d\phi'$$



$$|\vec{r}-\vec{r}'|=\sqrt{(r\sin\theta\cos\phi-a\cos\phi')^2+(r\sin\theta\sin\phi-a\sin\phi')^2+(r\cos\theta)^2}=\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}$$

① $a\ll r$

② $a \ll \frac{1}{k}$

$$|\vec{r}-\vec{r}'|\approx r-a\sin\theta\cos(\phi-\phi');\quad\frac{1}{|\vec{r}-\vec{r}'|}\approx\frac{1}{r}\left(1+\frac{a}{r}\sin\theta\cos(\phi-\phi')\right);\quad e^{-jk|\vec{r}-\vec{r}'|}\approx e^{-jkr}\left(1+jka\sin\theta\cos(\phi-\phi')\right)$$

$$\vec{A}(\vec{r})\approx\frac{\mu Ia}{4\pi}\frac{e^{-jkr}}{r}\underbrace{\int_0^{2\pi}(-\vec{1}_x\sin\phi'+\vec{1}_y\cos\phi')\left(1+\frac{a}{r}\sin\theta\cos(\phi-\phi')\right)\left(1+jka\sin\theta\cos(\phi-\phi')\right)d\phi'}_{\vec{1}_\phi\pi a\sin\theta}$$

$\vec{1}_{\phi'}=-\vec{1}_x\sin\phi'+\vec{1}_y\cos\phi'$

$Q=0\to\mathrm{grad}V=0$

$\omega\mu=\omega\sqrt{\mu\varepsilon}\sqrt{\frac{\mu}{\varepsilon}}=kZ_0$

$$\vec{A}(\vec{r})\approx\vec{1}_\phi\frac{\mu}{4\pi}I\pi a^2\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\sin\theta$$

$$\vec{E}=-j\omega\vec{A}-\mathrm{grad}V=\underline{\underline{\vec{1}_\phi\frac{-jkZ_0}{4\pi}IAe^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta}}$$

$$\underline{\underline{\vec{H}=\frac{j}{\omega\mu}\mathrm{rot}\vec{E}}}$$

Elektrodinamika #9 3.12.2012

VALOVNA ENAČBA BREZ IZVOROV

$$\omega^2\mu\varepsilon = k^2$$

$$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0 \qquad (x,y,z) \longrightarrow \Delta\vec{E} = \vec{1}_x\Delta E_x + \vec{1}_y\Delta E_y + \vec{1}_z\Delta E_z$$

$$\Delta\vec{H} + \omega^2\mu\varepsilon\vec{H} = 0 \qquad \Delta E_x = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}$$

$$\Delta\vec{E} = \mathrm{grad}(\mathrm{div}\vec{E}) - \mathrm{rot}(\mathrm{rot}\vec{E})$$

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0$$

$$E_x(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{-k_x^2} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{-k_y^2} + \underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{-k_z^2} + k^2 = 0$$

$$k_z^2 > 0 \longrightarrow Z(z) = \overbrace{C_1\cos k_z z + C_2\sin k_z z}^{\text{STOJNI VAL}} = \overbrace{C_3 e^{jk_z z}}^{\text{ODBITI}} + \overbrace{C_4 e^{-jk_z z}}^{\text{NAPREDUJOČI}}$$

$$k_z^2 < 0 \longrightarrow Z(z) = C_5\,\mathrm{ch}|k_z|z + C_6\,\mathrm{sh}|k_z|z = C_7 e^{|k_z|z} + \underbrace{C_8 e^{-|k_z|z}}_{\text{exp. usihanje}}$$

Zgledi: preveri $\mathrm{div}(\varepsilon\vec{E}) = 0$
izračunaj $\vec{H} = \frac{j}{\omega\mu}\mathrm{rot}\vec{E}$
preveri $\vec{J} = \mathrm{rot}\vec{H} - j\omega\varepsilon\vec{E} = 0$
izračunaj $\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*$

VALOVNI VEKTOR

za potujoči val $\vec{k} = \vec{1}_x k_x + \vec{1}_y k_y + \vec{1}_z k_z$

$$\left.\begin{aligned} E_x(x,y,z) &= E_{x0}\,e^{-jk_x x}e^{-jk_y y}e^{-jk_z z}\\ E_y(x,y,z) &= E_{y0}\,e^{-jk_x x}e^{-jk_y y}e^{-jk_z z}\\ E_z(x,y,z) &= E_{z0}\,e^{-jk_x x}e^{-jk_y y}e^{-jk_z z}\end{aligned}\right\}\ \vec{E}(x,y,z) = \vec{E}_0\,e^{-j\vec{k}\cdot\vec{r}}$$

$$|\vec{k}| = k = \omega^2\mu\varepsilon$$

$$\vec{k} = \vec{1}_k\,\omega^2\mu\varepsilon = \vec{1}_k k$$

$$\mathrm{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = -jk_xE_x - jk_yE_y - jk_zE_z = -j\vec{k}\cdot\vec{E} \longrightarrow \text{FIZIKALNA REŠITEV } \rho = 0 \rightarrow \vec{k} \perp \vec{E}$$

$$\mathrm{rot}\vec{E} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\vec{k}\times\vec{E} \longrightarrow \vec{H} = \frac{\vec{k}\times\vec{E}}{\omega\mu} = \frac{\vec{1}_k\times\vec{E}}{Z_0} \longrightarrow \vec{E}\perp\vec{H};\ \vec{H}\perp\vec{k};\ |\vec{H}| = \frac{|\vec{E}|}{Z_0}$$

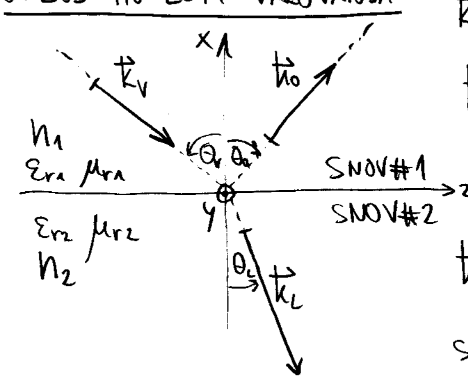
$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{E}_{eff}\times\vec{H}_{eff}^* = \vec{1}_s\frac{|\vec{E}|^2}{2Z_0} = \vec{1}_s\frac{|\vec{H}|^2 Z_0}{2}\ ;\ \vec{1}_s = \vec{1}_k$$

SNOV $\varepsilon_r \neq 1$, $\mu_r \neq 1$

Lomni količnik $n = \sqrt{\mu_r\varepsilon_r}$

$$\vec{k} = \vec{1}_k k = \vec{1}_k\,\omega\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0} = \vec{1}_k\frac{\omega}{c_0}\sqrt{\mu_r\varepsilon_r} = \vec{1}_k\frac{\omega}{c_0}n \qquad c = \frac{c_0}{n}$$

ODBOJ IN LOM VALOVANJA



$$\left.\begin{aligned}\vec{k}_v &= \vec{1}_x k_{vx} + \vec{1}_z\beta\\ \vec{k}_0 &= \vec{1}_x k_{0x} + \vec{1}_z\beta\\ \vec{k}_L &= \vec{1}_x k_{Lx} + \vec{1}_z\beta\end{aligned}\right\}\ \text{isti}\ \beta = k_z$$

isti pojav hkrati na meji snovi

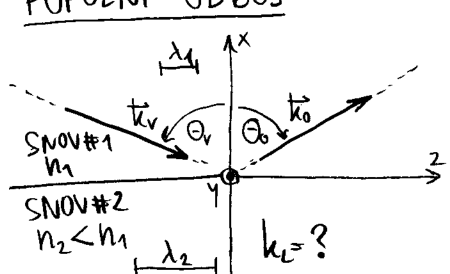
$$\underbrace{k_{vx}^2 + \beta^2 = k_{0x}^2 + \beta^2 = k_1^2 = n_1\omega\sqrt{\mu_0\varepsilon_0}}$$

$$\underbrace{\sin\theta_v = \frac{\beta}{k_1} \qquad \sin\theta_0 = \frac{\beta}{k_1}}_{\underline{\underline{\theta_v = \theta_0}}}$$

$$k_{Lx}^2 + \beta^2 = k_2^2 = n_2\omega\sqrt{\mu_0\varepsilon_0}$$

$$\sin\theta_L = \frac{\beta}{k_2} \longrightarrow \underline{\underline{n_2\sin\theta_L = n_1\sin\theta_v}}\ \text{Snell-ov lomni zakon}$$

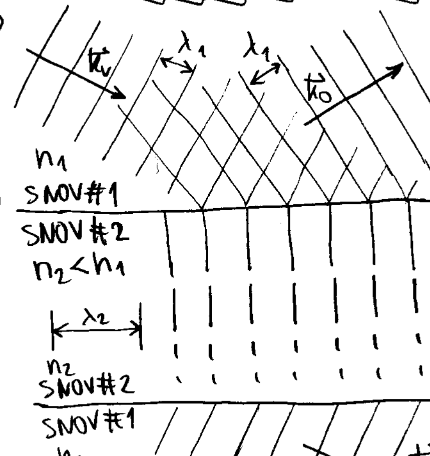
POPOLNI ODBOJ



$$\theta_L = \arcsin\left(\frac{n_1}{n_2}\sin\theta_v\right)$$

$$\frac{n_1}{n_2}\sin\theta_v > 1\ ?$$

TUNELIRANJE



$$k_{Lx}^2 < 0 \rightarrow k_{Lx} = j|k_{Lx}|$$

$$E_L = E_{L0}\,\underbrace{e^{|k_{Lx}|x}}_{}\,e^{-j\beta z}$$

EKSPONENTNO USIHANJE

TUNELIRANJE VALOVANJA $\vec{k}_T = \vec{k}_v$

Elektrodinamika #10   10.12.2012

Ponovitev $\Delta\vec{E}+k^2\vec{E}=0 \longrightarrow \underbrace{\vec{E}=\vec{E}_0 e^{-j\vec{k}\cdot\vec{r}}}_{\text{POTUJOČI VAL}}$, $\vec{k}\perp\vec{E}$, $\vec{E}\perp\vec{H}$, $\vec{H}\perp\vec{k}$, $\frac{|\vec{E}|}{|\vec{H}|}=Z_0=\sqrt{\frac{\mu}{\varepsilon}}$

1D STOJNI VAL

$\underbrace{\vec{E}=\vec{1}_x C\cos kz}_{\text{STOJNI VAL}}=\vec{1}_x\frac{C}{2}\left(\overbrace{e^{jkz}}^{\text{ODBITI VAL}}+\overbrace{e^{-jkz}}^{\text{NAPREDUJOČI VAL}}\right)$

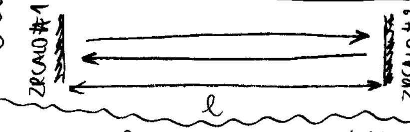
$\vec{H}=\frac{j}{\omega\mu}\mathrm{rot}\vec{E}=\frac{j}{\omega\mu}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ C\cos kz & 0 & 0\end{vmatrix}=\vec{1}_y\frac{-jkC}{\omega\mu}\sin kz=\underline{\vec{1}_y\frac{-j}{Z_0}C\sin kz}$

$\frac{\vec{E}}{\vec{H}}$ nestalno razmerje, kvadratura

$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\vec{1}_z\frac{j|C|^2}{2Z_0}\cos kz\sin kz$

JALOVA MOČ

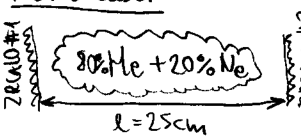
FABRY-PEROT-OV REZONATOR



$kl=m\pi$ $f=\frac{c_0}{2l}m$, $m=1,2,3,4\ldots$

$k=\frac{2\pi f}{c_0}$ TEM_{00m} PRIMERJAVA S STRUNO!

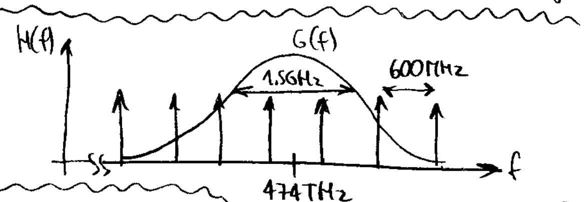
HeNe laser



$f \sim 474\,THz$ $c_0=3\cdot10^8\,m/s$

Doppler $\pm1.5\cdot10^{-6}$ (toplotno gibanje $\sim450\,m/s$)

$\Delta f=\frac{c_0}{2l}=\underline{600\,MHz}$

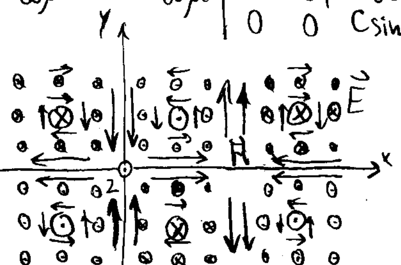


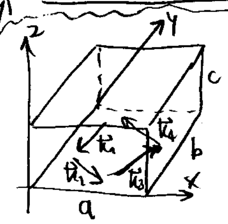
2D STOJNI VAL

$\vec{E}=\vec{1}_z C\sin k_x x\sin k_y y=\vec{1}_z\frac{C}{-4}\left(e^{jk_x x}-e^{-jk_x x}\right)\left(e^{jk_y y}-e^{-jk_y y}\right)=\vec{1}_z\frac{C}{-4}\left[\underbrace{e^{j(k_x x+k_y y)}}_{\vec{k}_1}-\underbrace{e^{j(-k_x x+k_y y)}}_{\vec{k}_2}-\underbrace{e^{j(k_x x-k_y y)}}_{\vec{k}_4}+\underbrace{e^{j(-k_x x-k_y y)}}_{\vec{k}_3}\right]$

$\vec{H}=\frac{j}{\omega\mu}\mathrm{rot}\vec{E}=\frac{j}{\omega\mu}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & 0 & C\sin k_x x\sin k_y y\end{vmatrix}=\underline{\vec{1}_x\frac{jk_y}{\omega\mu}C\sin k_x x\cos k_y y-\vec{1}_y\frac{jk_x}{\omega\mu}C\cos k_x x\sin k_y y}$

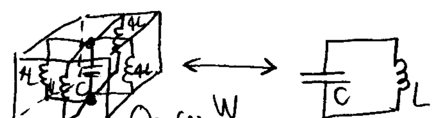
kvadratura $\vec{E},\vec{H}$ $\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=$ jalov!





VOTLINSKI REZONATOR

$TM_{110}: f=\frac{c_0}{2}\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2}$



$Q=\frac{\omega L}{R_L}$ $Q=\omega\frac{W}{P}$

$TM_{mn0}\rightarrow f_{mn}=\frac{c_0}{2}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$ $Q\uparrow\Rightarrow m\uparrow, n\uparrow=$ LASER visokim

3D STOJNI VAL

8 valovnih vektorjev

$f_{lmn}=\frac{c_0}{2}\sqrt{\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2+\left(\frac{n\pi}{c}\right)^2}$

PREPREČEVANJE VIŠJIH RODOV

$c<\frac{a}{2},\frac{b}{2}$

DIELEKTRIK

$\sqrt{\varepsilon_r}=n$ $f_{lmn}=\frac{c_0}{2n}\sqrt{\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2+\left(\frac{n\pi}{c}\right)^2}=\frac{c_0}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2+\left(\frac{n\pi}{c}\right)^2}$

1D STOJNI (y) + 1D POTUJOČI (z)

$\vec{E}=\vec{1}_x C\sin k_y y\, e^{-j\beta z}$

$k_y^2+\beta^2=k^2=\omega^2\mu\varepsilon$

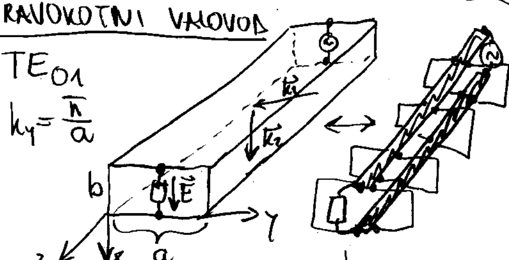
$\vec{H}=\frac{j}{\omega\mu}\mathrm{rot}\vec{E}=\frac{j}{\omega\mu}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ C\sin k_y y\, e^{-j\beta z} & 0 & 0\end{vmatrix}=\underbrace{\vec{1}_y\frac{\beta}{\omega\mu}C\sin k_y y\, e^{-j\beta z}}_{\text{V FAZI}\rightarrow \mathrm{Re}[\vec{S}]}-\underbrace{\vec{1}_z\frac{jk_y}{\omega\mu}C\cos k_y y\, e^{-j\beta z}}_{\text{V KVADRATURI}\rightarrow \mathrm{Im}[\vec{S}]}$

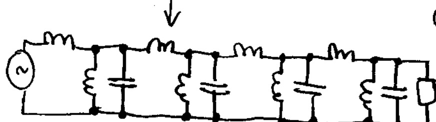
$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\frac{1}{2}\vec{1}_x C\sin k_y y\, e^{-j\beta z}\times\left[\vec{1}_y\frac{\beta}{\omega\mu}C^*\sin k_y y\, e^{+j\beta z}+\vec{1}_z\frac{jk_y}{\omega\mu}C^*\cos k_y y\, e^{+j\beta z}\right]=\vec{1}_z\frac{\beta|C|^2}{2\omega\mu}\sin^2 k_y y-\vec{1}_y\frac{jk_y|C|^2}{\omega\mu}\sin k_y y\cos k_y y$

PRAVOKOTNI VALOVOD

TE_{01}

$k_y=\frac{\pi}{a}$





VISOKOPREPUSTNO SITO

$TE_{01}\rightarrow\beta=\sqrt{\omega^2\mu\varepsilon-\left(\frac{\pi}{a}\right)^2}$

$\left(TE_{0m}\rightarrow\beta=\sqrt{\omega^2\mu\varepsilon-\left(\frac{m\pi}{a}\right)^2}\right)$

$\vec{E}=\vec{1}_x C\sin k_y y\, e^{-j\beta z}$

$\omega^2\mu\varepsilon>\left(\frac{\pi}{a}\right)^2\rightarrow\beta=$ realen (potujoči val)

$\omega^2\mu\varepsilon<\left(\frac{\pi}{a}\right)^2\rightarrow\beta=$ imaginaren (exponentno usihanje)

$\omega_k^2\mu\varepsilon=\left(\frac{\pi}{a}\right)^2$

$\vec{E}=\vec{1}_x C\sin k_y y\, e^{-|\beta|z}$

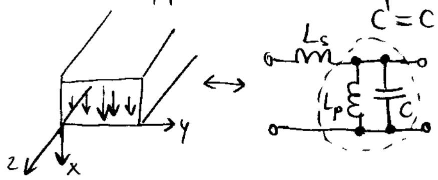
$\omega_k=\frac{\pi c_0}{a}\rightarrow f_k=\frac{c_0}{2a}$

Elektrodinamika #11 17.12.2012

PRAVOKOTNI VALOVOD

$\vec{E} = \vec{1}_x C \sin k_y y \, e^{-j\beta z}$

$k_y^2 + \beta_z^2 = k^2 = \omega^2 \mu\varepsilon \rightarrow \beta_z = \sqrt{\omega^2\mu\varepsilon - k_y^2} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$



$C' = C - \frac{1}{\omega^2 L_p}$

$Z_u = \sqrt{\frac{L_s}{C'}} = \sqrt{\frac{L_s}{C - \frac{1}{\omega^2 L_p}}}$

$\omega_k = \frac{1}{\sqrt{\mu\varepsilon}}\left(\frac{m\pi}{a}\right) = m\frac{\pi c_0}{a}$

$\beta_z \equiv$ realen

$\omega > \frac{1}{\sqrt{CL_p}} \rightarrow Z_u \equiv$ realen = prepust $\longleftarrow \omega > \omega_k$

$\omega < \frac{1}{\sqrt{CL_p}} \rightarrow Z_u \equiv$ imaginaren = zapora $\longleftarrow \omega < \omega_k$

$\beta \equiv$ imaginaren

FAZNA HITROST

$\vec{E} = \mathrm{Re}\left[\vec{1}_x C \sin k_y y \, e^{j(\omega t - \beta z)}\right]$

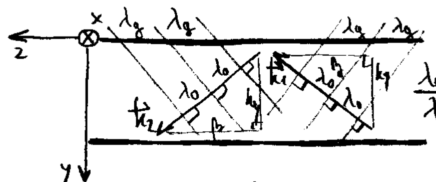
$\varphi = \omega t - \beta z = \text{konst} \;/\; \frac{d}{dt}$

$\omega - \beta\frac{dz}{dt} = 0 \rightarrow \underline{v_f = \frac{dz}{dt} = \frac{\omega}{\beta}}$

$v_f = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - k_y^2}} = \frac{c_0}{\sqrt{1 - \frac{k_y^2}{\omega^2\mu\varepsilon}}} = \underline{\frac{c_0}{\sqrt{1 - \left(\frac{\omega_k}{\omega}\right)^2}}} \geq c_0 \,!$

VALOVODNA VALOVNA DOLŽINA

$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - k_y^2}} = \underline{\frac{\lambda_0}{\sqrt{1 - \left(\frac{\omega_k}{\omega}\right)^2}}} \geq \lambda_0 \,!$



$\frac{\lambda_g}{\lambda_0} = \frac{k}{\beta}$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = \mathrm{Re}\left[\vec{1}_x \sin k_y y \left(C_1 e^{j\varphi_1} + C_2 e^{j\varphi_2}\right)\right] \qquad \Delta\varphi = \varphi_1 - \varphi_2$

SKUPINSKA HITROST

Vsota $\vec{E}$ dveh frekvenc ω_1, ω_2

$\varphi_1 = \omega_1 t - \beta_1 z$

$\varphi_2 = \omega_2 t - \beta_2 z$

$\vec{E} = \vec{1}_x \sin k_y y \, \mathrm{Re}\left[e^{j\frac{\varphi_1+\varphi_2}{2}}\left(C_1 e^{j\frac{\varphi_1-\varphi_2}{2}} + C_2 e^{j\frac{\varphi_2-\varphi_1}{2}}\right)\right]$

$\vec{E}_1 = \mathrm{Re}\left[\vec{1}_x C_1 \sin k_y y \, e^{j(\omega_1 t - \beta_1 z)}\right] = \mathrm{Re}\left[\vec{1}_x C_1 \sin k_y y \, e^{j\varphi_1}\right]$

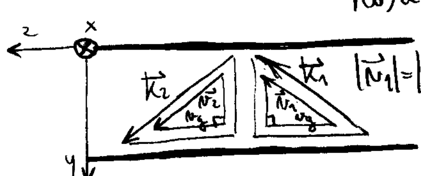
$\vec{E}_2 = \mathrm{Re}\left[\vec{1}_x C_2 \sin k_y y \, e^{j(\omega_2 t - \beta_2 z)}\right] = \mathrm{Re}\left[\vec{1}_x C_2 \sin k_y y \, e^{j\varphi_2}\right]$

amplituda $|\vec{E}| = \sin k_y y \left|C_1 e^{j\Delta\varphi} + C_2 e^{-j\Delta\varphi}\right|$

ovojnico sledim $\Delta\varphi = \text{konst} = \Delta\omega t - \Delta\beta z \;/\; \frac{d}{dt}$

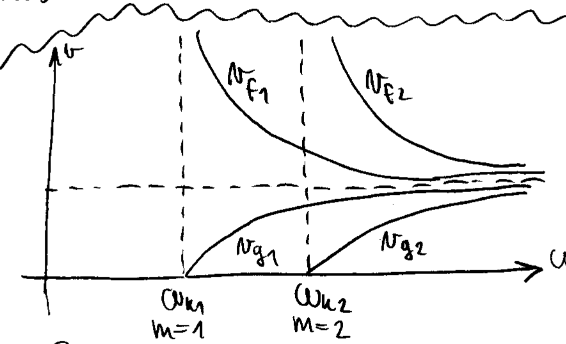
$\frac{dz}{dt} = v_g = \left.\frac{\Delta\omega}{\Delta\beta}\right|_{\Delta\omega \to 0} = \frac{d\omega}{d\beta} \qquad \boxed{v_f \geq c_0 \geq v_g}$

$\beta = \sqrt{\omega^2\mu\varepsilon - k_y^2} \rightarrow \frac{d\beta}{d\omega} = \frac{1}{2\sqrt{\omega^2\mu\varepsilon - k_y^2}} 2\omega\mu\varepsilon = \frac{1}{c_0\sqrt{1 - \left(\frac{\omega_k}{\omega}\right)^2}} \longrightarrow \underline{v_g = c_0\sqrt{1 - \left(\frac{\omega_k}{\omega}\right)^2} \leq c_0}$

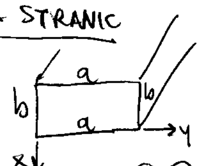


$|v_1| = |v_2| = c_0$

$\frac{v_g}{c_0} = \frac{\beta}{k}$



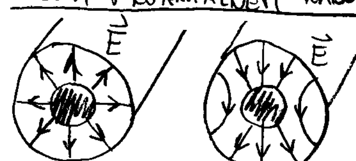
IZBIRA STRANIC



$b \leq \frac{a}{2} \longleftrightarrow \omega_{kTE10} \geq \omega_{kTE02}$

$\omega_{kTE02} = 2\,\omega_{kTE01}$

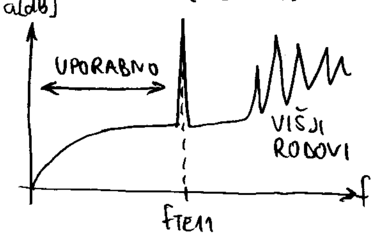
RODOVI V KOAKSIALNEM KABLU



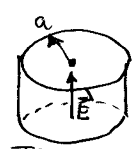
TEM $\omega_k = 0$

TE11 $\omega_k > 0$

$f_{kTE11} \approx \frac{c_0}{\pi(R_z + R_0)}$



VOTLINSKI REZONATOR (ρ, φ, z)



TM100

$\vec{E} = \vec{1}_z E_z$

$\Delta\vec{E} + k^2\vec{E} = 0 \qquad \vec{1}_z \cdot \Delta\vec{E} = \Delta E_z$

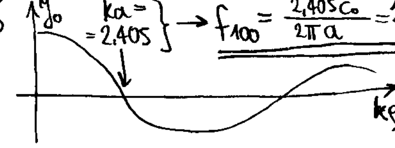
$\Delta E_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\varphi^2} + \frac{\partial^2 E_z}{\partial z^2}$

$E_z(\rho, \varphi, z) = R(\rho)F(\varphi)Z(z)$

$\frac{1}{\rho R}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial R}{\partial\rho}\right) + \frac{1}{\rho^2}\underbrace{\frac{1}{F}\frac{\partial^2 F}{\partial\varphi^2}}_{-m^2} + \underbrace{\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}}_{\pm\ell^2} + k^2 = 0$

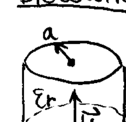
Zgled: $\frac{\partial}{\partial\varphi} = \frac{\partial}{\partial z} = 0$

$\frac{1}{\rho R}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial R}{\partial\rho}\right) + k^2 = 0 \rightarrow R(\rho) = J_0(k\rho)$



$\left.\begin{matrix} ka = \\ = 2{,}405 \end{matrix}\right\} \rightarrow \underline{\underline{f_{100} = \frac{2{,}405\, c_0}{2\pi a} = \frac{114{,}8\,\text{MHz}\cdot\text{m}}{a}}}$

DIELEKTRIČNI REZONATOR



$\varepsilon_r \gg 1 \rightarrow \vec{H}(\rho = a) \approx 0$

$\vec{H} = \vec{1}_z C J_0(k\rho)$

TE100 $\quad \Delta\vec{H} + k^2\vec{H} = 0$

$f_{100} \approx \frac{2{,}405\, c_0}{2\pi a\sqrt{\varepsilon_r}}$

$\underline{\underline{f_{100} \approx \frac{114.8\,\text{MHz}\cdot\text{m}}{a\sqrt{\varepsilon_r}}}}$

Elektrodinamika #12 7/1/2013

VALOVANJE V IZGUBNI SNOVI

$$\vec{E} = \vec{C}\, e^{-j\vec{k}\cdot\vec{r}}$$

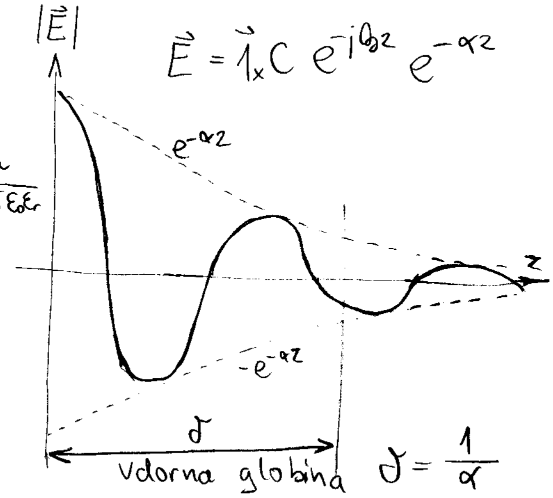
$$\vec{E} = \vec{1}_x C\, e^{-jkz}$$

$$\vec{E} = \vec{1}_x C\, e^{-j\beta z} e^{-\alpha z}$$

① ME: $\operatorname{rot}\vec{H} = \vec{J} + j\omega\varepsilon\vec{E} = (\gamma + j\omega\varepsilon_0\varepsilon_r)\vec{E} = j\omega\varepsilon_0 \underbrace{\left(\varepsilon_r - \frac{j\gamma}{\omega\varepsilon_0}\right)}_{\varepsilon_r'}\vec{E}$

$\vec{J} = \gamma\vec{E}$ $\quad \varepsilon = \varepsilon_0\varepsilon_r$

$$\Delta\vec{E} + k^2\vec{E} = 0 \longrightarrow k = \omega\sqrt{\mu\varepsilon_0\left(\varepsilon_r - \frac{j\gamma}{\omega\varepsilon_0}\right)} = \beta - j\alpha$$

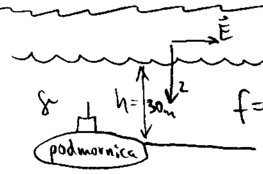


$\delta = \frac{1}{\alpha}$

$f_m = \frac{\gamma}{2\pi\varepsilon_0\varepsilon_r}$

baker $\gamma = 56\cdot10^6\,S/m$ $\varepsilon_r = 1$	morska voda $\gamma = 5\,S/m$ $\varepsilon_r = 80$	SiO_2 steklo $\varepsilon_r = 3{,}9$
$f_m \approx 10^{18}\,Hz$	$f_m = 1{,}13\cdot10^9\,Hz$	$f_m = 3\cdot10^{-10}\,Hz$
dober prevodnik $\alpha = \beta$	vmesni primer $0 < \alpha < \beta$	dober dielektrik $\alpha \to 0$

Dober prevodnik $\frac{\gamma}{\omega\varepsilon_0} \gg \varepsilon_r$ $\quad k = \omega\sqrt{\mu\left(-\frac{j\gamma}{\omega}\right)} = \sqrt{-j}\sqrt{\omega\mu\gamma} = \beta - j\alpha \longrightarrow \alpha = \beta = \sqrt{\frac{\omega\mu\gamma}{2}}$



$f = 10\,kHz \ll f_m \longrightarrow \alpha = \underline{0{,}44\,Np/m}$

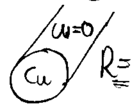
$a[dB] = \frac{20}{\ln 10}\alpha h = \underline{\underline{115\,dB}}$

f	δ_{Cu} ($\gamma_{Cu} = 56\cdot10^6\,S/m$, $\mu_{Cu} = 1$)	δ_{Fe} ($\gamma_{Fe} = \frac{1}{10}\gamma_{Cu}$, $\mu_{Fe} = 1000$)
1Hz	67mm	6,7mm
100Hz	6,7mm	0,67mm
10kHz	0,67mm	67µm
1MHz	67µm	6.7µm
100MHz	6,7µm	0,67µm
10GHz	0,67µm	67nm

Kovina $\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\gamma}} = \frac{1}{\beta}$

$$\vec{K} = \int_0^\infty \vec{J}\,dz = \gamma\int_0^\infty \vec{E}_0 e^{-j\beta z} e^{-\alpha z}\,dz = \frac{\gamma\vec{E}_0}{(\alpha + j\beta)}$$

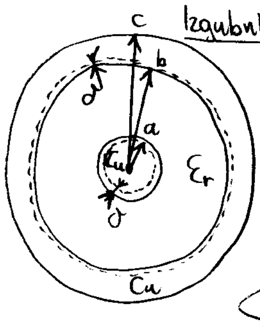
$$\vec{E}_0 = Z_p\vec{K} \longrightarrow Z_p = R_p + jX_p = \sqrt{\frac{\omega\mu}{2\gamma}} + j\sqrt{\frac{\omega\mu}{2\gamma}}$$

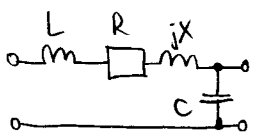


$R = \frac{l}{\gamma\pi r^2}$

$R \approx \frac{l}{2\pi r}R_p = \frac{l}{2\pi r}\sqrt{\frac{\omega\mu}{2\gamma}}$

Izgube koaksialnega kabla





$\omega L \gg X$

$L/l = \frac{\mu_0}{2\pi}\ln\frac{b}{a}$

$C/l = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}$ $\quad Z_K = \sqrt{\frac{L/l + \frac{R/l}{j\omega} + \frac{X/l}{\omega}}{C/l}} \approx \sqrt{\frac{L/l}{C/l}} = \underline{\frac{Z_0}{2\pi\sqrt{\varepsilon_r}}\ln\frac{b}{a}}$

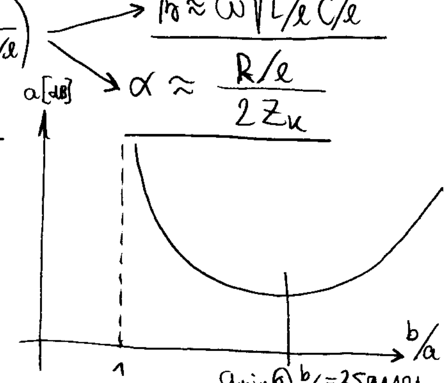
$$k = \beta - j\alpha = \sqrt{\omega\left(L/l + \frac{R/l}{j\omega} + \frac{X/l}{\omega}\right)\omega C/l} \approx \omega\sqrt{\left(L/l + \frac{R/l}{j\omega}\right)C/l} =$$

$$= \omega\sqrt{L/l\, C/l}\sqrt{1 - \frac{jR/l}{\omega C/l}} \approx \omega\sqrt{L/l\, C/l}\left(1 - \frac{jR/l}{2\omega C/l}\right)$$

$\longrightarrow \underline{\beta \approx \omega\sqrt{L/l\, C/l}}$

$\longrightarrow \underline{\alpha \approx \frac{R/l}{2Z_K}}$

$$a[dB]/l = \frac{20}{\ln 10}\alpha = \frac{10}{\ln 10}\frac{R/l}{Z_K} = \frac{10}{\ln 10}\frac{\sqrt{\frac{\omega\mu}{2\gamma}}}{Z_0}\sqrt{\varepsilon_r}\,\frac{1}{b}\,\frac{(b/a + 1)}{\ln b/a}$$



$$R/l = R_{\check{z}}/l + R_o/l = \frac{R_p}{2\pi a} + \frac{R_p}{2\pi b} = \frac{R_p}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$R_p = \sqrt{\frac{\omega\mu}{2\gamma}} = \underline{\underline{5{,}94\,m\Omega}}$

$f = 500\,MHz,\ \gamma = 56\cdot10^6\,S/m$

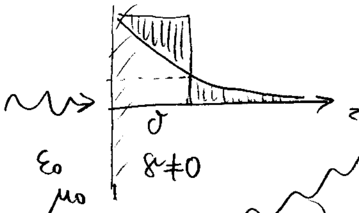
$\left.\begin{array}{l} a = 1\,mm \\ b = 3{,}5\,mm \\ \varepsilon_r = 2 \end{array}\right\}$ $\underline{\underline{Z_K = 53{,}2\,\Omega}}$ $\quad a[dB]/l = \frac{10}{\ln 10}\,\frac{5{,}94\,m\Omega}{377\,\Omega}\sqrt{2}\,\frac{1}{3{,}5\cdot10^{-3}\,m}\,\frac{3{,}5+1}{\ln 3{,}5} = \underline{\underline{0{,}0993\,dB/m = 99{,}3\,dB/km}}$

Elektrodinamika #13 14/1/2013

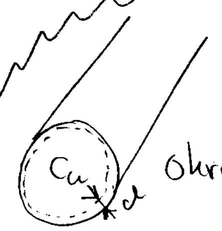
Ponovitev: DOBER PREVODNIK $\gamma \gg \omega\varepsilon$ (kovine $f <$ rentgen)

$$\delta = \sqrt{\frac{2}{\omega\mu\gamma}}$$

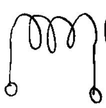
$$R_p = R_\square = \sqrt{\frac{\omega\mu}{2\gamma}}$$



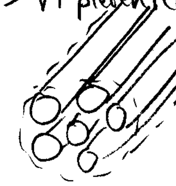
$$R = \rho\frac{l}{A} = \frac{l}{2\pi r \delta \gamma} = \frac{l}{2\pi r} R_p$$



Okrogel vodnik $\rightarrow$ VF pletenica

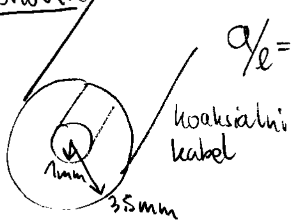


$$Q = \frac{\omega L}{R} < 100 \longrightarrow$$



Ponovitev:

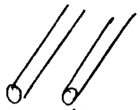
$$\alpha/l = \frac{10}{\ln 10}\frac{R/l}{Z_k}$$



$$\alpha/l = \frac{10}{\ln 10}\frac{\sqrt{\varepsilon_r}\,R_p}{Z_0}\frac{1}{b}\left(\frac{b/a+1}{\ln b/a}\right)$$

min @ $b/a = 3{,}591121 \approx 3{,}6$

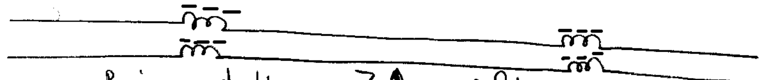
$\alpha/l \approx 0{,}1\,\mathrm{dB/m}$ @ 500MHz



SIMETRIČNI DVOVOD

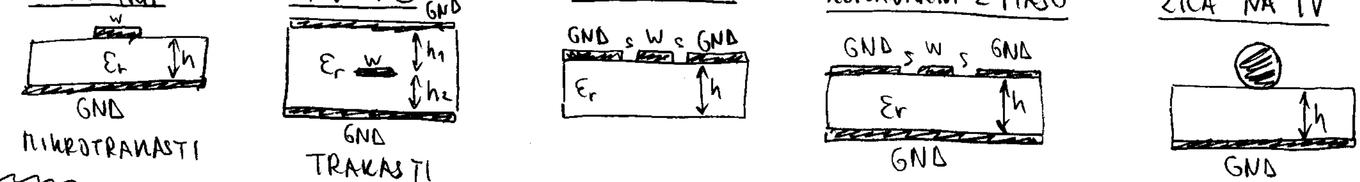
$Z_k \approx (2\times \ldots 6\times) Z_{koaks}$

$R/l \approx 2\times (R/l)_{koaks}$



Pupinove tuljave $Z_k\uparrow$, $\Delta f\downarrow$

MIKROSTRIP / STRIPLINE / KOPLANARNI / KOPLANARNI Z MASO / ŽICA NA TV

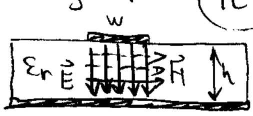


MIKROSTRIP BREZ STRESANJA

$L/l = \mu_0\frac{h}{w}$

zelo grob približek TEM

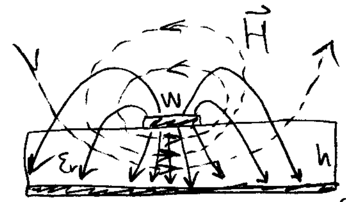
$C/l = \varepsilon_0\varepsilon_r\frac{w}{h}$

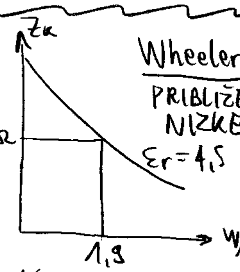


$$Z_k \approx \frac{Z_0}{\sqrt{\varepsilon_r}}\frac{h}{w}$$

$\varepsilon_r = 4{,}5$, $h = 1{,}6\,\mathrm{mm}$, $Z_k = 50\,\Omega \longrightarrow w = 6\,\mathrm{mm}$?

MIKROSTRIP S STRESANJEM





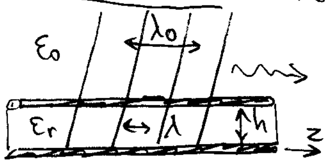
Wheeler TEM

PRIBLIŽEK ZA NIZKE f

$\varepsilon_r = 4{,}5$ $h \ll \lambda$

$\varepsilon_r = 4{,}5$, $h = 1{,}6\,\mathrm{mm}$, $Z_k = 50\,\Omega \rightarrow w = 3\,\mathrm{mm}$!

HIBRIDNI RODOVI $h \sim \lambda$



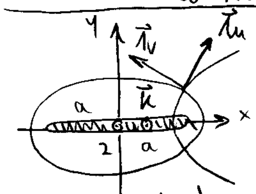
$E_z \neq 0$ TOČNA FIZIKALNA SLIKA

$H_z \neq 0$

$U \equiv$ nedefiniran

$Z_k \equiv$ nedefiniran

OSAMLJEN KOVINSKI TRAK za



(u, v, z)

$x = a\,\mathrm{ch}\,u\cos v$

$y = a\,\mathrm{sh}\,u\sin v$

$z = z$

$h_u = h_v = a\sqrt{\mathrm{sh}^2 u + \sin^2 v}$

Isti induciran $\vec{E} \rightarrow \vec{A} = \vec{1}_z A_z(u)$

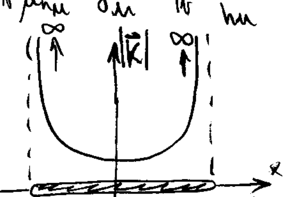
$$\vec{H} = \frac{1}{\mu}\mathrm{rot}\vec{A} = \frac{1}{\mu h_u h_v}\begin{vmatrix} h_u\vec{1}_u & h_v\vec{1}_v & \vec{1}_z \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial z} \\ h_u 0 & h_v 0 & A_z(u) \end{vmatrix}$$

$$\vec{H} = \vec{1}_v\frac{-1}{\mu h_u}\frac{\partial A_z(u)}{\partial u} = \vec{1}_v\frac{f(u)}{h_u}$$

TRAK: $u=0$ $\rightarrow \vec{H} = \vec{1}_v\frac{C}{a\sin v} = \vec{1}_v\frac{C}{\sqrt{a^2-x^2}}$

$\cos v = \frac{x}{a}$ $\sin v = \sqrt{1-\left(\frac{x}{a}\right)^2}$

$$\vec{K} = \vec{1}_z\,2|\vec{H}| = \vec{1}_z\frac{2C}{\sqrt{a^2-x^2}}$$



IZRIV TOKA NA ROB TRAKU!

IZGUBE MIKROTRAKASTEGA VODA

1. IZRIV TOKA NA POVRŠINO $\rightarrow \delta$!
2. IZRIV TOKA NA ROB TRAKU!
3. HRAPAVOST BAKRA ZA LEPLJENJE!

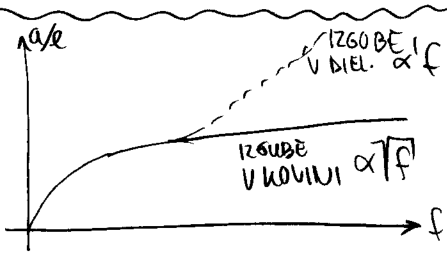
$(\alpha/l)_{mikrostrip} > 10\times(\alpha/l)_{koaks}$ (isti dielektrik, izmere, f)

IZGUBE V BAKRU / DIELEKTRIKU

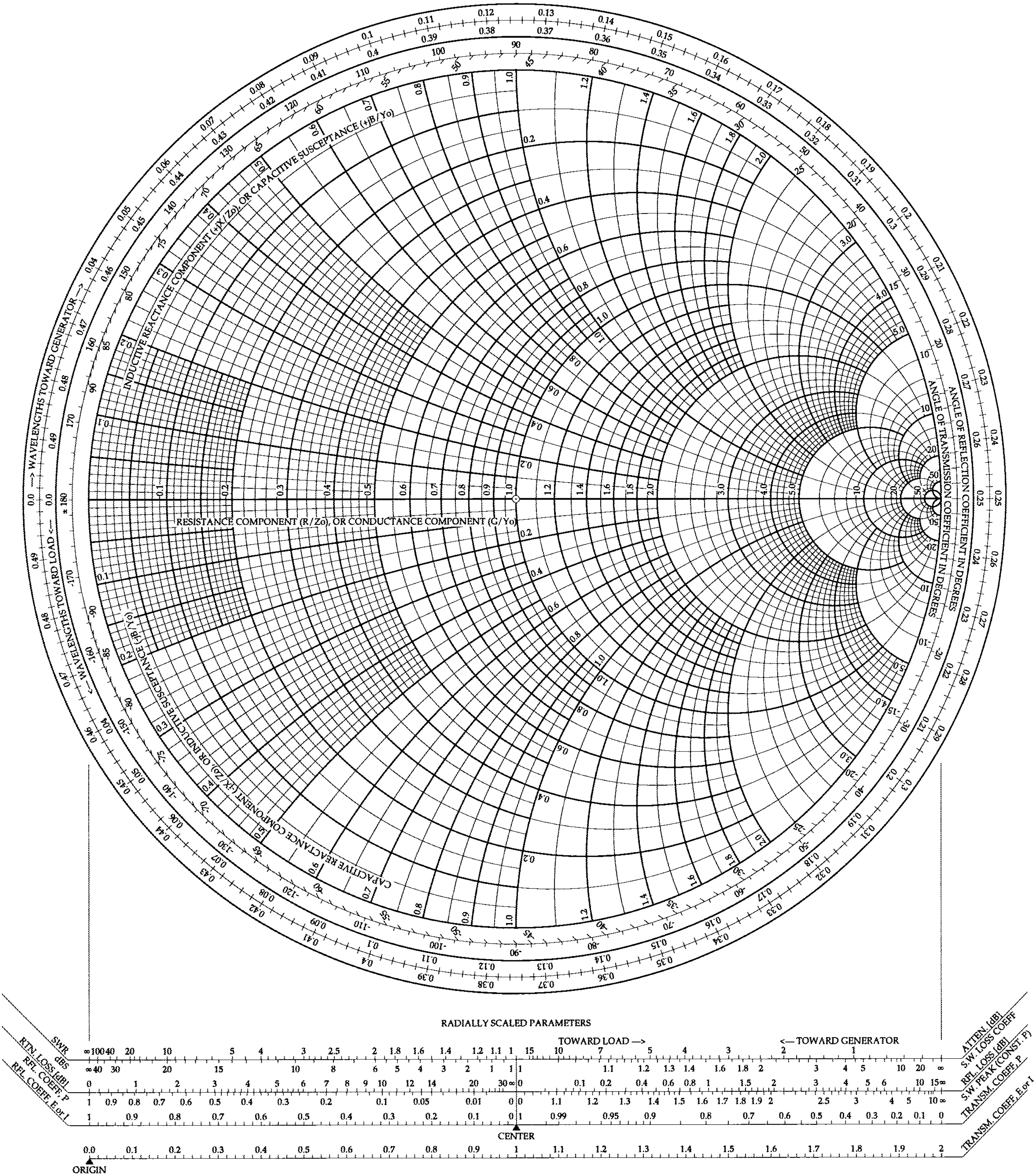
KOVINA: $R_p = \sqrt{\frac{\omega\mu}{2\gamma}} = \propto\sqrt{f}$

DIELEKTRIK Z UMAZANIJO:

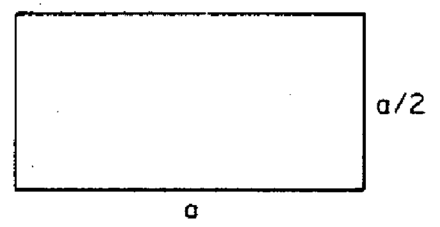
$\rightarrow \propto f$

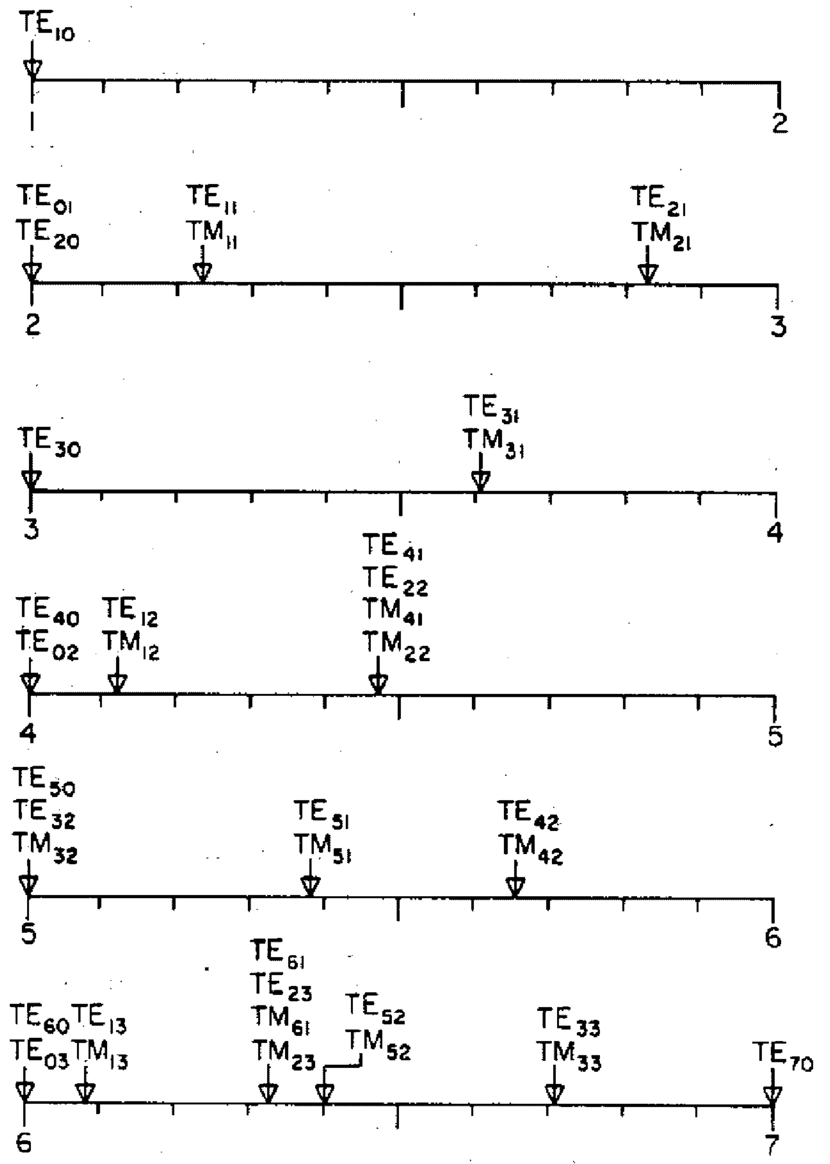


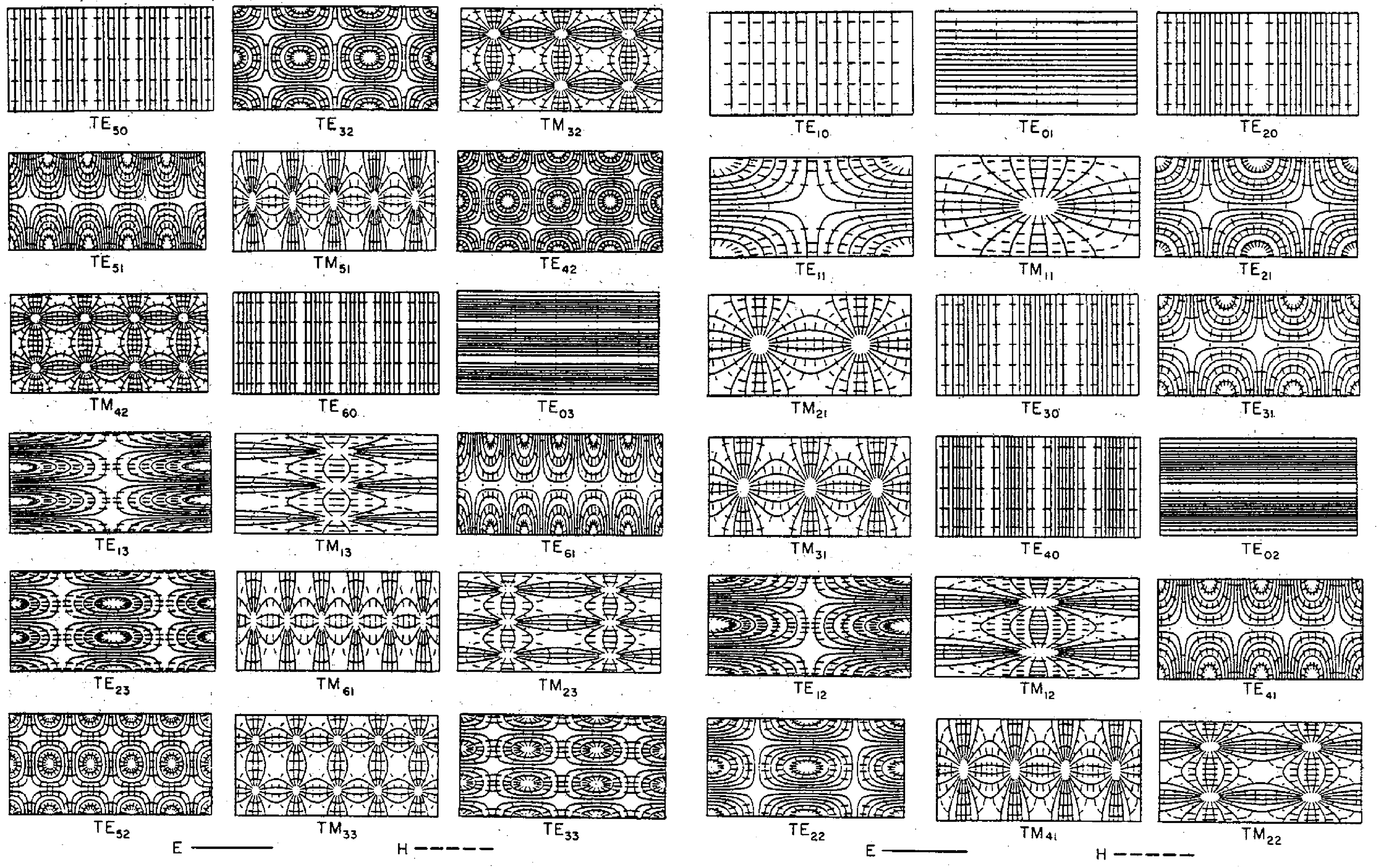
Smith-ov diagram:
impedanca/admitanca v merilu odbojnosti



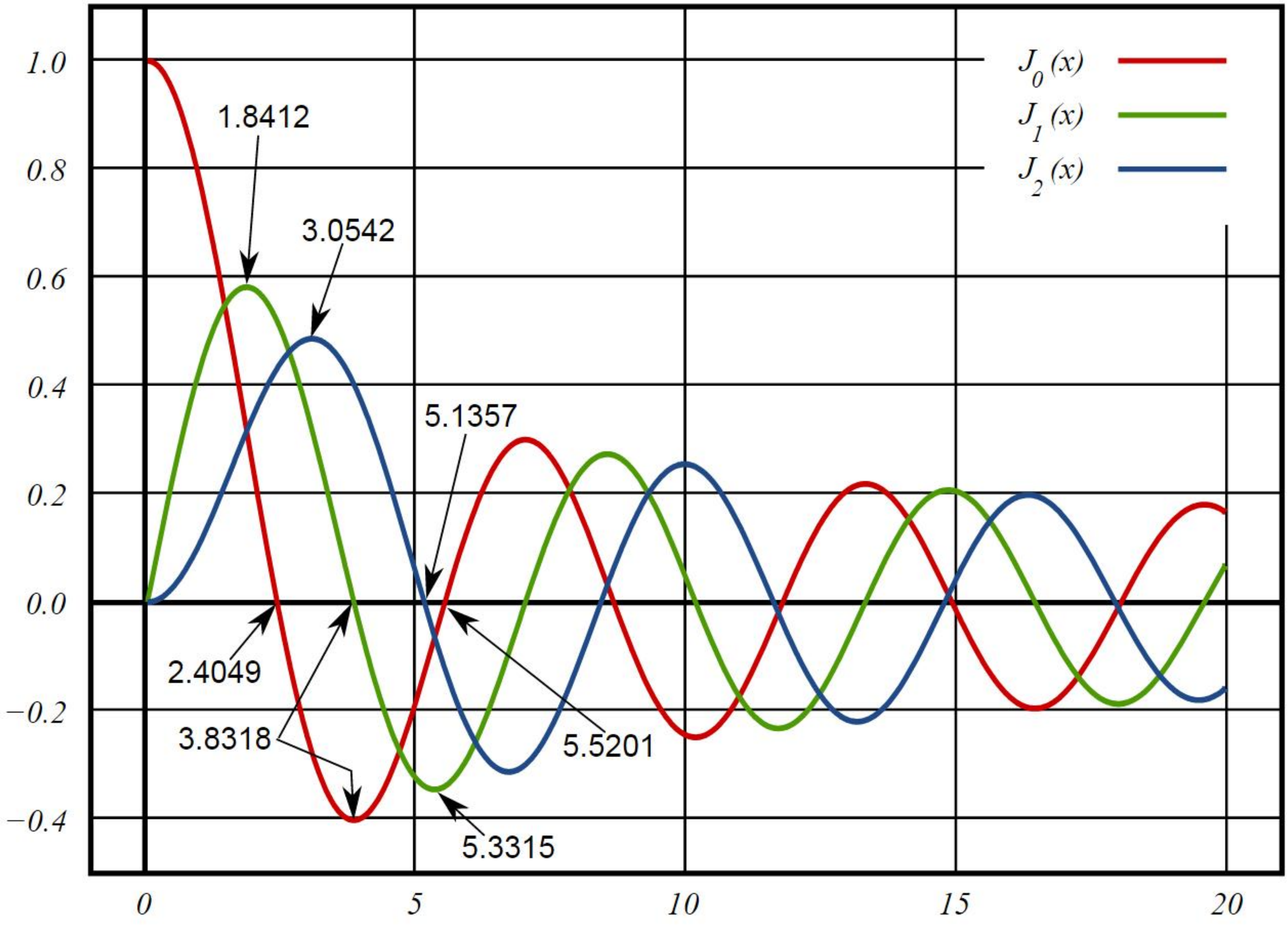
Pravokotni valovod



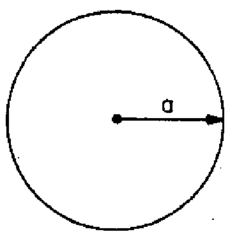


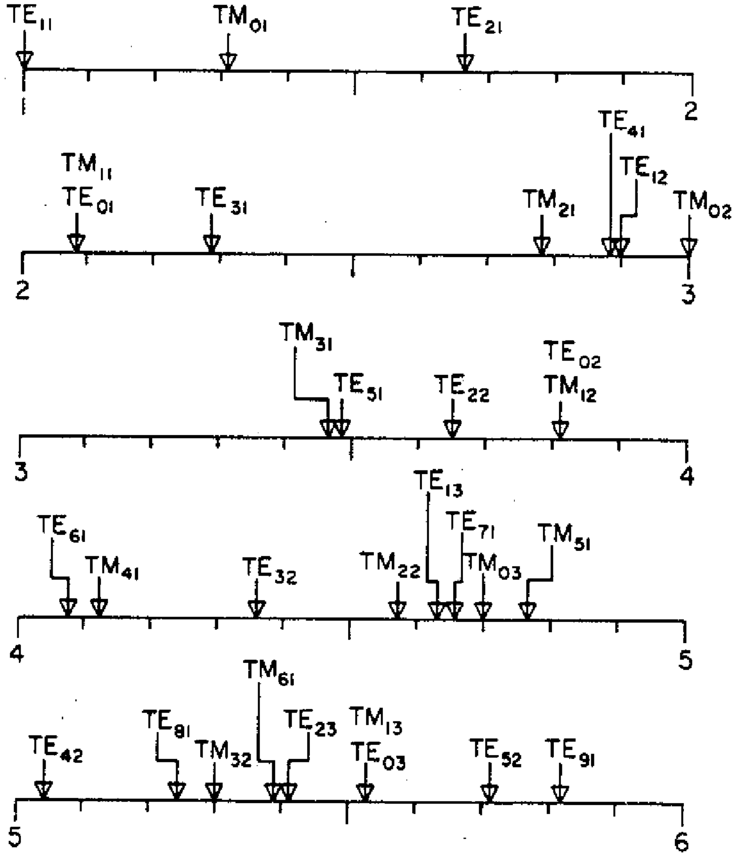


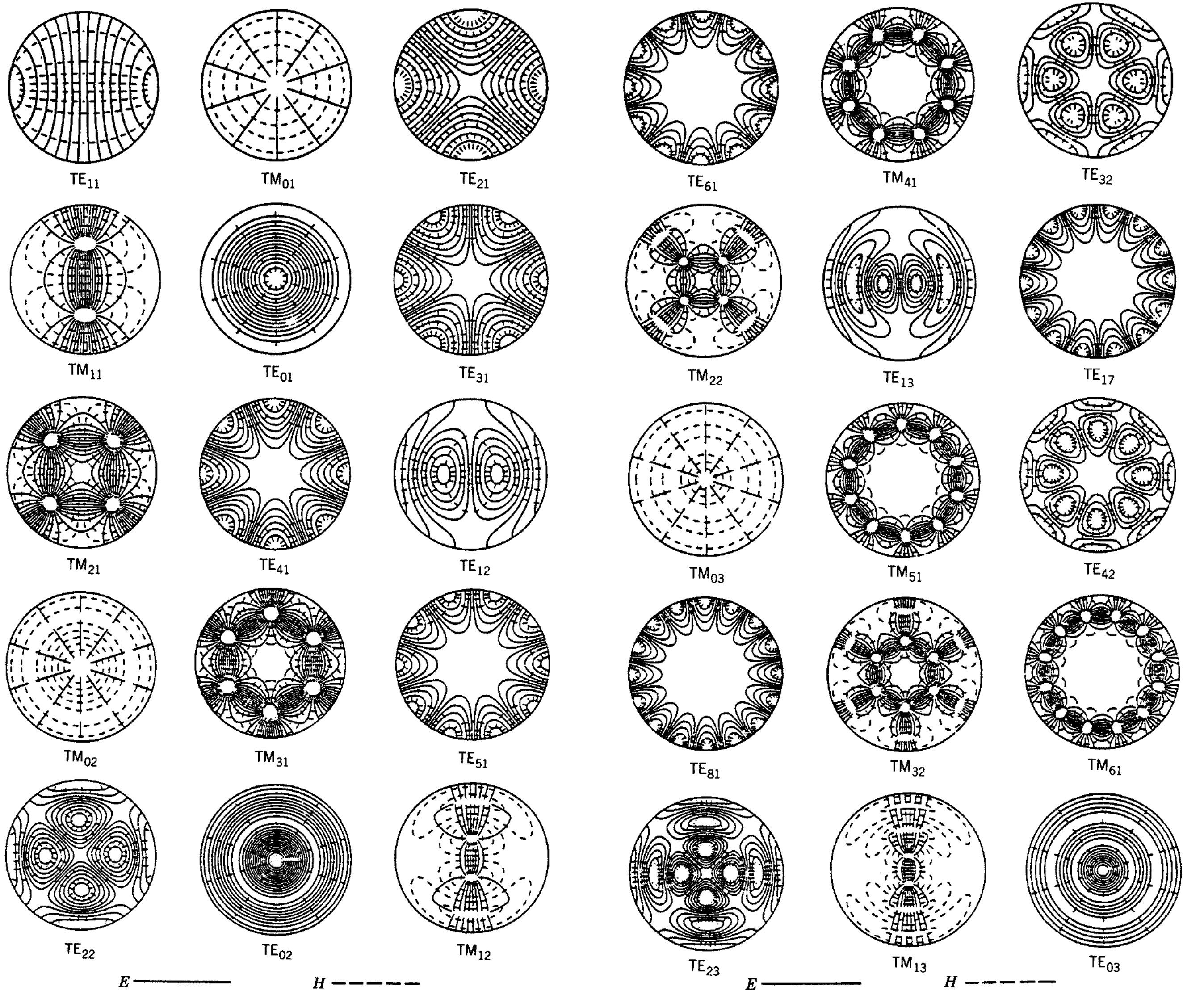
Bessel-ove funkcije



Krožni valovod

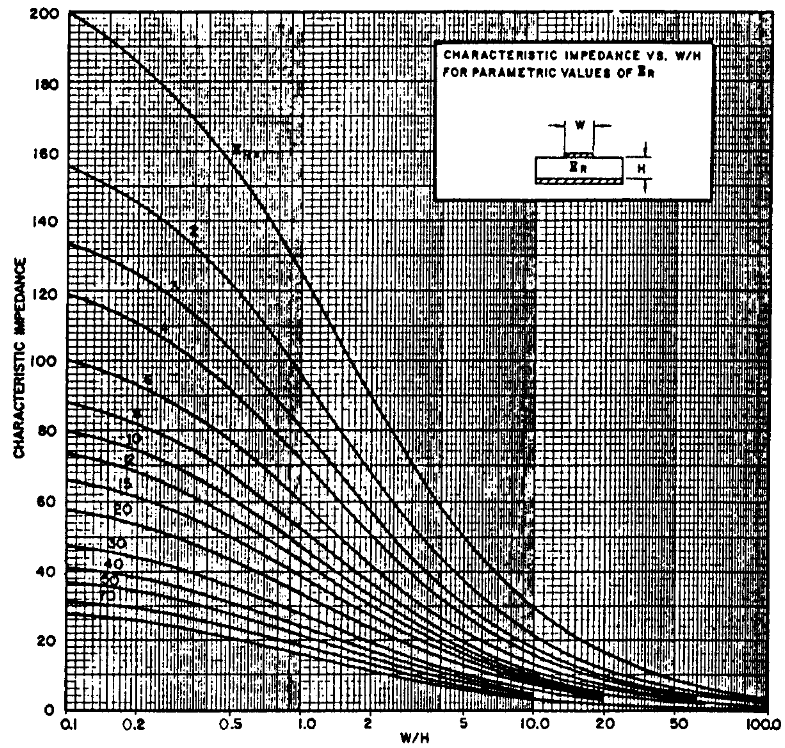






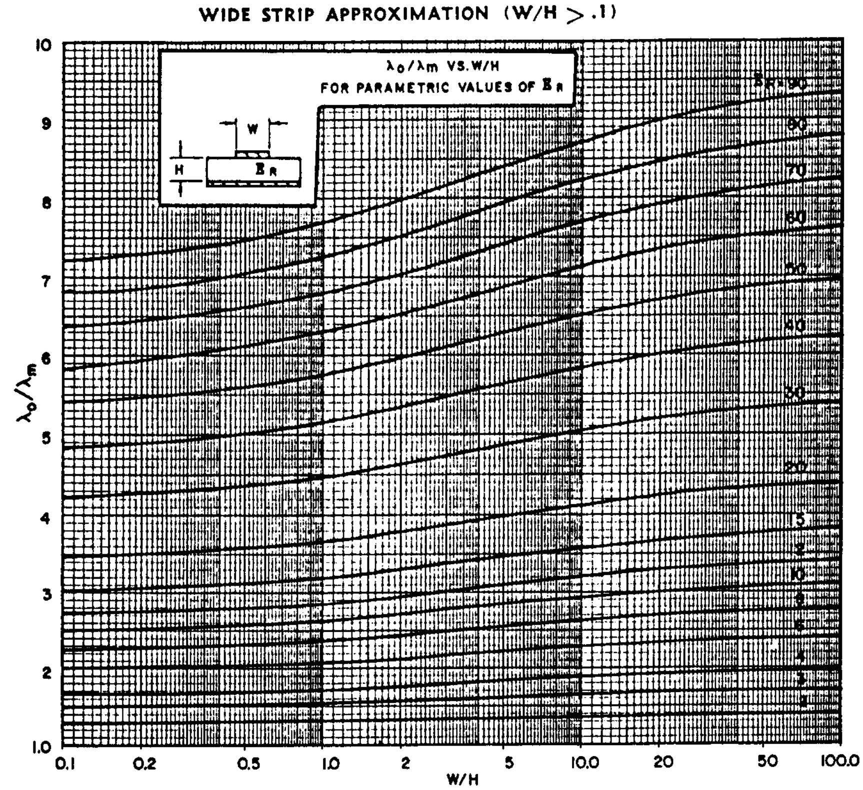
MICROSTRIP CHARACTERISTIC IMPEDANCE CALCULATED FROM WORK OF WHEELER

WIDE STRIP APPROXIMATION (W/H > .1)



RATIO OF FREE SPACE WAVELENGTH (λ_o) TO MICROSTRIP WAVELENGTH (λ_m) CALCULATED FROM WORK OF WHEELER

WIDE STRIP APPROXIMATION (W/H > .1)



Karakteristična impedanca Zk in navidezni lomni količnik n mikrotrakastega voda