Black body
$\Gamma = 0$

$B_f = \dfrac{2 k_B T}{\lambda^2}$

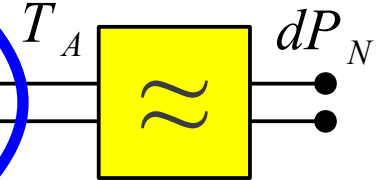dA'

Single polarization

Lossless antenna
$\eta = 1$ $A_{eff}(\Theta, \Phi)$

$dP_N = \dfrac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta \Omega$

T_A dP_N

$\approx$

Bandpass
filter Δf

r

$$\Delta \Omega = \frac{A_{eff}(\Theta, \Phi)}{r^2} = \frac{\lambda^2 D(\Theta, \Phi)}{4 \pi r^2} = \frac{\lambda^2 |F(\Theta, \Phi)|^2}{r^2 \iint\limits_{4\pi} |F(\Theta*, \Phi*)|^2 \, d\Omega*}$$

$dA' = r^2 \, d\Omega$

$$P_N = \iint\limits_{A'} \frac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta \Omega = \iint\limits_{4\pi} \frac{1}{2} \cdot \frac{2 k_B T(\Theta, \Phi)}{\lambda^2} \cdot \Delta f \cdot r^2 \, d\Omega \cdot \frac{\lambda^2 |F(\Theta, \Phi)|^2}{r^2 \iint\limits_{4\pi} |F(\Theta*, \Phi*)|^2 \, d\Omega*}$$

$$P_N = \Delta f \, k_B \frac{\iint\limits_{4\pi} T(\Theta, \Phi) |F(\Theta, \Phi)|^2 \, d\Omega}{\iint\limits_{4\pi} |F(\Theta, \Phi)|^2 \, d\Omega} = \Delta f \, k_B T_A$$

$$T_A = \frac{\iint\limits_{4\pi} T(\Theta, \Phi) |F(\Theta, \Phi)|^2 \, d\Omega}{\iint\limits_{4\pi} |F(\Theta, \Phi)|^2 \, d\Omega}$$