NOISE IN RADIO/OPTICAL COMMUNICATIONS

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List of figures: Noise in radio/optical communications

Fifth Solvay International Conference on Electrons and Photons (October 1927). The leading figures Albert Einstein and Niels Bohr disagreed:

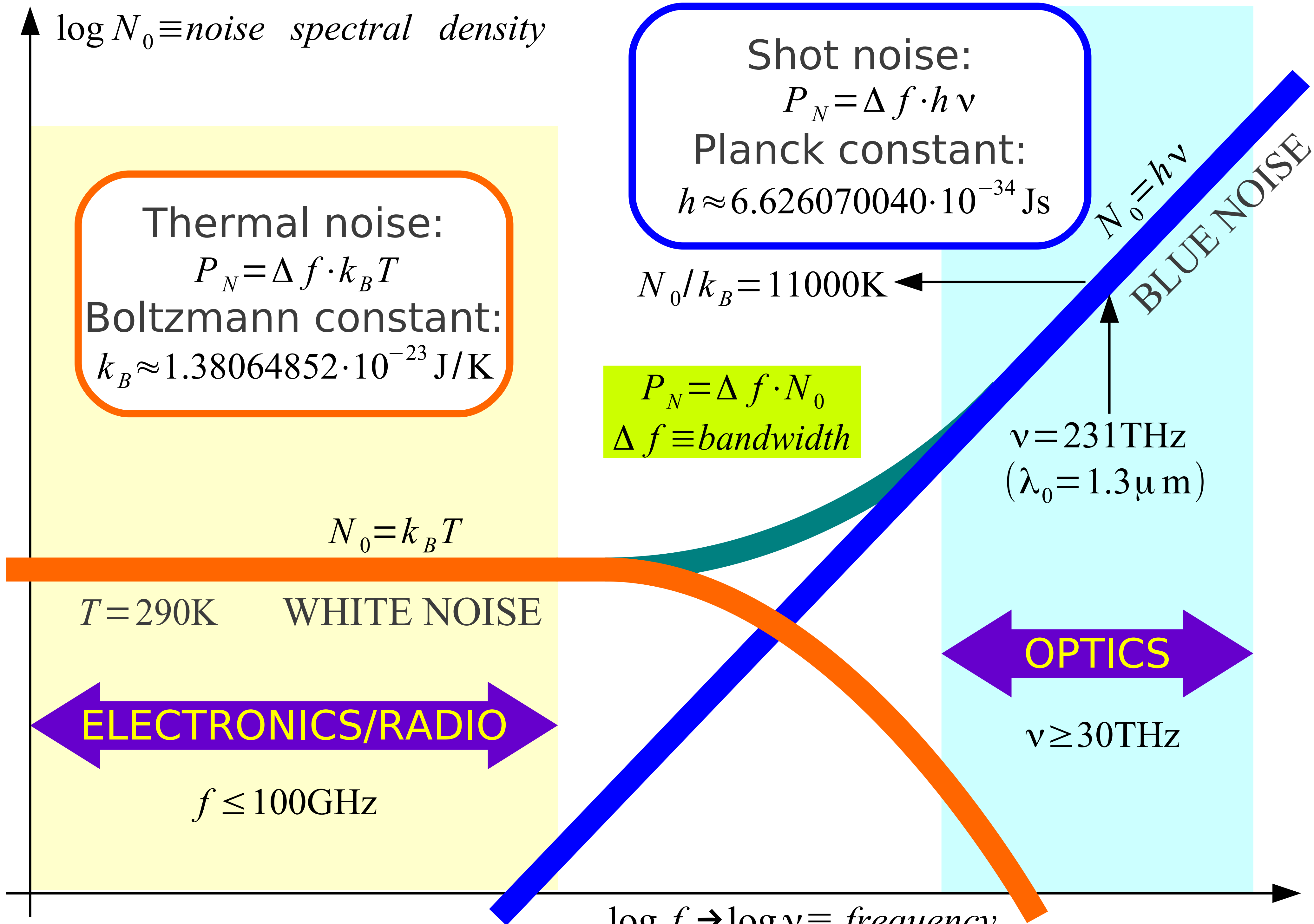
Albert Einstein: „God does not play dice!“

Niels Bohr: „Einstein, stop telling God what to do!“

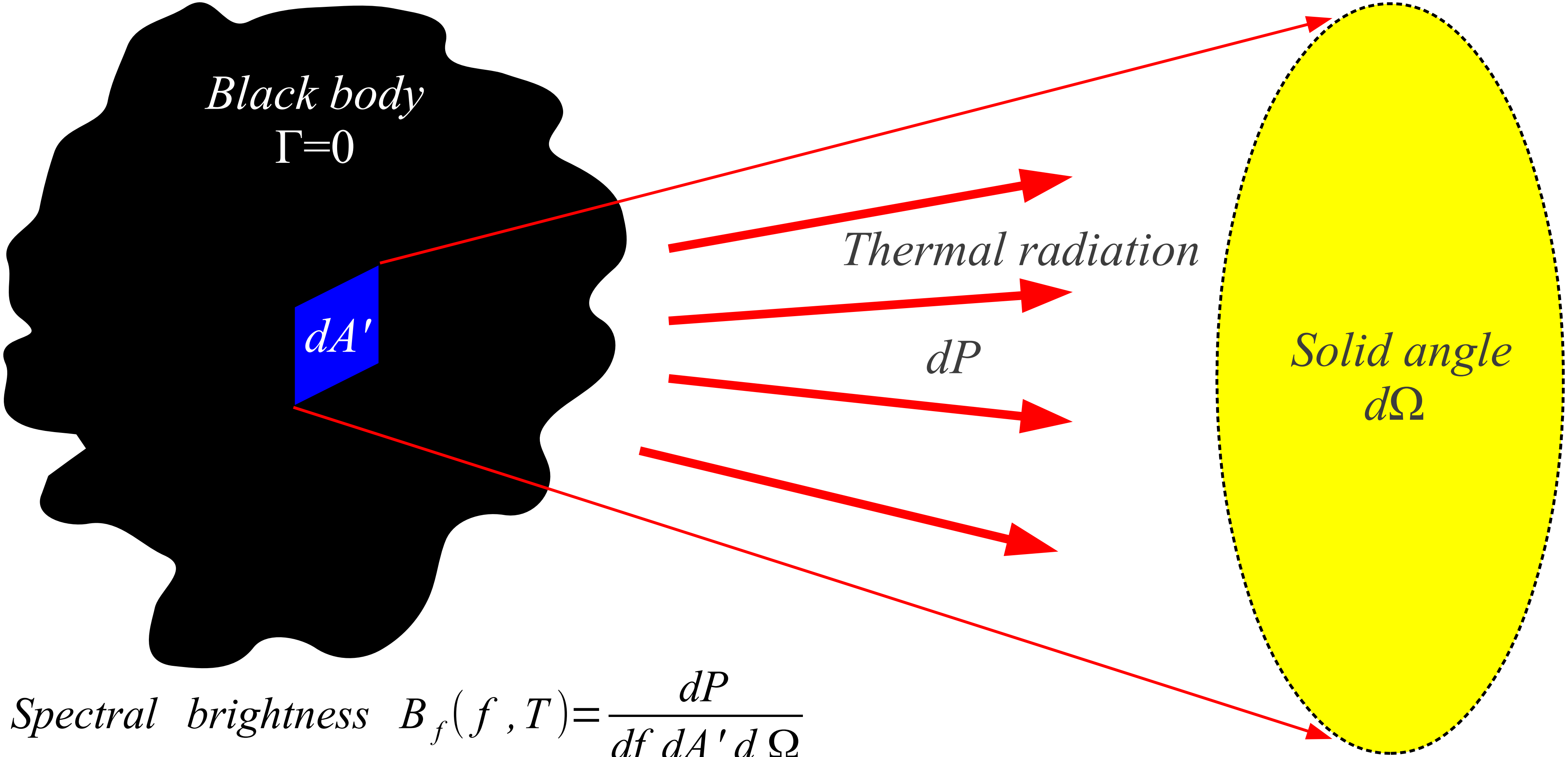
In telecommunications random signals are called noise. Noise impairs the performance of any communication link.

Noise is a macroscopic description of quantum effects!

1 - The dispute between famous scientists



2 - Noise spectral density



$$\textit{Spectral brightness}\quad B_f(f,T)=\frac{dP}{df\,dA'\,d\Omega}$$

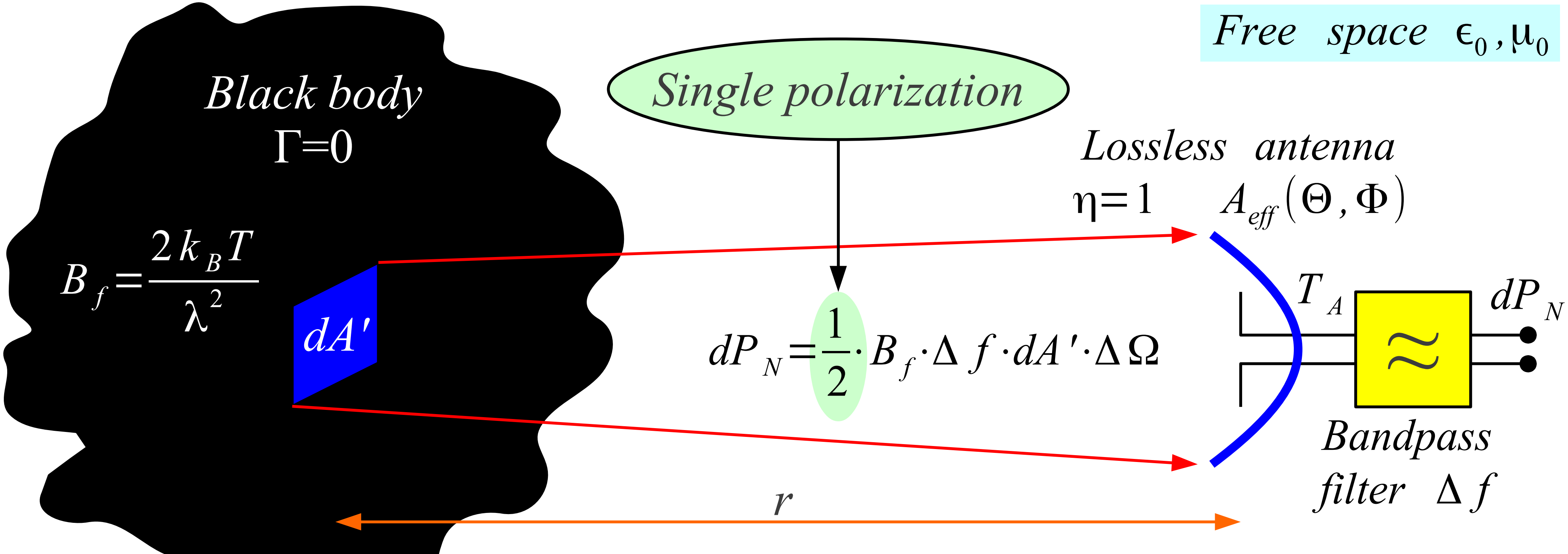
$$\textit{Planck law}\quad B_f(f,T)=\frac{2hf^3}{c_0^2}\cdot\frac{1}{e^{\frac{hf}{k_B T}}-1}$$

$$\textit{Free space}\quad \epsilon_0,\mu_0$$

$$c_0=299792458\,\mathrm{m/s}\approx 3\cdot 10^8\,\mathrm{m/s}$$

$$\textit{Radio}\quad hf\ll k_B T\ \rightarrow\ \textit{Rayleigh}-\textit{Jeans approximation}\quad B_f(f,T)\approx\frac{2k_B T f^2}{c_0^2}=\frac{2k_B T}{\lambda^2}$$

3 – Black-body thermal radiation



$$\Delta\Omega = \frac{A_{eff}(\Theta,\Phi)}{r^2} = \frac{\lambda^2 D(\Theta,\Phi)}{4\pi r^2} = \frac{\lambda^2 |F(\Theta,\Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*,\Phi^*)|^2 d\Omega^*}$$

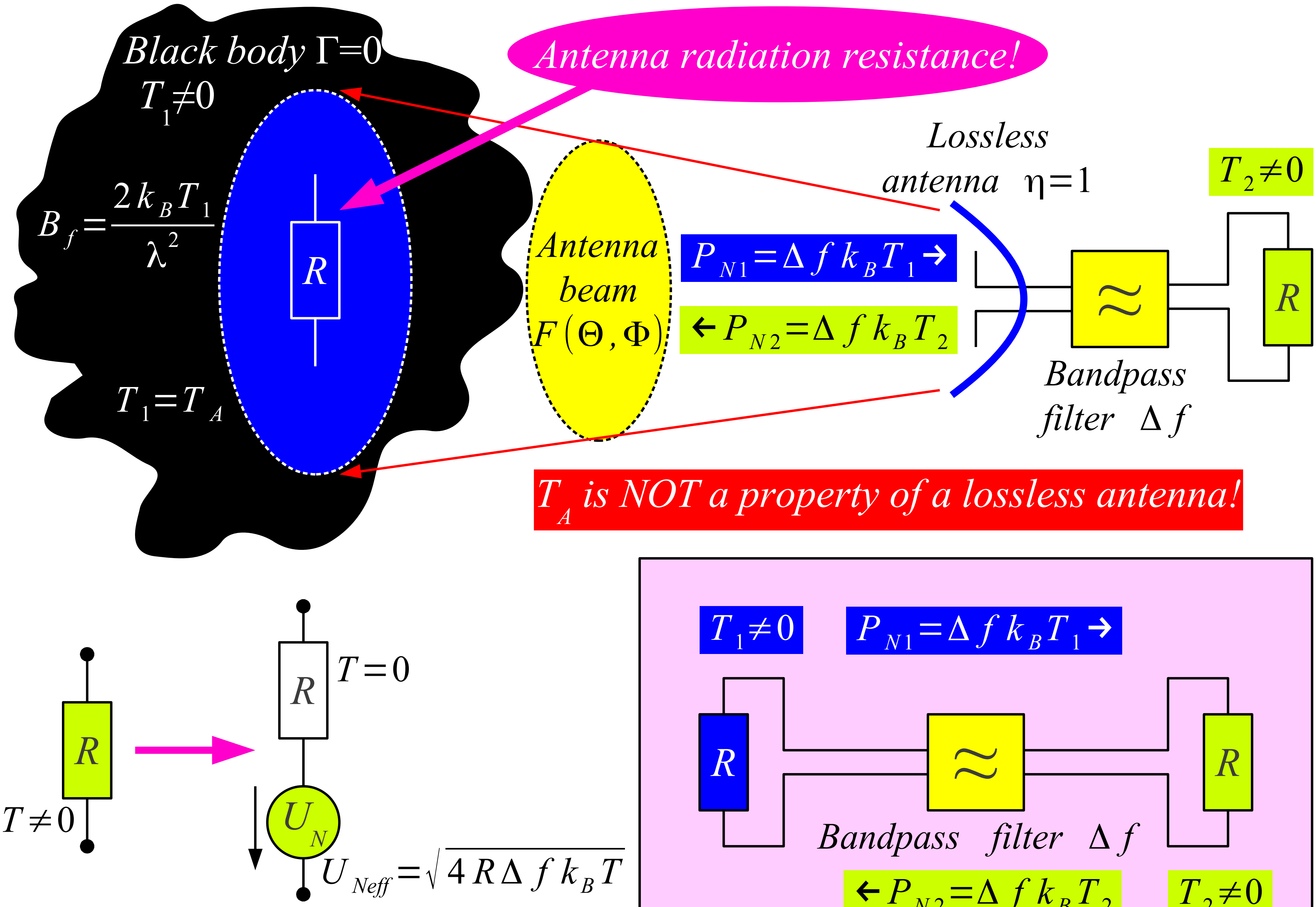
$$dA' = r^2 d\Omega$$

$$P_N = \iint_{A'} \frac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta\Omega = \iint_{4\pi} \frac{1}{2} \cdot \frac{2 k_B T(\Theta,\Phi)}{\lambda^2} \cdot \Delta f \cdot r^2 d\Omega \cdot \frac{\lambda^2 |F(\Theta,\Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*,\Phi^*)|^2 d\Omega^*}$$

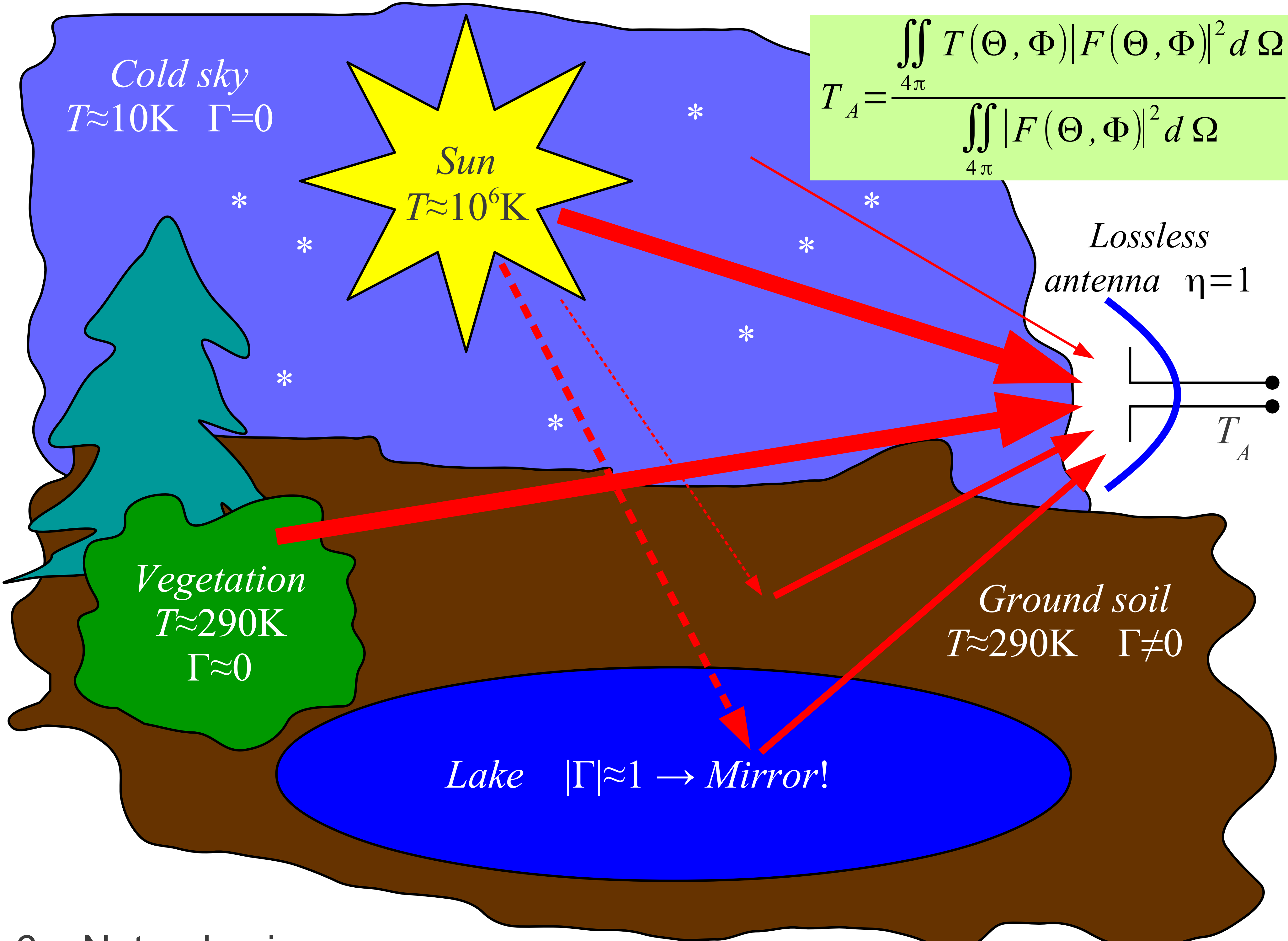
$$P_N = \Delta f\, k_B \frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega} = \Delta f\, k_B T_A$$

$$T_A = \frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

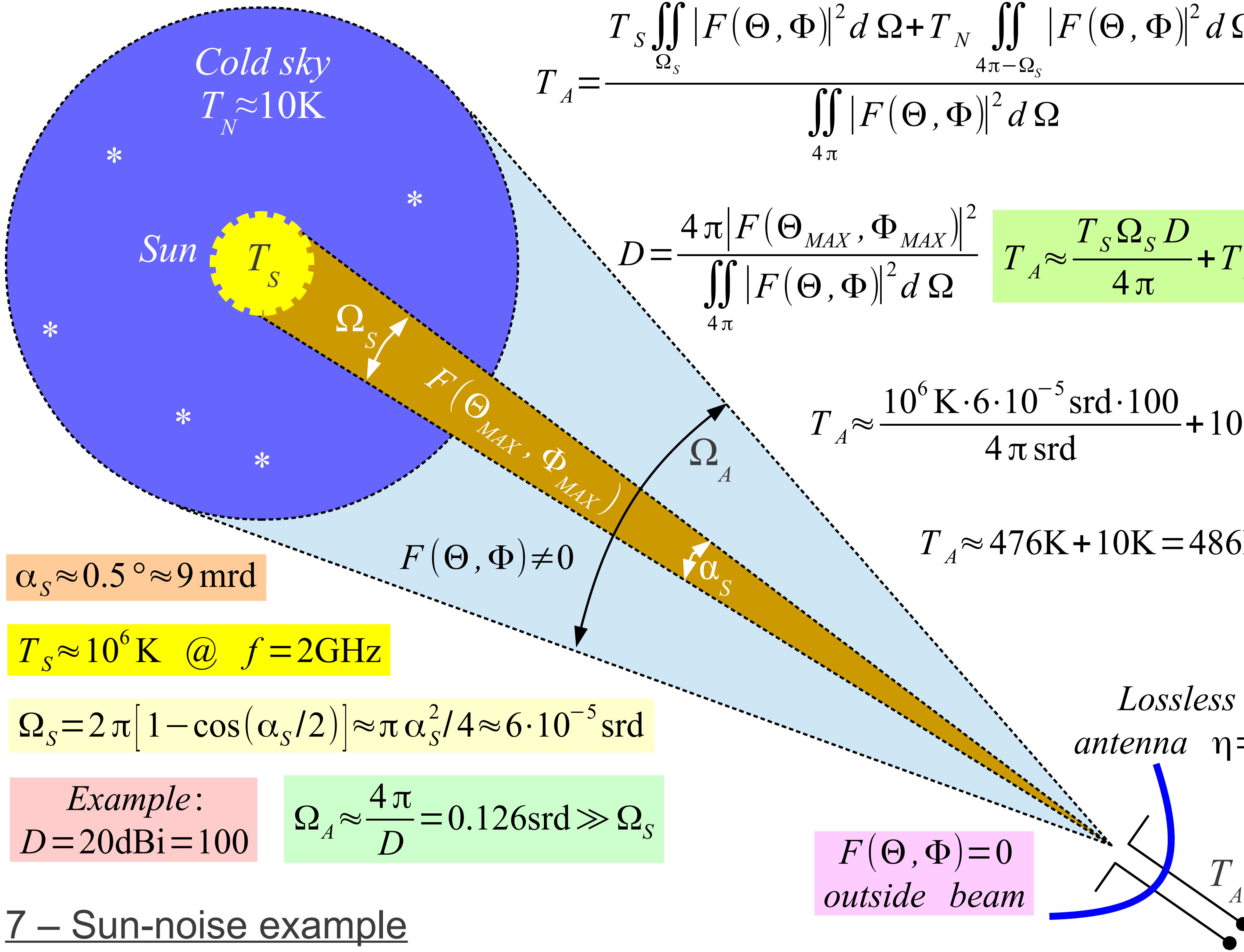
4 – Received thermal-noise power



5 – Thermal equilibrium



6 – Natural noise sources



$$T_A = \frac{T_S \iint_{\Omega_S} |F(\Theta,\Phi)|^2 d\Omega + T_N \iint_{4\pi - \Omega_S} |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$D = \frac{4\pi |F(\Theta_{MAX}, \Phi_{MAX})|^2}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$T_A \approx \frac{T_S \Omega_S D}{4\pi} + T_N$$

$$T_A \approx \frac{10^6 \text{K} \cdot 6 \cdot 10^{-5} \text{srd} \cdot 100}{4\pi \text{srd}} + 10\text{K}$$

$$T_A \approx 476\text{K} + 10\text{K} = 486\text{K}$$

$\alpha_S \approx 0.5° \approx 9 \text{mrd}$

$T_S \approx 10^6 \text{K}$ @ $f = 2\text{GHz}$

$\Omega_S = 2\pi[1 - \cos(\alpha_S/2)] \approx \pi\alpha_S^2/4 \approx 6 \cdot 10^{-5} \text{srd}$

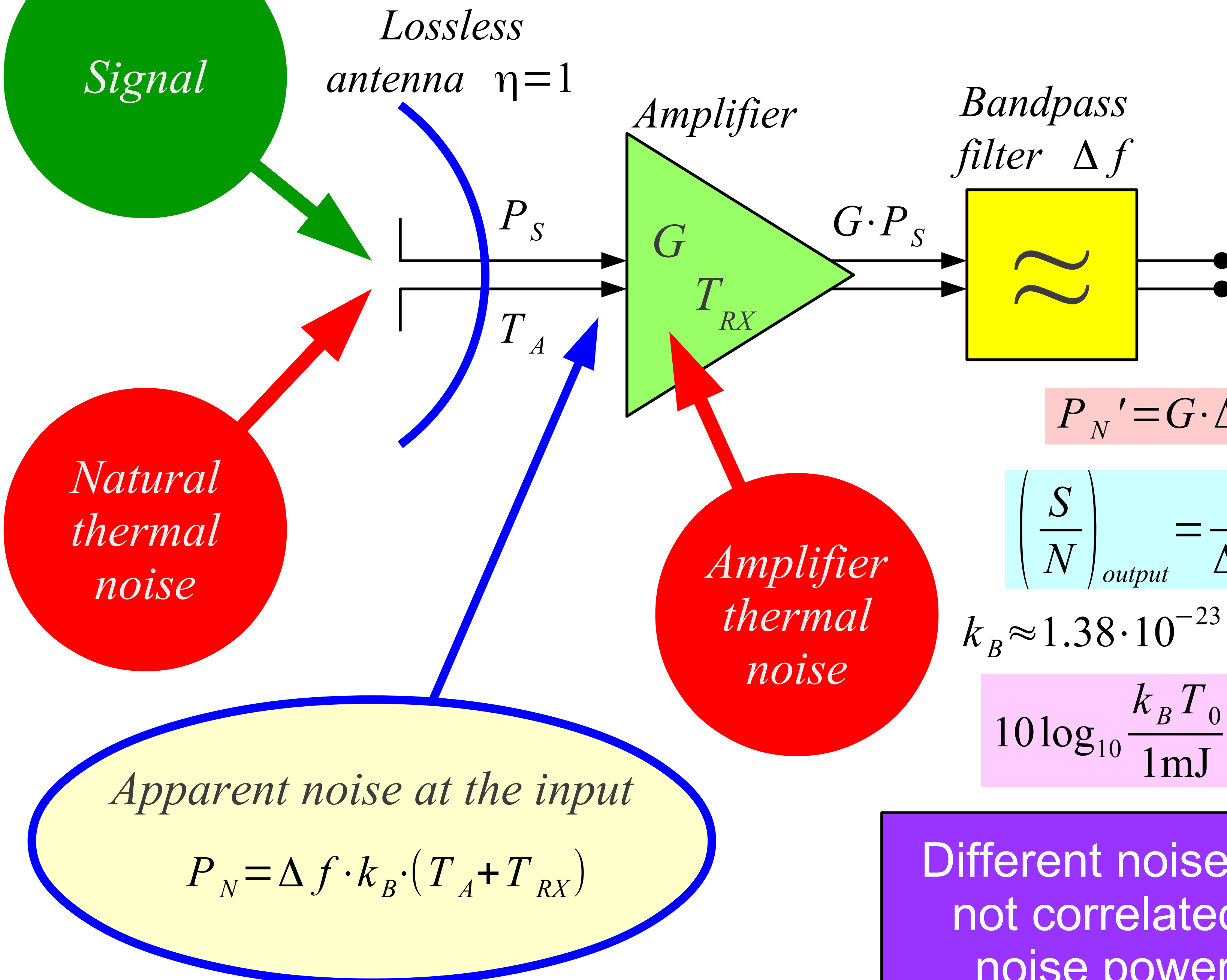
Example:
$D = 20\text{dBi} = 100$

$\Omega_A \approx \frac{4\pi}{D} = 0.126\text{srd} \gg \Omega_S$

$F(\Theta, \Phi) = 0$
outside beam

7 – Sun-noise example

$T_S \equiv$ *amplifier noise temperature reduced to the input!*



$$P_S' = G \cdot P_S$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S'}{P_N'}$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot (T_A + T_{RX})$$

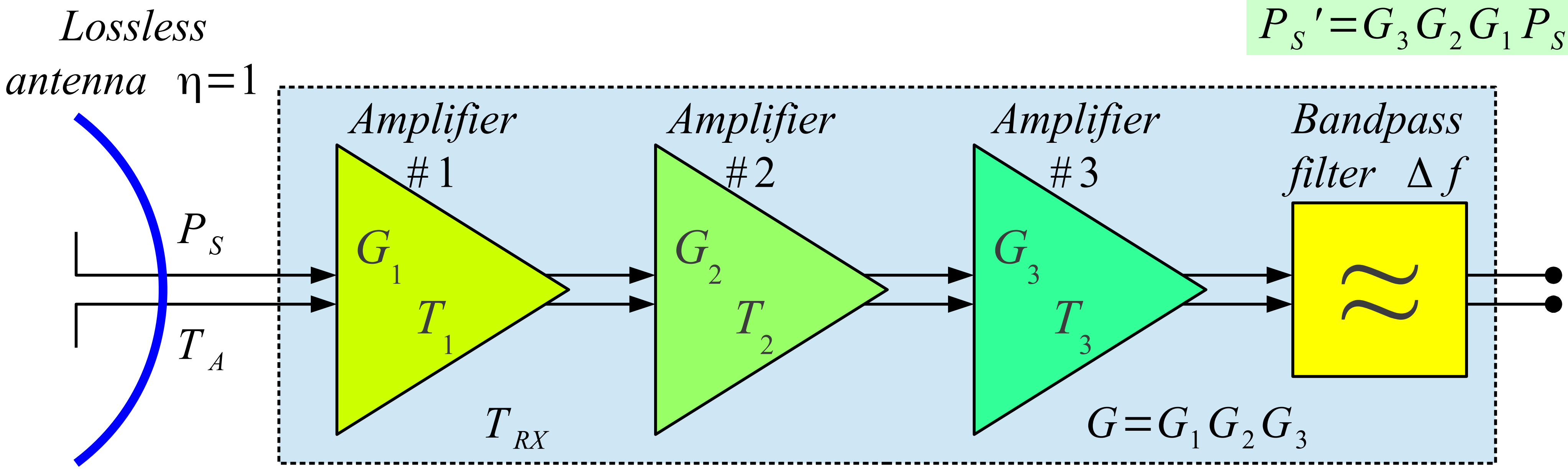
$$\left(\frac{S}{N}\right)_{output} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_{RX})}$$

$$k_B \approx 1.38 \cdot 10^{-23}\ \mathrm{J/K} \qquad T_0 = 290\mathrm{K}$$

$$10 \log_{10} \frac{k_B T_0}{1\mathrm{mJ}} \approx -174\ \mathrm{dBm/Hz}$$

Different noise sources are not correlated, therefore noise powers or noise temperatures are summed!

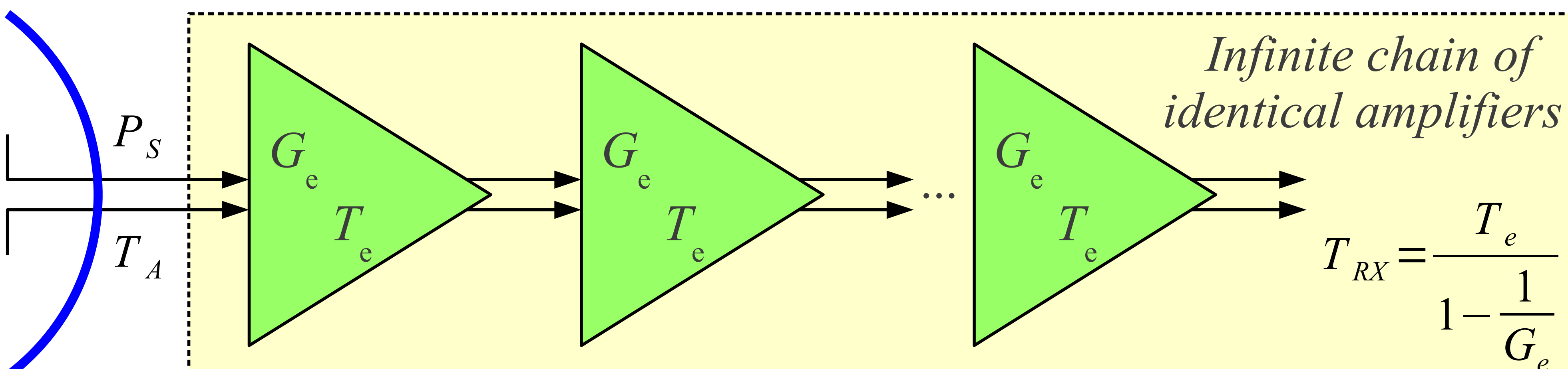
8 – Receiver signal-to-noise ratio



$$P_S' = G_3 G_2 G_1 P_S$$

$$P_N' = \Delta f\, k_B \left[G_3 G_2 G_1 (T_A + T_1) + G_3 G_2 T_2 + G_3 T_3 \right]$$

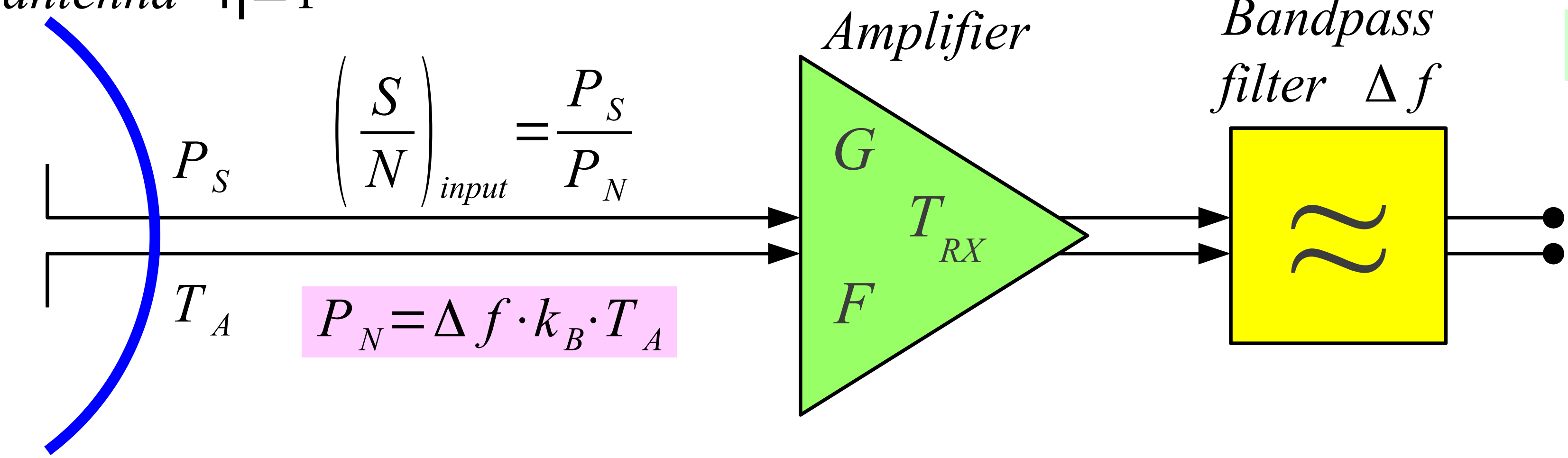
$$P_N' = G_3 G_2 G_1 \Delta f\, k_B (T_A + T_{RX}) \quad \rightarrow \quad T_{RX} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots$$



$$T_{RX} = \frac{T_e}{1 - \frac{1}{G_e}}$$

9 – Chain noise temperature

Lossless antenna $\eta=1$



$$\left(\frac{S}{N}\right)_{input}=\frac{P_S}{P_N}$$

$$P_N=\Delta f\cdot k_B\cdot T_A$$

$$P_N'=G\cdot\Delta f\cdot k_B\cdot(T_A+T_{RX})$$

$$P_S'=G\cdot P_S$$

$$\left(\frac{S}{N}\right)_{output}=\frac{P_S'}{P_N'}$$

Nonsense definition of the noise figure:

$$F=\frac{\left(\frac{S}{N}\right)_{input}}{\left(\frac{S}{N}\right)_{output}}=\frac{\dfrac{P_S}{\Delta f\, k_B T_A}}{\dfrac{G\,P_S}{G\,\Delta f\, k_B(T_A+T_{RX})}}=\frac{T_A+T_{RX}}{T_A}=1+\frac{T_{RX}}{T_A}$$

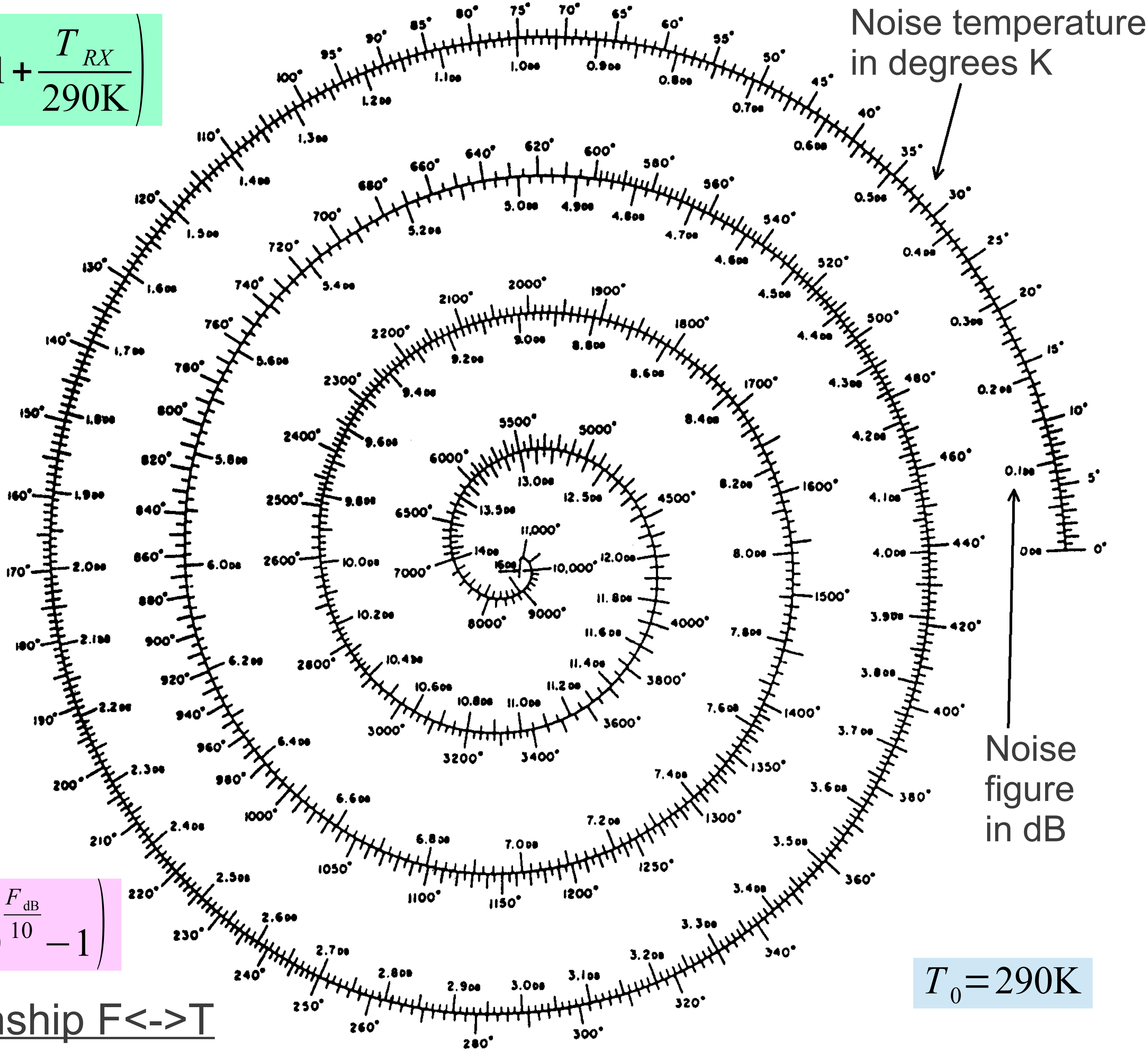
A property of an amplifier can not be a function of T_A!

Sensible definition $\quad F=1+\dfrac{T_{RX}}{T_0}$ @ $T_0=290\mathrm{K}$ $\leftrightarrow$ $T_{RX}=T_0(F-1)$

Logarithmic units $\quad F_{\mathrm{dB}}=10\log_{10}F=10\log_{10}\left(1+\dfrac{T_{RX}}{T_0}\right)$ $\leftrightarrow$ $T_{RX}=T_0\left(10^{\frac{F_{\mathrm{dB}}}{10}}-1\right)$

10 – Amplifier noise figure

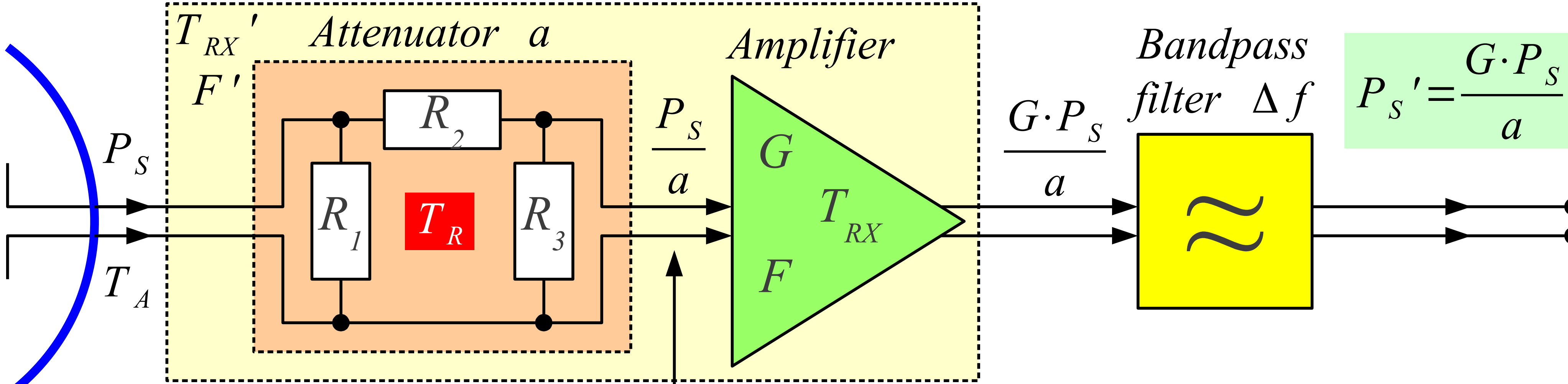
$$F_{\mathrm{dB}} = 10\log_{10}\left(1 + \frac{T_{RX}}{290\mathrm{K}}\right)$$



$$T_{RX} = 290\mathrm{K}\left(10^{\frac{F_{\mathrm{dB}}}{10}} - 1\right)$$

$$T_0 = 290\mathrm{K}$$

11 – Relationship F<->T



$$P_S' = \frac{G \cdot P_S}{a}$$

Lossless antenna $\eta = 1$

$$\frac{T_A}{a} + T_R\left(1 - \frac{1}{a}\right)$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot \left[\frac{T_A}{a} + T_R\left(1 - \frac{1}{a}\right) + T_{RX}\right]$$

$$T_{RX}' = T_R(a-1) + a\,T_{RX} \quad \longleftarrow \quad \left(\frac{S}{N}\right)_{output} = \frac{P_S'}{P_N'} = \frac{P_S}{\Delta f \cdot k_B \cdot \left[T_A + T_R(a-1) + a\,T_{RX}\right]}$$

$$F' = 1 + \frac{T_{RX}'}{T_0} = 1 + \frac{T_R}{T_0}(a-1) + a\frac{T_{RX}}{T_0}$$

Frequent case $T_R \approx T_0 = 290\text{K}$

$$F' \approx a + a\frac{T_{RX}}{T_0} = a\left(1 + \frac{T_{RX}}{T_0}\right) = a \cdot F$$

$$F_{dB}' \approx a_{dB} + F_{dB}$$

Attenuator examples $T_R \approx T_0 = 290\text{K}$

$$F' \approx a \cdot F \qquad F_{dB}' \approx a_{dB} + F_{dB}$$

(1) *lossy antenna* $a_{dB} = -10\log_{10}\eta$

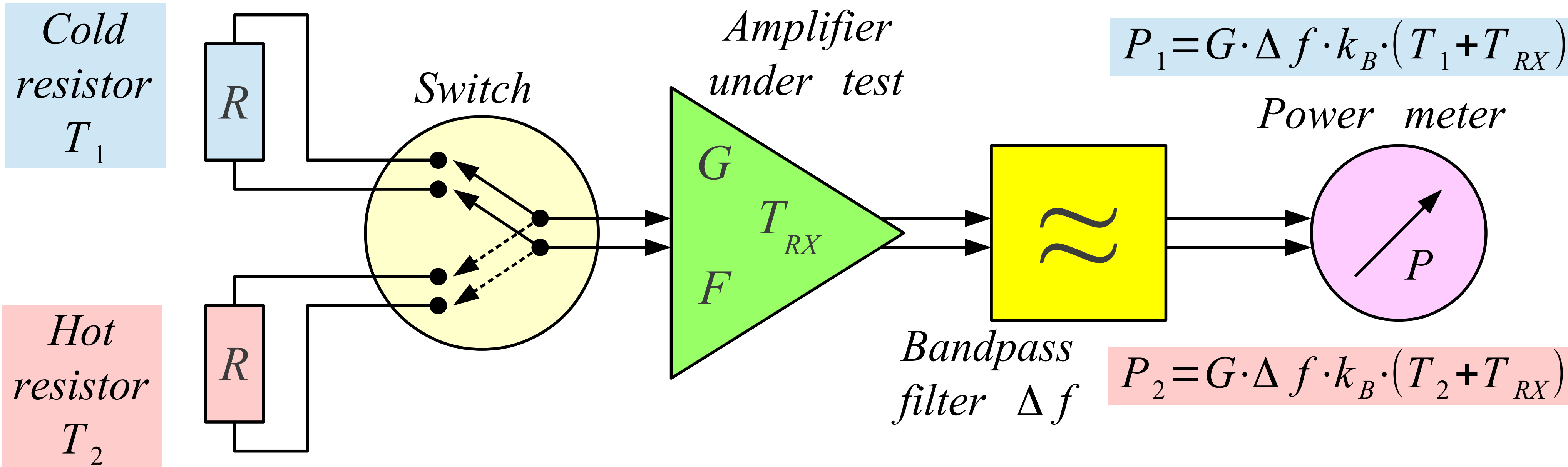
(2) *lossy transmission line* a_{dB}

(3) *lossy bandpass filter* a_{dB}

(4) *passive-mixer loss* a_{dB}

12 – Attenuator noise

Amplifier device	Gain G [dB]	Noise temperature T_{RX} [K]	Noise figure F_{dB} [dB]
Vacuum tube with control grid (triode, pentode)	10↔20	1600↔9000	8↔15
Vacuum tube with speed modulation (klystron, TWT)	20↔50	3000↔30000	10↔20
Parametric amplifier (room temperature)	10↔15	75↔300	1↔3
Si BJT, JFET or MOSFET (room temperature)	10↔20	75↔300	1↔3
GaAs FET or HEMT (room temperature)	10↔15	20↔120	0.3↔1.5
GaAs FET or HEMT (liquid-nitrogen 77K)	10↔15	7↔35	0.1↔0.5
Si or GaAs MMIC amplifier	10↔25	170↔1600	2↔8
Operational amplifier	40↔100	10^4↔10^9	16↔66



The unknowns $G\cdot\Delta f\cdot k_B$ *cancel in the Y ratio!*

$$Y = \frac{P_2}{P_1} = \frac{T_2 + T_{RX}}{T_1 + T_{RX}}$$

$$T_{RX} = \frac{T_2 - Y\cdot T_1}{Y-1}$$

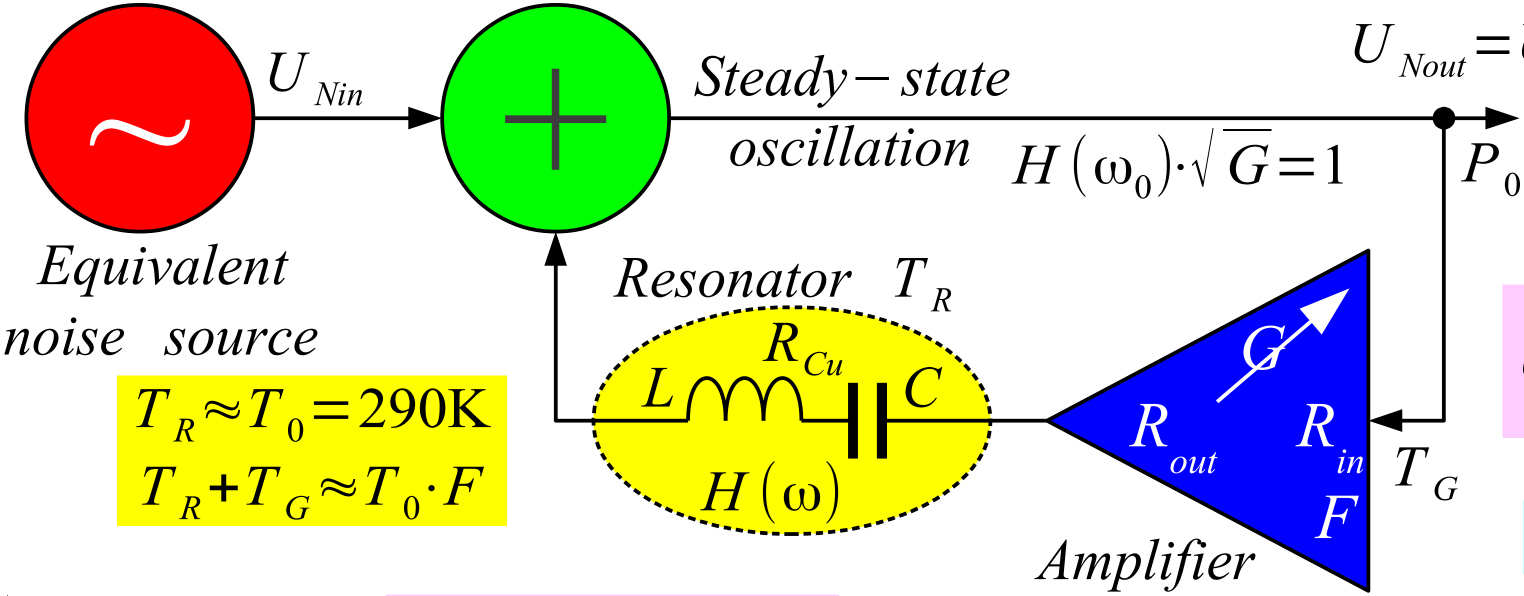
$$T_0 = 290\mathrm{K}$$

$$F_{dB} = 10\log_{10}\left[1 + \frac{T_2 - Y\cdot T_1}{(Y-1)\cdot T_0}\right]$$

Resistor type	Temperature
Antenna into cold sky	~20K
Liquid N_2 cooled R	~77K
Antenna into absorber	~290K
R at room temperature	~290K
Light-bulb filament as R	~2000K
Ionized gas as R	~10^4K
Avalanche breakdown	~10^6K

14 – Hot/cold method

15 – Oscillator phase noise



$$U_{Nout} = U_{Nin} + H(\omega)\cdot\sqrt{G}\cdot U_{Nout}$$

Valid at $P_{Nout} \ll P_0$

$$U_{Nout} = \frac{U_{Nin}}{1 - H(\omega)\cdot\sqrt{G}}$$

$$\Sigma R = R_{out} + R_{Cu} + R_{in}$$

$$H(\omega)\cdot\sqrt{G} = \frac{\Sigma R}{\Sigma R + j\omega L + \dfrac{1}{j\omega C}} \approx \frac{1}{1 + j2Q_L\dfrac{\Delta\omega}{\omega_0}}$$

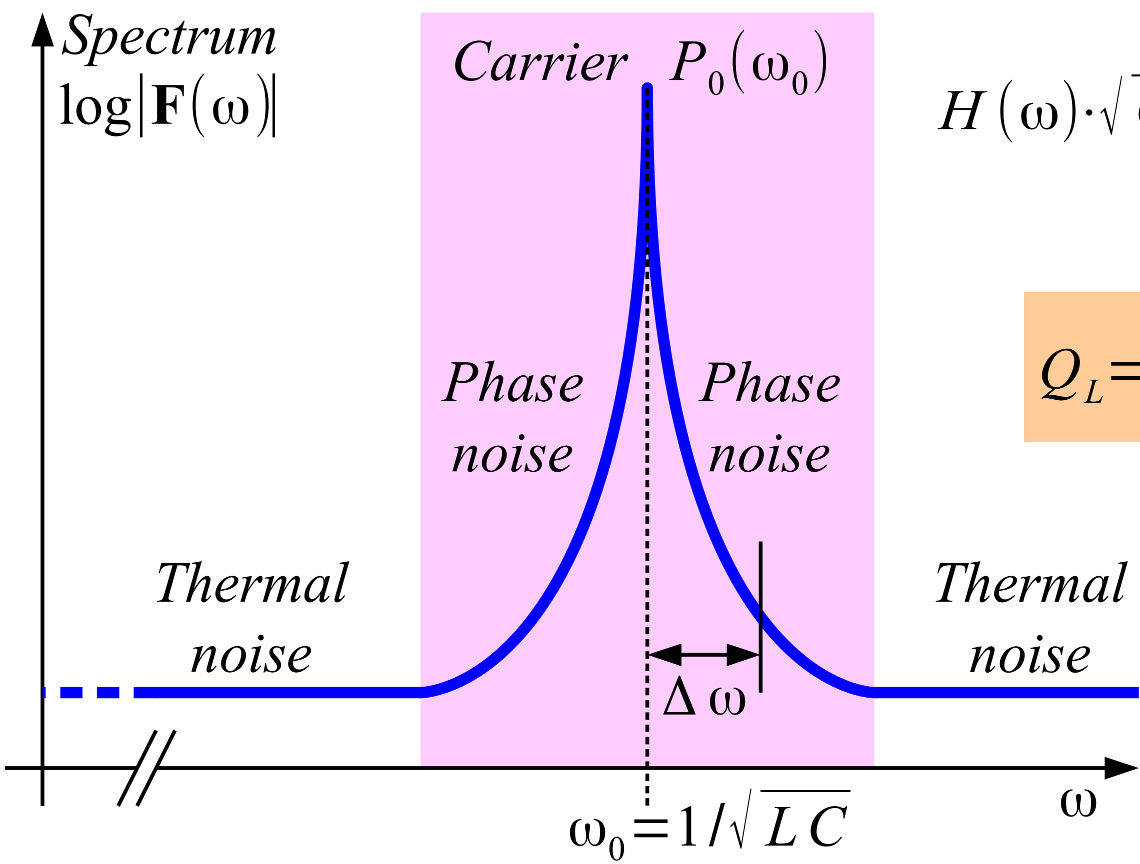
$$Q_L = \frac{\omega_0 L}{\Sigma R}$$

$$U_{Nout} \approx U_{Nin}\cdot\left(1 + \frac{\omega_0}{j2Q_L\Delta\omega}\right)$$

$$P_{Nout} \approx P_{Nin}\cdot\left[1 + \left(\frac{\omega_0}{2Q_L\Delta\omega}\right)^2\right]$$

Amplitude and phase noise

$$P_{Nout} \approx P_{Nin}\cdot\left[1 + \left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]$$



Normalized phase-noise spectral density

Saturation removes amplitude noise $P_\phi = P_{Nout}/2$

$$\frac{dP_{Nin}}{df} = N_0 = k_B(T_R + T_G) \approx k_B T_0 F$$

$$L(\Delta f) = \frac{1}{P_0}\cdot\frac{dP_\phi}{df} = \frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right) \quad [\mathrm{Hz}^{-1}]$$

Phase noise only

$P_0 \equiv$ *carrier power*

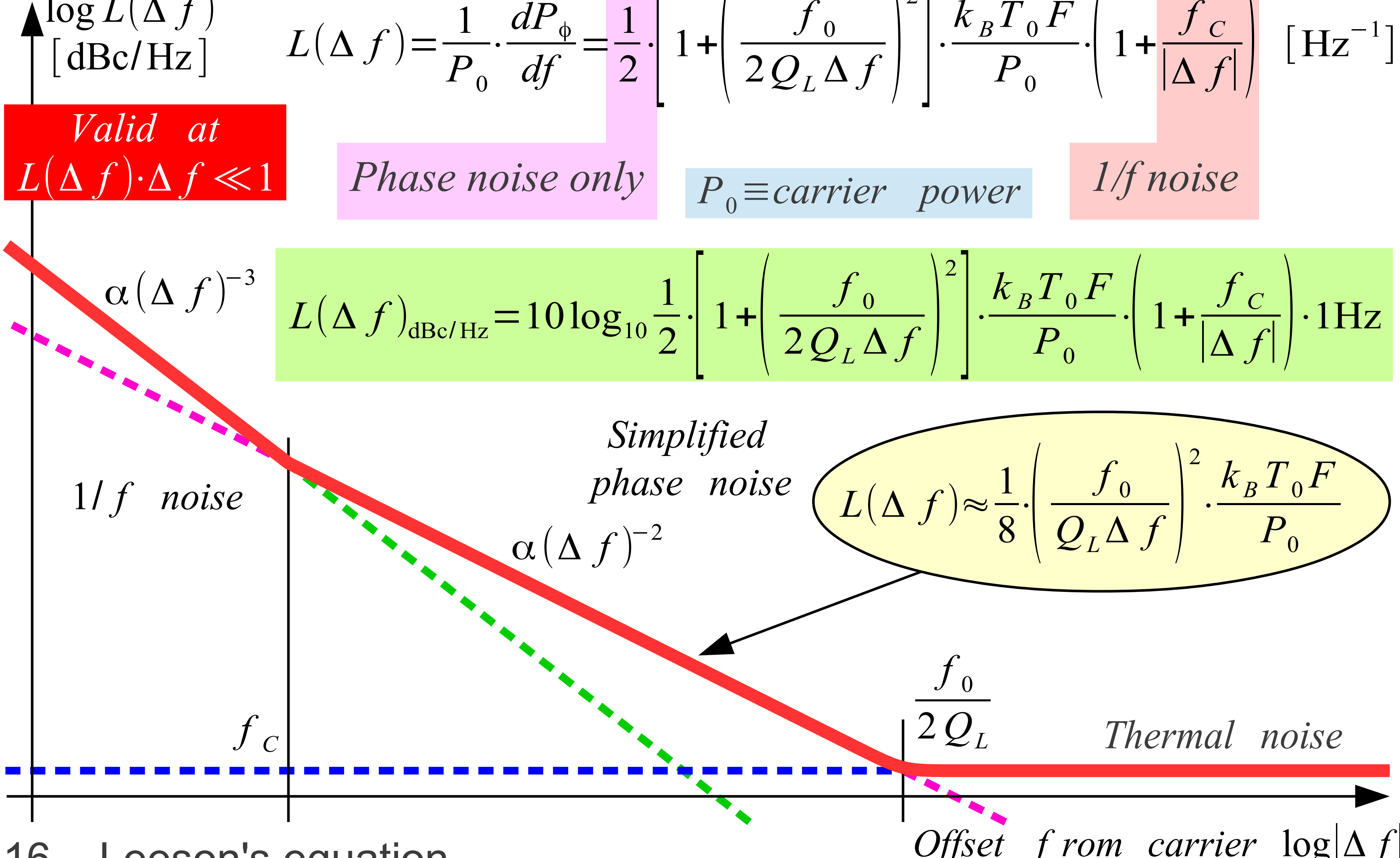
1/f noise

Valid at $L(\Delta f)\cdot\Delta f \ll 1$

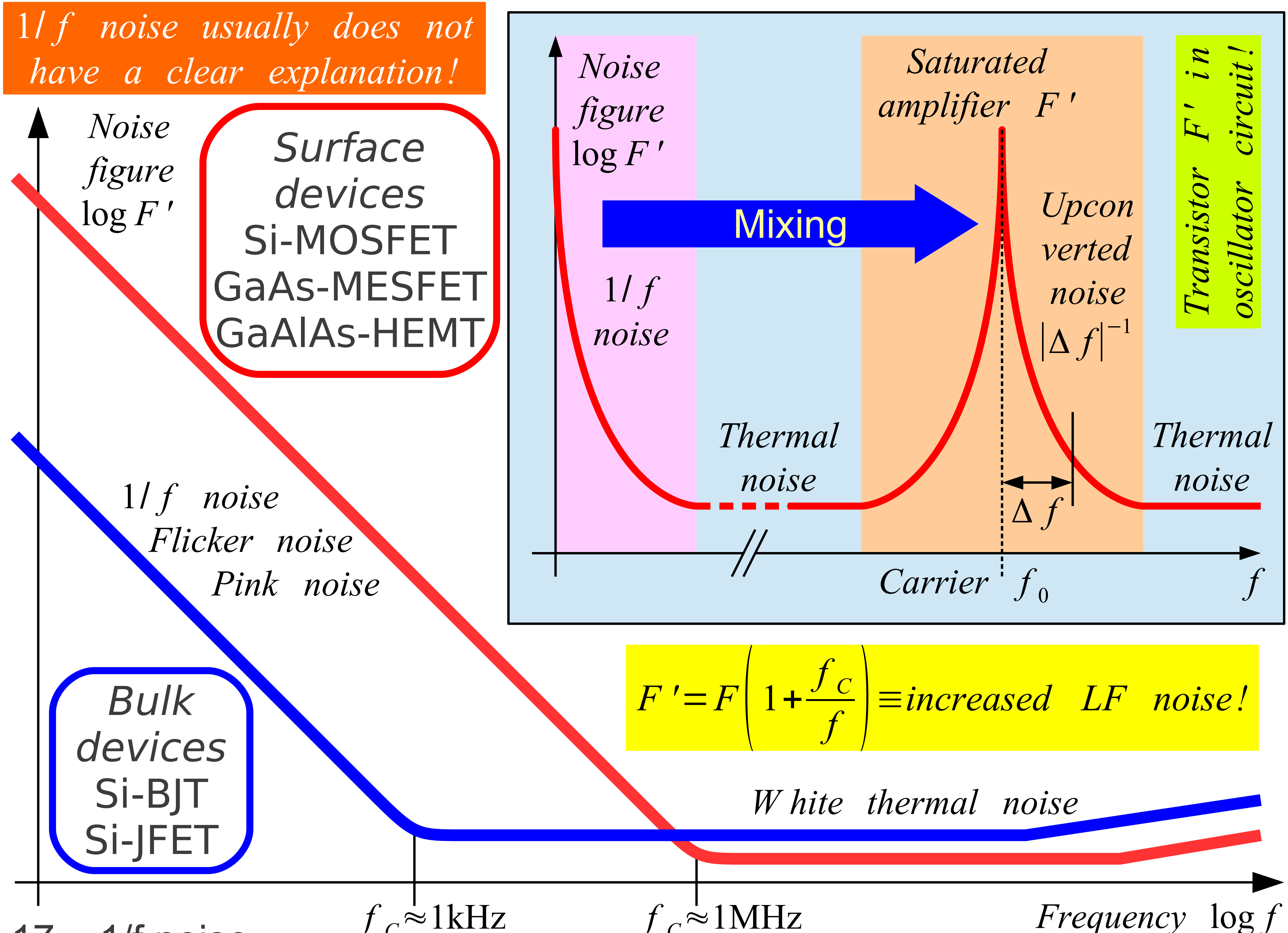
$$L(\Delta f)_{\mathrm{dBc/Hz}} = 10\log_{10}\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)\cdot 1\mathrm{Hz}$$

Simplified phase noise

$$L(\Delta f) \approx \frac{1}{8}\cdot\left(\frac{f_0}{Q_L\Delta f}\right)^2\cdot\frac{k_B T_0 F}{P_0}$$



16 – Leeson's equation


1/f noise usually does not have a clear explanation!
Noise figure log F'
Surface devices
Si-MOSFET
GaAs-MESFET
GaAlAs-HEMT
Noise figure log F'
Saturated amplifier F'
Transistor F' in oscillator circuit!
Mixing
1/f noise
Upconverted noise |Δf|^−1
Thermal noise
Thermal noise
Δf
Carrier f0
f
1/f noise
Flicker noise
Pink noise
F' = F(1 + fC/f) ≡ increased LF noise!
Bulk devices
Si-BJT
Si-JFET
White thermal noise
fC≈1kHz
fC≈1MHz
Frequency log f

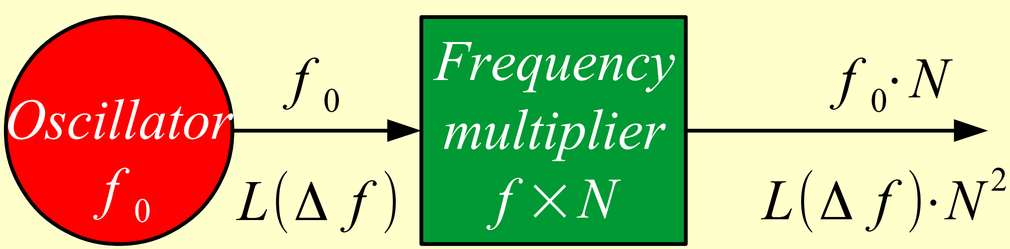
17 – 1/f noise

The loaded resonator quality Q_L defines the oscillator phase noise!

$$L(\Delta f)=\frac{1}{2}\cdot\left[1+\boxed{\left(\frac{f_0}{2Q_L\Delta f}\right)^2}\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)$$

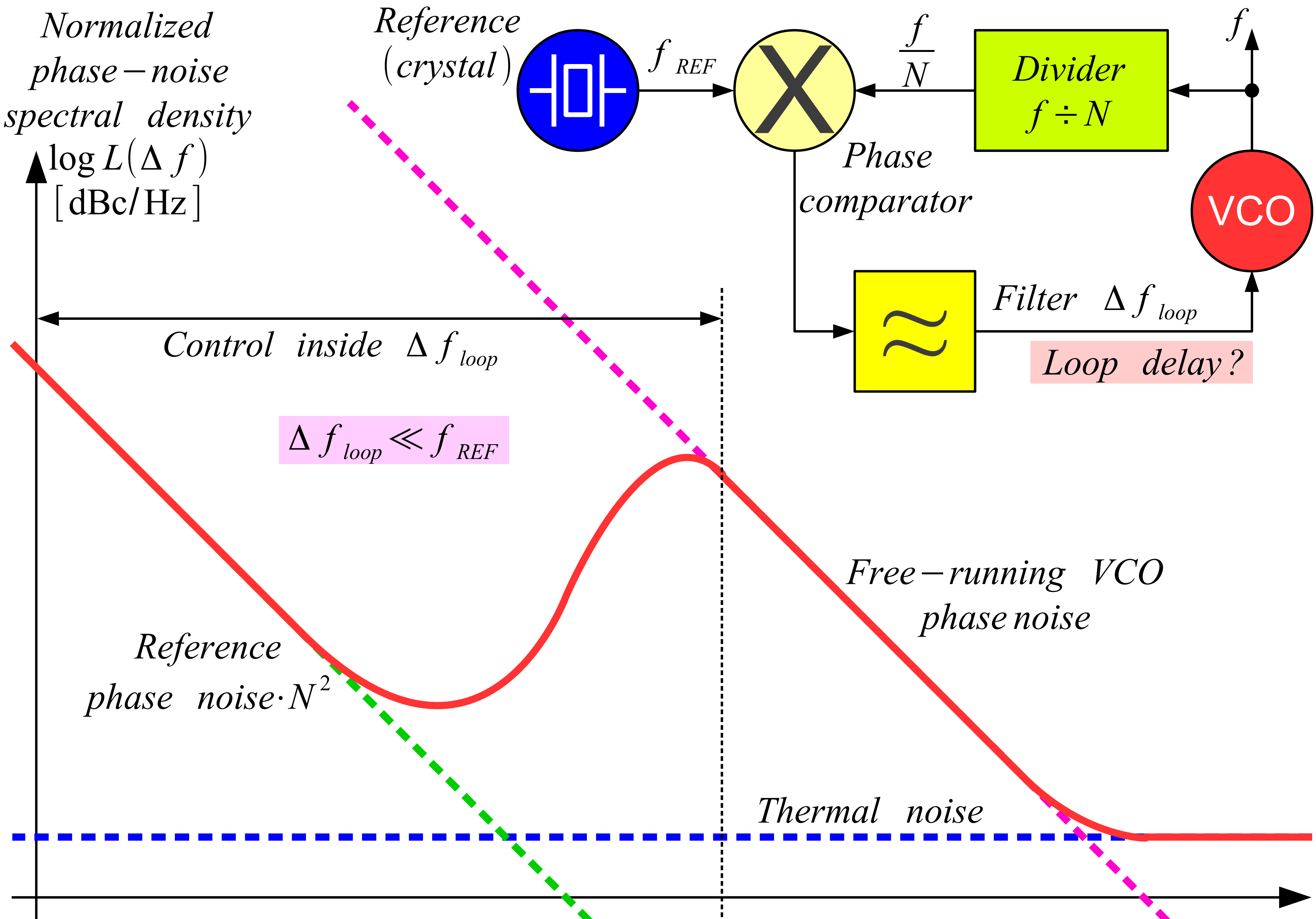
Variable-frequency oscillators	Q_L
RC VCO	~1
BWO tube	~1
Varactor-tuned LC VCO	10↔30
YIG ($Y_3Fe_5O_{12}$) oscillator	300↔1000

Fixed-frequency oscillators	Q_L
RC multivibrator	~1
LC resonator	30↔100
Cavity resonator	1000↔3000
Ceramic dielectric resonator	1000↔3000
AT-cut quartz crystal (fundamental mode)	3000↔10000
AT-cut quartz crystal (third/fifth overtone)	10000↔30000
Electro-optical delay line ($)	~10^6 (noisy!)
Sapphire dielectric resonator ($$$)	~3.10^5
Red HeNe LASER	~10^8

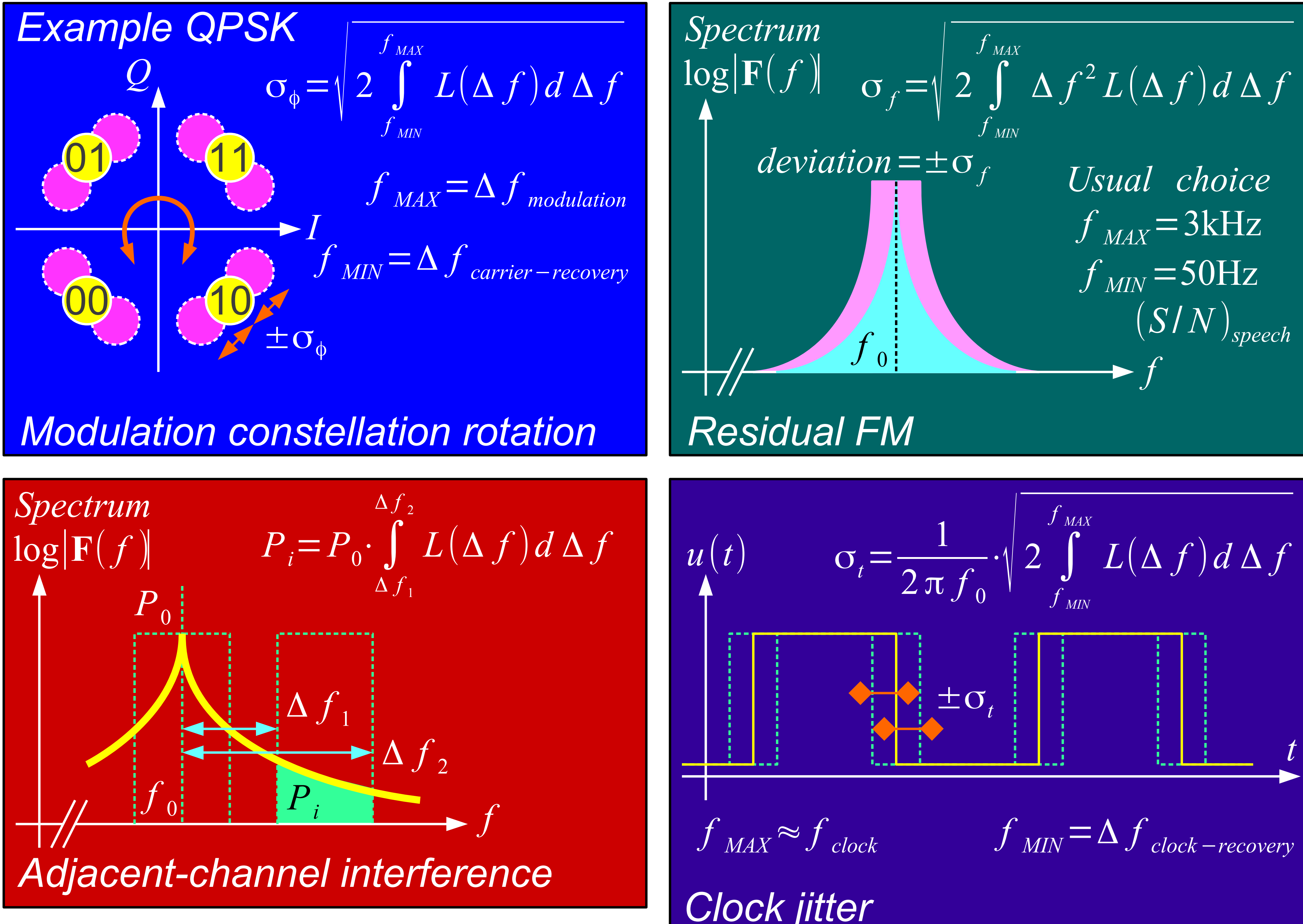


The phase noise multiplies with the square of the frequency multiplication factor!

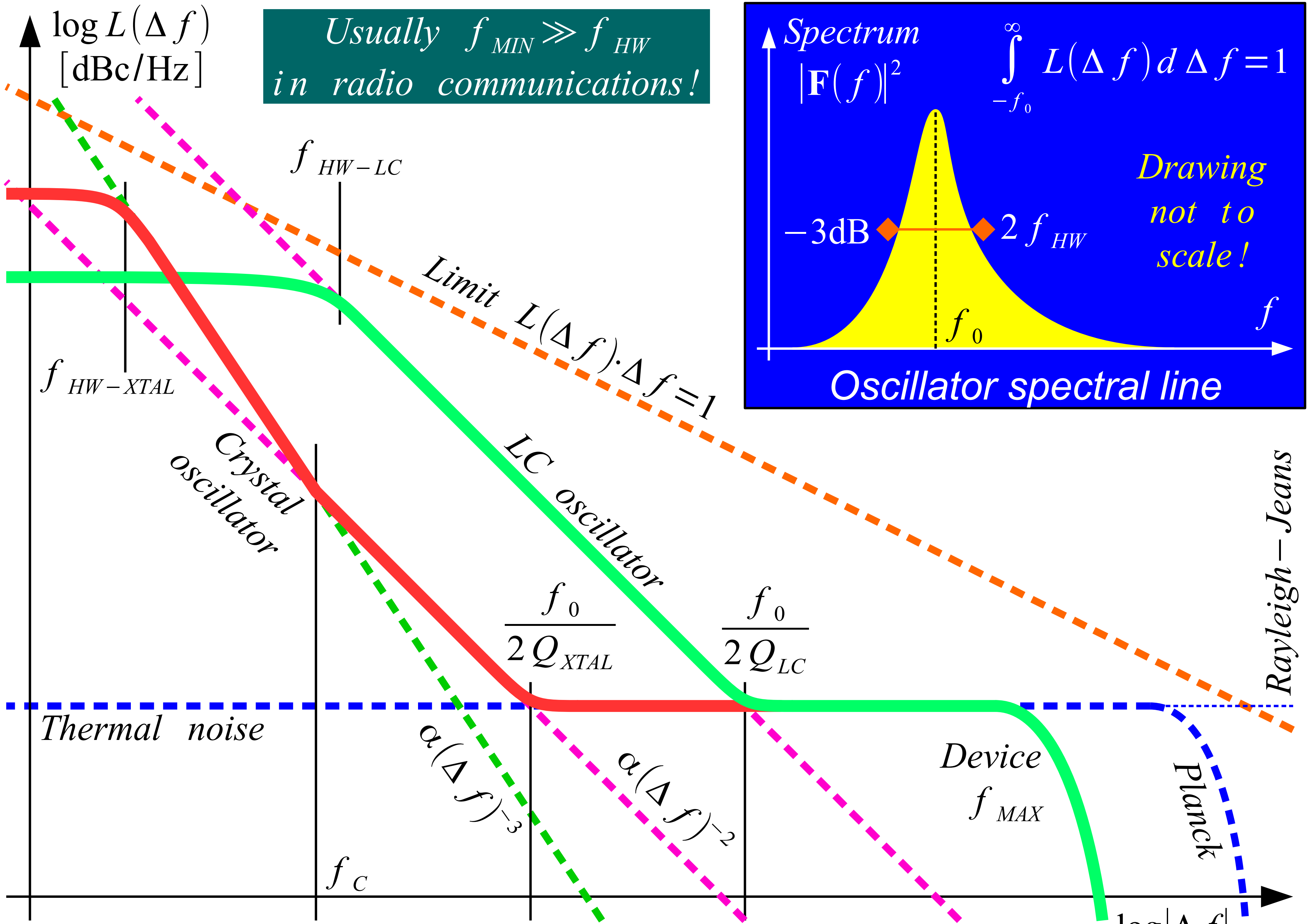
18 – Resonator quality Q



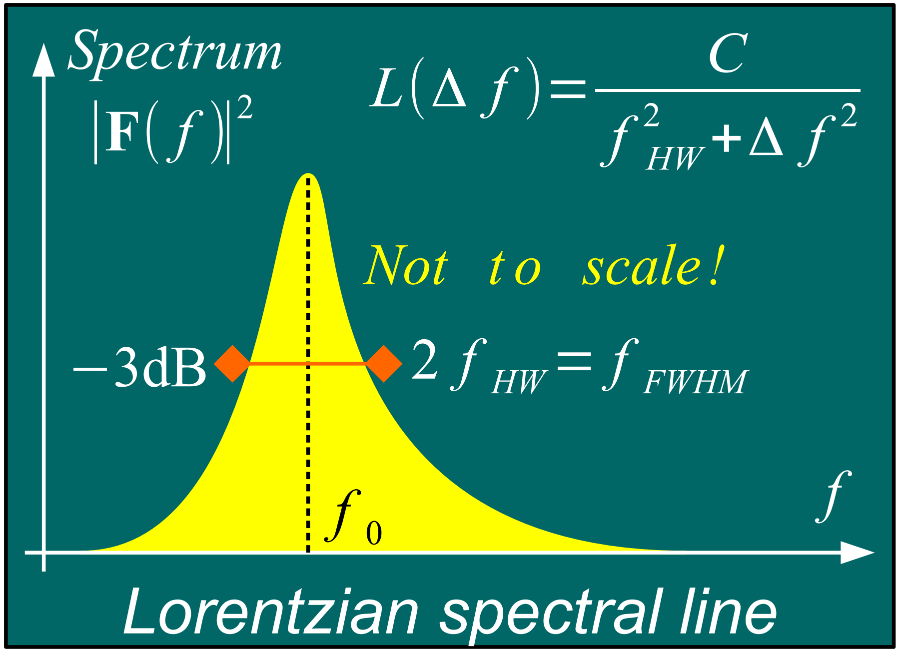
19 – Phase-Locked Loop (PLL)



20 – Effects of phase noise



21 – Phase noise without approximations



Flat thermal noise can be neglected: device f_{MAX} or Planck law

LC−oscillator $1/f$ noise can be neglected

$$L(\Delta f)=\frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{1}{f_{HW}^2+\Delta f^2}\cdot\frac{k_B T_0 F}{P_0}$$

Lorentzian line in Leeson's equation

$$\int_{-f_0}^{\infty} L(\Delta f)\,d\Delta f=1\approx\int_{-\infty}^{\infty} L(\Delta f)\,d\Delta f=\frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{k_B T_0 F}{P_0}\int_{-\infty}^{\infty}\frac{1}{f_{HW}^2+\Delta f^2}\,d\Delta f=$$

$$=\frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{k_B T_0 F}{P_0}\cdot\left[\frac{1}{f_{HW}}\cdot\arctan\frac{\Delta f}{f_{HW}}\right]_{\Delta f=-\infty}^{\Delta f=\infty}=\frac{k_B T_0 F}{8P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{\pi}{f_{HW}}$$

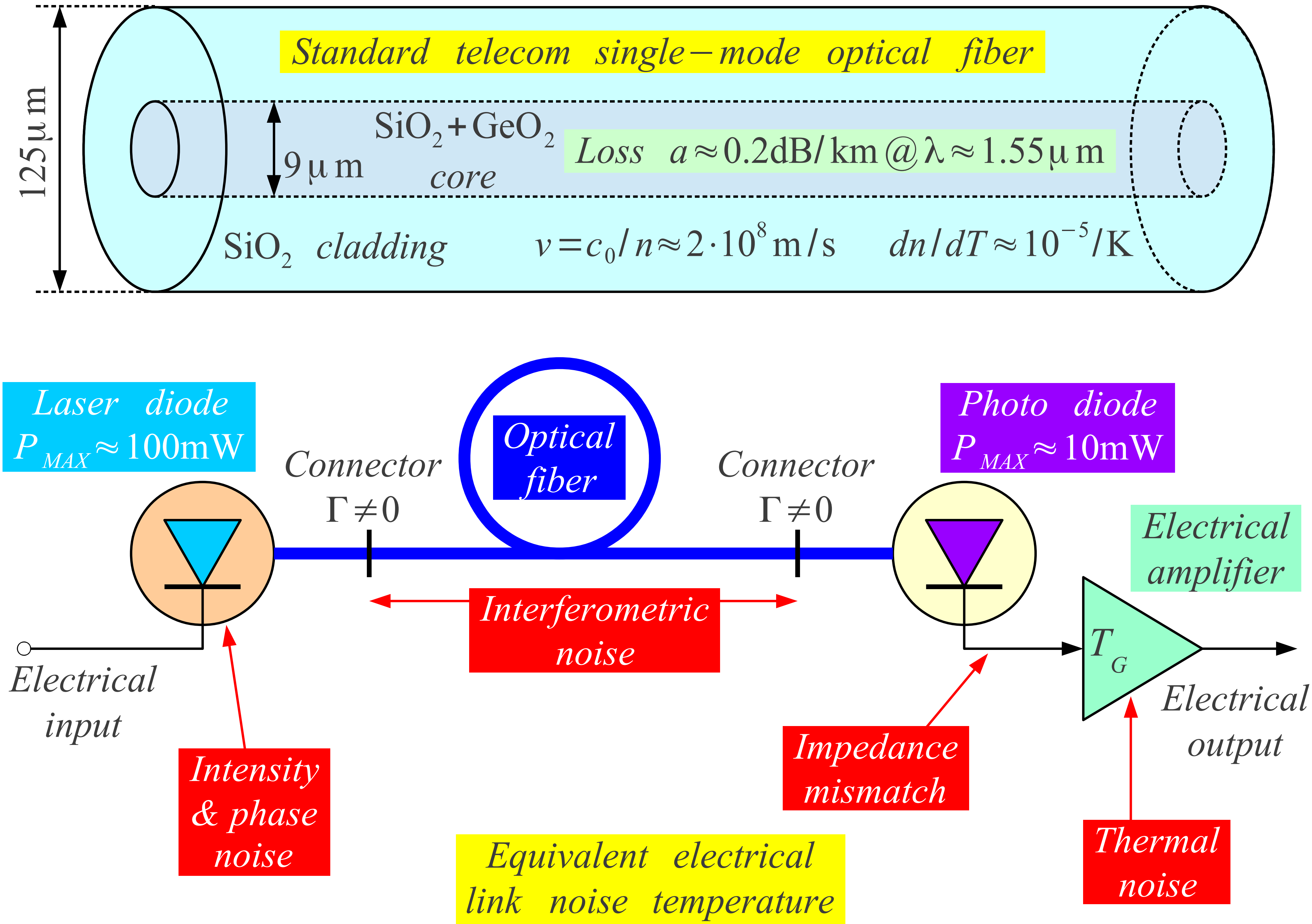
$$f_{HW}=\frac{\pi k_B T_0 F}{8P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2$$

Example $f_0=3\mathrm{GHz}$ $Q_L=10$
$P_0=0.1\mathrm{mW}$ $F=10\mathrm{dB}$
$f_{HW}=14\mathrm{Hz}$ $f_{FWHM}=28\mathrm{Hz}$

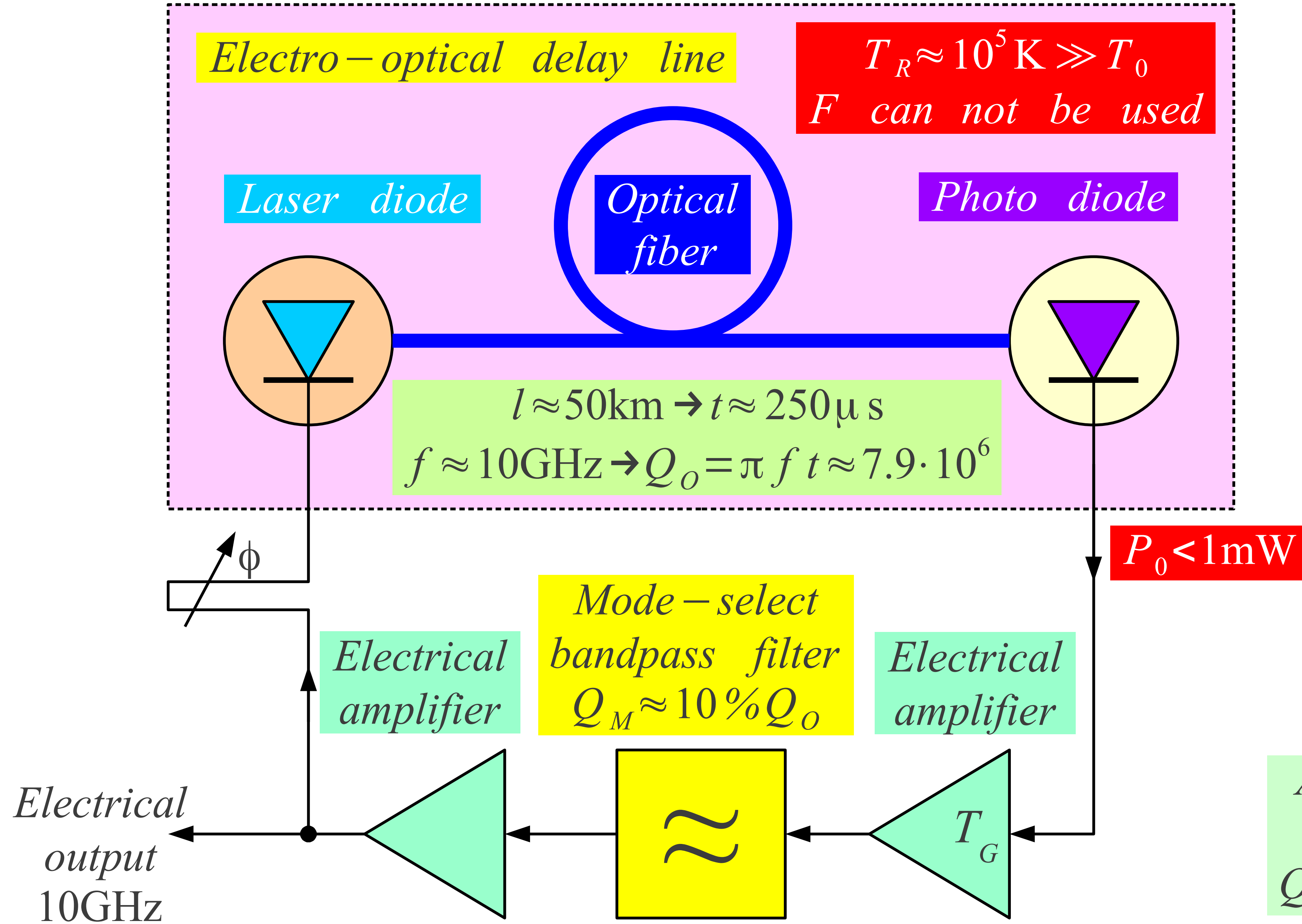
$$C=\frac{k_B T_0 F}{8P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2=\frac{f_{HW}}{\pi}$$

$$L(\Delta f)=\frac{f_{HW}/\pi}{f_{HW}^2+\Delta f^2}$$

22 – Lorentzian spectral-line width



23 – Optical-fiber link

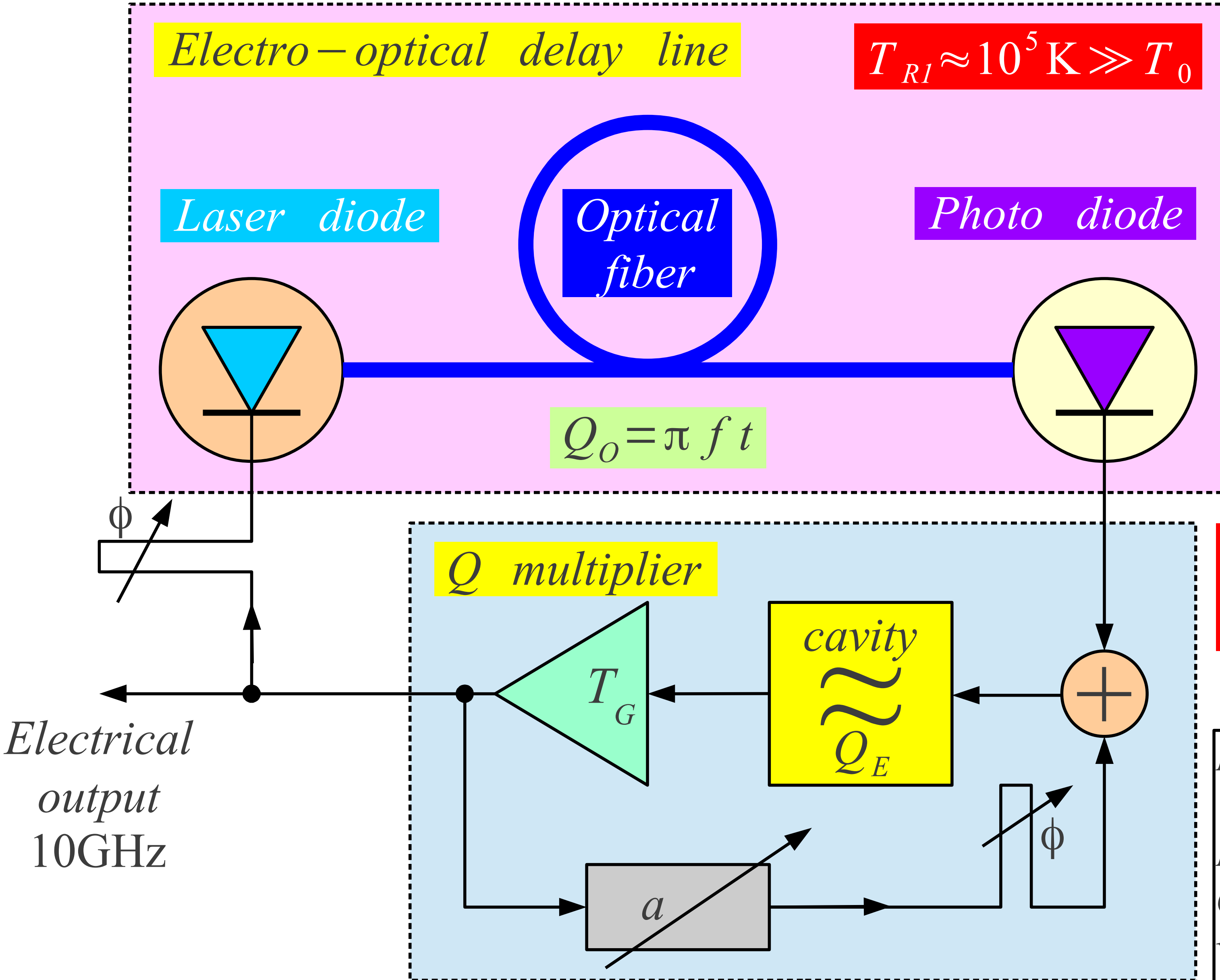


Simplified Leeson $L(\Delta f) \approx \frac{1}{8} \cdot \left(\frac{f_0}{Q_L \Delta f} \right)^2 \cdot \frac{k_B (T_G + T_R)}{P_0}$

Advantage:
Very high $Q_L \approx Q_O + Q_M$

Disdvantages:
Very high T_R
Low P_0
Difficult Q_M

24 – Opto-electronic oscillator



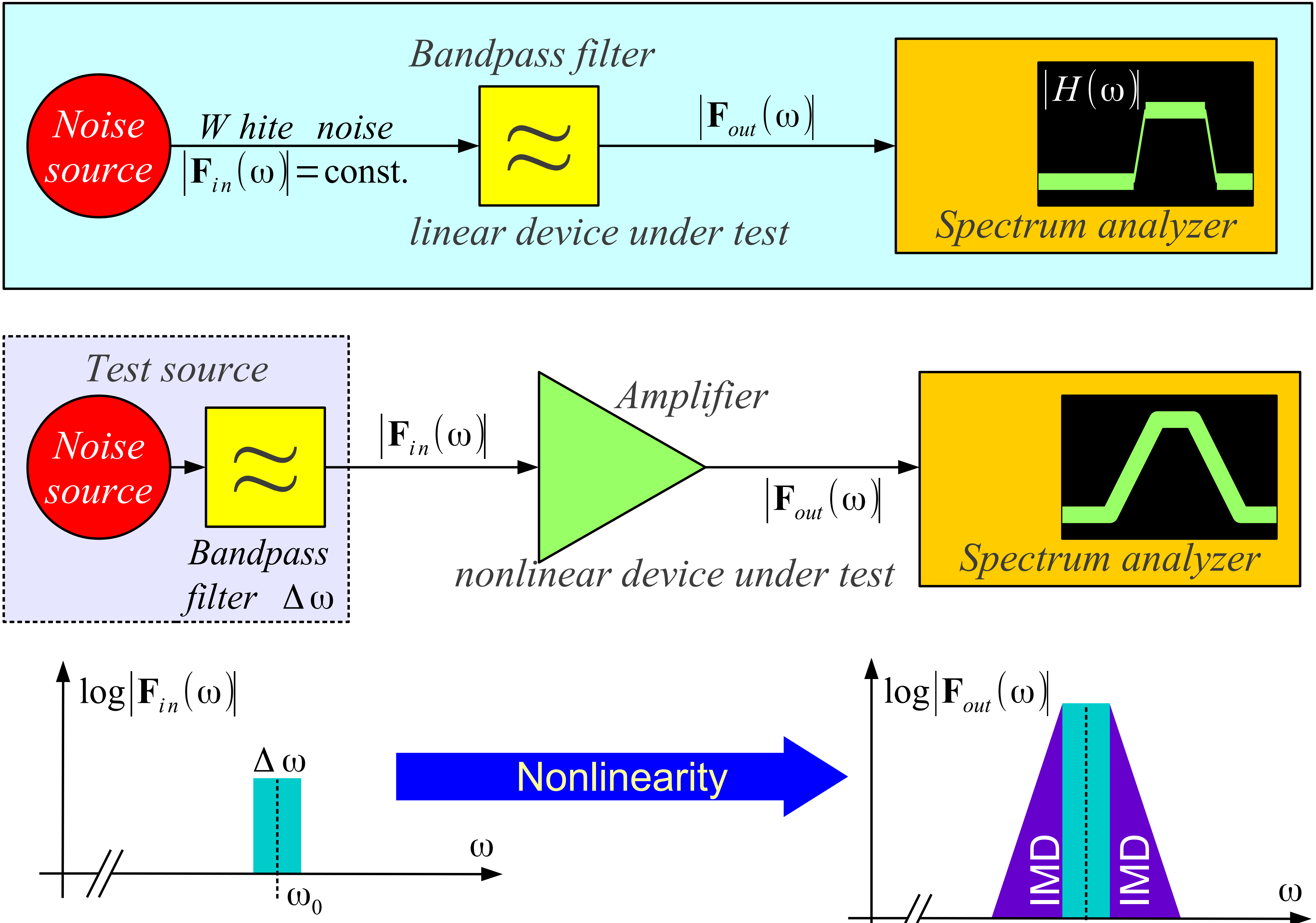
$$Q_M = m\,Q_E$$
$$T_{R2} \approx m^2 (T_0 + T_G)$$

Simplified Leeson $L(\Delta f) \approx \frac{1}{8} \cdot \left(\frac{f_0}{Q_L \Delta f} \right)^2 \cdot \frac{k_B (T_{R1} + T_{R2})}{P_0}$

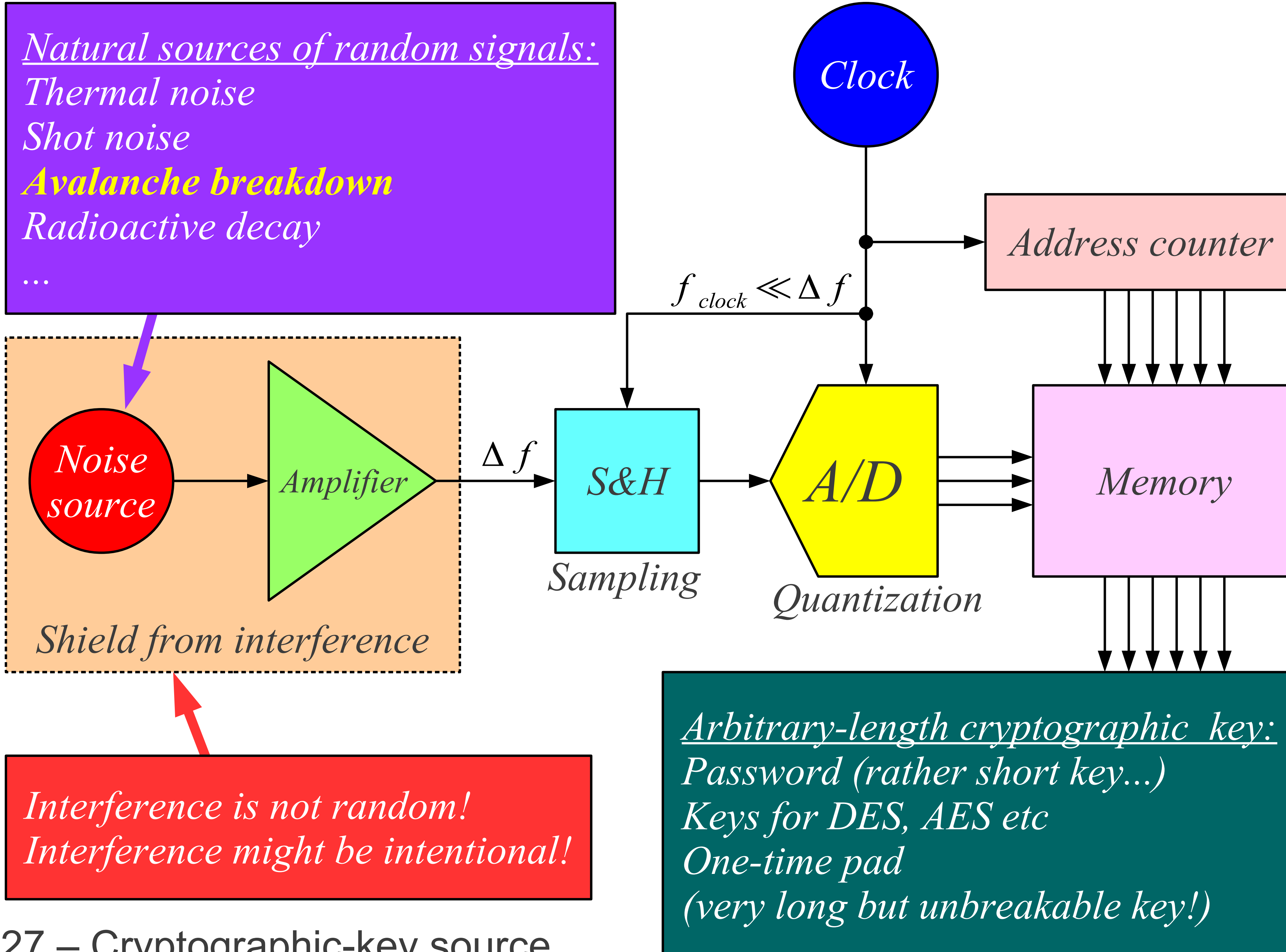
$$Q_L \approx Q_O + Q_M$$

BOGATAJ, Luka, VIDMAR, Matjaž, BATAGELJ, Boštjan: Opto-electronic oscillator with quality multiplier, IEEE transactions on microwave theory and Techniques, ISSN 0018-9480. Feb. 2016, vol. 64, no. 2, pp. 663-668.

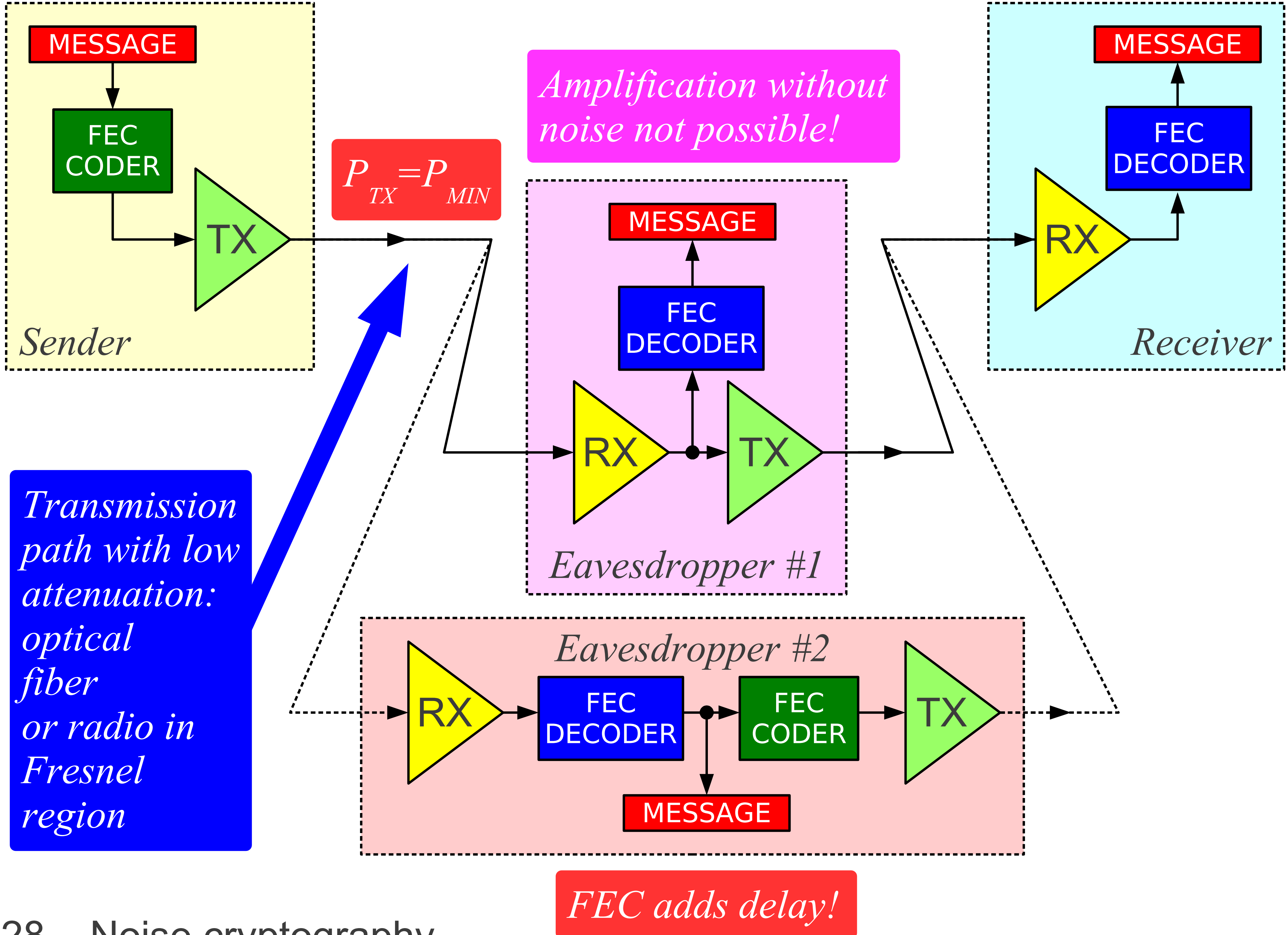
25 – Q multiplier

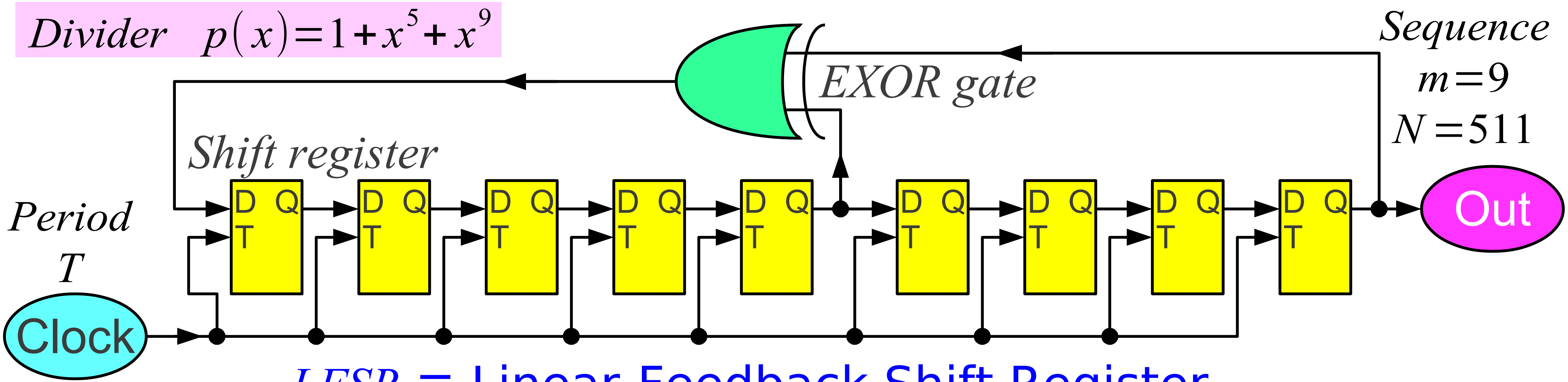


26 – Noise as test signal



27 – Cryptographic-key source


MESSAGE
FEC
CODER
TX
Sender
$P_{TX} = P_{MIN}$
Amplification without noise not possible!
MESSAGE
FEC
DECODER
RX
TX
Eavesdropper #1
MESSAGE
FEC
DECODER
RX
Receiver
Transmission path with low attenuation: optical fiber or radio in Fresnel region
Eavesdropper #2
RX
FEC
DECODER
FEC
CODER
TX
MESSAGE
FEC adds delay!



LFSR ≡ Linear-Feedback Shift Register

Primitive polynomial $p(x)=1+x^l+x^m \rightarrow$ *max sequnce length* $N=2^m-1$

2^{m-1} ones and 2^{m-1}-1 zeros arranged in groups of

1X m ones, m-1 zeros

1X m-2 ones and zeros

2X m-3 ones and zeros

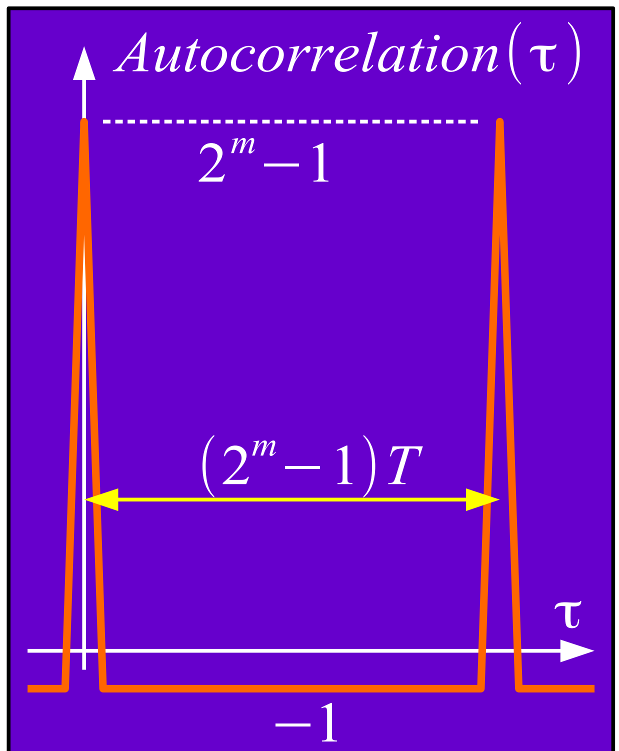
4X m-4 ones and zeros

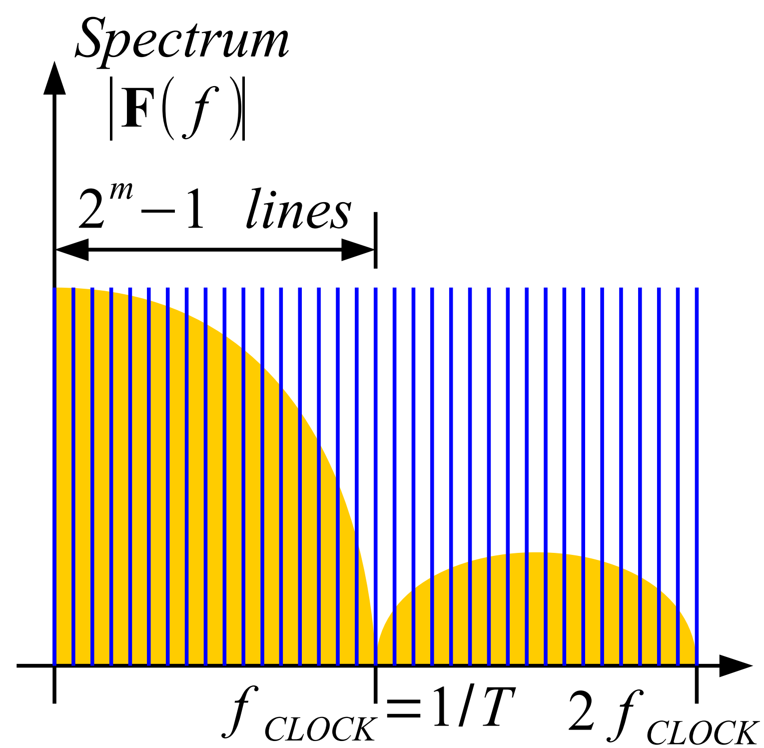
....

2^{m-5} groups 111 and 000

2^{m-4} groups 11 and 00

2^{m-3} individual 1 and 0





29 – LFSR pseudo-random sequences

Sound and appear as white noise!

Two-valued autocorrelation with a single very pronounced peak:

- synchronization headers for data frames
- spreading sequences in CDMA
- accurate time transfer in radio navigation (GPS, GLONASS)

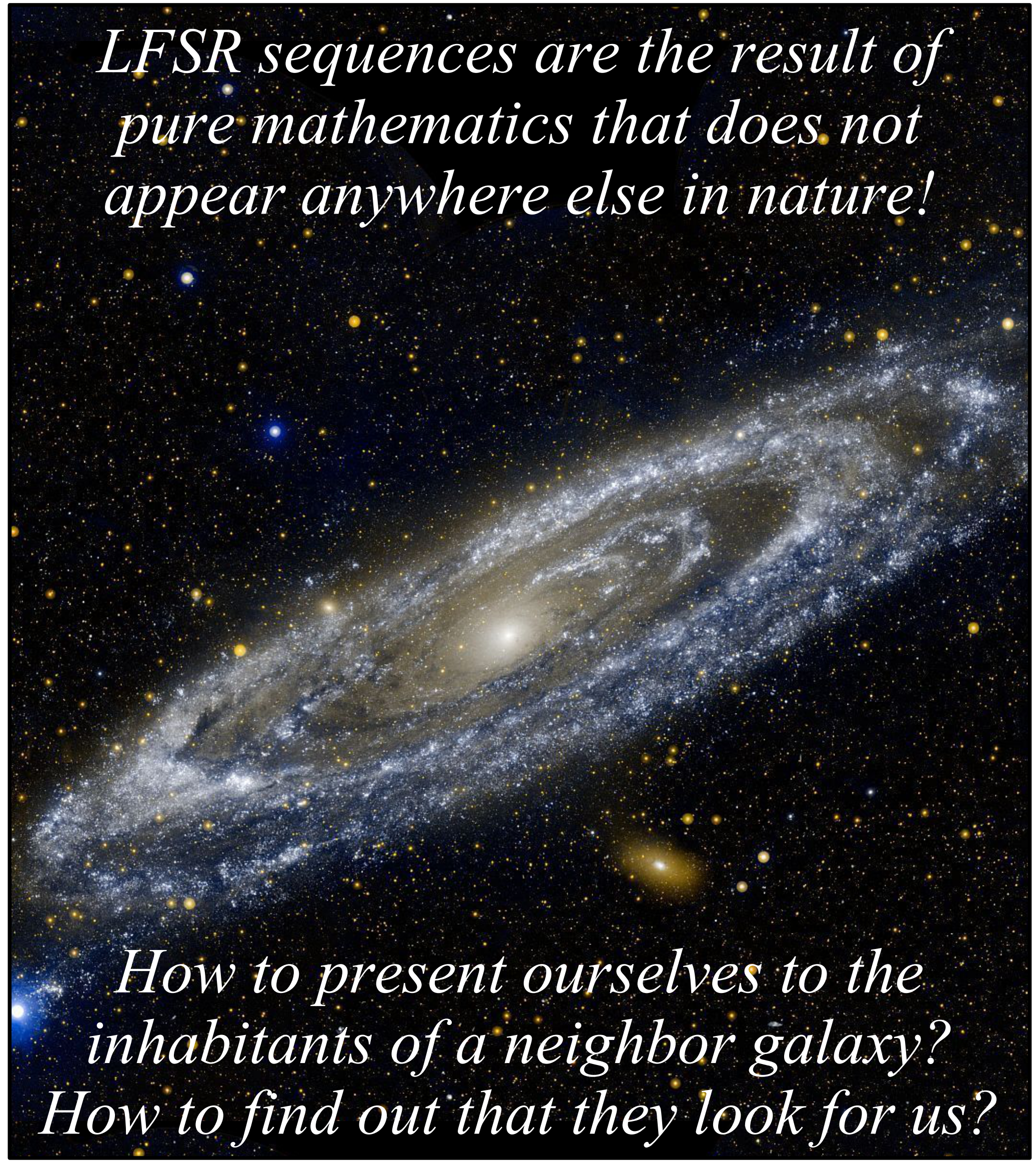
LFSR pseudo-random sequences are of NO cryptographic value: Berlekamp-Massey algorithm 1969

Perfect frequency spectrum of uniformly-spaced lines and simple generation/checking:

- test sequences for all kinds of telecommunication links
- data scrambling (randomization) as part of line coding

Peak-to-average power ratio:

$$LFSR: \frac{P_{MAX}}{\langle P \rangle} \approx 1 \qquad Noise: \frac{P_{MAX}}{\langle P \rangle} \to \infty$$



30 – Use of pseudo-random sequences