Retarded potential

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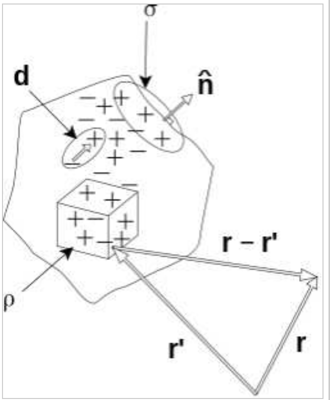
In electrodynamics, the **retarded potentials** are the electromagnetic potentials for the electromagnetic field generated by time-varying electric current or charge distributions in the past. The fields propagate at the speed of light *c*, so the delay of the fields connecting cause and effect at earlier and later times is an important factor: the signal takes a finite time to propagate from a point in the charge or current distribution (the point of cause) to another point in space (where the effect is measured), see figure below.[1]

Contents

Potentials in the Lorenz gauge

The starting point is Maxwell's equations in the potential formulation using the Lorenz gauge:

$$\Box\varphi = -\frac{\rho}{\epsilon_0}\,, \quad \Box\mathbf{A} = -\mu_0\mathbf{J}$$



Position vectors **r** and **r′** used in the calculation.

where φ(**r**, *t*) is the electric potential and **A**(**r**, *t*) is the magnetic potential, for an arbitrary source of charge density ρ(**r**, *t*) and current density **J**(**r**, *t*), and $\Box$ is the D'Alembert operator. Solving these gives the retarded potentials below.

Retarded and advanced potentials for time-dependent fields

For time-dependent fields, the retarded potentials are:[2][3]

$$\varphi(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}'$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}'\,.$$

where **r** is a point in space, *t* is time,

$$t_r = t - \frac{|\mathbf{r}-\mathbf{r}'|}{c}$$

is the retarded time, and d^3**r'** is the integration measure using **r'**.

From φ(**r**,t) and **A**(**r**, *t*), the fields **E**(**r**, *t*) and **B**(**r**, *t*) can be calculated using the definitions of the potentials:

$$-\mathbf{E} = \nabla\varphi + \frac{\partial\mathbf{A}}{\partial t}\,, \quad \mathbf{B} = \nabla\times\mathbf{A}\,.$$

and this leads to Jefimenko's equations. The corresponding advanced potentials have an identical form, except the advanced time

$$t_a = t + \frac{|\mathbf{r}-\mathbf{r}'|}{c}$$

replaces the retarded time.

Comparison with static potentials for time-independent fields

In the case the fields are time-independent (electrostatic and magnetostatic fields), the time derivatives in the $\Box$ operators of the fields are zero, and Maxwell's equations reduce to

$$\nabla^2\varphi = -\frac{\rho}{\epsilon_0}\,, \quad \nabla^2\mathbf{A} = -\mu_0\mathbf{J}\,,$$

where ∇^2 is the Laplacian, which take the form of Poisson's equation in four components (one for φ and three for **A**), and the solutions are:

$$\varphi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}'$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}'\,.$$

These also follow directly from the retarded potentials.

Potentials in the Coulomb gauge

In the Coulomb gauge, Maxwell's equations are[4]

$$\nabla^2\varphi = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{J} + \frac{1}{c^2}\nabla\left(\frac{\partial\varphi}{\partial t}\right)\,,$$

although the solutions contrast the above, since **A** is a retarded potential yet φ changes *instantly*, given by:

$$\varphi(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}^3\mathbf{r}'$$

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\nabla\times\int \mathrm{d}^3\mathbf{r}'\int_0^{|\mathbf{r}-\mathbf{r}'|/c}\mathrm{d}t_r\frac{t_r\mathbf{J}(\mathbf{r}',t-t_r)}{|\mathbf{r}-\mathbf{r}'|^3}\times(\mathbf{r}-\mathbf{r}')\,.$$

This presents an advantage and a disadvantage of the coulomb gauge - φ is easily calculable from the charge distribution ρ but **A** is not so easily calculable from the current distribution **j**. However, provided we require that the potentials vanish at infinity, they can be expressed neatly in terms of fields:

$$\varphi(\mathbf{r},t) = \frac{1}{4\pi}\int \frac{\nabla\cdot\mathbf{E}(r',t)}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}^3\mathbf{r}'$$

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{4\pi}\int \frac{\nabla\times\mathbf{B}(r',t)}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}^3\mathbf{r}'$$

Occurrence and application

A many-body theory which includes an average of retarded and *advanced* Liénard–Wiechert potentials is the Wheeler–Feynman absorber theory also known as the Wheeler–Feynman time-symmetric theory.

References

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