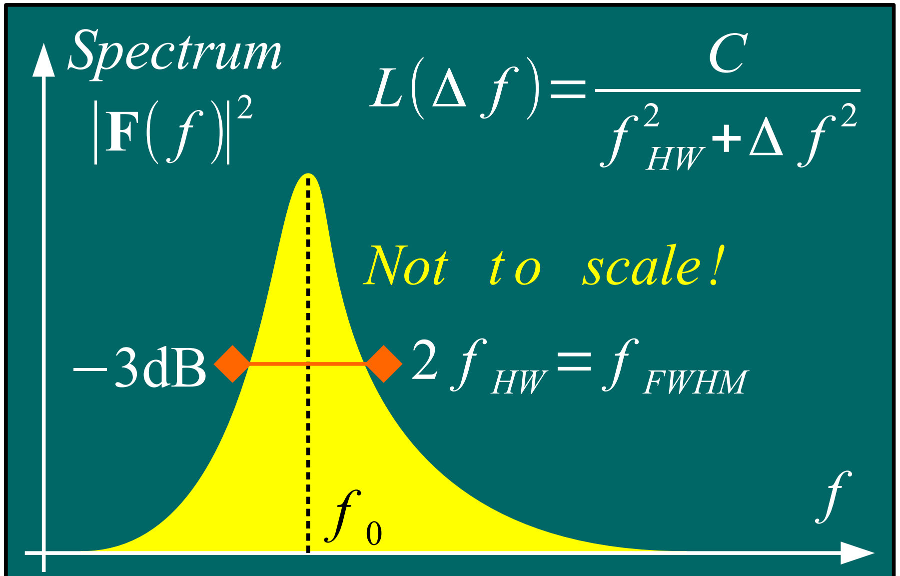

Lorentzian spectral line

Flat thermal noise can be neglected: device f_{MAX} or Planck law

LC-oscillator $1/f$ noise can be neglected

$$L(\Delta f) = \frac{1}{8} \cdot \left(\frac{f_0}{Q_L} \right)^2 \cdot \frac{1}{f_{HW}^2 + \Delta f^2} \cdot \frac{k_B T_0 F}{P_0}$$

Lorentzian line in Leeson's equation

$$\int_{-f_0}^{\infty} L(\Delta f)\, d\Delta f = 1 \approx \int_{-\infty}^{\infty} L(\Delta f)\, d\Delta f = \frac{1}{8} \cdot \left(\frac{f_0}{Q_L} \right)^2 \cdot \frac{k_B T_0 F}{P_0} \int_{-\infty}^{\infty} \frac{1}{f_{HW}^2 + \Delta f^2}\, d\Delta f =$$

$$= \frac{1}{8} \cdot \left(\frac{f_0}{Q_L} \right)^2 \cdot \frac{k_B T_0 F}{P_0} \cdot \left[\frac{1}{f_{HW}} \cdot \arctan \frac{\Delta f}{f_{HW}} \right]_{\Delta f = -\infty}^{\Delta f = \infty} = \frac{k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L} \right)^2 \cdot \frac{\pi}{f_{HW}}$$

$$f_{HW} = \frac{\pi k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L} \right)^2$$

Example $f_0 = 3\text{GHz}$ $\quad Q_L = 10$

$P_0 = 0.1\text{mW}$ $\quad F = 10\text{dB}$

$f_{HW} = 14\text{Hz}$ $\quad f_{FWHM} = 28\text{Hz}$

$$C = \frac{k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L} \right)^2 = \frac{f_{HW}}{\pi}$$

$$L(\Delta f) = \frac{f_{HW}/\pi}{f_{HW}^2 + \Delta f^2}$$