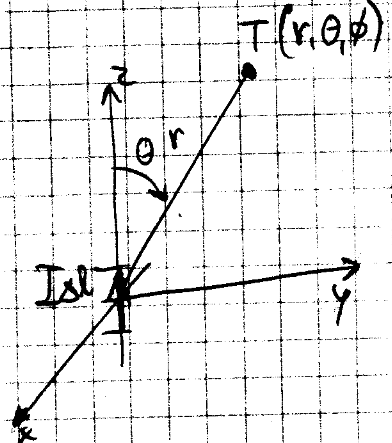
Bližnje polje tokovnega elementa



$$\vec{A} = \frac{\mu}{4\pi}\int_V \frac{\vec{J}(\vec{r}\,')e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\,dV$$

$$\Delta l \ll \lambda = \frac{2\pi}{k}\ ;\ \Delta l \ll r$$

$$\vec{A} = \frac{\mu}{4\pi}\vec{1}_z I\Delta l \frac{e^{-jkr}}{r} \qquad \vec{1}_z = \vec{1}_r\cos\theta - \vec{1}_\theta\sin\theta$$

$$\vec{H} = \frac{1}{\mu}\,\mathrm{rot}\vec{A} = \frac{I\Delta l}{4\pi}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \frac{e^{-jkr}}{r}\cos\theta & -e^{-jkr}\sin\theta & 0 \end{vmatrix} = \frac{I\Delta l}{4\pi}\vec{1}_\phi\frac{1}{r}\left(jke^{-jkr}\sin\theta + \frac{e^{-jkr}}{r}\sin\theta\right)$$

$$\vec{H} = \vec{1}_\phi\frac{I\Delta l}{4\pi}\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\sin\theta$$

$$\vec{E} = \frac{1}{j\omega\varepsilon}\,\mathrm{rot}\vec{H} = \frac{1}{j\omega\varepsilon}\frac{I\Delta l}{4\pi}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & r\sin\theta\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\sin\theta \end{vmatrix} =$$

$$= \frac{1}{j\omega\varepsilon}\frac{I\Delta l}{4\pi}\left[\vec{1}_r\frac{e^{-jkr}}{r}\left(\frac{2jk}{r}+\frac{1}{r^2}\right)\cos\theta + \vec{1}_\theta\frac{e^{-jkr}}{r}\left(jk\left(jk+\frac{1}{r}\right)+\frac{1}{r^2}\right)\sin\theta\right] =$$

$$= \frac{I\Delta l}{4\pi}Z_0\left[\vec{1}_r\frac{e^{-jkr}}{r}\left(\frac{1}{r}+\frac{1}{jkr^2}\right)\cos\theta + \vec{1}_\theta\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}+\frac{1}{jkr^2}\right)\sin\theta\right]$$

Poyntingov vektor, moč in sevalna upornost tokovnega elementa

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\frac{I\Delta l}{4\pi}Z_0\left[\vec{1}_r\frac{e^{-jkr}}{r}\left(\frac{1}{r}+\frac{1}{jkr^2}\right)\cos\theta +\right.$$

$$\left.+\vec{1}_\theta\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}+\frac{1}{jkr^2}\right)\sin\theta\right]\times\vec{1}_\phi\frac{I^*\Delta l}{4\pi}\frac{e^{+jkr}}{r}\left(-jk+\frac{1}{r}\right)\sin\theta =$$

$$= \frac{1}{2}\frac{II^*\Delta l^2 Z_0}{(4\pi r)^2}\left[-\vec{1}_\theta\left(\frac{-2jk}{r}+\frac{1}{r^2}-\frac{1}{r^2}+\frac{1}{jkr^3}\right)\sin\theta\cos\theta +\right.$$

$$\left.+\vec{1}_r\left(k^2+\frac{jk}{r}-\frac{jk}{r}+\frac{1}{r^2}-\frac{1}{r^2}+\frac{1}{jkr^3}\right)\sin^2\theta\right]$$

$$\vec{S} = \frac{II^*\Delta l^2 Z_0}{2(4\pi r)^2}\left[\vec{1}_\theta\left(\frac{2jk}{r}-\frac{1}{jkr^3}\right)\sin\theta\cos\theta + \vec{1}_r\left(k^2+\frac{1}{jkr^3}\right)\sin^2\theta\right]$$

$$P = \oint_{r=R}\mathrm{Re}[\vec{S}]\,d\vec{A} = \int_0^\pi\int_0^{2\pi}\mathrm{Re}[\vec{S}]\cdot\vec{1}_r R^2\sin\theta\,d\phi\,d\theta = 2\pi R^2\int_0^\pi\mathrm{Re}[\vec{S}]\cdot\vec{1}_r\sin\theta\,d\theta =$$

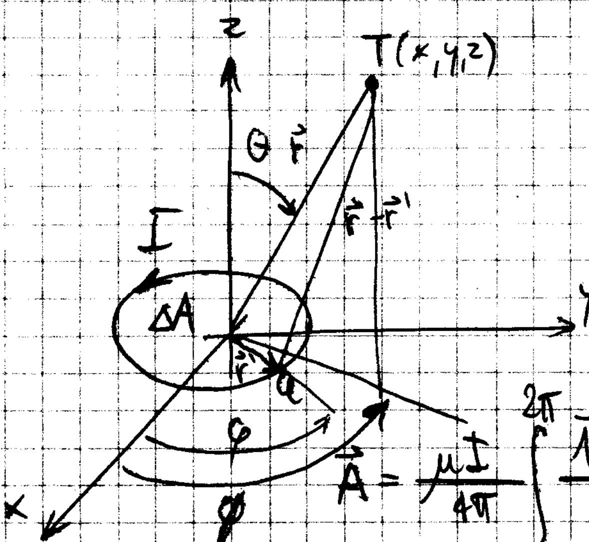
$$= 2\pi R^2\frac{II^*\Delta l^2 Z_0}{2(4\pi R)^2}\mathrm{Re}\left[k^2+\frac{1}{jkR^3}\right]\cdot\int_0^\pi\sin^3\theta\,d\theta =$$

$$= \frac{II^*\Delta l^2 Z_0 k^2}{16\pi}\int_{\theta=0}^{\theta=\pi}\left[1-\cos^2\theta\right](-d\cos\theta) = \frac{II^*\Delta l^2 Z_0 k^2}{16\pi}\int_{-1}^{1}(1-u^2)\,du =$$

$$= \frac{II^*\Delta l^2 Z_0 k^2}{16\pi}\left(2-\frac{2}{3}\right) = \frac{II^*\Delta l^2 Z_0 k^2}{12\pi} = \frac{II^*\Delta l^2\pi Z_0}{3\lambda^2}$$

$$R = \frac{2P}{II^*} = \frac{2\pi\Delta l^2 Z_0}{3\lambda^2}$$

Bližnje polje kolovne zanke



$$\vec{A} = \frac{\mu}{4\pi}\int_v \frac{\vec{J}(\vec{r}\,')\, e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\, dv$$

$$a \ll \lambda = \frac{2\pi}{k} \; ; \; a \ll r$$

$$\vec{A} = \frac{\mu I}{4\pi}\int_0^{2\pi} \frac{\vec{1}_\varphi\, e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\, a\, d\varphi$$

$$\vec{1}_\varphi = -\vec{1}_x \sin\varphi + \vec{1}_y \cos\varphi$$

$$|\vec{r}-\vec{r}\,'| = \sqrt{(x-a\cos\varphi)^2 + (y-a\sin\varphi)^2 + z^2} \approx \sqrt{x^2+y^2+z^2 - 2xa\cos\varphi - 2ya\sin\varphi} \approx$$

$$\approx r - \frac{xa}{r}\cos\varphi - \frac{ya}{r}\sin\varphi = r - a\sin\theta\,\underbrace{(\cos\phi\cos\varphi + \sin\phi\sin\varphi)}_{\cos(\phi-\varphi)}$$

$$e^{-jk|\vec{r}-\vec{r}\,'|} \approx e^{-jkr}\cdot e^{jka\sin\theta\cos(\phi-\varphi)} = e^{-jkr}\cdot\left(\cos(ka\sin\theta\cos(\phi-\varphi)) + j\sin(ka\sin\theta\cdot\cos(\phi-\varphi))\right) \approx$$

$$\approx e^{-jkr}\cdot\left(1 + jka\sin\theta\cos(\phi-\varphi)\right)$$

$$\frac{1}{|\vec{r}-\vec{r}\,'|} \approx \frac{1}{r - a\sin\theta\cos(\phi-\varphi)} \approx \frac{1}{r}\left(1 + \frac{a}{r}\sin\theta\cos(\phi-\varphi)\right)$$

$$\vec{A} \approx \frac{\mu I}{4\pi}\int_0^{2\pi}\left(-\vec{1}_x\sin\varphi + \vec{1}_y\cos\varphi\right)\frac{e^{-jkr}}{r}\left(1 + \left(jka + \frac{a}{r}\right)\sin\theta\cos(\phi-\varphi)\right) a\, d\varphi =$$

$$= \frac{\mu I a^2}{4\pi}\frac{e^{-jkr}}{r}\sin\theta\left(jk + \frac{1}{r}\right)\int_0^{2\pi}\left(-\vec{1}_x\sin\varphi + \vec{1}_y\cos\varphi\right)\left(\cos\phi\cos\varphi + \sin\phi\sin\varphi\right) d\varphi =$$

$$= \frac{\mu I a^2}{4\pi}\frac{e^{-jkr}}{r}\left(jk + \frac{1}{r}\right)\sin\theta\cdot\pi\,\underbrace{\left(-\vec{1}_x\sin\phi + \vec{1}_y\cos\phi\right)}_{\vec{1}_\phi} =$$

$$=\vec{1}_\phi \frac{\mu I\Delta A}{4\pi}\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\sin\theta$$

$$\vec{H}=\frac{1}{\mu}\operatorname{rot}\vec{A}=\frac{I\Delta A}{4\pi}\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & 0 & r\sin\theta\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\sin\theta\end{vmatrix}=$$

$$=\frac{I\Delta A}{4\pi}\left[\vec{1}_r\frac{e^{-jkr}}{r}\left(\frac{2jk}{r}+\frac{2}{r^2}\right)\cos\theta+\vec{1}_\theta\frac{e^{-jkr}}{r}\left(-k^2+\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta\right]$$

$$\vec{E}=\frac{1}{j\omega\varepsilon}\operatorname{rot}\vec{H}=\frac{1}{j\omega\varepsilon}\frac{I\Delta A}{4\pi}\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r\ ; & r\vec{1}_\theta\ ; & r\sin\theta\vec{1}_\phi\\ \frac{\partial}{\partial r}\ ; & \frac{\partial}{\partial\theta}\ ; & \frac{\partial}{\partial\phi}\\ \frac{e^{-jkr}}{r}\left(\frac{2jk}{r}+\frac{2}{r^2}\right)\cos\theta\ ; & \frac{e^{-jkr}}{r}\left(-k^2+\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta\cdot r\ ; & 0\end{vmatrix}=$$

$$=\frac{1}{j\omega\varepsilon}\frac{I\Delta A}{4\pi}\frac{e^{-jkr}}{r}\vec{1}_\phi\left(-jk\left(-k^2+\frac{jk}{r}+\frac{1}{r^2}\right)-\frac{jk}{r^2}-\frac{2}{r^3}+\frac{2jk}{r^2}+\frac{2}{r^3}\right)\sin\theta=$$

$$=\frac{1}{j\omega\varepsilon}\frac{I\Delta A}{4\pi}\frac{e^{-jkr}}{r}\sin\theta\,\vec{1}_\phi\left(jk^3+\frac{k^2}{r}\right)$$

$$\vec{E}=\vec{1}_\phi\frac{\omega\mu}{j}\frac{I\Delta A}{4\pi}\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\sin\theta$$

Pretok moči za tokovno zanko

$$\vec{H} = \frac{I\Delta A}{4\pi}\frac{e^{-jkr}}{r}\left[\vec{1}_r 2\left(\frac{jk}{r}+\frac{1}{r^2}\right)\cos\theta + \vec{1}_\theta\left(-k^2+\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta\right]$$

$$\vec{E} = \vec{1}_\phi\,\omega\mu_0\frac{I\Delta A}{4\pi}\frac{e^{-jkr}}{r}\left(k-\frac{j}{r}\right)\sin\theta$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{\omega\mu_0}{2}\left(\frac{II^*\Delta A}{4\pi}\right)^2\frac{1}{r^2}\left[\vec{1}_r\left(k^3 + \frac{j}{r^3}\right)^{\sin^2\theta} + \vec{1}_\theta 2\left(-\frac{jk^2}{r} \mp \frac{j}{r^3}\right)^{\sin\theta\cos\theta}\right]$$

Moč

$$P = \int_{r=R}\mathrm{Re}[\vec{S}]\cdot d\vec{A} = \int_0^\pi\int_0^{2\pi}\mathrm{Re}[\vec{S}]\cdot\vec{1}_r R^2\sin\theta\, d\phi\, d\theta = 2\pi R^2\int_0^\pi \mathrm{Re}[\vec{S}]\cdot\vec{1}_r$$

$$P = 2\pi R^2\frac{\omega\mu_0}{2}\frac{II^*\Delta A^2}{(4\pi R)^2}k^3\int_0^\pi\sin^3\theta\, d\theta =$$

$$\boxed{\omega\mu_0 = kZ_0}$$

$$P = \frac{\omega\mu_0 II^*\Delta A^2 k^3}{16\pi}\cdot\frac{4}{3} = \frac{\omega\mu_0 k^3 II^*\Delta A^2}{12\pi}$$

$$P = \frac{Z_0 k^4 II^*\Delta A^2}{12\pi} = II^* Z_0\frac{4\pi^3\Delta A^2}{3\lambda^4}$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

Sevalna upornost zanke

$$R = \frac{2P}{II^*} = \frac{8\pi^3 Z_0\Delta A^2}{3\lambda^4}$$

$f = 100\,\mathrm{MHz}$

$\lambda = 3\,\mathrm{m}$

$r = 22\,\mathrm{cm} \rightarrow \Delta A = 0.152\,\mathrm{m}^2$

DUALNE VELIČINE — RAČUNANJE

$$\mathrm{rot}\,\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$$
$$\mathrm{rot}\,\vec{E} = -j\omega\mu\vec{H}$$
$$\mathrm{div}\,\vec{D} = \rho$$
$$\mathrm{div}\,\vec{B} = 0$$

$$\mathrm{rot}\,\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$$
$$\mathrm{rot}\,\vec{E} = -\vec{J}_m - j\omega\mu\vec{H}$$
$$\mathrm{div}\,\vec{D} = \rho$$
$$\mathrm{div}\,\vec{B} = \rho_m$$

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(2)
$$\vec{H} = \frac{1}{\mu}\mathrm{rot}\,\vec{A}$$
$$\vec{E} = -j\omega\vec{A} - \mathrm{grad}\,V$$

(2)
$$\vec{H} = -j\omega\vec{F} - \mathrm{grad}\,V_m + \frac{1}{\mu}\mathrm{rot}\,\vec{A}$$
$$\vec{E} = -j\omega\vec{A} - \mathrm{grad}\,V - \frac{1}{\varepsilon}\mathrm{rot}\,\vec{F}$$

(1)
$$\vec{E} = \frac{1}{j\omega\varepsilon\mu}\mathrm{rot}(\mathrm{rot}\,\vec{A})$$

(1)
$$\vec{E} = \frac{1}{j\omega\varepsilon\mu}\mathrm{rot}(\mathrm{rot}\,\vec{A}) - \frac{1}{\varepsilon}\mathrm{rot}\,\vec{F}$$
$$\vec{H} = \frac{1}{j\omega\varepsilon\mu}\mathrm{rot}(\mathrm{rot}\,\vec{F}) + \frac{1}{\mu}\mathrm{rot}\,\vec{A}$$

(3)
$$\mathrm{div}\,\vec{A} + j\omega\varepsilon\mu V = 0$$
$$\vec{E} = -j\omega\vec{A} + \frac{1}{j\omega\varepsilon\mu}\mathrm{grad}(\mathrm{div}\,\vec{A})$$

(3)
$$\mathrm{div}\,\vec{F} + j\omega\varepsilon\mu V_m = 0$$
$$\vec{E} = -j\omega\vec{A} + \frac{1}{j\omega\varepsilon\mu}\mathrm{grad}(\mathrm{div}\,\vec{A}) - \frac{1}{\varepsilon}\mathrm{rot}\,\vec{F}$$
$$\vec{H} = -j\omega\vec{F} + \frac{1}{j\omega\varepsilon\mu}\mathrm{grad}(\mathrm{div}\,\vec{F}) + \frac{1}{\mu}\mathrm{rot}\,\vec{A}$$

$r \gg \frac{1}{k}$ $\boxed{\text{POENOSTAVITVE}}$

$$\mathrm{div}\,\vec{A} = \frac{1}{r^2\sin\theta}\left(\frac{\partial(r^2\sin\theta A_r)}{\partial r} + \frac{\partial(r\sin\theta A_\theta)}{\partial\theta} + \frac{\partial(rA_\phi)}{\partial\phi}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) +$$
$$+ \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi} \approx \frac{\partial A_r}{\partial r} \approx -jkA_r$$
$$\mathrm{grad}(\mathrm{div}\,\vec{A}) \approx \vec{1}_r\frac{\partial(-jkA_r)}{\partial r} + \vec{1}_\theta\frac{1}{r}\frac{\partial(-jkA_r)}{\partial\theta} + \vec{1}_\phi\frac{1}{r\sin\theta}\frac{\partial(-jkA_r)}{\partial\phi} \approx -k^2A_r$$
$$\vec{E} \approx -j\omega\vec{A} + \frac{\vec{1}_r}{j\omega\varepsilon\mu}(-k^2A_r) = -j\omega\vec{A} + \vec{1}_r j\omega A_r = \underline{-j\omega\left(\vec{A} - \vec{1}_r(\vec{1}_r\cdot\vec{A})\right)}$$

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prehod na računske formule

RECIPROČNOST

$$\mathrm{rot}\,\vec{H}_1 = j\omega\varepsilon\vec{E}_1 + \vec{J}_1 \;/\cdot E_2 \qquad \mathrm{rot}\,\vec{H}_2 = j\omega\varepsilon\vec{E}_2 + \vec{J}_2 \;/\cdot E_1$$

$$\mathrm{rot}\,\vec{E}_1 = -j\omega\mu\vec{H}_1 - \vec{J}_{m1} \;/\cdot H_2 \qquad \mathrm{rot}\,\vec{E}_2 = -j\omega\mu\vec{H}_2 - \vec{J}_{m2} \;/\cdot H_1$$

$\ominus$

$$\vec{E}_2\cdot\mathrm{rot}\,\vec{H}_1 - \vec{E}_1\cdot\mathrm{rot}\,\vec{H}_2 = \vec{J}_1\cdot E_2 - \vec{J}_2\cdot\vec{E}_1$$

$$\vec{H}_2\cdot\mathrm{rot}\,\vec{E}_1 - \vec{H}_1\cdot\mathrm{rot}\,\vec{E}_2 = -\vec{J}_{m1}\cdot\vec{H}_2 + \vec{J}_{m2}\cdot\vec{H}_1$$

$\oplus$

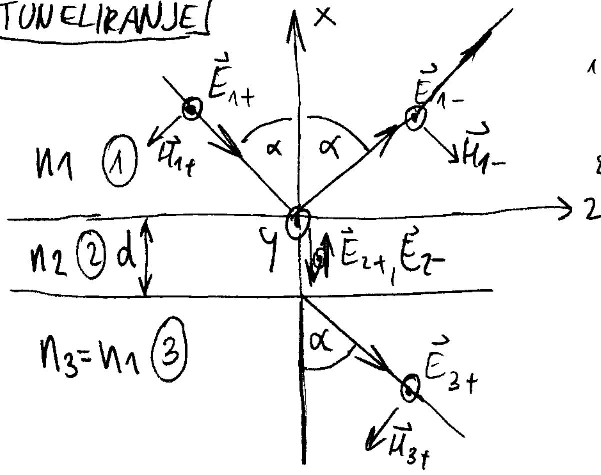
$$-\mathrm{div}(\vec{E}_2\times\vec{H}_1) + \mathrm{div}(\vec{E}_1\times\vec{H}_2) = \vec{J}_1\cdot\vec{E}_2 - \vec{J}_2\cdot\vec{E}_1 - \vec{J}_{m1}\cdot\vec{H}_2 + \vec{J}_{m2}\cdot\vec{H}_1$$

$$-\oint_A \underbrace{(\vec{E}_2\times\vec{H}_1 - \vec{E}_1\times\vec{H}_2)}_{\text{člena v } \infty \text{ enaka}}\cdot d\vec{A} = \int_V (\vec{J}_1\cdot\vec{E}_2 - \vec{J}_2\cdot\vec{E}_1 - \vec{J}_{m1}\cdot\vec{H}_2 + \vec{J}_{m2}\cdot\vec{H}_1)\,dV$$

$$\int \vec{J}_2\,dV = \vec{1}_s I_s \Delta l_s$$

$$\vec{E}\cdot\vec{1}_s = \frac{1}{I_s\Delta l_s}\int_V (\vec{J}\cdot\vec{E}_s - \vec{J}_m\cdot\vec{H}_s)\,dV$$

TUNELIRANJE



TE $\vec{E} = \vec{1}_y E$

1 (1,2) $E_{1+} + E_{1-} = E_{2+} + E_{2-}$

2 (2,3) $E_{2+} e^{+|k_{2x}|d} + E_{2-} e^{-|k_{2x}|d} = E_{3+}$

$$\vec{H} = \frac{-1}{j\omega\mu} \operatorname{rot} \vec{E} = \frac{j}{kZ} \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \frac{j}{kZ}\left(-\vec{1}_x E_y (j\beta) + \vec{1}_z E_y (-jk_x)\right)$$

$$H_z = \frac{k_x E_y}{kZ} = \frac{E_y}{Z}(\pm\cos\alpha) \quad (1) \text{ ali } (3)$$

$$H_z = \frac{\pm j|k_{2x}|}{k_2}\frac{E_y}{Z_2} = \pm j u \frac{E_y}{Z_2}$$

$$\beta = k_1 \sin\alpha = n_1 k_0 \sin\alpha$$

$$k_{2x} = \sqrt{k_2^2 - \beta^2} = \sqrt{n_2^2 k_0^2 - n_1^2 k_0^2 \sin^2\alpha} = k_0\sqrt{n_2^2 - n_1^2 \sin^2\alpha}$$

$$\frac{k_{2x}}{k_2} = \frac{1}{n_2}\sqrt{n_2^2 - n_1^2\sin^2\alpha} = \boxed{\sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\alpha} = u}$$

$$\boxed{|k_{2x}|d = |u|k_2 d = v}$$

3 (1,2) $\left(E_{1+} - E_{1-}\right)\frac{\cos\alpha}{Z_1} = \left(E_{2+} - E_{2-}\right)\frac{u}{Z_2}$

4 (2,3) $\left(E_{2+}e^{v} - E_{2-}e^{-v}\right)\frac{u}{Z_2} = E_{3+}\frac{\cos\alpha}{Z_1}$

$1+3 \rightarrow \left[2E_{1+} - (E_{2+}+E_{2-})\right]\frac{\cos\alpha}{Z_1} = \left(E_{2+}-E_{2-}\right)\frac{u}{Z_2}$

$2+4 \rightarrow \left(E_{2+}e^{v} - E_{2-}e^{-v}\right)\frac{u}{Z_2} = \left(E_{2+}e^{v} + E_{2-}e^{-v}\right)\frac{\cos\alpha}{Z_1}$

$2E_{1+} - (E_{2+}+E_{2-}) = (E_{2+}-E_{2-})\cdot w$; $\boxed{w = \frac{u Z_1}{\cos\alpha Z_2}}$; $(E_{2+}e^{2v} - E_{2-})w = E_{2+}e^{2v} + E_{2-}$

$2E_{1+} - E_{2+}\left(1 + e^{2v}\frac{w-1}{w+1}\right) = E_{2+}\left(1 - e^{2v}\frac{w-1}{w+1}\right)\cdot w$ $E_{2-} = E_{2+}e^{2v}\frac{w-1}{1+w}$

$$2E_{1+} = E_{2+}\left[(w+1) + (1-w)e^{2v}\frac{w-1}{w+1}\right]$$

$$E_{2+} = \frac{2E_{1+}}{(w+1)+(1-w)e^{2v}\frac{w-1}{w+1}} \; ; \; E_{2-} = e^{2v}\frac{w-1}{w+1}\frac{2E_{1+}}{(w+1)+(1-w)e^{2v}\frac{w-1}{w+1}}$$

$$E_{3+} = E_{2+}\left(e^{v} + e^{-v}e^{2v}\frac{w-1}{1+w}\right) = E_{2+}e^{v}\left(1+\frac{w-1}{1+w}\right) = E_{2+}e^{v}\frac{2w}{1+w}$$

$$E_{3+} = e^{v}\frac{2w}{1+w}\frac{2E_{1+}(w+1)}{(w+1)^2-(w-1)^2e^{2v}} = E_{1+}\frac{4w e^{v}}{(w+1)^2-(w-1)^2e^{2v}}$$

$$\frac{E_{3+}}{E_{1+}} = \frac{4w}{-4w\operatorname{ch}v - 2(w^2+1)\operatorname{sh}v} = \frac{1}{\operatorname{ch}v - \frac{w^2+1}{2w}\operatorname{sh}v}$$

$$v = k_2 d\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\alpha - 1}$$

$$|w| = \frac{Z_1\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\alpha - 1}}{Z_2\cos\alpha}$$

$$\left|\frac{E_{3+}}{E_{1+}}\right| = \frac{1}{\sqrt{\operatorname{ch}^2 v + \left(\frac{1-|w|^2}{2|w|}\right)^2\operatorname{sh}^2 v}} = \frac{1}{\sqrt{1+\frac{(1+|w|^2)^2}{4|w|^2}\operatorname{sh}^2 v}}$$

$$t = |w|^2 = \left(\frac{n_2}{n_1}\right)^2\frac{\left(\frac{n_1}{n_2}\right)^2\sin^2\alpha - 1}{\cos^2\alpha}$$

$$\left|\frac{E_{3+}}{E_{1+}}\right| = \frac{1}{\sqrt{1+\frac{(1+t)^2}{4t}\operatorname{sh}^2 v}}$$

TM $t = \left(\frac{n_1}{n_2}\right)^2\frac{\left(\frac{n_1}{n_2}\right)^2\sin^2\alpha - 1}{\cos^2\alpha}$

Maxwell-ove enačbe:

1. $\text{rot}\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$ $\qquad \text{rot}\vec{H} = \vec{J} + j\omega\vec{D}$
2. $\text{rot}\vec{E} = -j\omega\mu\vec{H}$ $\qquad \text{rot}\vec{E} = -j\omega\vec{B}$
3. $\text{div}\vec{D} = \rho$
4. $\text{div}\vec{B} = 0$

Kontinuiteta toka in elektrine:

$$\text{div}(\text{rot}\vec{H}) = 0 = \text{div}\vec{J} + j\omega\rho$$

Skalarni električni potencial: $\vec{E} = -\text{grad}\,V$ pri $\text{rot}\vec{E} = 0$ statika

Skalarni magnetni potencial: $\vec{H} = -\text{grad}\,V_m$ pri $\text{rot}\vec{H} = 0$; $\vec{J} = 0$

Vektorski potencial $\vec{A}$ ($\vec{V}_m$) : $\vec{B} = \text{rot}\vec{A}$

$$\text{rot}\vec{E} = -j\omega\,\text{rot}\vec{A} \rightarrow \text{rot}(\vec{E} + j\omega\vec{A}) = 0 \longrightarrow \vec{E} = -j\omega\vec{A} - \text{grad}\,V \text{ (sklepam)}$$

Diferencialna enačba za $\vec{A}$:

$$\frac{1}{\mu}\text{rot}(\text{rot}\vec{A}) = \vec{J} + j\omega\varepsilon(-j\omega\vec{A} - \text{grad}\,V) \qquad \text{(uporabim ME 1)}$$

$$\text{grad}(\text{div}\vec{A}) - \Delta\vec{A} = \mu\vec{J} - j\omega\mu\varepsilon\,\text{grad}\,V + \omega^2\mu\varepsilon\vec{A}$$

$$\text{grad}\underbrace{(\text{div}\vec{A} + j\omega\mu\varepsilon V)}_{=0} - \mu\vec{J} = \Delta\vec{A} + \omega^2\mu\varepsilon\vec{A}$$

Lorentz-ov pogoj za $\text{div}\vec{A}$

$$\boxed{\text{div}\vec{A} = -j\omega\mu\varepsilon V}$$

$$\vec{E} = -j\omega\vec{A} + \frac{1}{j\omega\mu\varepsilon}\text{grad}(\text{div}\vec{A})$$

$$\boxed{\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J}} \qquad \Delta\vec{A} + k^2\vec{A} = -\mu\vec{J}$$

Diferencialna enačba za V: $\qquad$ (uporabim ME 3)

$$\rho = \text{div}\vec{D} = \text{div}(\varepsilon\vec{E}) = \text{div}(-j\omega\varepsilon\vec{A} - \varepsilon\,\text{grad}\,V) = -j\omega\varepsilon\,\text{div}\vec{A} - \varepsilon\Delta V$$

$$\rho = -j\omega\varepsilon(-j\omega\mu\varepsilon V) - \varepsilon\Delta V$$

$$\frac{\rho}{\varepsilon} = -\omega^2\mu\varepsilon V - \Delta V \longrightarrow \boxed{\Delta V + \omega^2\mu\varepsilon V = -\frac{\rho}{\varepsilon}}$$

Tok in napetost:

$$I = \int_A (\vec{J} + j\omega\vec{D})\,d\vec{A} \qquad U_{21} = -\int_1^2 \vec{E}\,d\vec{l} = V_2 - V_1 + j\omega\int_1^2 \vec{A}\,d\vec{l}$$

kdaj je $\oint \vec{A}\,d\vec{l} = 0$?

Diferencialna enačba za $\vec{E}$:

$$\operatorname{rot}(\text{ME } ②) \longrightarrow \operatorname{rot}(\operatorname{rot}\vec{E}) = -j\omega\mu \operatorname{rot}\vec{H} \overset{\text{ME } ①}{=} -j\omega\mu\left(\vec{J} + j\omega\varepsilon\vec{E}\right)$$

$$\operatorname{grad}(\operatorname{div}\vec{E}) - \Delta\vec{E} = -j\omega\mu\vec{J} + \omega^2\mu\varepsilon\vec{E}$$

$$\boxed{\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = +j\omega\mu\vec{J} + \frac{1}{\varepsilon}\operatorname{grad}\rho}$$

Diferencialna enačba za $\vec{H}$:

$$\operatorname{rot}(\text{ME } ①) \rightarrow \operatorname{rot}(\operatorname{rot}\vec{H}) = \operatorname{rot}\vec{J} + j\omega\varepsilon \operatorname{rot}\vec{E} \overset{\text{ME } ②}{=} \operatorname{rot}\vec{J} + j\omega\varepsilon(-j\omega\mu\vec{H})$$

$$\operatorname{grad}(\underbrace{\operatorname{div}\vec{H}}_{=0}) - \Delta\vec{H} = \operatorname{rot}\vec{J} + \omega^2\mu\varepsilon\vec{H}$$

$$\boxed{\Delta\vec{H} + \omega^2\mu\varepsilon\vec{H} = -\operatorname{rot}\vec{J}}$$

Energija v elektrostatiki:

$$W = \frac{1}{2}\int_v \vec{E}\cdot\vec{D}\,dv \qquad \Delta V = -\frac{\rho}{\varepsilon}$$

$$\vec{E} = -\operatorname{grad}V \;;\; \vec{D} = -\varepsilon\operatorname{grad}V \;;\; \vec{E}\cdot\vec{D} = \varepsilon\operatorname{grad}V\cdot\operatorname{grad}V$$

$$\operatorname{div}(\varepsilon V\operatorname{grad}V) = \varepsilon\operatorname{grad}V\cdot\operatorname{grad}V + \varepsilon V\Delta V = \varepsilon\operatorname{grad}V\cdot\operatorname{grad}V - \rho V$$

$$\int_v \operatorname{div}(\varepsilon V\operatorname{grad}V)\,dv = \oint_A (\varepsilon V\operatorname{grad}V)\cancel{d\vec{A}} = \oint_A V\vec{D}\cdot d\vec{A}$$

$$\underbrace{\oint_A V\vec{D}\cdot d\vec{A}}_{=0} = \int_v \vec{E}\cdot\vec{D}\,dv - \int_v \rho V\,dv \longrightarrow W = \frac{1}{2}\int_v \rho V\,dv$$

Energija v magnetostatiki:

$$W = \frac{1}{2}\int_v \vec{H}\cdot\vec{B}\,dv \qquad \Delta\vec{A} = -\mu\vec{J}$$

$$\vec{H} = \frac{1}{\mu}\operatorname{rot}\vec{A} \;;\; \vec{B} = \operatorname{rot}\vec{A} \;;\; \vec{H}\cdot\vec{B} = \frac{1}{\mu}\operatorname{rot}\vec{A}\cdot\operatorname{rot}\vec{A} = \vec{H}\cdot\operatorname{rot}\vec{A}$$

$$\operatorname{div}(\vec{H}\times\vec{A}) = \vec{A}\cdot\operatorname{rot}\vec{H} - \vec{H}\cdot\operatorname{rot}\vec{A} = \vec{A}\cdot\vec{J} - \vec{H}\cdot\vec{B}$$

$$\underbrace{\oint_A (\vec{H}\times\vec{A})\,d\vec{A}}_{=0} = \int_v \vec{A}\cdot\vec{J}\,dv - \int_v \vec{H}\cdot\vec{B}\,dv \longrightarrow W = \frac{1}{2}\int_v \vec{J}\cdot\vec{A}\,dv$$

Poynting:

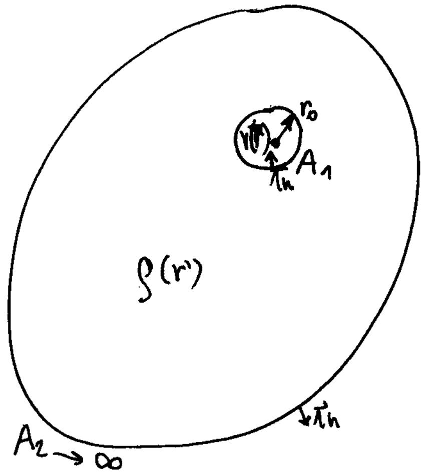
$$W = \frac{1}{2}\int_v \vec{E}\cdot\vec{D}\,dv + \frac{1}{2}\int_v \vec{H}\cdot\vec{B}\,dv \;;\; \frac{d\vec{D}}{dt} = \operatorname{rot}\vec{H} - \vec{J} \;;\; \frac{d\vec{B}}{dt} = -\operatorname{rot}\vec{E}$$

$$\frac{dW}{dt} = 2\cdot\frac{1}{2}\int_v\left[\vec{E}\cdot(\operatorname{rot}\vec{H} - \vec{J}) - \vec{H}\cdot\operatorname{rot}\vec{E}\right]dv = -\int_v \vec{E}\cdot\vec{J}\,dv + \int_v(\vec{E}\cdot\operatorname{rot}\vec{H} - \vec{H}\cdot\operatorname{rot}\vec{E})\,dv =$$

$$= -\int_v \vec{E}\cdot\vec{J}\,dv + \int_v \operatorname{div}(\vec{H}\times\vec{E})\,dv = -\int_v \vec{E}\cdot\vec{J}\,dv + \oint_A (\vec{H}\times\vec{E})\cdot d\vec{A}$$

$$\oint_A (\vec{E}\times\vec{H})\cdot d\vec{A} = -\int_v \vec{E}\cdot\vec{J}\,dv - \frac{dW}{dt}$$

$$\Delta V + k^2 V = -\frac{\rho}{\varepsilon} \qquad \oint_A \left(U\cdot \mathrm{grad}\, V - V\cdot \mathrm{grad}\, U\right) d\vec{A} = \int_v \left(U\Delta V - V\Delta U\right) dv$$



$$U = \frac{e^{-jkr}}{r} \; ; \; \mathrm{grad}\, U = \vec{1}_r \frac{e^{-jkr}}{r}\left(-jk - \frac{1}{r}\right)$$

$$\Delta U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{e^{-jkr}}{r}\left(-jk - \frac{1}{r}\right)\cdot r^2\right) = \frac{-jke^{-jkr} - k^2 r e^{-jkr} + jke^{-jkr}}{r^2}$$

$$\Delta U = -k^2 U$$

$$\int_v \left[U\left(-k^2 V - \frac{\rho}{\varepsilon}\right) - V\left(-k^2 U\right)\right] dv = -\int_v \frac{\rho}{\varepsilon} U\, dv \qquad \oint_{A_2} ?\cdot d\vec{A} = 0$$

$$r_0 \to 0 \implies \mathrm{grad}\, U \approx -\vec{1}_r \frac{1}{r^2}$$

$$\oint_{A_1} V\cdot \mathrm{grad}\, U \cdot d\vec{A} \approx \oint_{A_1} V\cdot\left(-\vec{1}_r \frac{1}{r^2}\right)\cdot \vec{1}_r\, r^2 d\Omega = -4\pi V$$

$$\oint_{A_1} U\cdot \mathrm{grad}\, V\cdot d\vec{A} \longrightarrow 0 \implies \boxed{V = \frac{1}{4\pi\varepsilon}\int_v \rho U\, dv}$$

$\Delta\vec{A} + k^2\vec{A} = -\mu\vec{J} \longrightarrow$ *Kartezični sistem, po komponentah*

$\Delta V = 0$ v kartezičnih koordinatah (x, y, z)

$$V(x,y,z) = X(x)\cdot Y(y)\cdot Z(z)$$

$$\frac{d^2X}{dx^2} + k_x^2 X = 0 \longrightarrow \text{rešitev } X = \sum_i C_i x^i, \text{ zveza } C_n\cdot n\cdot(n-1) + k_x^2 C_{n-2} = 0$$

$$C_n = \frac{-k_x^2 C_{n-2}}{n\cdot(n-1)}$$

$$X = A\sin k_x x + B\cos k_x x = Ce^{jk_x x} + De^{-jk_x x}$$

$$\frac{d^2X}{dx^2} - k_x^2 X = 0 \longrightarrow X = A\,\mathrm{sh}\, k_x x + B\,\mathrm{ch}\, k_x x = Ce^{k_x x} + De^{-k_x x}$$

$$X = \sum_i C_i x^i \qquad C_n = \frac{k_x^2 C_{n-2}}{n(n-1)}$$

$$J_m(x) \longrightarrow \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4} - \frac{m\pi}{2}\right) \qquad x \gg 1$$

$$N_m(x) \longrightarrow \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4} - \frac{m\pi}{2}\right) \qquad x \gg 1$$

$$H_m^{(1)}(x) = J_m(x) + j N_m(x)$$

$$H_m^{(2)}(x) = J_m(x) - j N_m(x)$$

Hankel-ove funkcije (ustrezajo $e^{\pm jx}$)

PRIREJENE (MODIFICIRANE) BESSELOVE FUNKCIJE

$$I_m(x) = j^{-m} J_m(jx) \qquad (\text{pri } -l^2)$$

$$K_m(x) = \frac{\pi\left[I_{-m}(x) - I_m(x)\right]}{2\sin(m\pi)}$$

ORTONORMALNOST

$$\int_0^a \rho\, J_m(\lambda_1\rho)\, J_m(\lambda_2\rho)\, d\rho = 0 \quad ; \quad \lambda_1 \neq \lambda_2 \text{ različna ničla pri } \rho = a$$

$$J_m(\lambda_1 a) = J_m(\lambda_2 a) = 0$$

$$\int_0^a \rho\, J_m^2(\lambda\rho)\, d\rho = \frac{1}{2}\left[a\, J_m'(\lambda a)\right]^2 \quad ; \; J_m(\lambda a) = 0$$

INTEGRALI

$$\int_0^a \rho\, J_m(\lambda\rho)\, J_m(\mu\rho)\, d\rho = a\,\frac{\lambda J_m'(\lambda a)\, J_m(\mu a) - \mu J_m'(\mu a) \cdot J_m(\lambda a)}{\mu^2 - \lambda^2}$$

$\mu = 0$; $m = 0$

$$\int_0^a \rho\, J_0(\lambda\rho)\, d\rho = \frac{a\, J_0'(\lambda a)}{-\lambda^2}$$

$\Delta V = 0$ v valjnih koordinatah (ϱ, φ, z)

$$\Delta V = \frac{1}{\varrho}\frac{\partial}{\partial \varrho}\left(\varrho \frac{\partial V}{\partial \varrho}\right) + \frac{1}{\varrho^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad ; \quad V = R(\varrho)\cdot F(\varphi)\cdot Z(z)$$

$$\Delta V = FZ \frac{1}{\varrho}\frac{d}{d\varrho}\left(\varrho \frac{dR}{d\varrho}\right) + RZ\frac{1}{\varrho^2}\frac{d^2F}{d\varphi^2} + RF\frac{d^2Z}{dz^2} = 0 \quad / \cdot \frac{1}{RFZ}$$

$$\frac{1}{\varrho R}\frac{d}{d\varrho}\left(\varrho\frac{dR}{d\varrho}\right) + \frac{1}{\varrho^2}\underbrace{\frac{1}{F}\frac{d^2F}{d\varphi^2}}_{-m^2} + \underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{\pm \ell^2 ; 0} = 0$$

$$F = A\sin m\varphi + B\cos m\varphi \quad ; \quad Z = Ce^{\sqrt{\pm \ell^2}\, z} + De^{-\sqrt{\pm \ell^2}\, z}$$

$$\frac{1}{\varrho R}\frac{d}{d\varrho}\left(\varrho\frac{dR}{d\varrho}\right) \pm \ell^2 - \frac{m^2}{\varrho^2} = 0 \quad ; \quad R = \sum_i C_i \varrho^i$$

$$\frac{1}{\varrho}\frac{d}{d\varrho}\left(\varrho\frac{dR}{d\varrho}\right) + R\left(\pm\ell^2 - \frac{m^2}{\varrho^2}\right) = 0$$

$$C_n n^2 + C_{n-2}(\pm \ell^2) - C_n m^2 = 0$$

$$\pm\ell^2 = 0 \longrightarrow C_n n^2 - C_n m^2 = 0 \rightarrow \boxed{n = \pm m} \quad ; \quad R = C_m \varrho^m + C_{-m}\varrho^{-m}$$

$$\pm \ell^2 C_{n-2} = (m^2 - n^2) C_n \longrightarrow C_n = \frac{\pm \ell^2}{m^2 - n^2} C_{n-2}$$

$n = 2k + m$

$$R = \alpha \sum_{k=0}^{\infty} \frac{(\pm \ell^2)^k}{4^k k! (k+m)!} \varrho^{2k+m}$$

$$m^2 - n^2 = m^2 - 4k^2 - 4km - m^2 = -4k^2 - 4km = -4k(k+m)$$

BESSEL

$$+\ell^2: \; J_m(\ell\varrho) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\ell\varrho}{2}\right)^{2k+m}}{k!\,(k+m)!}$$

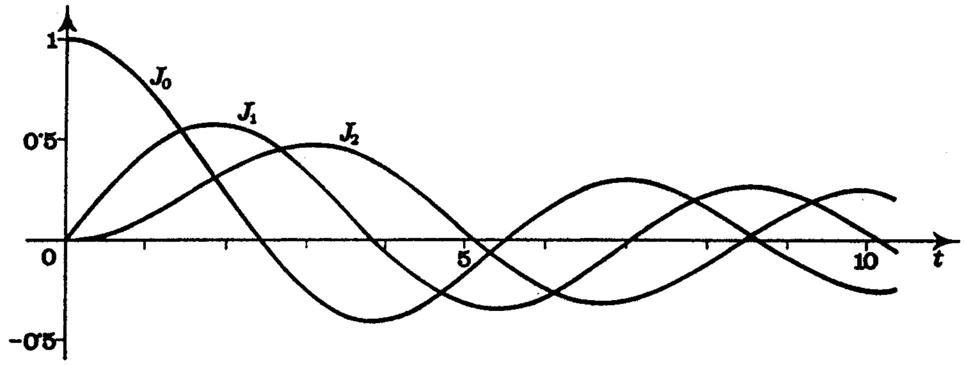
$$-\ell^2: \; I_m(\ell\varrho) = \sum_{k=0}^{\infty} \frac{\left(\frac{\ell\varrho}{2}\right)^{2k+m}}{k!\,(k+m)!}$$

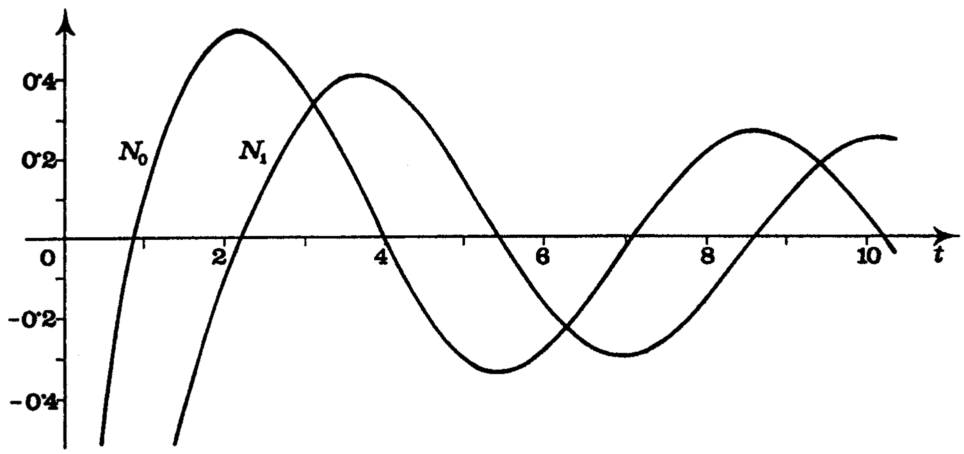
NEUMANN

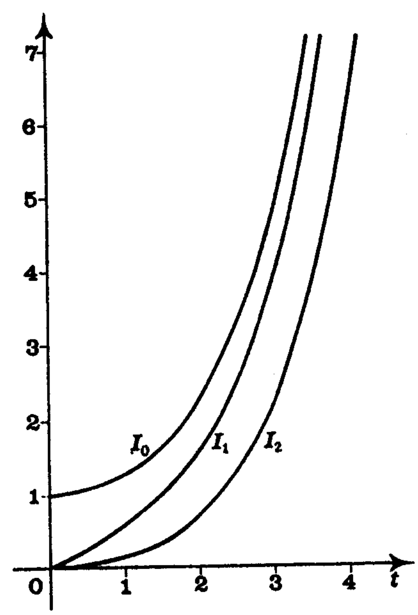
$$N_m(\ell\varrho) = \frac{\cos(m\pi) J_m(\ell\varrho) - J_{-m}(\ell\varrho)}{\sin(m\pi)}$$

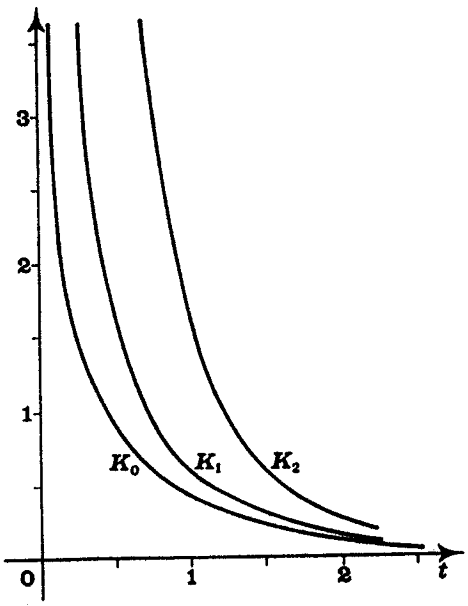
(ponavadi limita)

Valjne funkcije :





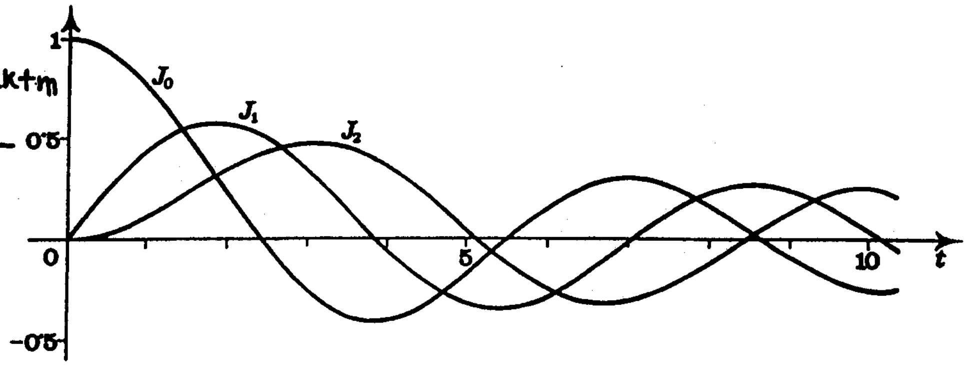




Valjne funkcije:

$J_m(t)$

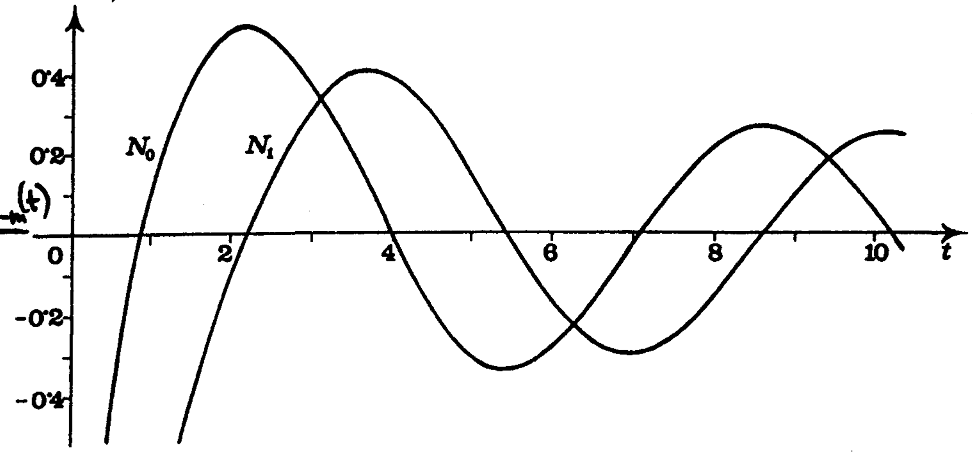
$$J_m(t) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{t}{2}\right)^{2k+m}}{k!\,(k+m)!}$$



$N_m(t)$

$$N_m(t) = \frac{\cos(m\pi) J_m(t) - J_{-m}(t)}{\sin(m\pi)}$$

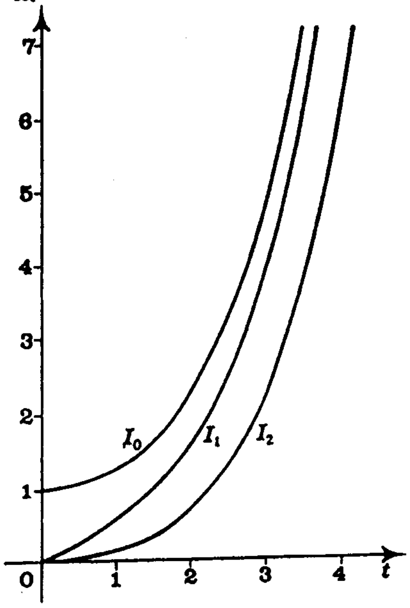
(ponavadi limita)



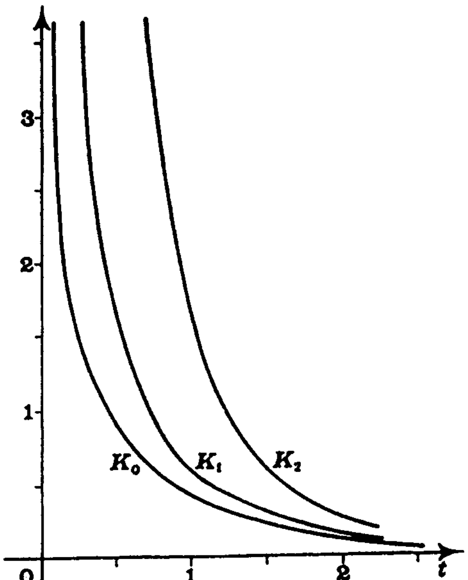
$I_m(t)$

$$I_m(t) = \sum_{k=0}^{\infty} \frac{\left(\frac{t}{2}\right)^{2k+m}}{k!\,(k+m)!}$$

$$I_m(t) = j^{-m} J_m(jt)$$



$K_m(t)$



$$K_m(t) = \frac{\pi \left[I_{-m}(t) - I_m(t) \right]}{2 \sin(m\pi)} \quad \text{(ponavadi limita)}$$

$\Delta\vec{E} + k^2\vec{E} = 0$ v valjnih koordinatah

$\vec{E} = \vec{1}_z E_z \longrightarrow \Delta E_z + k^2 E_z = 0$ poenostavitev

$E_z = RFZ \quad R(\varrho);\ F(\varphi);\ Z(z)$

$$FZ\frac{1}{\varrho}\frac{d}{d\varrho}\left(\varrho\frac{dR}{d\varrho}\right) + RZ\frac{1}{\varrho^2}\frac{d^2F}{d\varphi^2} + RF\frac{d^2Z}{dz^2} + RFZk^2 = 0 \quad / :RFZ$$

$$\frac{1}{R\varrho}\frac{d}{d\varrho}\left(\varrho\frac{dR}{d\varrho}\right) + \frac{1}{\varrho^2}\underbrace{\frac{1}{F}\frac{d^2F}{d\varphi^2}}_{-m^2} + \underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{-k_z^2 = -\beta^2} + k^2 = 0$$

$Z = A\cos k_z z + B\sin k_z z$

$Z = A'e^{jk_z z} + B'e^{-jk_z z}$

$F = C\cos m\varphi + D\sin m\varphi$

$$\frac{1}{\varrho}\frac{d}{d\varrho}\left(\varrho\frac{dR}{d\varrho}\right) + R\left(\underbrace{k^2 - k_z^2}_{\pm l^2} - \frac{m^2}{\varrho^2}\right) = 0$$

$+l^2$: $R = J_m(l\varrho)$ ali

$\boxed{l^2 = k^2 - \beta^2}$ $R = N_m(l\varrho)$ ali

$R = H_m^{(1)}(l\varrho)$ ali $R = H_m^{(2)}(l\varrho)$

$-l^2$: $R = I_m(l\varrho)$ ali

$\boxed{l^2 = \beta^2 - k^2}$ $R = K_m(l\varrho)$

$$E_z = J_m(l\varrho)\cdot(C\cos m\varphi + D\sin m\varphi)(A'e^{j\beta z} + B'e^{-j\beta z})$$

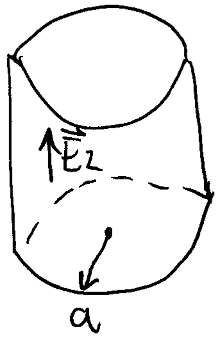
ZGLED

$\beta = 0,\ m = 0$

$E_z = J_0(k\varrho)$

$J_0(ka) = 0$

$ka = 2{,}405$



$E_z(\varrho = a) = 0$

$\omega\sqrt{\mu_0\varepsilon_0}\,a = 2{,}405$

$$f = \frac{2{,}405\,c_0}{a\cdot 2\pi}$$

$$f = \frac{114.8\,\text{MHz}\cdot\text{m}}{a}$$

DIELEKTRIČNI REZONATOR

$H_z = J_0(k\varrho)$

$H_z(\varrho a) \Longrightarrow 0$

$$f = \frac{2{,}405\,c}{a\cdot 2\pi} = \frac{2{,}405\,c_0}{a\cdot 2\pi\sqrt{\varepsilon_r}}$$

$$f = \frac{114.8\,\text{MHz}\cdot\text{m}}{a\sqrt{\varepsilon_r}}$$

zelo približno.....

$\Delta V = 0$ v krogelnih koordinatah

$$0 = \Delta V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$V(r,\theta,\phi) = R(r)\cdot T(\cos\theta)\cdot F(\phi) \quad ; \quad T(t) = T(\cos\theta) \ ; \ t = \cos\theta$$

$$0 = \underbrace{\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)}_{n(n+1)} + \underbrace{\frac{1}{T\sin\theta}\left(\frac{d}{d\theta}\left(\sin\theta\frac{dT}{d\theta}\right)\right) + \frac{1}{F\sin^2\theta}\frac{d^2F}{d\phi^2}}_{-n(n+1)}$$

$$R = A r^n + B r^{-(n+1)}$$

$$\frac{1}{T\sin\theta}\left(\frac{d}{d\theta}\left(\sin\theta\frac{dT}{d\theta}\right)\right) + \frac{1}{\sin^2\theta}\underbrace{\frac{1}{F}\frac{d^2F}{d\phi^2}}_{-m^2} + n(n+1) = 0$$

$$F = C\cos m\phi + D\sin m\phi \qquad\qquad \sin^2\theta = 1-t^2$$

$$\frac{1}{T}\frac{d}{dt}\left((1-t^2)\frac{dT}{dt}\right) + \frac{-m^2}{1-t^2} + n(n+1) = 0 \qquad -1\leq t\leq 1$$

$$T(t) = P_n^m(t) = \frac{(1-t^2)^{\frac{m}{2}}\left[(1-t^2)^n\right]^{(n)}}{2^n\, n!} \qquad m = 0,1,2,\dots,n-1,n$$

prirejena Legendrova funkcija

PREDZNAK?

$$m = 0 \rightarrow T(t) = P_n^0(t) = P_n(t) = \frac{\left[(1-t^2)^n\right]^{(n)}}{2^n\, n!}$$

ORTONORMALNOST

$$\int_{-1}^{1} P_n(t)\,P_m(t)\,dt = \begin{cases} \frac{2}{2n+1} & ; \ n = m \\ 0 & ; \ n \neq m \end{cases}$$

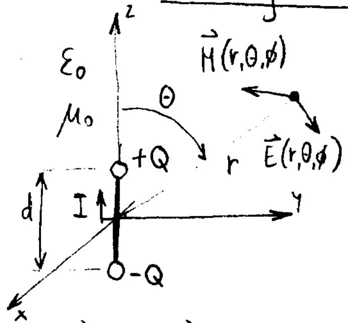
INTEGRALI

$$\int P_n(t)\,dt = \frac{(t^2-1)}{n(n+1)}\dot{P}_n(t)$$

$$\int_0^{\pi/2} P_m(\cos\theta)\sin\theta\,d\theta = \begin{cases} 1, & m=0 \\ 0, & m = \text{sodo število} \\ \frac{2}{2m+2}\cdot\frac{1\cdot 3\cdots(m-2)}{2\cdot 4\cdots(m-1)}; & m = \text{liho število} \end{cases}$$

$$P_0(t) = 1$$
$$P_1(t) = \cos\theta = t$$
$$P_2(t) = \frac{1}{2}(3\cos^2\theta - 1) = \frac{3t^2-1}{2}$$
$$P_3(t) = \frac{1}{2}(5t^3 - 3t)$$
$$P_4(t) = \frac{1}{8}(35t^4 - 30t^2 + 3)$$

Sevanje kratke žice



① $r \gg d$

② $d \ll \frac{1}{k}$

$k = \omega\sqrt{\mu_0\varepsilon_0}$

$$\vec{A}(r,\theta,\phi) = \frac{\mu_0}{4\pi}\int_{V'} \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} e^{-jk|\vec{r}-\vec{r}'|} dv'$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\int_{-\frac{d}{2}}^{+\frac{d}{2}} \frac{\vec{1}_z}{|\vec{r}-\vec{r}'|} e^{-jk|\vec{r}-\vec{r}'|} dz \approx \vec{1}_z \frac{\mu_0 I d}{4\pi}\frac{e^{-jkr}}{r}$$

$$\vec{1}_z = \vec{1}_r\cos\theta - \vec{1}_\theta\sin\theta \longrightarrow \underline{\vec{A} = \left(\vec{1}_r\cos\theta - \vec{1}_\theta\sin\theta\right)\frac{\mu_0 I d}{4\pi}\frac{e^{-jkr}}{r}}$$

$\vec{B} = \operatorname{rot}\vec{A}$

$$\vec{H} = \frac{1}{\mu_0}\operatorname{rot}\vec{A} = \frac{Id}{4\pi}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\,\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{e^{-jkr}}{r}\cos\theta & -r\frac{e^{-jkr}}{r}\sin\theta & r\sin\theta\, 0 \end{vmatrix} = \underline{\underline{\vec{1}_\phi \frac{Id}{4\pi}\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\sin\theta}}$$

$$V(r,\theta,\phi) = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}e^{-jk|\vec{r}-\vec{r}'|}dv' = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{|\vec{r}-\vec{r}_i|}e^{-jk|\vec{r}-\vec{r}_i|} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{e^{-jkr_A}}{r_A} - \frac{e^{-jkr_B}}{r_B}\right)$$

$$r_A = |\vec{r}-\vec{r}_1| = \sqrt{r^2 + (d/2)^2 - 2r\,d/2\cos\theta} \approx r - \frac{d}{2}\cos\theta \qquad r_B = |\vec{r}-\vec{r}_2| \approx r + \frac{d}{2}\cos\theta$$

$$V \approx \frac{Q}{4\pi\varepsilon_0}\left(\frac{e^{-jkr}e^{jk\frac{d}{2}\cos\theta}}{r-\frac{d}{2}\cos\theta} - \frac{e^{-jkr}e^{-jk\frac{d}{2}\cos\theta}}{r+\frac{d}{2}\cos\theta}\right) = \frac{Q}{4\pi\varepsilon_0}e^{-jkr}\frac{r\,2j\sin\left(\frac{kd}{2}\cos\theta\right) + \frac{d}{2}\cos\theta\,2\cos\left(\frac{kd}{2}\cos\theta\right)}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2} \approx$$

$$\approx \underline{\frac{Qd}{4\pi\varepsilon_0}\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\cos\theta} \qquad I = \frac{dQ}{dt} = j\omega Q \longrightarrow \underline{V = \frac{Id}{j\omega 4\pi\varepsilon_0}\frac{e^{-jkr}}{r}\left(jk+\frac{1}{r}\right)\cos\theta}$$

$$\vec{E} = -j\omega\vec{A} - \operatorname{grad}V = \left(-\vec{1}_r\cos\theta + \vec{1}_\theta\sin\theta\right)\frac{j\omega\mu_0 Id}{4\pi}\frac{e^{-jkr}}{r} + \frac{jId}{4\pi\omega\varepsilon_0}\frac{e^{-jkr}}{r}\left[\vec{1}_r\left(k^2 - \frac{2jk}{r} - \frac{2}{r^2}\right)\cos\theta - \right.$$

$$\left. - \vec{1}_\theta\left(\frac{jk}{r} + \frac{1}{r^2}\right)\sin\theta\right] = \underline{\underline{\frac{Id}{4\pi}Z_0\frac{e^{-jkr}}{r}\left[\vec{1}_r\left(\frac{1}{r} - \frac{j}{kr^2}\right)2\cos\theta + \vec{1}_\theta\left(jk + \frac{1}{r} - \frac{j}{kr^2}\right)\sin\theta\right]}}$$

$$\omega\mu_0 = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\frac{\mu_0}{\varepsilon_0}} = kZ_0 \qquad \frac{1}{\omega\varepsilon_0} = \frac{1}{\omega\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{Z_0}{k}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \underline{\underline{\frac{|I|^2 d^2 Z_0}{32\pi^2 r^2}\left[\vec{1}_r\left(k^2 - \frac{j}{kr^3}\right)\sin^2\theta + \vec{1}_\theta\left(\frac{jk}{r} + \frac{j}{kr^3}\right)2\sin\theta\cos\theta\right]}}$$

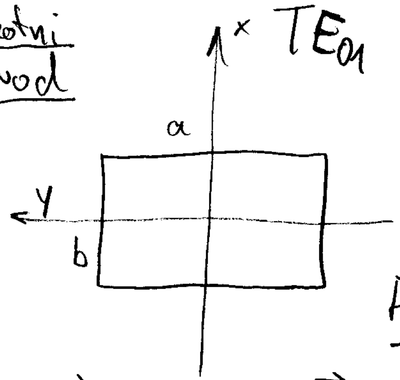
$$P = \oint_{A\to\infty}\vec{S}\cdot\vec{1}_n dA = \int_0^\pi\int_0^{2\pi}\vec{S}\cdot\vec{1}_r r^2\sin\theta\,d\theta\,d\phi\Big|_{r\to\infty} = \frac{|I|^2 d^2 Z_0}{32\pi^2}k^2\int_0^\pi\sin^3\theta\,d\theta\int_0^{2\pi}d\phi$$

$$\int_0^\pi\sin^3\theta\,d\theta = \int_0^\pi(1-\cos^2\theta)(-d\cos\theta) = \int_{-1}^{+1}(1-u^2)\,du = 2 - \frac{2}{3} = \frac{4}{3} \qquad \underline{\underline{P = \frac{|I|^2 d^2 Z_0 k^2}{12\pi}}}$$

$$P = \frac{1}{2}|I|^2 R_s \longrightarrow \underline{\underline{R_s = \frac{Z_0 d^2 k^2}{6\pi} = \frac{2\pi Z_0}{3}\left(\frac{d}{\lambda}\right)^2}}$$

$$k = \frac{2\pi}{\lambda}$$

Pravokotni valovod



TE01

$$\vec{E} = \vec{1}_x C \cos\frac{\pi}{a}y \, e^{-j\beta z}$$

$$k_y = \frac{\pi}{a} \qquad k_y^2 + k_z^2 = k^2$$

$$k_z = \beta \qquad \left(\frac{\pi}{a}\right)^2 + \beta^2 = \left(\frac{\omega}{c_0}\right)^2$$

$$\vec{E} = \vec{1}_x \frac{C}{2}\left(e^{+j\frac{\pi}{a}y} + e^{-j\frac{\pi}{a}y}\right) e^{-j\beta z}$$

$$\vec{k}_1 = \vec{1}_y \frac{\pi}{a} + \vec{1}_z \beta \, , \quad \vec{k}_2 = -\vec{1}_y \frac{\pi}{a} + \vec{1}_z \beta$$

$$\vec{H} = \vec{1}_y \frac{\beta}{\omega\mu} C \cos\frac{\pi}{a}y \, e^{-j\beta z} + \vec{1}_z \frac{j\pi}{a\omega\mu} C \sin\frac{\pi}{a}y \, e^{-j\beta z}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{1}_z \frac{\beta}{\omega\mu}|C|^2 \cos^2\frac{\pi}{a}y + \vec{1}_y \frac{j\pi |C|^2}{a\omega\mu} \cos\frac{\pi}{a}y \sin\frac{\pi}{a}y =$$

$$= \vec{1}_z \frac{\beta}{k}\frac{|C|^2}{Z_0}\cos^2\frac{\pi}{a}y + \vec{1}_y j\frac{\pi}{ak}\frac{|C|^2}{Z_0}\cos\frac{\pi}{a}y \sin\frac{\pi}{a}y$$

$$k = \frac{\omega}{c_0}$$

$$\beta = \frac{\omega}{v_f} \rightarrow v_f = \frac{\omega}{\beta} = c_0\frac{k}{\beta} = c_0\frac{k}{\sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}} \geq c_0 \qquad \lambda_g = \frac{v_f}{f} = \frac{k}{\beta}\lambda$$

$$v_g = \lim_{\Delta\omega\to 0}\frac{\Delta\omega}{\Delta\beta} = \frac{d\omega}{d\beta} \qquad \frac{d\beta}{d\omega} = \frac{d}{d\omega}\left(\sqrt{\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{\pi}{a}\right)^2}\right) = \frac{2\frac{\omega}{c_0^2}}{2\sqrt{\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{\pi}{a}\right)^2}} = \frac{k}{c_0\sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}}$$

$$v_g = c_0\frac{\sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}}{k} \leq c_0 \qquad v_g = c_0\frac{\beta}{k}$$