Rešitve nalog 1. kolokvija iz TEORIJE ELEKTROMAGNETIKE 13/12/1989

1.

$\Delta V = -\frac{\rho(r)}{\varepsilon}$; $\frac{\partial}{\partial\theta}=0$; $\frac{\partial}{\partial\phi}=0$; $\rho_N=\rho_0\frac{r^4}{a^4}$ $\rho_z=0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)=-\frac{\rho(r)}{\varepsilon}$$

$$r^2\frac{\partial V}{\partial r}=-\int r^2\frac{\rho(r)}{\varepsilon}dr+C_1$$

$$V=\int r^{-2}\left[-\int r^2\frac{\rho(r)}{\varepsilon}dr+C_1\right]dr+C_2$$

$$V_z=-C_{1z}r^{-1}+C_{2z} \qquad C_{2z}=0 \longleftarrow V(\infty)=0$$

$$V_N=-\frac{\rho_0}{42\varepsilon a^4}r^6-C_{1N}r^{-1}+C_{2N} \qquad C_{1N}=0 \longleftarrow V(0)<\infty$$

$$V_z(a)=V_N(a) \qquad \left.\frac{\partial V_z}{\partial r}\right|_{r=a}=\left.\frac{\partial V_N}{\partial r}\right|_{r=a}$$

$$-C_{1z}a^{-1}=-\frac{\rho_0a^2}{42\varepsilon}+C_{2N} \qquad C_{1z}a^{-2}=\frac{-\rho_0 a}{7\varepsilon}$$

$$\frac{\rho_0a^2}{7\varepsilon}=\frac{-\rho_0a^2}{42\varepsilon}+C_{2N}$$

$$C_{2N}=\frac{7\rho_0a^2}{42\varepsilon}$$

$$\underline{\underline{V_z=\frac{\rho_0a^3}{7\varepsilon}r^{-1}}} \qquad \underline{\underline{\vec{E}_z=\vec{1}_r\frac{\rho_0a^3}{7\varepsilon}r^{-2}}}$$

$$\underline{\underline{V_N=\frac{\rho_0}{42\varepsilon}\left[-\frac{r^6}{a^4}+7a^2\right]}} \qquad \underline{\underline{\vec{E}_N=\vec{1}_r\frac{\rho_0}{7\varepsilon a^4}r^5}}$$

$$W=\frac{1}{2}\int_v\rho V\,dv=\frac{1}{2}\int_0^a\rho V_N\,4\pi r^2dr=2\pi\int_0^a\rho_0\frac{r^4}{a^4}\cdot\frac{\rho_0}{42\varepsilon}\left[-\frac{r^6}{a^4}+7a^2\right]r^2dr=\frac{\pi\rho_0^2}{21\varepsilon a^4}\left[-\frac{a^9}{13}+\frac{7a^9}{7}\right]=\underline{\underline{\frac{4\pi\rho_0^2a^5}{91\varepsilon}}}$$

2.

$$V=\left(Ar^n+Br^{-(n+1)}\right)P_n(\cos\theta) \qquad V=V_0\cos\theta\rightarrow n=1$$

$$\underline{\underline{V_N=\frac{V_0}{a}r\cos\theta}} \;;\; B_N=0\longleftarrow V(r=0)<\infty \;;\; \vec{E}_N=\frac{V_0}{a}\left(-\vec{1}_r\cos\theta+\vec{1}_\theta\sin\theta\right)=\underline{-\vec{1}_z V_0a^{-1}}$$

$$\underline{V_z=V_0a^2r^{-2}\cos\theta} \;;\; A_z=0\longleftarrow V(r\to\infty)=0 \;;\; \underline{\vec{E}_z=V_0a^2\left(\vec{1}_r\cdot 2r^{-3}\cos\theta+\vec{1}_\theta r^{-3}\sin\theta\right)}$$

$$W=\frac{\varepsilon}{2}\int_v E^2dv \qquad W_N=\frac{\varepsilon}{2}\int_v\left(-\vec{1}_z\frac{V_0}{a}\right)^2dv=\frac{\varepsilon}{2}\frac{V_0^2}{a^2}\frac{4\pi a^3}{3}=\underline{\underline{\frac{2\pi\varepsilon V_0^2a}{3}}}$$

$$W_z=\frac{\varepsilon}{2}\int_v\left[V_0a^2\left(\vec{1}_r\cdot 2r^{-3}\cos\theta+\vec{1}_\theta r^{-3}\sin\theta\right)\right]^2dv=\frac{\varepsilon V_0^2a^4}{2}\int_v\left(4r^{-6}\cos^2\theta+r^{-6}\sin^2\theta\right)dv=\frac{\varepsilon V_0^2a^4}{2}\int_a^\infty r^{-6}r^2dr\int_0^\pi\left[4\cos^2\theta+\sin^2\theta\right]2\pi\sin\theta\,d\theta$$

$$W_z=\pi\varepsilon V_0^2a^4\frac{1}{3a^3}\int_0^\pi\left[3\cos^2\theta+1\right](-d\cos\theta)=\frac{\pi\varepsilon V_0^2a}{3}\int_{-1}^1(3u^2+1)du=\underline{\underline{\frac{4\pi\varepsilon V_0^2a}{3}}}$$

3.

$$\left.\begin{array}{l}4V_1=2V_2+V\\4V_2=V_1+V_3+V_4\\4V_3=2V_2\\4V_4=2V_2+2V\end{array}\right\} \left.\begin{array}{l}4V_1=4V_3+V\\8V_3=V_1+V_3+V_4\\4V_4=4V_3+2V\end{array}\right\} \left.\begin{array}{l}4V_1=4V_3+V\\16V_3=2V_1+2V_3+2V_3+V\\ \downarrow\\12V_3=2V_1+V\end{array}\right\} \begin{array}{l}24V_3=(4V_3+V)+2V\\ \downarrow\\ \underline{\underline{V_3=\frac{3}{20}V=0.15V}}\end{array}$$

$$\underline{\underline{V_2=\frac{6}{20}V=0.3V}}$$

$$\underline{\underline{V_1=\frac{8}{20}V=0.4V}}$$

$$\underline{\underline{V_4=\frac{13}{20}V=0.65V}}$$

4.

$\gamma=\gamma_0\frac{\rho^2}{a^2}$ $\vec{J}=\gamma\cdot\vec{E}$; $I=2\pi\rho l J$; $\vec{J}=\vec{1}_\rho\cdot J$; $\vec{E}=\vec{1}_\rho E$

$$\vec{E}=\vec{1}_\rho\frac{I}{2\pi\rho l\gamma} \qquad U=\int_a^b\frac{I}{2\pi\rho l\gamma}d\rho=\int_a^b\frac{Ia^2}{2\pi l\gamma_0}\frac{d\rho}{\rho^3}=\frac{Ia^2}{2\pi l\gamma_0}\cdot\frac{1}{2}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$$

$$P=U\cdot I=\frac{4\pi l\gamma_0U^2}{a^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)} \qquad \underline{\underline{P/l=\frac{4\pi\gamma_0U^2}{a^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)}=\frac{4\pi\gamma_0U^2b^2}{b^2-a^2}}}$$

10.01.1990

Rešitve 2. kolokvija iz TEORIJE ELEKTROMAGNETIKE (1. List)

(1.) $\Delta V_m = 0$ v vseh treh prostorih

$V_n = A_n \varrho \sin\varphi$

$V_v = (A_v \varrho + B_v \varrho^{-1}) \sin\varphi$

$V_z = (A_z \varrho + B_z \varrho^{-1}) \sin\varphi$

$\vec{H} = -\operatorname{grad} V_m$

$\vec{H}_n = -A_n(\vec{1}_\varrho \sin\varphi + \vec{1}_\varphi \cos\varphi)$

$\vec{H}_v = -\left[\vec{1}_\varrho \left(A_v - \frac{B_v}{\varrho^2}\right)\sin\varphi + \vec{1}_\varphi\left(A_v + \frac{B_v}{\varrho^2}\right)\cos\varphi\right]$

$\vec{H}_z = -\left[\vec{1}_\varrho \left(A_z - \frac{B_z}{\varrho^2}\right)\sin\varphi + \vec{1}_\varphi\left(A_z + \frac{B_z}{\varrho^2}\right)\cos\varphi\right]$

tangencialni $\vec{H}$: normalni $\vec{B}$:

(1) $A_n = A_v + \frac{B_v}{a^2}$; (3) $A_n = \mu_r\left(A_v - \frac{B_v}{a^2}\right)$

(2) $A_v + \frac{B_v}{b^2} = A_z + \frac{B_z}{b^2}$; (4) $\mu_r\left(A_v - \frac{B_v}{b^2}\right) = A_z - \frac{B_z}{b^2}$

(2)+(4) $\longrightarrow A_v(1+\mu_r) + \frac{B_v}{b^2}(1-\mu_r) = 2A_z$ (5)

(1)+(3) $\longrightarrow A_v = \frac{A_n(1+\mu_r)}{2\mu_r}$ (6)

$\longrightarrow B_v = \frac{-a^2 A_n(1-\mu_r)}{2\mu_r}$ (7)

(5) (6)+(7) $\longrightarrow A_n \frac{(1+\mu_r)^2}{2\mu_r} - A_n \frac{a^2}{b^2}\frac{(1-\mu_r)^2}{2\mu_r} = 2A_z$

$$\underline{\underline{\frac{A_n}{A_z} = \frac{H_n}{H_z(\infty)} = \frac{4\mu_r}{(1+\mu_r)^2 - \frac{a^2}{b^2}(1-\mu_r)^2}}}$$

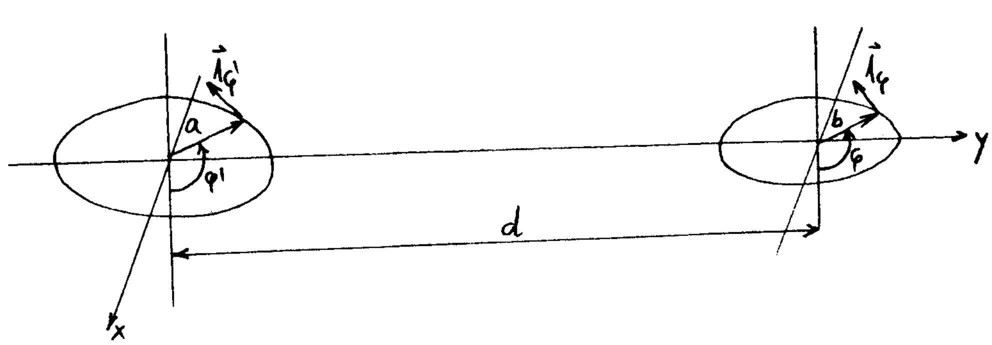
(2.) $a, b \ll d$

$$\vec{V}_m = \frac{\mu}{4\pi}\int_{v'} \frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}dv' = \frac{\mu}{4\pi}\int_0^{2\pi} \frac{\vec{1}_{\varphi'} I}{r - a\sin\varphi'} a\,d\varphi' = \frac{\mu I a}{4\pi}\int_0^{2\pi}\frac{-\vec{1}_x \sin\varphi' + \vec{1}_y\cos\varphi'}{r - a\sin\varphi'}d\varphi'$$

$$\vec{V}_m \approx \frac{\mu I a}{4\pi}\int_0^{2\pi}(-\vec{1}_x \sin\varphi' + \vec{1}_y\cos\varphi')\frac{1}{r}\left(1+\frac{a}{r}\sin\varphi'\right)d\varphi' = \frac{\mu I a}{4\pi r}(-\vec{1}_x)\pi\frac{a}{r} = \underline{-\vec{1}_x\frac{\mu I a^2}{4r^2}}$$

$$M = \frac{1}{I}\int_A \vec{B}\,d\vec{A} = \frac{1}{I}\int_A (\operatorname{rot}\vec{V}_m)\,d\vec{A} = \frac{1}{I}\oint \vec{V}_m\,d\vec{s} = \frac{1}{I}\frac{\mu I a^2}{4}\int_0^{2\pi}(-\vec{1}_x)\cdot(\vec{1}_\varphi)\frac{1}{r^2}b\,d\varphi =$$

$$= \frac{\mu a^2 b}{4}\int_0^{2\pi}\frac{\sin\varphi}{(d+b\sin\varphi)^2}d\varphi \approx \frac{\mu a^2 b}{4d^2}\int_0^{2\pi}\sin\varphi\left(1-\frac{2b}{d}\sin\varphi\right)d\varphi = \underline{\underline{-\frac{\pi\mu a^2 b^2}{2d^3}}}$$



10.01.1990

Rešitve 2. kolokvija iz TEORIJE ELEKTROMAGNETIKE (2. list)

(3.) $\vec{E} = \vec{1}_z E_0 J_0(k\rho)$; $k = \frac{2.405}{a}$

$$\vec{H} = \frac{-1}{j\omega\mu} \operatorname{rot}\vec{E} = \frac{-1}{j\omega\mu}\frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & 0 & E_0 J_0(k\rho) \end{vmatrix} = \frac{-1}{j\omega\mu\rho}\rho\vec{1}_\varphi\left(-E_0 k J_0'(k\rho)\right) = \underline{\underline{\vec{1}_\varphi \frac{E_0 k}{j\omega\mu} J_0'(k\rho)}}$$

$$\vec{E} = \frac{1}{j\omega\varepsilon}\operatorname{rot}\vec{H} = \frac{1}{j\omega\varepsilon}\frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \rho\frac{E_0 k}{j\omega\mu}J_0'(k\rho) & 0 \end{vmatrix} = \vec{1}_z \frac{1}{j\omega\varepsilon}\cdot\frac{E_0 k}{j\omega\mu}\frac{1}{\rho}\left(J_0'(k\rho) + k\rho J_0''(k\rho)\right) =$$

$$= \vec{1}_z \frac{E_0 k^2}{\omega^2\mu\varepsilon} J_0(k\rho) \longrightarrow k^2 = \omega^2\mu\varepsilon \; ; \; \omega = \frac{k}{\sqrt{\mu\varepsilon}} \; ; \; \underline{\underline{f = \frac{2.405}{2\pi a\sqrt{\mu\varepsilon}} = \frac{2.405\, c}{2\pi a}}}$$

$\vec{1}_n \times \vec{H} = \vec{K}$ na steni rezonatorja.

Na spodnji steni : $\vec{1}_n = \vec{1}_z$; $z = 0$ $\quad \vec{K} = \vec{1}_z \times \vec{1}_\varphi \frac{E_0 k}{j\omega\mu}\left(J_0'(k\rho)\right) = \underline{\underline{\vec{1}_\rho \frac{jE_0 k}{\omega\mu} J_0'(k\rho)}}$

Na obodu valja : $\vec{1}_n = -\vec{1}_\rho$; $\rho = a$ $\quad \vec{K} = -\vec{1}_\rho \times \vec{1}_\varphi \frac{E_0 k}{j\omega\mu} J_0'(ka) = \underline{\underline{\vec{1}_z \frac{jE_0 k}{\omega\mu} J_0'(2.405)}}$

Na zgornji steni : $\vec{1}_n = -\vec{1}_z$; $z = b$ $\quad \vec{K} = -\vec{1}_z \times \vec{1}_\varphi \frac{E_0 k}{j\omega\mu} J_0'(k\rho) = \underline{\underline{-\vec{1}_\rho \frac{jE_0 k}{\omega\mu} J_0'(k\rho)}}$

(4.) $\vec{E}_1 = \vec{1}_x E_1 e^{-jk_0 z}$ $\qquad \vec{H}_1 = \vec{1}_y \frac{E_1}{Z_0} e^{-jk_0 z}$

$\vec{E}_2 = \vec{1}_x\left(E_{2+} e^{-jkz} + E_{2-} e^{jkz}\right)$ $\qquad \vec{H}_2 = \vec{1}_y\left(\frac{E_{2+}}{Z} e^{-jkz} - \frac{E_{2-}}{Z} e^{+jkz}\right)$

$\vec{E}_3 = \vec{1}_x E_3 e^{-jk_0 z}$ $\qquad \vec{H}_3 = \vec{1}_y \frac{E_3}{Z_0} e^{-jk_0 z}$

Na sprednji steni: $\vec{E}_1 = \vec{E}_2$; $\vec{H}_1 = \vec{H}_2$

$$E_1 = E_{2+} + E_{2-} \; ; \; \frac{E_1}{Z_0} = \frac{E_{2+}}{Z} - \frac{E_{2-}}{Z}$$

$$E_{2+}(Z - Z_0) = -E_{2-}(Z + Z_0)$$

$$\frac{E_{2-}}{E_{2+}} = \frac{Z_0 - Z}{Z_0 + Z}$$

Na zadnji steni: $\vec{E}_2 = \vec{E}_3$; $\vec{H}_2 = \vec{H}_3$

$$E_{2+} e^{-jkd} + E_{2-} e^{+jkd} = E_3 e^{-jk_0 d}$$

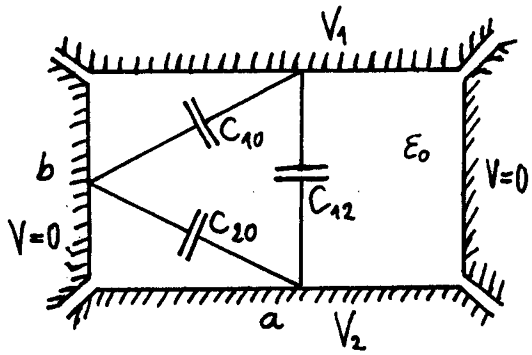
$$\frac{E_{2+}}{Z} e^{-jkd} - \frac{E_{2-}}{Z} e^{+jkd} = \frac{E_3}{Z_0} e^{-jk_0 d}$$

$$\frac{E_{2-}}{E_{2+}} e^{2jkd} = \frac{Z_0 - Z}{Z_0 + Z}$$

$$e^{2jkd} = 1 \longrightarrow 2kd = m\cdot 2\pi \; ; \; m = 0, 1, 2 \ldots$$

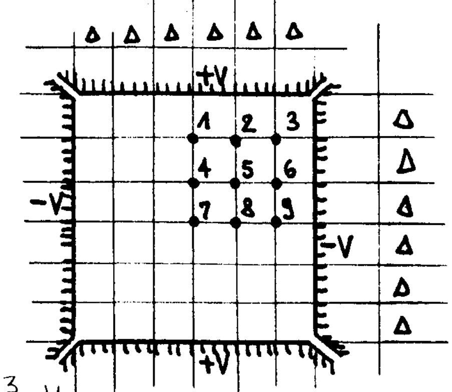
$$\underline{\underline{d = \frac{m\cdot 2\pi}{2k} = \frac{m\cdot 2\pi\cdot\lambda}{2\cdot 2\pi} = m\cdot\frac{\lambda}{2}}} \qquad (\lambda \text{ v dielektriku!})$$

1. Kolokvij iz TEORIJE ELEKTROMAGNETIKE - 06/12/1990



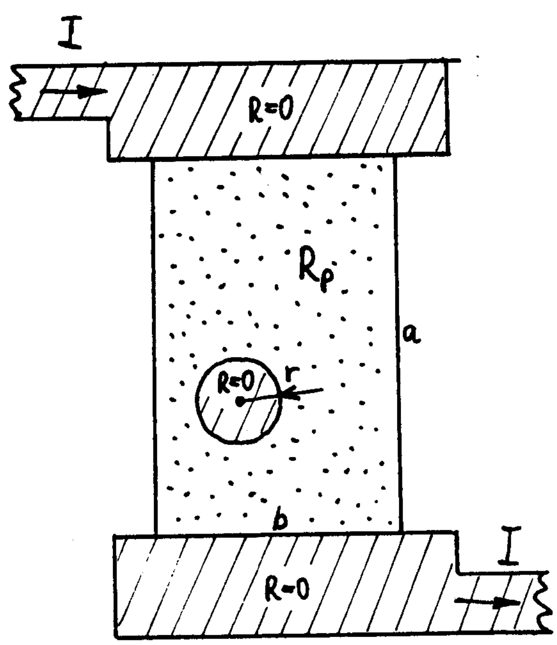
1) Izračunajte delno kapacitivnost na enoto dolžine med gornjo in spodnjo elektrodo v kovinskem žlebu (C12), če so stranske eklektrode ozemljene!

$$\frac{C_{12}}{l} = 8\varepsilon_0 \sum_{k=0}^{\infty} \frac{1}{(2k+1)\pi \, \mathrm{sh} \frac{(2k+1)\pi}{a} b}$$



2) Izračunajte potenciale v točkah 1, 2, 3, 4, 5, 6, 7, 8 in 9 z metodo končnih diferenc, če se (neskončno dolga) gornja in spodnja elektroda nahajata na potencialu +V, stranski elektrodi pa na potencialu -V.

$$V_1 = \frac{6}{13} V, \quad V_2 = \frac{19}{52} V, \quad V_4 = \frac{3}{26} V$$



3) Skozi tankoslojni upor dolžine a in širine b teče tok I. Upor je izdelan iz snovi s plastno upornostjo Rp. Zaradi tehnološke napake je sredi upora ostal majhen okrogel otok polmera r (r<<a in r<<b) iz zelo dobro prevodnega materiala (za elektrode upora). Izračunajte električno polje v okolici napake! Kolikokrat je gostota moči na enoto ploskve tu večja od tiste v brezhibnem uporu?

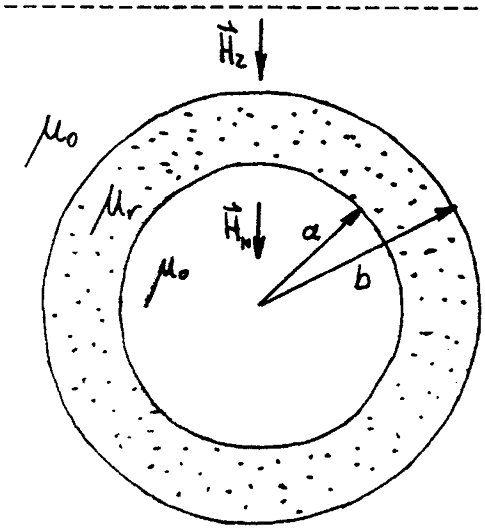
$$\vec{E} = \frac{I R_p}{b} \left(-\vec{1}_\rho \left(1 + \frac{r^2}{\rho^2}\right) \sin\varphi - \vec{1}_\varphi \left(1 - \frac{r^2}{\rho^2}\right) \cos\varphi \right)$$

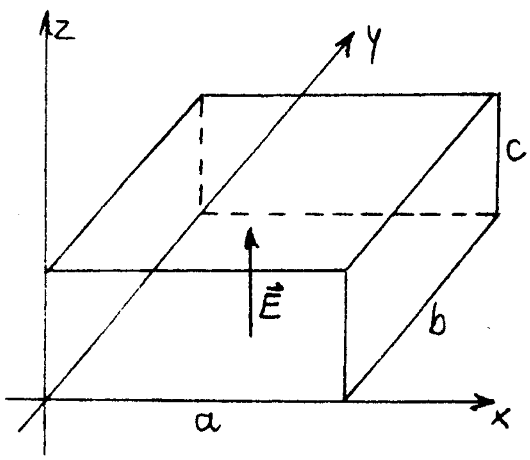
$p_{max} = 4 p_0$ pri $\rho = r$ in $\varphi = \pm\frac{\pi}{2}$

2. Kolokvij iz TEORIJE ELEKTROMAGNETIKE - 17/01/1991 **1. LIST**

1) Izračunajte učinkovitost magnetnega oklopa (razmerje med zunanjim in notranjim poljem) v obliki krogelne lupine z notranjim polmerom a in zunanjim polmerom b, ki je narejen iz feromagnetne snovi z relativno permeabilnostjo μ_r.

$$\frac{A_z}{A_N} = \frac{H_z}{H_N} = \frac{(\mu_r+2)(2\mu_r+1)-(1-\mu_r)^2\,\frac{2a^3}{b^3}}{9\mu_r}$$





2) V pravokotnem rezonatorju z dimenzijami a, b in c je dano električno polje:

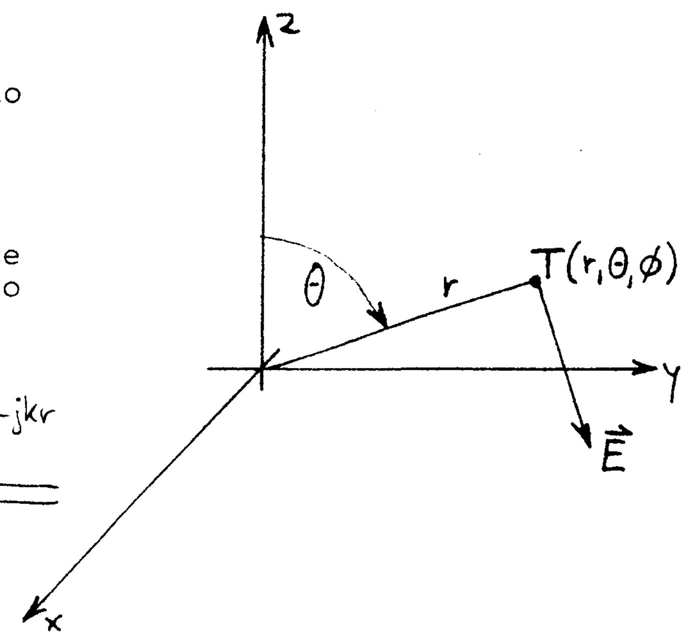
$$\vec{E} = \vec{1}_z A \sin\frac{2\pi}{a}x \,\sin\frac{3\pi}{b}y$$

Izračunajte pripadajoče magnetno polje, rezonančno frekvenco, tokove v stenah rezonatorja in celotno energijo, ki jo vsebuje rezonator.

3) V prostoru je podano električno polje:

$$\vec{E} = \vec{1}_\theta A \frac{\sin\theta}{r} e^{-jkr}$$

Izračunajte pripadajoče magnetno polje, gostoto električnega toka in Poyntingov vektor.



$$\vec{H} = \frac{j}{\omega\mu}\,\mathrm{rot}\,\vec{E} = \vec{1}_\phi \frac{A}{Z_0}\frac{\sin\theta}{r} e^{-jkr}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{1}_r \frac{AA^*}{2Z_0}\frac{\sin^2\theta}{r^2}$$

$$\vec{J} = \mathrm{rot}\vec{H} - j\omega\varepsilon\vec{E} = \vec{1}_r \frac{1}{r^2\sin\theta}\frac{A}{Z_0} 2\sin\theta\cos\theta\, e^{-jkr} + \vec{1}_\theta \frac{-jk}{r\sin\theta}\frac{A}{Z_0}\sin^2\theta\, e^{-jkr} - \vec{1}_\theta A\frac{\sin\theta}{r} e^{-jkr} j\omega\varepsilon$$

$$\vec{J} = \vec{1}_r \frac{2A}{Z_0}\frac{\cos\theta}{r^2} e^{-jkr}$$

2.LIST REŠITEV
2.K ELEKTROMAGNETIKA
17/1/1991

① $\Delta V_m = 0$

$$V_{mN} = A_N\, r\cos\theta$$
$$V_{mv} = A_v\, r\cos\theta + B_v r^{-2}\cos\theta$$
$$V_{mz} = A_z\, r\cos\theta + B_z r^{-2}\cos\theta$$

$$\vec H = -\operatorname{grad} V_m$$
$$\vec H_N = -\vec 1_r A_n\cos\theta + \vec 1_\theta A_n\sin\theta$$
$$\vec H_v = -\vec 1_r\left(A_v - \frac{2}{r^3}B_v\right)\cos\theta + \vec 1_\theta\left(A_v + \frac{B_v}{r^3}\right)\sin\theta$$
$$\vec H_z = -\vec 1_r\left(A_z - \frac{2B_z}{r^3}\right)\cos\theta + \vec 1_\theta\left(A_z + \frac{B_z}{r^3}\right)\sin\theta$$

① $A_n = \left(A_v - \dfrac{2B_v}{a^3}\right)\mu_r$ ③ $A_n = A_v + \dfrac{B_v}{a^3}$

② $\mu_r\left(A_v - \dfrac{2B_v}{b^3}\right) = A_z - \dfrac{2B_z}{b^3}$ ④ $A_v + \dfrac{B_v}{b^3} = A_z + \dfrac{B_z}{b^3}$

① + ③ $\cdot 2\mu_r \Rightarrow A_n(1+2\mu_r) = 3\mu_r A_v$

$$A_v = \frac{1+2\mu_r}{3\mu_r}A_N$$

② + ④ $\cdot 2 \Rightarrow (\mu_r+2)A_v + (1-\mu_r)\dfrac{2B_v}{b^3} = 3A_z$

① − ③ $\mu_r \Rightarrow A_N(1-\mu_r) = -3\mu_r\dfrac{B_v}{a^3}$

$$B_v = -\frac{1-\mu_r}{3\mu_r}a^3 A_N$$

$$(\mu_r+2)\frac{1+2\mu_r}{3\mu_r}A_N - (1-\mu_r)\frac{1-\mu_r}{3\mu_r}\frac{2a^3}{b^3}A_n = 3A_z$$

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② $\vec E = \vec 1_z A \sin\frac{2\pi}{a}x \sin\frac{3\pi}{b}y$

pogoj za rezonanco $\operatorname{rot}\vec H = j\omega\varepsilon\vec E$

$$\frac{jA}{\omega\mu}\left(\left(\frac{3\pi}{b}\right)^2 + \left(\frac{2\pi}{a}\right)^2\right) = j\omega\varepsilon A$$

$$\boxed{\omega = \frac{\sqrt{\left(\frac{3\pi}{b}\right)^2 + \left(\frac{2\pi}{a}\right)^2}}{\sqrt{\mu\varepsilon}}}$$

$$W = \frac{\varepsilon}{2}\int_V E_{max}^2\, dv = \frac{\varepsilon|A|^2 abc}{8}$$

$$\vec H = \frac{j}{\omega\mu}\operatorname{rot}\vec E = \vec 1_x\frac{jA}{\omega\mu}\frac{3\pi}{b}\sin\frac{2\pi}{a}x\cos\frac{3\pi}{b}y - \vec 1_y\frac{jA}{\omega\mu}\frac{2\pi}{a}\cos\frac{2\pi}{a}x\sin\frac{3\pi}{a}y$$

$\vec K = \vec 1_n \times \vec H$ $\vec 1_n$ gleda v škatlo!

$z=0;\ \vec 1_n = \vec 1_z;\ \vec K(z=0) = \vec 1_y\frac{jA}{\omega\mu}\frac{3\pi}{b}\sin\frac{2\pi}{a}x\cos\frac{3\pi}{b}y + \vec 1_x\frac{jA}{\omega\mu}\frac{2\pi}{a}\cos\frac{2\pi}{a}x\sin\frac{3\pi}{b}y$

$z=c,\ \vec 1_n = -\vec 1_z;\ \vec K(z=c) = -\vec 1_y\frac{jA}{\omega\mu}\frac{3\pi}{b}\sin\frac{2\pi}{a}x\cos\frac{3\pi}{b}y - \vec 1_x\frac{jA}{\omega\mu}\frac{2\pi}{a}\cos\frac{2\pi}{a}x\sin\frac{3\pi}{b}y$

$x=0;\ \vec 1_n = \vec 1_x;\ \vec K(x=0) = -\vec 1_z\frac{jA}{\omega\mu}\frac{2\pi}{a}\sin\frac{3\pi}{a}y$

$x=a,\ \vec 1_n = -\vec 1_x;\ \vec K(x=a) = \vec 1_z\frac{jA}{\omega\mu}\frac{2\pi}{a}\sin\frac{3\pi}{a}y$

$y=0;\ \vec 1_n = \vec 1_y;\ \vec K(y=0) = -\vec 1_z\frac{jA}{\omega\mu}\frac{3\pi}{b}\sin\frac{2\pi}{a}x$

$y=b,\ \vec 1_n = -\vec 1_y;\ \vec K(y=b) = -\vec 1_z\frac{jA}{\omega\mu}\frac{3\pi}{b}\sin\frac{2\pi}{a}x$

Rešitve nalog 1. Kolokvija iz TEMAG 19/11/1991

1\. LIST.

(1.) $$V = \sum_n \left(A_n \rho^n + B_n \rho^{-n}\right) \sin n\varphi$$

$$V(\rho = a) = 0 = \sum_n \left(A_n a^n + B_n a^{-n}\right) \sin n\varphi \longrightarrow \underline{B_n = -A_n a^{2n}}$$

$$V = \sum_n A_n \left(\rho^n - a^{2n} \rho^{-n}\right) \sin n\varphi$$

$$V(\rho = b) = \begin{cases} +V_0 \ ; \ 0 < \varphi < \pi \\ -V_0 \ ; \ \pi < \varphi < 2\pi \end{cases} \longrightarrow \underline{A_n = \frac{2V_0 (1 - \cos n\pi)}{n\pi \left(b^n - a^{2n} b^{-n}\right)}}$$

$$\underline{\underline{V(\rho, \varphi) = \sum_{k=0}^{\infty} \frac{4V_0}{(2k+1)\pi} \frac{1}{b^{(2k+1)} - a^{4k+2} b^{-(2k+1)}} \left(\rho^{2k+1} - a^{4k+2} \rho^{-(2k+1)}\right) \sin(2k+1)\varphi}}$$

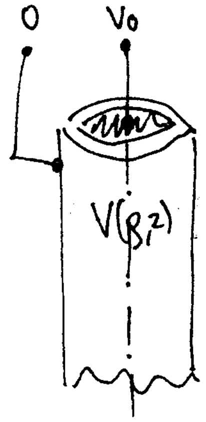
$$\vec{E} = -\operatorname{grad} V$$

$$\underline{\underline{\vec{E} = -\vec{1}_\rho \frac{4V_0}{\pi} \sum_{k=0}^{\infty} \frac{1}{b^{2k+1} - a^{4k+2} b^{-(2k+1)}} \left(\rho^{2k} + a^{4k+2} \rho^{-(2k+2)}\right) \sin(2k+1)\varphi -}}$$

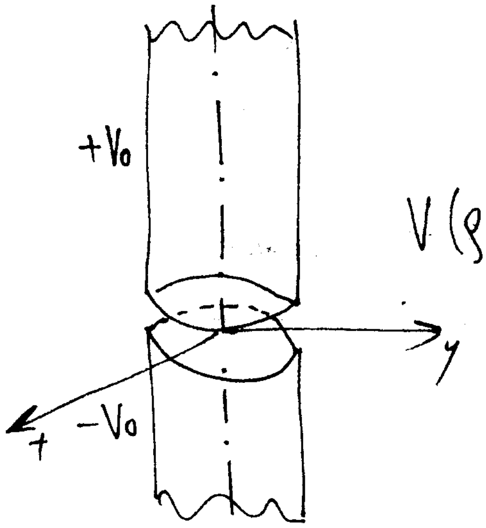
$$\underline{\underline{- \vec{1}_\varphi \frac{4V_0}{\pi} \sum_{k=0}^{\infty} \frac{1}{b^{2k+1} - a^{4k+2} b^{-(2k+1)}} \left(\rho^{2k} - a^{4k+2} \rho^{-(2k+2)}\right) \cos(2k+1)\varphi}}$$

REŠITVE 1.k TEMAG 19/11/1991
2. LIST

(2.) Poznamo rešitev problema:



$$V(\rho,z)=\sum_{k=1}^{\infty}\frac{-2V_0}{\lambda_k a\, J_0'(\lambda_k a)}\,J_0(\lambda_k \rho)\cdot e^{\lambda_k z}=f(\rho,z)$$



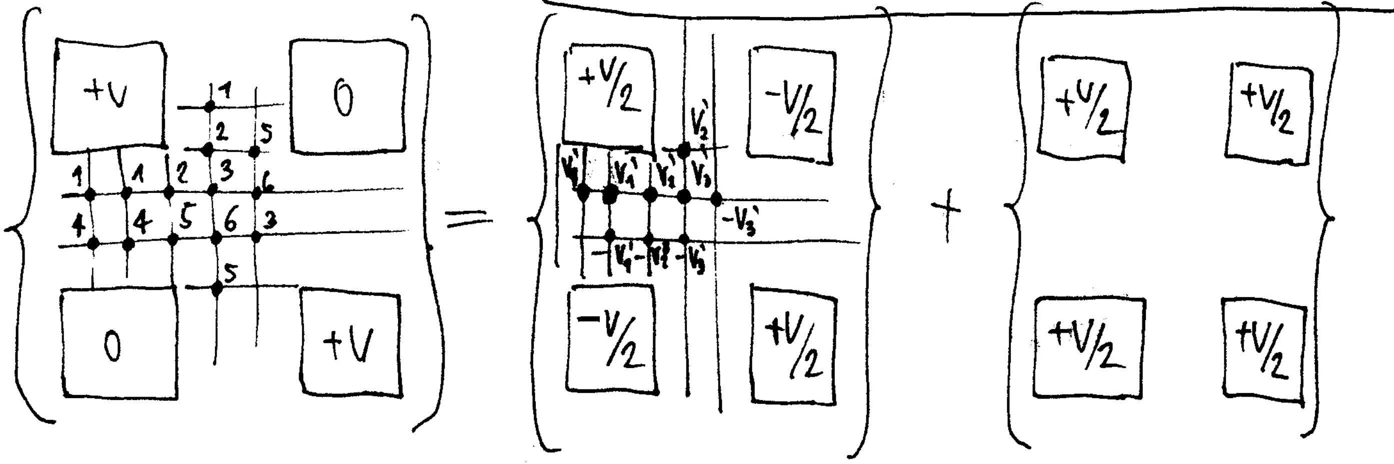
$$V(\rho,z)=\begin{cases} f(\rho,z)-V_0 & ;\quad z<0 \\ & \\ V_0-f(\rho,-z) & ;\quad z>0 \end{cases}$$

na ravnini x-y je potencial 0!

$$V(\rho,z)=\begin{cases} \displaystyle\sum_{k=1}^{\infty}\frac{-2V_0}{\lambda_k a J_0'(\lambda_k a)}J_0(\lambda_k \rho)\,e^{\lambda_k z}-V_0 & ;\quad z<0 \\ & \\ \displaystyle V_0-\sum_{k=1}^{\infty}\frac{-2V_0}{\lambda_k a J_0'(\lambda_k a)}J_0(\lambda_k \rho)\,e^{-\lambda_k z} & ;\quad z>0 \end{cases}$$

REŠITVE 1.k TEMAG 19/11/1991. 3.LIST

3.



$$4V_1' = V/2 + V_1' - V_1' + V_2' \longrightarrow 8V_1' = V + 2V_2'$$

$$4V_2' = V/2 + V_1' - V_2' + V_3' \longrightarrow 5V_2' = V/2 + V_1' + V_3' \rightarrow 10V_2' = V + 2V_1' + 2$$

$$4V_3' = V_2' + V_2' - V_3' - V_3' \longrightarrow 6V_3' = 2V_2' \rightarrow 3V_3' = V_2'$$

$$8V_1' = V + 2V_2'$$

$$30V_2' = 3V + 6V_1' + 2V_2'$$

$$28V_2' = 3V + 6V_1'$$

$$2V_2' = 8V_1' - V$$

$$112V_1' - 14V = 3V + 6V_1'$$

$$106V_1' = 17V$$

$$V_1' = \frac{17}{106}V \qquad V_2' = \frac{15}{106}V \qquad V_3' = \frac{5}{106}V$$

$$V_1 = V_1' + V/2 = \frac{70}{106}V \qquad V_4 = -V_1' + V/2 = \frac{36}{106}$$

$$V_2 = V_2' + V/2 = \frac{68}{106}V \qquad V_5 = -V_2' + V/2 = \frac{38}{106}$$

$$V_3 = V_3' + V/2 = \frac{58}{106}V \qquad V_6 = -V_3' + V/2 = \frac{48}{106}$$

REŠITVE 1.K TEMAG
19/11/1991 4. LIST

4.

$$\frac{1}{dR} = dG = \int_a^b \frac{d\cdot\gamma}{r\,d\varphi}\,dr = \frac{d\cdot\gamma}{d\varphi}\ln\frac{b}{a}$$

$$dR = \frac{1}{\gamma d \ln\frac{b}{a}}\,d\varphi$$

$$R(\varphi) = R \cdot \frac{U}{U_0} = R \cdot \frac{e^{k\varphi}-1}{e^{k\pi}-1}$$

$$\frac{dR(\varphi)}{d\varphi} = R\,\frac{k\,e^{k\varphi}}{e^{k\pi}-1} = \frac{1}{\gamma d \ln\frac{b}{a}}$$

$$\gamma = \frac{e^{k\pi}-1}{Rk\,d\ln\frac{b}{a}}\,e^{-k\varphi}$$

Rešitve nalog 2. kolokvija iz TEORIJE ELEKTROMAGNETIKE 14/1/1992

1. Polje same zankice: $\vec{H}_0 = \frac{\mu_0 I r_0^2}{4r^3}\left(\vec{1}_r \cdot 2\cos\Theta + \vec{1}_\Theta \cdot \sin\Theta\right)$

pogoji:

(A) $\vec{H} \approx \vec{H}_0$ pri $r \to 0$

(B) $\vec{1}_\Theta \cdot \vec{H} = 0$ pri $r = a$ in $\mu_r \to \infty$

$$\Delta V_m = 0 \longrightarrow V_m = \left(Ar + Br^{-2}\right)\cos\Theta$$

$$\vec{H} = -\operatorname{grad} V_m = +\vec{1}_r\left(-A + 2Br^{-3}\right)\cos\Theta + \vec{1}_\Theta\left(A + Br^{-3}\right)\sin\Theta$$

$$(A) \rightarrow \underline{\underline{B = \frac{\mu_0 I r_0^2}{4}}} \quad ; \quad (B) \longrightarrow \underline{\underline{A = -Ba^{-3}}}$$

$$\underline{\underline{\vec{H} = \vec{1}_r \frac{\mu_0 I r_0^2}{4}\left(a^{-3} + 2r^{-3}\right)\cos\Theta + \vec{1}_\Theta\left(r^{-3} - a^{-3}\right)\sin\Theta}}$$

---

2. $$M = \frac{1}{I}\oint_b \vec{V}_{ma} \cdot d\vec{s} = 4 \cdot \left[\frac{1}{I}\int_{-\frac{b}{2}}^{+\frac{b}{2}} \vec{V}_{ma} \cdot \vec{1}_x dx\right] \approx \underline{\frac{4}{I}\left(\vec{V}_{ma} \cdot \vec{1}_x\right) b}$$

$$\vec{V}_{ma} \cdot \vec{1}_x = \left[\frac{\mu_0}{4\pi}\oint_a \frac{\vec{1}_I \cdot I}{|\vec{r} - \vec{r}\,'|} ds\right] \cdot \vec{1}_x = \frac{\mu_0}{4\pi}\int_{-\frac{a}{2}}^{+\frac{a}{2}}\left[\underbrace{\frac{I}{\sqrt{d^2 + \left(\frac{a-b}{2}\right)^2}}}_{\text{prednja stranica}} - \underbrace{\frac{I}{\sqrt{d^2 + \left(\frac{a+b}{2}\right)^2}}}_{\text{zadnja stranica}}\right] dx =$$

$$\frac{1}{\sqrt{1+\varepsilon}} \approx 1 - \frac{\varepsilon}{2} \quad \varepsilon \ll 1$$

$$= \frac{\mu_0 I a}{4\pi d}\left[1 - \frac{(a-b)^2}{8d^2} - 1 + \frac{(a+b)^2}{8d^2}\right] = \underline{\frac{\mu_0 I a}{4\pi d}\frac{4ab}{8d^2}}$$

$$M = \frac{4}{I}\left(\frac{\mu_0 I a}{4\pi d} \cdot \frac{4ab}{8d^2}\right) \cdot b = \underline{\underline{\frac{\mu_0 a^2 b^2}{2\pi d^3}}}$$

---

3. $$\vec{E} = \vec{1}_E \cdot E \quad \vec{1}_E = \frac{\vec{S} \times \vec{1}_z}{|\vec{S} \times \vec{1}_z|} = \frac{-\vec{1}_y + \vec{1}_x}{|-\vec{1}_y + \vec{1}_x|} = \underline{\underline{\vec{1}_x\frac{1}{\sqrt{2}} - \vec{1}_y\frac{1}{\sqrt{2}}}} \quad E = \sqrt{2|\vec{S}|Z_0} = \sqrt{2\sqrt{3}\,120\pi} = \underline{\underline{36.14\,\frac{V}{m}}}$$

$$\vec{H} = \vec{1}_H \cdot H \quad \vec{1}_H = \frac{\vec{S} \times \vec{1}_E}{|\vec{S} \times \vec{1}_E|} = \frac{1/\sqrt{2}\left(-\vec{1}_z - \vec{1}_z + \vec{1}_x + \vec{1}_y\right)}{1/\sqrt{2}\sqrt{4+1+1}} = \underline{\underline{\vec{1}_x\frac{1}{\sqrt{6}} + \vec{1}_y\frac{1}{\sqrt{6}} - \vec{1}_z\frac{2}{\sqrt{6}}}} \quad \underline{\underline{H = \frac{E}{Z_0} = 0.096\,\frac{A}{m}}}$$

$$\vec{k} = \vec{1}_k \cdot k \quad \vec{1}_k = \vec{1}_s = \frac{\vec{S}}{|\vec{S}|} = \underline{\underline{\vec{1}_x\frac{1}{\sqrt{3}} + \vec{1}_y\frac{1}{\sqrt{3}} + \vec{1}_z\frac{1}{\sqrt{3}}}}$$

$$2\pi = \vec{k} \cdot \vec{1}_z(z_1 - z_2) = \frac{k}{\sqrt{3}}(1m - 0m) \rightarrow k = 2\pi\sqrt{3} = \underline{\underline{10.88\,rd/m}}$$

$$f = \frac{c}{\lambda} \;;\; k = \frac{2\pi}{\lambda} \rightarrow f = \frac{c \cdot k}{2\pi} = 3 \cdot 10^8 \cdot \sqrt{3} = \underline{\underline{519.6\,MHz}}$$

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4. $$P = \frac{1}{2}|I|^2 Z_k \;;\; Z_k = \frac{Z_0}{\pi}\ln\frac{2d - r_0}{r_0}$$

$$dP = -\frac{1}{2}|I|^2 dR \;;\; dR = 2R_p\frac{1}{2\pi r_0}dl \;;\; R_p = \sqrt{\frac{\omega\mu_0}{2\sigma}} = \underline{3.755\,m\Omega}$$

$$\frac{dP}{P} = -\frac{dR}{Z_k} = -\frac{2R_p}{2\pi r_0 Z_k}dl = -\frac{R_p}{\pi r_0 Z_k}dl$$

$$\ln P = -\frac{R_p}{\pi r_0 Z_k}l + C$$

$$A = 10\log\frac{P(0)}{P(l)} = \frac{10}{\ln 10}\ln\frac{P(0)}{P(l)} = \frac{10}{\ln 10}\frac{R_p l}{\pi r_0 Z_k} = \frac{10}{\ln 10}\frac{l\sqrt{\frac{\omega\mu_0}{2\sigma}}}{r_0 Z_0 \ln\frac{2d - r_0}{r_0}} = \underline{\underline{0.734\,dB}}$$

Rešitve nalog 1. Kolokvija iz Teorije Elektromagnetike - 30/11/1993

(1.) $V = \sum_n A_n \rho^{2n} \cos 2n\varphi$

Na pokrovu: $\rho = a$; $\int_{-\pi/4}^{+\pi/4} V_0 \cos 2n\varphi \, d\varphi = \int_{-\pi/4}^{+\pi/4} \left(\sum_m A_m a^{2m} \cos 2m\varphi \right) \cos 2n\varphi \, d\varphi$

$$\frac{V_0}{2n}\left(\sin\frac{\pi}{2}n - \sin\left(-\frac{\pi}{2}n\right)\right) = A_n a^{2n} \frac{\pi}{4}$$

$$A_n = \frac{4V_0 \sin\left(\frac{\pi}{2}n\right)}{n\pi \, a^{2n}} \qquad V(\rho,\varphi,z) = \sum_{n=1}^{\infty} \frac{4V_0 \sin\left(\frac{\pi}{2}n\right)}{n\pi \, a^{2n}} \rho^{2n} \cos 2n\varphi$$

$$\vec{E} = -\operatorname{grad} V = -\vec{1}_\rho \sum_{n=1}^{\infty} \frac{8V_0 \sin\left(\frac{\pi}{2}n\right)}{\pi a^{2n}} \rho^{2n-1} \cos 2n\varphi + \vec{1}_\varphi \sum_{n=1}^{\infty} \frac{8V_0 \sin\left(\frac{\pi}{2}n\right)}{\pi a^{2n}} \rho^{2n-1} \sin 2n\varphi$$

Pri praktičnem računu vzamemo: $n = 2k+1$; $k = 0,1,2 \ldots$; $\sin\left(\frac{\pi}{2}n\right) = (-1)^k$

(2.) Iz geometrije sledi: $\vec{E} = \vec{1}_r E(r)$; $\vec{D} = \vec{1}_r \varepsilon(r) E(r)$

$$\frac{dW}{dv} = \text{konst.} = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\varepsilon(r)E^2(r) = \frac{W}{v} = \frac{\frac{Q^2}{2C}}{\frac{4\pi}{3}(r_2^3 - r_1^3)} \; ; \; Q = \oint \vec{D}\cdot d\vec{A} = 4\pi r^2 \varepsilon(r) E(r)$$

$$\frac{1}{2}\varepsilon(r)E^2(r) = \frac{(4\pi r^2)^2 \varepsilon^2(r) E^2(r)}{2C \, \frac{4\pi}{3}(r_2^3 - r_1^3)} \longrightarrow \varepsilon(r) = \frac{r_2^3 - r_1^3}{12\pi r^4} C$$

(3.) Uporabimo rešitev iz elektrostatike: $\gamma \to \varepsilon$; $I/l \to q$

Valj: $V_v(R) = -\frac{q}{2\pi\varepsilon} \ln\frac{R}{r_0} + V_0$; Trak: $V_T(R) = -\frac{q}{2\pi\varepsilon} u + V_0$; $x = a \operatorname{ch} u \cos v$, $y = a \operatorname{sh} u \sin v$, $z = z$

$$R = \sqrt{x^2+y^2} = \sqrt{a^2\operatorname{ch}^2 u \cos^2 v + a^2 \operatorname{sh}^2 u \sin^2 v} \approx \frac{a}{2} e^u \longrightarrow u \approx \ln\frac{2R}{a} \text{ pri } R \gg a$$

$$V_v(R) = V_T(R) \longrightarrow \ln\frac{R}{r_0} = \ln\frac{2R}{a} \longrightarrow a = 2r_0 \; ; \; 2a = 4r_0$$

(4.) Iz simetrije sledi: $V_1 = V_6 = V_{11} = V_{16} = \frac{V_0}{2}$; $V_{12} = V_2$; $V_8 = V_3$; $V_5 = V_0 - V_2 = V_{15}$; $V_9 = V_0 - V_3 = V_{14}$

$V_{13} = V_0 - V_4$; $V_{10} = V_0 - V_7$

Ostanejo 4 neznanke: V_2; V_3; V_4; V_7

$$4V_2 = V_0 + \frac{V_0}{2} + \frac{V_0}{2} + V_3$$
$$4V_3 = V_0 + V_2 + V_7 + V_4$$
$$4V_4 = V_0 + V_3 + V_3 + V_0$$
$$4V_7 = V_3 + \frac{V_0}{2} + \frac{V_0}{2} + V_3$$

$$4V_2 = 2V_0 + V_3$$
$$4V_3 = V_0 + V_2 + V_4 + V_7 \quad / \cdot 4$$
$$4V_4 = 2V_0 + 2V_3$$
$$4V_7 = V_0 + 2V_3$$

$$16V_3 = 4V_0 + 2V_0 + V_3 + 2V_0 + 2V_3 + V_0 + 2V_3$$

$$V_3 = \frac{9}{11}V_0 = V_8 \qquad V_9 = \frac{2}{11}V_0 = V_{14}$$

$$V_2 = \frac{31}{44}V_0 = V_{12} \qquad V_5 = \frac{13}{44}V_0 = V_{15}$$

$$V_4 = \frac{10}{11}V_0 \qquad V_{13} = \frac{1}{11}V_0$$

$$V_7 = \frac{29}{44}V_0 \qquad V_{10} = \frac{15}{44}V_0$$

Rešitve nalog 2. kolokvija iz TEORIJE ELEKTROMAGNETIKE - 19/1/94

(1)

$$\vec V_m = \frac{\mu_0}{4\pi}\int_V \frac{\vec J(\vec r\,')}{|\vec r-\vec r\,'|}\,dV = \frac{\mu_0 I}{4\pi}\int \frac{\vec{ds}}{|\vec r-\vec r\,'|} \qquad \vec{ds} = \left(\vec 1_\varphi a + \vec 1_z \frac{h}{2\pi}\right)d\varphi$$

$$|\vec r-\vec r\,'| = \sqrt{(r\sin\Theta\cos\phi - a\cos\varphi)^2 + (r\sin\Theta\sin\phi - a\sin\varphi)^2 + \left(r\cos\Theta - \frac{h}{2\pi}\varphi\right)^2}$$

$$|\vec r-\vec r\,'| \approx r\left(1 - \frac{a}{r}\sin\Theta\cos(\phi-\varphi) - \frac{h}{2\pi r}\varphi\cos\Theta\right)$$

$$\vec V_m \approx \frac{\mu_0 I}{4\pi r}\int_0^{2\pi}\left(\underline{\underline{1}} + \frac{a}{r}\sin\Theta\cos(\phi-\varphi) + \frac{h}{2\pi r}\varphi\cos\Theta\right)\left(\vec 1_\varphi a + \underline{\underline{\vec 1_z \frac{h}{2\pi}}}\right)d\varphi$$

največja člena, produkt ne upada z $\frac{a}{r}$ ali $\frac{h}{r}$!

$$\vec V_m \approx \frac{\mu_0 I}{4\pi r}\int_0^{2\pi}\vec 1_z \frac{h}{2\pi}\,d\varphi = \underline{\underline{\vec 1_z \frac{\mu_0 I h}{4\pi r}}}$$

rezultat upada z r, ker tok ni zaključen!

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(2) Znotraj: $V_{mn} = (A_n\varrho + B_n\varrho^{-1})\sin\varphi$ $\qquad \vec H_n = -\left[\vec 1_\varrho(A_n - B_n\varrho^{-2})\sin\varphi + \vec 1_\varphi(A_n + B_n\varrho^{-2})\cos\varphi\right]$

V steni: $V_{mv} = (A_v\varrho + B_v\varrho^{-1})\sin\varphi$ $\qquad \vec H_v = -\left[\vec 1_\varrho(A_v - B_v\varrho^{-2})\sin\varphi + \vec 1_\varphi(A_v + B_v\varrho^{-2})\cos\varphi\right]$

Zunaj: $V_{mz} = B_z\varrho^{-1}\sin\varphi$ $\qquad \vec H_z = -\left[-\vec 1_\varrho B_z\varrho^{-2}\sin\varphi + \vec 1_\varphi B_z\varrho^{-2}\cos\varphi\right]$

Zunaj ni polja v ∞, zato je $A_z = 0$

Učinkovitost oklopa $= \frac{B_n}{B_z}$

TANGENCIALNI $\vec H$: φ komp. $\qquad$ NORMALNI $\vec B$: ϱ komp.

$\varrho = a$: (1) $A_n + B_n a^{-2} = A_v + B_v a^{-2}$ $\qquad$ (3) $A_n - B_n a^{-2} = \mu_r(A_v - B_v a^{-2})$

$\varrho = b$: (2) $A_v + B_v b^{-2} = B_z b^{-2}$ $\qquad$ (4) $\mu_r(A_v - B_v b^{-2}) = -B_z b^{-2}$

Izločimo A_n: (1)−(3) $\quad 2B_n a^{-2} = A_v(1-\mu_r) + B_v a^{-2}(1+\mu_r)$

Izrazimo A_v in B_v z B_z: μ_r(2)±(4) $\quad 2\mu_r A_v = B_z b^{-2}(\mu_r - 1) \qquad A_v = B_z b^{-2}\frac{\mu_r - 1}{2\mu_r}$

$2\mu_r B_v b^{-2} = B_z b^{-2}(\mu_r + 1) \qquad B_v = B_z\frac{\mu_r + 1}{2\mu_r}$

$$2B_n a^{-2} = B_z b^{-2}\frac{\mu_r - 1}{2\mu_r}(1-\mu_r) + B_z\frac{\mu_r + 1}{2\mu_r}a^{-2}(1+\mu_r) \longrightarrow \underline{\underline{\frac{B_n}{B_z} = \frac{(1+\mu_r)^2 - a^2/b^2\,(1-\mu_r)^2}{4\mu_r}}}$$

---

(3) $\vec k = \vec 1_k\cdot k \qquad \vec 1_k = \pm\frac{\vec E\times\vec 1_z}{|\vec E\times\vec 1_z|} = \pm\frac{(-\vec 1_x + \vec 1_y\cdot 2 + \vec 1_z\cdot 2)\times\vec 1_z}{\sqrt 5} = \underline{\underline{\pm\frac{\vec 1_x\cdot 2 + \vec 1_y}{\sqrt 5}}} \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c} = \underline{\underline{3.33\,\mathrm{m}^{-1}}}$

$\vec H = \vec 1_H\cdot H \qquad \vec 1_H = \vec 1_k\times\vec 1_E \qquad \vec H = \vec 1_k\times\vec E\frac{1}{Z_0} = \pm\frac{\vec 1_x\cdot 2 + \vec 1_y}{\sqrt 5}\times(-\vec 1_x + \vec 1_y\cdot 2 + \vec 1_z\cdot 2)\frac{\mathrm{A}}{120\pi\,\mathrm{m}} = \underline{\underline{\pm(\vec 1_x\cdot 2 - \vec 1_y\cdot 4 + \vec 1_z\cdot 5)\frac{\mathrm{A/m}}{\sqrt 5\,120\pi}}}$

$\vec S = \vec 1_S\cdot S \qquad \underline{\underline{\vec 1_S = \vec 1_k}} \qquad S = \frac{|E|^2}{2Z_0} = \underline{\underline{\frac{9}{2\cdot 120\pi}\frac{\mathrm{W}}{\mathrm{m}^2} = 0.012\,\mathrm{W/m^2}}}$

---

(4) $U = E\cdot d$; $I = H\cdot w = \frac{E\sqrt{\varepsilon_r}}{Z_0}\cdot w$; $Z_k = \frac{U}{I} = \frac{d\,Z_0}{w\sqrt{\varepsilon_r}}$; $P = \frac{1}{2}|I|^2 Z_k$

$R_p = \sqrt{\frac{\mu_0\omega}{2\gamma_{Cu}}}$; $dR = 2R_p\frac{dl}{w} = \sqrt{\frac{\mu_0\omega}{2\gamma_{Cu}}}\,2\frac{dl}{w}$; $dP = -\frac{1}{2}|I|^2 dR = \frac{1}{2}|I|^2\sqrt{\frac{\mu_0\omega}{2\gamma_{Cu}}}\,2\frac{dl}{w}$

$$\frac{dP}{P} = -\sqrt{\frac{\mu_0\omega}{2\gamma_{Cu}}}\frac{2\sqrt{\varepsilon_r}}{d\,Z_0}\,dl \ ; \quad \ln\frac{P_1}{P_2} = \frac{l}{Z_0 d}\sqrt{\frac{2\mu_0\omega\varepsilon_r}{\gamma_{Cu}}}$$

$$A = 10\log_{10}\frac{P_1}{P_2} = \frac{10}{\ln 10}\ln\frac{P_1}{P_2} = \underline{\underline{\frac{10}{\ln 10}\frac{l}{Z_0 d}\sqrt{\frac{2\mu_0\omega\varepsilon_r}{\gamma_{Cu}}} = 12.2\,\mathrm{dB}}}$$

Rešitev 1. kolokvija iz TEORIJE ELEKTROMAGNETIKE – 29/11/1994

① $0 = \mathrm{div}\,\vec{D} = \mathrm{div}(\varepsilon_r \varepsilon_0 \vec{E}) = \varepsilon_0 \vec{E} \cdot \mathrm{grad}\,\varepsilon_r + \varepsilon_r \varepsilon_0\, \mathrm{div}\,\vec{E}$ $\qquad \vec{E} = \text{konst.} = \vec{1}_\varrho \cdot E_0$

$$0 = \varepsilon_0 \vec{1}_\varrho E_0 \cdot \vec{1}_\varrho \frac{\partial \varepsilon_r}{\partial \varrho} + \varepsilon_r \varepsilon_0 \frac{E_0}{\varrho} \longrightarrow \frac{d\varepsilon_r}{\varepsilon_r} = -\frac{d\varrho}{\varrho} \longrightarrow \ln \varepsilon_r = -\ln \varrho + \ln A$$

$$\underline{\varepsilon_r = \frac{A}{\varrho}}$$

$$q = \int_0^{2\pi} \vec{D} \cdot \vec{1}_\varrho\, \varrho\, d\varphi = 2\pi \frac{A}{\varrho} \varepsilon_0 E_0 \varrho = 2\pi A \varepsilon_0 E_0 \qquad U = (b-a)E_0$$

$$C/\ell = \frac{q}{U} = \frac{2\pi A \varepsilon_0 E_0}{(b-a)E_0} = \frac{2\pi \varepsilon_0 A}{b-a} \longrightarrow A = \frac{C/\ell\,(b-a)}{2\pi\varepsilon_0} = \underline{54\,\mathrm{mm}} \qquad \underline{\underline{\varepsilon_r = \frac{54\,\mathrm{mm}}{\varrho}}}$$

---

② $$\sigma(x, y=0) = -\frac{4V_0\varepsilon_0}{a} \sum_{k=0}^{\infty} \frac{\sin (2k+1)\frac{\pi}{a}x}{\mathrm{sh}\,(2k+1)\frac{\pi}{a}b} \longrightarrow \begin{cases} k=0: & -30\cdot 10^{-3}\frac{\mathrm{As}}{\mathrm{m}^2} = -\frac{4V_0\varepsilon_0}{a}\frac{1}{\mathrm{sh}\,\pi\frac{b}{a}} \\ k=1: & -1\cdot 10^{-3}\frac{\mathrm{As}}{\mathrm{m}^2} = -\frac{4V_0\varepsilon_0}{a}\frac{1}{\mathrm{sh}\,3\pi\frac{b}{a}} \end{cases}$$

Vzamemo samo $k=0,1$; členi $k \geq 2 \rightarrow$ majhna napaka!

$$30 = \frac{\mathrm{sh}\,3\pi\frac{b}{a}}{\mathrm{sh}\,\pi\frac{b}{a}} = \frac{u^3 - u^{-3}}{u - u^{-1}} = u^2 + 1 + u^{-2}\ ;\ u = e^{\pi\frac{b}{a}} \rightarrow u^4 - 29u^2 + 1 = 0\ ;\ u^2 = \frac{29 \pm \sqrt{29^2-4}}{2}$$

$$u^2 = 28.965\ ;\ u = 5.382\ ;\ b = \frac{a}{\pi}\ln u = \underline{\underline{0.536\,\mathrm{m}}}$$

$$V_0 = \frac{a}{4\varepsilon_0}\,\mathrm{sh}\,\pi\frac{b}{a} \cdot 30\cdot 10^{-3}\frac{\mathrm{As}}{\mathrm{m}^2} = \underline{\underline{22\,\mathrm{kV}}}$$

---

③ $$\vec{E}_0 = -\vec{1}_z E_0 = (-\vec{1}_r\cos\Theta + \vec{1}_\Theta \sin\Theta)E_0 \qquad \sigma = \vec{1}_r \cdot (\vec{D}_Z - \vec{D}_N) = \vec{1}_r \cdot (\varepsilon_0 \vec{E}_0 - \varepsilon_r\varepsilon_0 \vec{E}_0)$$

$$\sigma = \vec{1}_r \cdot \vec{E}_0 \varepsilon_0 (1-\varepsilon_r) = \underline{\underline{E_0 \varepsilon_0 (\varepsilon_r - 1)\cos\Theta = 79.6\,\mathrm{nAs/m^2}\cdot\cos\Theta}}$$

---

④ $$V = A\ln\left(\mathrm{tg}\frac{\Theta}{2}\right)\ ;\ \vec{E} = -\mathrm{grad}\,V = -\vec{1}_\Theta \frac{A}{r\sin\Theta}\ ;\ \vec{J} = \gamma\vec{E} = -\vec{1}_\Theta \gamma_0 e^{-\frac{r}{r_0}} \frac{A}{r\sin\Theta}$$

$$I = \int_0^\infty\int_0^{2\pi} \vec{J}\cdot(-\vec{1}_\Theta)\, r\sin\Theta\, d\phi\, dr = 2\pi \int_0^\infty \gamma_0 e^{-\frac{r}{r_0}} A\, dr = \underline{2\pi\gamma_0 r_0 A}$$

$$U = V_B - V_A = A\ln\left(\mathrm{tg}\frac{\beta}{2}\right) - A\ln\left(\mathrm{tg}\frac{\alpha}{2}\right) = \underline{A\ln\frac{\mathrm{tg}\,5\pi/12}{\mathrm{tg}\,\pi/12}}$$

$$R = \frac{U}{I} = \frac{\ln\frac{\mathrm{tg}\,5\pi/12}{\mathrm{tg}\,\pi/12}}{2\pi\gamma_0 r_0} = \underline{\underline{0.419\,\Omega}}$$

Rešitve 2. kolokvija iz TEORIJE ELEKTROMAGNETIKE – 10/1/95

① $I = \oint \vec{H}\,d\vec{s} \rightarrow \vec{H} = \vec{1}_\varphi \frac{I}{2\pi\rho}$ $\qquad \vec{1}_z = \vec{1}_r \cos\Theta - \vec{1}_\Theta \sin\Theta$; $\rho = r\sin\Theta$

$\vec{H} = \frac{1}{\mu_0}\,\mathrm{rot}\,\vec{V}_m$; $\vec{V}_m = \vec{1}_z V_m(\rho)$; $\mathrm{rot}\,\vec{V}_m = -\vec{1}_\varphi \frac{\partial V_m(\rho)}{\partial \rho} = \vec{1}_\varphi \frac{\mu_0 I}{2\pi\rho}$

$$\vec{V}_m = -\vec{1}_z\left(\frac{\mu_0 I}{2\pi}\ln\rho + C\right) = \underline{\underline{\left(-\vec{1}_r\cos\Theta + \vec{1}_\Theta\sin\Theta\right)\left(\frac{\mu_0 I}{2\pi}\ln(r\sin\Theta) + C\right)}}$$

---

② $V_{m0} = \frac{IN}{l} z = \frac{IN}{l} r\cos\Theta$; rešujemo $\Delta V_m = 0$

$V_{mz} = \left(\frac{IN}{l} r + \frac{B}{r^2}\right)\cos\Theta$; $\vec{H}_z = -\vec{1}_r\left(\frac{IN}{l} - \frac{2B}{r^3}\right)\cos\Theta + \vec{1}_\Theta\left(\frac{IN}{l} + \frac{B}{r^3}\right)\sin\Theta$

$V_{mN} = Ar\cos\Theta$; $\vec{H}_N = -\vec{1}_r A\cos\Theta + \vec{1}_\Theta A\sin\Theta$

$H_{tz} = H_{tN} \longrightarrow \frac{IN}{l} + \frac{B}{r_0^3} = A$; $B_{nz} = B_{nN} \longrightarrow \left(\frac{IN}{l} - \frac{2B}{r_0^3}\right)\mu_0\mu_r = A\mu_0$

$\frac{IN}{l} 3\mu_r = A(2\mu_r + 1) \longrightarrow A = \frac{IN}{l}\frac{3\mu_r}{2\mu_r+1}$; $\underline{\underline{\vec{H}_N = -\vec{1}_z \frac{IN}{l}\frac{3\mu_r}{2\mu_r+1}}}$

---

③ $\Delta V = 0 \longrightarrow V = E_0 e^{j\omega t}\left(\rho - \frac{a^2}{\rho}\right)\sin\varphi$

$\vec{E} = -\mathrm{grad}\,V = -\vec{1}_\rho E_0 e^{j\omega t}\left(1 + \frac{a^2}{\rho^2}\right)\sin\varphi - \vec{1}_\varphi E_0 e^{j\omega t}\left(1 - \frac{a^2}{\rho^2}\right)\cos\varphi$

$\vec{1}_n = \vec{1}_\rho \qquad \sigma = \vec{1}_n \cdot \varepsilon_0 \vec{E} = -2\varepsilon_0 E_0 e^{j\omega t}\sin\varphi$

$\mathrm{div}\,\vec{J} + j\omega\rho = 0 \rightarrow \mathrm{div}\,\vec{K} + j\omega\sigma = 0$ $\qquad \vec{K} = \vec{1}_\varphi K(\varphi)$

$\mathrm{div}\,\vec{K} = \frac{1}{a}\left(\frac{\partial K}{\partial \varphi}\right) = -j\omega\sigma$

$\frac{\partial K}{\partial\varphi} = a\,2j\omega\varepsilon_0 E_0 e^{j\omega t}\sin\varphi \longrightarrow \underline{\underline{\vec{K} = -\vec{1}_\varphi\left(2j\omega\varepsilon_0 E_0 a e^{j\omega t}\cos\varphi + C\right)}}$

---

④ $(\alpha + j\beta)^2 = j\omega\mu_0(\gamma + j\omega\varepsilon_0\varepsilon_r)$

$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu_0\gamma - \omega^2\mu_0\varepsilon_0\varepsilon_r$

$\alpha^2 - \beta^2 = -\omega^2\mu_0\varepsilon_0\varepsilon_r \qquad 2\alpha\beta = \omega\mu_0\gamma$

$\beta = \frac{\omega\mu_0\gamma}{2\alpha}$

$\alpha^2 - \frac{\omega^2\mu_0^2\gamma^2}{4\alpha^2} = -\omega^2\mu_0\varepsilon_0\varepsilon_r$

$\alpha^4 + \alpha^2\omega^2\mu_0\varepsilon_0\varepsilon_r - \frac{\omega^2\mu_0^2\gamma^2}{4} = 0$

$\alpha^2 = \frac{-\omega^2\mu_0\varepsilon_0\varepsilon_r \pm \sqrt{\omega^4\mu_0^2\varepsilon_0^2\varepsilon_r^2 + \omega^2\mu_0^2\gamma^2}}{2}$

$\alpha = \sqrt{\frac{\omega\mu_0\gamma}{2}} \cdot \sqrt{\sqrt{1 + \frac{\omega^2\varepsilon_0^2\varepsilon_r^2}{\gamma^2}} - \frac{\omega\varepsilon_0\varepsilon_r}{\gamma}}$

uvedemo $u = \frac{\omega\varepsilon_0\varepsilon_r}{\gamma}$

1% napaka $\longrightarrow \sqrt{\sqrt{1+u^2} - u} = 0.99$

$u \approx 0.02$

$f = \frac{\omega}{2\pi} = \frac{u\gamma}{2\pi\varepsilon_0\varepsilon_r} = \underline{\underline{22.5\,\mathrm{MHz}}}$

Rešitev 1. kolokvija iz TEORIJE ELEKTROMAGNETIKE – 28/11/1995

① $h_{q_i} = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2}$

$$\underline{h_u = a\sqrt{sh^2 u + \sin^2 v}}$$

$$\underline{h_v = a\sqrt{sh^2 u + \sin^2 v}}$$

$$\underline{h_w = a\, ch\, u \cos v}$$

$$\operatorname{rot}\vec{F} = \frac{1}{a^3(sh^2u+\sin^2v)\, ch\, u\cos v}\begin{vmatrix} a\sqrt{sh^2u+\sin^2v}\,\vec{1}_u ; & a\sqrt{sh^2u+\sin^2v}\,\vec{1}_v ; & a\, ch\, u\cos v\,\vec{1}_w \\ \frac{\partial}{\partial u} ; & \frac{\partial}{\partial v} ; & \frac{\partial}{\partial w} \\ a\sqrt{sh^2u+\sin^2v}\, F_u ; & a\sqrt{sh^2u+\sin^2v}\, F_v ; & a\, ch\, u\cos v\, F_w \end{vmatrix}$$

$$\operatorname{div}\vec{F} = \frac{1}{a^3(sh^2u+\sin^2v)\, ch\, u\cos v}\left[\frac{\partial}{\partial u}\left(a^2\sqrt{sh^2u+\sin^2v}\, ch\, u\cos v\, F_u\right) + \frac{\partial}{\partial v}\left(a^2\sqrt{sh^2u+\sin^2v}\, ch\, u\cos v\, F_v\right) + \frac{\partial}{\partial w}\left(a^2(sh^2u+\sin^2v)\right.\right.$$

---

② $\vec{E} = -\operatorname{grad} V = \underline{-\vec{1}_r V_0 \cos 3\theta + \vec{1}_\theta V_0\, 3\sin 3\theta}$ $\qquad \vec{D} = \varepsilon_0 \vec{E} \qquad \rho = \operatorname{div}\vec{D} = \operatorname{div}(\varepsilon_0\vec{E})$

$$\rho = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\sin\theta\left(-\varepsilon_0 V_0\cos 3\theta\right)\right) + \frac{\partial}{\partial\theta}\left(r\sin\theta\,\varepsilon_0 V_0\, 3\sin 3\theta\right)\right] = -\frac{2\varepsilon_0 V_0}{r}\cos 3\theta +$$

$$+\frac{3\varepsilon_0 V_0\cos\theta\,\sin 3\theta}{r\sin\theta} + \frac{3\varepsilon_0 V_0}{r}\,3\cos 3\theta = \underline{\underline{\frac{\varepsilon_0 V_0}{r}\left(7\cos 3\theta + 3\operatorname{ctg}\theta\,\sin 3\theta\right)}}$$

---

③ $V(\rho,\varphi,z) = \sum_n C_n \rho^n \cos n\varphi \qquad \int_{-\pi}^{+\pi}\left[\sum_n C_n a^n \cos n\varphi\right]\cos m\varphi\, d\varphi = \int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}} V_0 \cos m\varphi\, d\varphi = \frac{2V_0}{m}\sin m\frac{\pi}{4} = \underbrace{\pi C_m a^m}$

Velja le pri $m \neq 0$

$$\underline{C_m = \frac{V_0}{2a^m}\,\frac{\sin\frac{m\pi}{4}}{\frac{m\pi}{4}} \text{ pri } m\neq 0 ; \quad C_0 = \frac{V_0}{4}}$$

$$\underline{\underline{V(\rho,\varphi,z) = \frac{V_0}{4} + \frac{V_0}{2}\sum_{m=1}^{\infty}\frac{\rho^m}{a^m}\,\frac{\sin\frac{m\pi}{4}}{\frac{m\pi}{4}}\cos m\varphi}}$$

---

④ $r \ll l \rightarrow V = -V_0 \ln\left(\operatorname{tg}\frac{\theta}{2}\right) ; \quad \underline{\vec{E} = \vec{1}_\theta V_0 \frac{1}{r\sin\theta}} ; \quad \vec{J} = \vec{1}_\theta \gamma V_0 \frac{1}{r\sin\theta}$

$$\frac{dI}{dz} = \int_0^{2\pi}\vec{J}\cdot\vec{1}_\theta\, r\sin\theta\, d\phi = 2\pi\gamma V_0 \longrightarrow V_0 = \frac{\frac{dI}{dz}}{2\pi\gamma} = \underline{\frac{I}{2\pi\gamma l}}$$

$$\underline{\underline{\vec{E} = \vec{1}_\theta \frac{I}{2\pi\gamma l}\,\frac{1}{r\sin\theta}}}$$

---

⑤ $U = \frac{I}{\Omega\gamma a} ; \quad R = \frac{1}{\Omega\gamma a} ; \quad \Omega = 2\pi\left(1-\cos\frac{\alpha}{2}\right)$

$$\underline{\underline{R = \frac{1}{2\pi\left(1-\cos\frac{\alpha}{2}\right)\gamma a}}}$$

Rešitev 2. kolokvija iz TEORIJE ELEKTROMAGNETIKE – 17/01/1996

(1)

$$\vec{H} = \frac{1}{\mu_0}\operatorname{rot}\vec{A} = \frac{1}{\mu_0\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \rho\cdot 0 & C\ln\rho \end{vmatrix} = -\vec{1}_\varphi\frac{C}{\mu_0\rho} \qquad \vec{J} = \operatorname{rot}\vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & -\rho\cdot\frac{C}{\mu_0\rho} & 0 \end{vmatrix} = 0$$

$$I = \oint \vec{H}\cdot d\vec{s} = \int_0^{2\pi} -\vec{1}_\varphi\frac{C}{\mu_0\rho}\cdot\vec{1}_\varphi\,\rho\,d\varphi = -\frac{2\pi C}{\mu_0}$$ tok v osi z!

(2)

$$\Delta W = \pm 2\cdot 2\frac{1}{2}\int_V \vec{J}\cdot\vec{A}\,dv = \pm 2\int_0^{2\pi} I\vec{1}_\varphi\cdot\vec{1}_\phi\frac{C}{a^2}\sin\frac{\pi}{2}\,a\,d\varphi = \pm\frac{4\pi IC}{a}$$

(3)

$$\operatorname{div}\vec{J} = -j\omega\rho \rightarrow \operatorname{div}\vec{K} = -j\omega\sigma$$

$$\sigma = \frac{j}{\omega}\operatorname{div}\vec{K} = \frac{j}{\omega}\frac{\partial}{\partial x}\left(\sin\alpha x\right) = \frac{j\alpha}{\omega}\cos\alpha x$$

(4)

$$\vec{H} = \frac{j}{\omega\mu_0}\operatorname{rot}\vec{E} = \frac{j}{\omega\mu_0}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\,\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & rC\frac{e^{-jkr}}{r}\sin\theta & 0 \end{vmatrix} = \vec{1}_\phi\frac{j}{\omega\mu_0 r}C(-jk)e^{-jkr}\sin\theta = \vec{1}_\phi\frac{C}{Z_0}\frac{e^{-jkr}}{r}\sin\theta$$

$$\vec{J} = \operatorname{rot}\vec{H} - j\omega\varepsilon_0\vec{E} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\,\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & r\sin\theta\frac{C}{Z_0}\frac{e^{-jkr}}{r}\sin\theta \end{vmatrix} - \vec{1}_\theta j\omega\varepsilon_0 C\frac{e^{-jkr}}{r}\sin\theta = \vec{1}_r\frac{C}{Z_0}\frac{e^{-jkr}}{r^2}2\cos\theta$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^{*} = \vec{1}_r\frac{|C|^2}{2Z_0}\frac{\sin^2\theta}{r^2}$$

(5)

$$P = \frac{1}{2}A\,|\vec{K}|^2 R_p \quad ; \quad R_p = \sqrt{\frac{\omega\mu}{2\gamma}}$$

$$|\vec{K}|^2 = \frac{2P}{A\sqrt{\frac{\omega\mu}{2\gamma}}} = |\vec{H}|^2 \qquad |\vec{H}| = \sqrt{\frac{2P}{A\sqrt{\frac{\omega\mu}{2\gamma}}}}$$

Rešitev 1. kolokvija iz TEORIJE ELEKTROMAGNETIKE - 26/11/1996

(1) $\vec{E}\cdot\vec{D}=\alpha$; $\vec{D}=\varepsilon_r\varepsilon_0\vec{E}$; $\vec{D}=\vec{1}_\varrho \frac{q}{2\pi\varrho} \longrightarrow \frac{|\vec{D}|^2}{\varepsilon_0\varepsilon_r}=\alpha$; $\varepsilon_r=\frac{q^2}{(2\pi)^2\varrho^2\varepsilon_0\alpha}$

$\alpha \equiv$ konstanta

$$U=-\int_b^a \vec{E}d\vec{s}=\int_a^b|\vec{E}|d\varrho=\int_a^b\frac{\alpha}{|\vec{D}|}d\varrho=\int_a^b\frac{\alpha 2\pi\varrho}{q}d\varrho=\frac{2\pi\alpha}{q}\,\frac{b^2-a^2}{2} \ ; \ \frac{C}{\ell}=\frac{q}{U}=\frac{2q^2}{2\pi\alpha(b^2-a^2)}$$

$$\alpha=\frac{q^2}{\pi\frac{C}{\ell}(b^2-a^2)} \ ; \ \underline{\underline{\varepsilon_r=\frac{C}{\ell}\,\frac{b^2-a^2}{4\pi\varepsilon_0}\,\frac{1}{\varrho^2}=\frac{6.75\cdot10^{-4}m^2}{\varrho^2}}} \qquad \varepsilon_0=\frac{1}{4\pi\cdot9\cdot10^9}\frac{As}{Vm}$$

---

(2) $V=\sum_k C_k\varrho^{-k}\sin k\varphi$; $\vec{E}=+\vec{1}_\varrho\frac{kC_k}{\varrho^{k+1}}\sin k\varphi-\vec{1}_\varphi\frac{kC_k}{\varrho^{k+1}}\cos k\varphi$; $\vec{E}(\varrho\gg a)\approx\vec{1}_\varrho\frac{C_1}{\varrho^2}\sin\varphi-\vec{1}_\varphi\frac{C_1}{\varrho^2}\cos\varphi$

$$\int_0^\pi V_0\sin\varphi d\varphi=\int_0^\pi\frac{C_1}{a}\sin^2\varphi d\varphi\longrightarrow C_1=\frac{4aV_0}{\pi} \ ; \ \vec{E}(\varrho\gg a)\approx\vec{1}_\varrho\frac{4aV_0}{\pi\varrho^2}\sin\varphi-\vec{1}_\varphi\frac{4aV_0}{\pi\varrho^2}\cos\varphi$$

Premi dipol $\vec{m}=\vec{1}_y m$: $\vec{E}=\frac{m}{2\pi\varepsilon_0}\left(\vec{1}_\varrho\frac{\sin\varphi}{\varrho^2}-\vec{1}_\varphi\frac{\cos\varphi}{\varrho^2}\right)\longrightarrow \underline{\underline{\vec{m}=\vec{1}_y\,8aV_0\varepsilon_0}}$

---

(3) Koordinate (u,v,z): $x=a\,\mathrm{ch}u\cos v$; $y=a\,\mathrm{sh}u\sin v$, $z=z$

Ekvipotencialna ploskev : $\underline{\underline{u=1}}$

$$V=-\frac{q}{2\pi\varepsilon_0}u\longrightarrow u=\frac{2\pi\varepsilon_0(-V)}{q}=\frac{2\pi As\cdot360V\,m}{4\pi\cdot9\cdot10^9Vm\,2\cdot10^{-8}As}=\underline{\underline{1}}$$

$x=a\,\mathrm{ch}1\cos v$

$y=a\,\mathrm{sh}1\sin v$

$$\frac{x}{a\,\mathrm{ch}1}=\cos v \ ; \ \frac{y}{a\,\mathrm{sh}1}=\sin v \ ; \ \underline{\underline{\frac{x^2}{a^2\mathrm{ch}^21}+\frac{y^2}{a^2\mathrm{sh}^21}=1}} \qquad \underline{\underline{\frac{x^2}{1.488\cdot10^{-3}m^2}+\frac{y^2}{8.632\cdot10^{-4}m^2}=1}}$$

Enačba elipse

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(4) $\vec{E}=-\vec{1}_r E_0\left(1+\frac{2a^3}{r^3}\right)\cos\Theta+\vec{1}_\Theta E_0\left(1-\frac{a^3}{r^3}\right)\sin\Theta$; $\vec{J}=\frac{\vec{E}}{\varrho}$; $\underline{J_r(r=a)=-\frac{3E_0}{\varrho}\cos\Theta}$

$$I=\int_0^\Theta\int_0^{2\pi}J_r a^2\sin\vartheta d\vartheta d\varphi=-\frac{3\pi E_0a^2}{\varrho}\int_0^\Theta 2\cos\vartheta\sin\vartheta d\vartheta=-\frac{3\pi E_0a^2}{\varrho}\left(-\frac{1}{2}\cos2\vartheta\right)\Big|_0^\Theta=\frac{3\pi E_0a^2}{2\varrho}(\cos2\Theta-1)=\underline{-\frac{3\pi E_0a^2}{\varrho}\sin^2\Theta}$$

$$\vec{K}=-\vec{1}_\Theta\frac{I}{2\pi a\sin\Theta}=\underline{\underline{\vec{1}_\Theta\frac{3E_0a}{2\varrho}\sin\Theta=\vec{1}_\Theta0.15\frac{A}{m}\sin\Theta}}$$

---

(5) Ozka vzdolžna razpoka polja NE MOTI!

$$\vec{K}=\underline{\underline{-\vec{1}_y\frac{I}{w}=-\vec{1}_y\,2\frac{A}{m}}}$$

Rešitev 2. kolokvija iz TEORIJE ELEKTROMAGNETIKE – 14/1/1997

①

$$\vec{H} = \frac{1}{\mu}\,\mathrm{rot}\,\vec{V}_m = \frac{1}{\mu r^2 \sin\Theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\Theta & r\sin\Theta\,\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \Theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & r\sin\Theta\, C\frac{\sin\Theta}{r^2} \end{vmatrix} = \underline{\underline{\vec{1}_r\, 2C\frac{\cos\Theta}{\mu r^3} + \vec{1}_\Theta\, C\frac{\sin\Theta}{\mu r^3}}}$$

$$\vec{J} = \mathrm{rot}\,\vec{H} = \frac{1}{r^2\sin\Theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\Theta & r\sin\Theta\,\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \Theta} & \frac{\partial}{\partial \phi} \\ 2C\frac{\cos\Theta}{\mu r^3} & rC\frac{\sin\Theta}{\mu r^3} & 0 \end{vmatrix} = \frac{1}{r}\vec{1}_\phi\left(-2C\frac{\sin\Theta}{\mu r^3} + 2C\frac{\sin\Theta}{\mu r^3}\right) = \underline{\underline{0}}$$; razen v izhodišču!

Magnetni dipol $I\Delta A \rightarrow \vec{V}_m = \vec{1}_\phi \frac{\mu}{4\pi r^2} I\Delta A \sin\Theta \longrightarrow$ Dipol $\underline{\underline{I\Delta A = \frac{4\pi C}{\mu}}}$ v izhodišču

---

② $\vec{V}_{m1} = \vec{1}_\phi \frac{\mu_0 I_1 r_1^2}{4}\frac{\sin\Theta}{r^2}$ pri $r \gg r_1$

$$M = \frac{1}{I_1}\oint_{2\pi} \vec{V}_{m1}\cdot d\vec{s}_2 = \frac{1}{I_1}\int_0^{2\pi} \vec{1}_\phi \frac{\mu_0 I_1 r_1^2}{4}\frac{1}{r_2^2}\cdot \vec{1}_\phi\, r_2\, d\phi = \underline{\underline{\frac{\pi \mu_0 r_1^2}{2 r_2}}}$$

---

③

$$\vec{H} = \frac{j}{\omega\mu}\,\mathrm{rot}\,\vec{E} = \frac{j}{\omega\mu}\frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\phi & \vec{1}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \frac{C}{\rho}e^{-jkz} & 0 & 0 \end{vmatrix} = \vec{1}_\varphi \frac{Ck}{\omega\mu_0}\frac{1}{\rho}e^{-jkz} = \underline{\vec{1}_\varphi \frac{C}{Z}\frac{1}{\rho}e^{-jkz}} \quad ; \; Z = \sqrt{\frac{\mu_0}{\varepsilon}}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times H^* = \underline{\underline{\vec{1}_z \frac{|C|^2}{2Z}\frac{1}{\rho^2}}} \qquad P = \int_A \vec{S}\cdot d\vec{A} = \int_a^b\!\!\int_0^{2\pi} \vec{1}_z \frac{|C|^2}{2Z}\frac{1}{\rho^2}\cdot\vec{1}_z\,\rho\, d\rho\, d\varphi = \underline{\underline{\frac{\pi |C|^2}{Z}\ln\frac{b}{a}}}$$

---

④

$$f = \frac{c_0}{2}\sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$$

kjer so l, m, n cela števila, vsaj dva različna od 0!

$l=1, m=1, n=0 \longrightarrow \underline{\underline{f_{110} = 5.02\ \mathrm{GHz}}}$

$l=1, m=0, n=1 \longrightarrow \underline{\underline{f_{101} = 5.43\ \mathrm{GHz}}}$

$l=0, m=1, n=1 \longrightarrow \underline{\underline{f_{011} = 5.70\ \mathrm{GHz}}}$

$l=1, m=1, n=1 \longrightarrow f_{111} = 6.60\ \mathrm{GHz}$

$l=2, m=1, n=0 \longrightarrow f_{210} = 7.65\ \mathrm{GHz}$

---

⑤ $\vec{S} = \vec{1}_z \cdot 1\,\mathrm{W/m^2}$; $\vec{E} = \frac{\vec{1}_x + \vec{1}_y}{\sqrt{2}}E$; $\vec{1}_S = \vec{1}_E \times \vec{1}_H \longrightarrow \vec{1}_H = \vec{1}_S \times \vec{1}_E = \frac{-\vec{1}_x + \vec{1}_y}{\sqrt{2}}$

$H = \frac{E}{Z_0}$; $\vec{H} = \vec{1}_H H = \frac{-\vec{1}_x + \vec{1}_y}{\sqrt{2}}\frac{E}{Z_0}$; $\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{1}_z \frac{|E|^2}{2Z_0}$

$|E| = \sqrt{2Z_0 S} = \underline{27.46\ \mathrm{V/m}}$ $\qquad |H| = \frac{|E|}{Z_0} = \underline{0.073\ \mathrm{A/m}}$

$$\underline{\underline{\vec{E} = \frac{\vec{1}_x + \vec{1}_y}{\sqrt{2}}\cdot 27.46\ \mathrm{V/m}\cdot e^{j\varphi}}}$$

kjer je φ poljuben fazni kot!

$$\underline{\underline{\vec{H} = \frac{-\vec{1}_x + \vec{1}_y}{\sqrt{2}}\cdot 0.073\ \mathrm{A/m}\cdot e^{j\varphi}}}$$

Rešitev 1. kolokvija iz TEORIJE ELEKTROMAGNETIKE – 28/11/1997

① $\vec{E} = -\operatorname{grad} V = \vec{1}_r V_0 \frac{2\sin\Theta\cos\phi}{r^3} - \vec{1}_\Theta V_0 \frac{\cos\Theta\cos\phi}{r^3} + \vec{1}_\phi V_0 \frac{\sin\phi}{r^3}$

$$\rho = \operatorname{div}(\varepsilon_0\vec{E}) = \frac{\varepsilon_0}{r^2\sin\Theta}\left(-V_0\frac{2\sin^2\Theta\cos\phi}{r^2} - V_0\frac{(-\sin^2\Theta+\cos^2\Theta)\cos\phi}{r^2} + V_0\frac{\cos\phi}{r^2}\right) = \underline{\underline{0}}$$

Odvisnost $r^{-2} \longrightarrow$ točkasti dipol v izhodišču! $\sin\Theta\cos\phi = \cos\Theta_x \longrightarrow$ dipol v smeri x!

$$V = V_0\frac{\sin\Theta\cos\phi}{r^2} = V_0\frac{\cos\Theta_x}{r^2} = \frac{Qd}{4\pi\varepsilon_0}\frac{\cos\Theta_x}{r^2} \longrightarrow \underline{\underline{m = Q\cdot d = 4\pi\varepsilon_0 V_0}}$$

---

② Koordinate (u, v, z) : $x = a\,\mathrm{ch}\,u\cos v$; $y = a\,\mathrm{sh}\,u\sin v$; $z = z$

$\Delta V = 0 \rightarrow \underline{V = C_1 u + C_2}$ (glej vaje na tabli dne 17/10/1997)

$u \gg 1 \rightarrow \rho \approx \frac{a}{2}e^u$; $V \approx -\frac{q}{2\pi\varepsilon_0}\ln\rho + C_3 \approx -\frac{q}{2\pi\varepsilon_0}u + C_2 \longrightarrow \underline{C_1 = -\frac{q}{2\pi\varepsilon_0}}$

$x = 0; z = 0 \longrightarrow v = \frac{\pi}{2}$; $y = a\,\mathrm{sh}\,u$; $u = \mathrm{arsh}\frac{y}{a}$; $V = -\frac{q}{2\pi\varepsilon_0}\mathrm{arsh}\frac{y}{a} + C_2$

$$\vec{E} = -\operatorname{grad} V = -\vec{1}_y\frac{d}{dy}\left(-\frac{q}{2\pi\varepsilon_0}\mathrm{arsh}\frac{y}{a} + C_2\right) = \underline{\underline{\vec{1}_y\frac{q}{2\pi\varepsilon_0}\frac{1}{\sqrt{y^2+a^2}}}} \quad \text{pri } y > 0$$

pri $y < 0 \rightarrow v = \frac{3\pi}{2} \longrightarrow$ obraten predznak $\vec{E}$

---

③ $\Delta V = 0 \rightarrow V = \sum_m C_m\rho^m\cos m\varphi$; $V(\varphi = \frac{\alpha}{2}) = 0 \longrightarrow m\frac{\alpha}{2} = (2k+1)\frac{\pi}{2}$; $k = 0, 1, 2, 3, 4 \ldots\ldots$

$m = \frac{\pi}{\alpha}(2k+1)$

$$\rho = r_0 \rightarrow \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} V_0\cos n\varphi\, d\varphi = \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}}\left[\sum_m C_m r_0^m\cos m\varphi\right]\cos n\varphi\, d\varphi$$

$$V_0\alpha\frac{\sin\frac{\pi}{2}(2k+1)}{\frac{\pi}{2}(2k+1)} = C_m r_0^{\frac{\pi}{\alpha}(2k+1)}\frac{\alpha}{2} \longrightarrow \underline{\underline{V = \sum_{k=0}^{\infty} 2V_0\frac{\sin\frac{\pi}{2}(2k+1)}{\frac{\pi}{2}(2k+1)}\frac{\rho^{\frac{\pi}{\alpha}(2k+1)}}{r_0^{\frac{\pi}{\alpha}(2k+1)}}\cos\frac{\pi}{\alpha}(2k+1)\varphi}}$$

---

④ $\underline{V_z(r \geq r_0) = \frac{I}{A\gamma}\left(r + \frac{r_0^3}{2r^2}\right)\cos\Theta}$ (glej vaje na tabli dne 21/11/1997)

$z = r\cos\Theta$

Znotraj mehurčka $(r \leq r_0)$: $\Delta V_N = 0 \rightarrow \underline{V_N = Cr\cos\Theta}$

$r = r_0 \rightarrow V_z = V_N$; $\frac{I}{A\gamma}\left(r_0 + \frac{r_0^3}{2r_0^2}\right)\cos\Theta = C r_0\cos\Theta \longrightarrow C = \frac{3}{2}\frac{I}{A\gamma}$; $V_n = \frac{3I}{2A\gamma}r\cos\Theta = \underline{\frac{3I}{2A\gamma}z}$

$\vec{E} = -\operatorname{grad} V_N = \underline{\underline{-\vec{1}_z\frac{3I}{2A\gamma}}}$

---

⑤ $4V_1 = 10 + 10 + V_5 + V_2$

$4V_2 = 10 + V_1 + V_6 + V_2$

$4V_5 = V_1 + 10 + (10 - V_5) + V_6$

$4V_6 = V_2 + V_5 + (10 - V_6) + V_6$

① $4V_1 = 20 + V_5 + V_2$

② $3V_2 = 10 + V_1 + V_6$

③ $5V_5 = 20 + V_1 + V_6$

④ $4V_6 = 10 + V_2 + V_5$

$\underbrace{①-④}$ $V_1 - V_6 = 2.5$

$V_1 = V_6 + \frac{5}{2}$

②' $6V_2 = 25 + 4V_6$

③' $10V_5 = 45 + 4V_6$ $/\cdot 3$

③'-②' $10V_5 - 6V_2 = 20$

$V_2 = \frac{5}{3}V_5 - \frac{10}{3}$

④' $12V_6 = 20 + 8V_5$ $/ +3③'$

$22V_5 = 155$

$\underline{\underline{V_5 = \frac{155}{22} = V_8}}$

$\underline{\underline{V_6 = \frac{1}{12}(20 + 8V_5) = \frac{140}{22} = V_7}}$

$\underline{\underline{V_1 = \frac{140}{22} + \frac{5}{2} = \frac{195}{22} = V_4}}$

$\underline{\underline{V_2 = \frac{5}{3}\frac{155}{22} - \frac{10}{3} = \frac{185}{22} = V_3}}$

$\underline{\underline{V_9 = V_{12} = 10 - V_5 = \frac{65}{22}}}$

$\underline{\underline{V_{10} = V_{11} = 10 - V_6 = \frac{80}{22}}}$

$\underline{\underline{V_{13} = V_{16} = 10 - V_1 = \frac{25}{22}}}$

$\underline{\underline{V_{14} = V_{15} = 10 - V_2 = \frac{35}{22}}}$

Rešitev 2. kolokvija iz ELEKTROMAGNETIKE – 16/1/1998

① $\vec{H}_N = \vec{1}_z \frac{N}{l} I$, $\vec{H}_z = 0$ } iz OE II

predpostavimo: $\vec{A} = \vec{1}_\varphi A_\varphi(\varrho)$ (iz smeri toka)

$$\vec{H} = \frac{1}{\mu}\,\mathrm{rot}\,\vec{A} = \frac{1}{\mu}\frac{1}{\varrho}\begin{vmatrix} \vec{1}_\varrho & \varrho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\varrho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \varrho A_\varphi & 0 \end{vmatrix} = \vec{1}_z \frac{1}{\mu\varrho}\frac{\partial}{\partial\varrho}(\varrho A_\varphi)$$

$$|\vec{H}| = \frac{1}{\mu\varrho}\frac{\partial}{\partial\varrho}(\varrho A_\varphi)$$

$$\mu\varrho|\vec{H}| = \frac{\partial}{\partial\varrho}(\varrho A_\varphi) \; / \int d\varrho$$

$$\frac{\mu\varrho^2|\vec{H}|}{2} + C = \varrho A_\varphi$$

$$A_\varphi = \frac{\mu\varrho|\vec{H}|}{2} + C/\varrho$$

znotraj: $A_{\varphi N} = \frac{\mu\varrho NI}{2l} + C_N/\varrho$

zunaj: $A_{\varphi z} = C_z/\varrho$

Zvezen prehod: $A_{\varphi N}(r) = A_{\varphi z}(r)$

$$\frac{\mu r NI}{2l} + \frac{C_N}{r} = \frac{C_z}{r} \rightarrow C_z = \frac{\mu r^2 NI}{2l} + C_N$$

$$\vec{A}_N = \vec{1}_\varphi\left(\frac{\mu\varrho NI}{2l} + \frac{C_N}{\varrho}\right); \quad \vec{A}_z = \vec{1}_\varphi\left(\frac{\mu r^2 NI}{2l\varrho} + \frac{C_N}{\varrho}\right)$$

---

② brez jeder: $M' = \frac{\pi\mu_0 r_1^2 r_2^2}{2d^3}$ (vaje 9/1/98) ; $\vec{B}$ v feromag. kroglici: $\vec{B}_N = \frac{3\mu_r}{2+\mu_r}\vec{B}_0$ (vaje strani 152/153)

$\mu_r \to \infty \Rightarrow \vec{B}_N = 3\vec{B}_0$ eno jedro: $M'' = 3M' = \frac{3\pi\mu_0 r_1^2 r_2^2}{2d^3}$

dve jedri (recipročnost M!): $M = 3M'' = 9M' = \frac{9\pi\mu_0 r_1^2 r_2^2}{2d^3}$

---

③ Prestopni pogoji: $E_x = E_{x0}\cos\frac{\pi}{a}x\,\sin\frac{\pi}{b}y\,\sin\frac{\pi}{c}z$; $E_y = E_{y0}\sin\frac{\pi}{a}x\,\cos\frac{\pi}{b}y\,\sin\frac{\pi}{c}z$ (isti $\vec{k}$!)

Prostorske elektrine ni: $0 = \mathrm{div}\vec{D} = \mathrm{div}\,\varepsilon_0\vec{E}$; $0 = \mathrm{div}\vec{E} = E_{x0}\frac{\pi}{a}\sin\frac{\pi}{a}x\sin\frac{\pi}{b}y\sin\frac{\pi}{c}z + E_{y0}\frac{\pi}{b}\sin\frac{\pi}{a}x\sin\frac{\pi}{b}y\sin\frac{\pi}{c}z + E_0\frac{\pi}{c}\sin\frac{\pi}{a}x\sin\frac{\pi}{b}y\sin\frac{\pi}{c}z$

$$0 = E_{x0}\frac{\pi}{a} + E_{y0}\frac{\pi}{b} + E_0\frac{\pi}{c}$$

$$E_{x0} = -a\left(\frac{E_{y0}}{b} + \frac{E_0}{c}\right)$$

$$k^2 = \omega^2\cdot\mu_0\varepsilon_0 = k_x^2 + k_y^2 + k_z^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 \quad ; \quad \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c_0$$

$$\omega = c_0\pi\sqrt{1/a^2 + 1/b^2 + 1/c^2} \; ; \; f = \frac{c_0}{2}\sqrt{1/a^2 + 1/b^2 + 1/c^2}$$

---

④ $|\vec{H}| = 2|\vec{H}_v| = 2\sqrt{\frac{2|\vec{S}|}{Z_0}}$; $|\vec{S}| = \frac{1}{2}|\vec{H}|^2 Z_0$; $|\vec{K}| = |\vec{H}| = 2\sqrt{\frac{2S}{Z_0}}$; $p = \frac{dP}{dA} = \frac{1}{2}|\vec{K}|^2 R_p$

$R_p = \sqrt{\frac{\omega\mu_0}{2\gamma}}$

$$p = \frac{1}{2}\left(4\,\frac{2S}{Z_0}\right)R_p = \frac{4R_p}{Z_0}S = \frac{4S}{Z_0}\sqrt{\frac{\omega\mu_0}{2\gamma}} = 4S\sqrt{\frac{\pi f\varepsilon_0}{\gamma}} = 0.282\,\mathrm{W/m^2}$$

---

⑤ $$\vec{A} = \frac{\mu}{4\pi}\int_V \frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}e^{-jk|\vec{r}-\vec{r}\,'|}dV \approx \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\int_A \vec{K}(\vec{r}\,')dA = \vec{1}_x\frac{\mu_0}{4\pi}K_0a^2\frac{e^{-jkr}}{r}$$

$\vec{1}_x = \vec{1}_r\sin\Theta\cos\phi + \vec{1}_\Theta\cos\Theta\cos\phi - \vec{1}_\phi\sin\phi$; $\vec{A} = \left(\vec{1}_r\sin\Theta\cos\phi + \vec{1}_\Theta\cos\Theta\cos\phi - \vec{1}_\phi\sin\phi\right)\frac{\mu_0}{4\pi}K_0a^2\frac{e^{-jkr}}{r}$

$$\vec{H} = \frac{1}{\mu_0}\mathrm{rot}\vec{A} = \frac{K_0a^2}{4\pi}\frac{1}{r^2\sin\Theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\Theta & r\sin\Theta\,\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\Theta} & \frac{\partial}{\partial\phi} \\ \frac{e^{-jkr}}{r}\sin\Theta\cos\phi ; & e^{-jkr}\cos\Theta\cos\phi ; & -e^{-jkr}\sin\Theta\sin\phi \end{vmatrix} = \frac{K_0a^2}{4\pi}\frac{e^{-jkr}}{r}\left(\frac{1}{r}+jk\right)\left[-\vec{1}_\Theta\sin\phi - \vec{1}_\phi\cos\Theta\cos\phi\right]$$

Elektromagnetika 1.kolokvij 9.12.1998, rešitve

1) $\vec{E} = \vec{1}_\theta C \dfrac{e^{-jkr}}{r\sin\theta} \qquad h_r = 1 \qquad h_\theta = r \qquad h_\phi = r\sin\theta$

Prostorske veličine $\rightarrow$ *izračunamo iz Maxwellovih enačb*

$$\vec{H} = -\frac{1}{j\omega\mu_0} rot\vec{E} = -\frac{C}{j\omega\mu_0}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & \frac{e^{-jkr}}{sin\theta} & 0 \end{vmatrix} = \underline{\underline{\vec{1}_\phi \frac{C}{Z_0}\frac{e^{-jkr}}{rsin\theta}}}$$

$$rot\vec{H} = \frac{1}{r^2\sin\theta}\frac{C}{Z_0}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & e^{-jkr} \end{vmatrix} = \vec{1}_\theta jC\varepsilon_0\omega\frac{e^{-jkr}}{sin\theta}$$

$$\vec{J} = rot\vec{H} - j\omega\varepsilon_0\vec{E} = \vec{1}_\theta jC\varepsilon_0\omega\frac{e^{-jkr}}{r\,sin\theta} - \vec{1}_\theta jC\varepsilon_0\omega\frac{e^{-jkr}}{r\,sin\theta} = \underline{\underline{0}}$$

$$\rho = div(\varepsilon\vec{E}) = C\varepsilon_0\frac{1}{r^2\sin\theta}\left(\frac{\partial}{\partial\theta}r\sin\theta\frac{e^{-jkr}}{rsin\theta}\right) = \underline{\underline{0}}$$

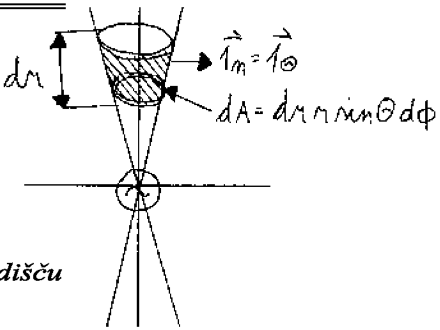
Ploskovne veličine

$\vec{K} = \underline{\underline{0}}$ *in* $\sigma = -\dfrac{1}{j\varpi}div\vec{K} = \underline{\underline{0}} \rightarrow$ *Ni singularnih ploskev*

3. *Preme veličine, singularnost pri* $\theta = 0,\pi$ *na osi* z *različno za* $+/-z$

$$I = \oint \vec{H}\vec{ds} = \frac{C}{Z_0}\int_0^{2\pi}\frac{e^{-jkr}}{rsin\theta}r\sin\theta\,d\phi = \underline{\underline{\frac{C}{Z_0}2\pi e^{-jkr}}}$$

$dQ = \oint \vec{D}\,\vec{dA}$



$$dQ = \int_0^{2\pi}\vec{1}_\theta\varepsilon_0 C\frac{e^{-jkr}}{rsin\theta}dr\,\vec{1}_\theta r\sin\theta d\phi$$

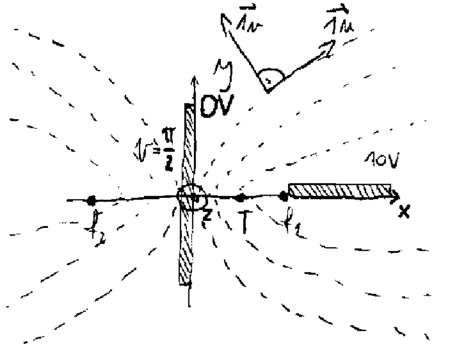
$$dQ = \varepsilon_0 C\frac{e^{-jkr}}{1}dr\,2\pi = q\,dr$$

$q = \underline{\underline{2\pi\varepsilon_0 Ce^{-jkr}}}$

4. *Točkasta veličina v koordinatnem izhodišču*

$$Q|_{r=0} = \int_{k\to 0}\vec{D}\,\vec{dA} = \int_v div\vec{D}\,dv = \underline{\underline{0}}$$

2) *Valjni eliptični koordinatni sistem* (u,v,z)



$x = f\cosh u\cos v$

$y = f\sinh u\sin v$

$z = z$

$\Delta V = 0 \qquad \dfrac{\partial}{\partial u} = 0 \qquad \dfrac{\partial}{\partial z} = 0$

$hu = f\sqrt{\sinh u^2 + \sin v^2}$

$hv = f\sqrt{\sinh u^2 + \sin v^2}$

$hz = 1$

$$\Delta V = div(gradV) = \frac{1}{f^2(\sinh u^2 + \sin v^2)}\left[\frac{\partial^2 V}{\partial^2 u} + \frac{\partial^2 V}{\partial^2 v}\right] + \frac{\partial^2 V}{\partial^2 z} = 0$$

$$0 = \frac{1}{f^2(\sinh u^2 + \sin v^2)}\frac{\partial^2 V}{\partial^2 v} \rightarrow V = C_1 v + C_2$$

$10V = C_1 0 + C_2 \rightarrow C_2 = 10V$

$0V = C_1\dfrac{\pi}{2} + C_2$

$C_1 = -\dfrac{2}{\pi}C_2 = -\dfrac{20V}{\pi}$

$V = -\dfrac{20V}{\pi}v + 10V$

Točka $T:\ u = 0 \quad x = \dfrac{f}{2}$

$\dfrac{f}{2} = f*1*\cos v \rightarrow v = \dfrac{\pi}{3}$

$V(T) = \dfrac{20V}{\pi}\dfrac{\pi}{3} + 10V = \underline{\underline{3{,}33V}}$

Elektromagnetika 1.kolokvij 9.12.1998, rešitve

3) *Najprej določimo polje v levi polovici korita. V desni polovici je porazdelitev potenciala zrcalno simetrična*

$\Delta V = 0 \qquad V(a-x,y) = -V(x,y)$

$a)\; 0 \le x \le \frac{a}{2}$

$V(x,y) = \sum_m C_m \sin\left(\frac{m\pi}{a/2}x\right)\sinh\left(\frac{m\pi}{a/2}y\right) \;\rightarrow\; V(x,y=b) = \sum_m C_m \sin\left(\frac{m\pi}{a/2}x\right)\sinh\left(\frac{m\pi}{a/2}b\right) = V_0$

S Fourierjevo analizo določimo koeficiente C_m

$\int_0^{a/2} V_0 \sin\left(\frac{n\pi}{a/2}x\right)dx + \int_{a/2}^{a}(-V_0)\sin\left(\frac{n\pi}{a/2}x\right)dx = \int_0^a \sum_m \left(C_m \sin\left(\frac{m\pi}{a/2}x\right)\sinh\left(\frac{m\pi}{a/2}b\right)\right)\sin\left(\frac{n\pi}{a/2}x\right)dx$

$\left(\frac{V_0}{2m\pi}\left(2\sin\left(\frac{\pi m}{2}\right)^2 - (\cos(m\pi)-\cos(2m\pi))\right)\right) = C_m \sinh\left(\frac{m\pi}{a}b\right)\frac{a}{8}\left(\frac{4m\pi - \sin(4m\pi)}{m\pi}\right)$

$\frac{2V_0 a}{m\pi} = \frac{a}{2}C_m \sinh\left(\frac{m\pi}{a/2}b\right) \qquad \rightarrow C_m = \begin{cases} 0 & \textit{če je m sodo} \\ \frac{4V_0}{m\pi}\sinh\left(\frac{m\pi}{a/2}b\right)^{-1} & \textit{če je m liho} \end{cases}$

b) Upoštevamo zrcalno simetrijo, saj je $\sin\left(\frac{(2k+1)\pi(a-x)}{a/2}\right) = -\sin\left(\frac{(2k+1)\pi x}{a/2}\right)$

$$\underline{\underline{V(x,y) = \sum_{k=0}^{\infty}\frac{4V_0}{(2k+1)\pi}\sinh\left(\frac{(2k+1)\pi}{a/2}b\right)^{-1}\sin\left(\frac{(2k+1)\pi}{a/2}x\right)\sinh\left(\frac{(2k+1)\pi}{a/2}y\right)}}$$

4) $\Delta V(r,\theta,\Phi) = 0 \qquad \frac{\partial}{\partial\Phi} = 0 \qquad V(\infty) \rightarrow \infty$

$V(r,\theta) = \sum_{n=0}^{\infty} B_n r^{-(n+1)} P_n(\cos\theta) \qquad za \;\; r > r_0$

$B_n = \frac{1}{r_0^{\,n}}\frac{\int_{-1}^{1} V(\cos\theta)P_n(\cos\theta)\, d\cos\theta}{\int_{-1}^{1} P_n^{\,2}(\cos\theta)\, d\cos\theta} \qquad V(\cos\theta) = \begin{cases} +V_0;\; 1 > \cos\theta > 0 \\ -V_0;\; 0 > \cos\theta \ge -1 \end{cases}$

$B_n = \frac{1}{r_0^{\,n}}\frac{-V_0\int_{-1}^{0} P_n(t)\, dt + V_0\int_0^1 P_n(t)\, dt}{\int_{-1}^{1} P_n^{\,2}(t)\, dt} \quad ; \quad t = \cos\theta \;\; \rightarrow B_n = \frac{2n+1}{n(n+1)a^n}V_0 P_n(0)$

Koeficienti $B_n:\; B_{2K} = 0 \quad K = 1,2,\ldots$

$B_{2K-1} = V_0 r_0^{\,2K}\left[P_{2K}(0) - P_{2K-2}(0)\right]$

Funkcija potenciala za $r > r_0$

$V(r,\theta) = V_0 \sum_{K=1}^{\infty}\frac{P_{2K}(0) - P_{2K-2}(0)}{r^{2K}} r_0^{\,2K} P_{2K-1}(\cos\theta)$

Poenostavimo za $r >> r_0$

za $K = 1 \quad V(r,\theta) = V_0 \frac{3}{2}\frac{r_0^{\,2}}{r^2}\cos\theta$

za $K = 2 \quad V(r,\theta) = V_0 \frac{7}{8}\frac{r_0^{\,4}}{r^4}(5\cos\theta^3 - 3\cos\theta)$

za $K = 3 \quad V(r,\theta) = V_0 \frac{3}{16}\frac{r_0^{\,6}}{r^6}(63\cos\theta^5 - 70\cos\theta^3 + 15\cos\theta)$

$\underline{\underline{V(r,\theta) = V_0 \frac{3}{2}\frac{r_0^{\,2}}{r^2}\cos\theta}} \qquad za\; r >> r_0$

5) $\vec{E} = -\vec{1}_y \frac{IRp}{a} \quad V = \frac{IRp}{a} y + V_0$

$P = \gamma E^2 \quad P_1 = 2P_2 \quad \gamma E_1^{\,2} = 2\gamma E_2^{\,2} \qquad E_2 = \frac{E_1}{\sqrt{2}}$

Nastavek :

$V = (A\rho + B\rho^{-1})\sin\varphi \rightarrow V = A\rho\sin\varphi \approx \frac{IRp}{a}y \qquad A = \frac{IRp}{a}$

$\rho = r \rightarrow V = 0 = (A\rho + B\rho^{-1})\sin\varphi\big|_{\rho=r}$

$B = -Ar^2 = \frac{IRp}{a}r^2$

$\vec{K}_0 = -\vec{1}_y \frac{I}{W} \qquad E_0 = \vec{K}_0 Rp = \vec{1}_y \frac{IRp}{a} \qquad V = \frac{IRp}{a}(\rho - \frac{r^2}{\rho})\sin\varphi$

$V_1 = (-E_0\rho + \frac{B}{\rho})\sin\varphi \quad V_2 = A\rho\sin\varphi$

$\vec{E}_1 = -grad\, V_1 = \vec{1}_\rho(E_0 + \frac{B_1}{\rho^2})\sin\varphi + \vec{1}_\varphi(E_0 - \frac{B1}{\rho^2})\cos\varphi$

$\vec{E}_2 = -grad\, V_2 = -\vec{1}_\rho A\sin\varphi - \vec{1}_\varphi A\cos\varphi = -\vec{1}_y A$

Prestopni pogoji : $\begin{cases} E_{\varphi1} = E_{\varphi2} \rightarrow (E_0 - \frac{B_1}{r_0^{\,2}})\cos\varphi = -A_2\cos\varphi \rightarrow E_0 - \frac{B_1}{r_0^{\,2}} = -A \\ K_{\rho1} = K_{\rho2} \rightarrow (E_0 + \frac{B_1}{r_0^{\,2}})\sin\varphi\frac{1}{R_{p1}} = \frac{A_2}{R_{p2}}\sin\varphi \rightarrow E_0 + \frac{B_1}{r_0^{\,2}} = -A\frac{R_{p1}}{R_{p2}} \end{cases}$

$\vec{E}_1 = \vec{1}_\rho(-A\frac{R_{p1}}{R_{p2}})\sin\varphi + \vec{1}_\varphi(-A)\cos\varphi = -\vec{1}_\rho A\frac{R_{p1}}{R_{p2}}\sin\varphi - \vec{1}_\varphi A\cos\varphi$

$\vec{E}_2 = -\vec{1}_\rho A\sin\varphi - \vec{1}_\varphi A\cos\varphi$

$2E_0 = -A(1 + \frac{R_{p1}}{R_{p2}}) = E_2(1 + \frac{R_{p1}}{R_{p2}})$

$\frac{2E_0}{E_2} = 1 + \frac{R_{p1}}{R_{p2}} \rightarrow R_{p2} = R_{p1}\frac{1}{\frac{2E_0}{E_0} - 1} = \frac{100\Omega}{\frac{2E_1\sqrt{2}}{E_1} - 1} = \underline{\underline{54{,}69\Omega}}$

Elektromagnetika 2.kolokvij 20.1.1999, rešitve:

1) $r_0 = 1m \quad d = 30m \quad I = 10A \quad {}_\Delta V = ?$

$A_m = I_\Delta \phi = -{}_\Delta W \quad d << r_0$

$M_{12} = \frac{\phi_{12}}{I_L} \qquad {}_\Delta \phi = -2\phi_D$

$M = \frac{\pi\mu_0 N_1 N_2 a^2 a^2}{2d^3} = \frac{\pi\, 4\pi 10^{-7} 1H}{2\cdot 30^3} = 7{,}31\cdot 10^{-11} H$

$\Delta W = I\cdot 2\phi_D = I^2 2M = \underline{\underline{1{,}462\cdot 10^{-8} Ws}}$

2) $r_o = 1mm \quad l = 10m \quad f = 1MHz \quad \gamma_{cu} = 56\cdot 10^6 S/m$

$\delta << r_0 \quad in \quad l << \lambda \qquad \frac{R_\sim}{R_=} = ?$

$R_= = \frac{l}{\gamma A} = \frac{l}{\gamma\pi r_0} = \frac{10\Omega}{56\cdot 10^{-6}\pi(1\cdot 10^{-3})^2} = 56{,}8\cdot 10^{-3}\Omega$

$\delta = \sqrt{\frac{2}{\omega\mu_0\gamma}} = 67{,}25\cdot 10^{-6} m$

$R_\sim = \frac{l}{\gamma A_\sim} = \frac{l}{\gamma 2\pi r\delta} = 422{,}6\cdot 10^{-3}\Omega$

$\frac{R_\sim}{R_=} = \underline{\underline{7{,}44}}$

3) $W = 20mm \quad d = 5mm \quad f = 100MHz \quad \vec{H} = \vec{1}_x H_0 e^{-jk_0 z} \quad H_0 = 100A/m$

$\vec{E} = ? \quad \vec{S} = ? \quad \vec{P} = ? \quad \vec{U} = ? \quad \vec{I} = ?$

$$\vec{E} = \frac{1}{j\omega\varepsilon_0} rot\vec{H} = \frac{1}{j\omega\varepsilon_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_0 e^{-jk_0 z} & 0 & 0 \end{vmatrix} = \frac{1}{j\omega\varepsilon_0}(\vec{1}_y H_0 e^{-jk_0 z}(-jk)) = -\frac{H_0 k}{\omega\varepsilon_0}\vec{1}_y e^{-jk_0 z} =$$

$$-\vec{1}_y H_0 Z_0 e^{-jk_0 z} = -\vec{1}_y E_0 e^{-jk_0 z} = \underline{\underline{-\vec{1}_y 37699\frac{V}{m}e^{-jk_0 z}}}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 0 & -H_0 Z_0 e^{-jk_0 z} & 0 \\ H_0^* e^{+jk_0 z} & 0 & 0 \end{vmatrix} = \vec{1}_z \frac{Z_0 H_0^2}{2} = \vec{1}_z S_0 = \underline{\underline{\vec{1}_z 1{,}88M\frac{W}{m^2}}}$$

$$P = \int_A S\,dA = \int_0^w\int_0^d \frac{Z_0 H_0^2}{2} dxdy = \frac{Z_0 H_0^2}{2} wd = \underline{\underline{188{,}496W}}$$

$$\vec{K} = \vec{1}_n \times \vec{H}\Big|_{y=0} = \vec{1}_y \times \vec{1}_x H_0 e^{-jk_0 z} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 0 & 1 & 0 \\ H_0 e^{-jkoz} & 0 & 0 \end{vmatrix} = -\vec{1}_z H_0 e^{-jk_0 z} = \underline{\underline{-\vec{1}_z 100\frac{A}{m}e^{-jk_0 z}}}$$

$$I = \int_o^w \vec{K}\vec{ds} = \int_o^w H_0 e^{-jk_0 z} dx = H_0 w e^{-jk_0 z} = I_0 e^{-jk_0 z} = \underline{\underline{\pm 2A e^{-jk_0 z}}} \quad \rightarrow \pm\frac{gornja\ plošča}{spodnja\ plošča}$$

$$U = -\int \vec{E}\vec{ds} = -\int_0^d (-\vec{1}y)\frac{H_0 k}{\omega\varepsilon_0}e^{-jk_0 z}\vec{1}y\,dy = \frac{H_0 k}{\omega\varepsilon_0} d e^{-jk_0 z} = H_0 Z_0 d e^{-jk_0 z} = U_0 e^{-jk_0 z} = \underline{\underline{188{,}4V e^{-jk_0 z}}}$$

Elektromagnetika 2.kolokvij 20.1.1999, rešitve:

4) $a = 1cm \quad b = 2cm \quad c = 3cm \quad f_{MIN} = 1GHz \qquad \varepsilon_r = ?$

$$f = \frac{1}{2\sqrt{\varepsilon_0 \varepsilon_r \mu_0}} = \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$f_{\min} = f_{011}$ $\quad m, n, p \rightarrow$ *cela števila, vsaj 2 različna od 0*

$$f^2 = \frac{1}{4\varepsilon_0 \varepsilon_r \mu_0}\left[\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2\right]$$

poiščemo 2 največja izmed a, b, c

$$\varepsilon_r = \frac{c_o^{\,2}}{4f}\left(\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2\right) =$$

$$\frac{3*10^8}{4(1*10^9)^2}\left[\left(\frac{1}{0,02}\right)^2 + \left(\frac{1}{0,03}\right)^2\right] = \underline{\underline{81,176}}$$

5) $\varepsilon_r = 2 \quad f = 10GHz \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \rightarrow Z = \sqrt{\frac{\mu_0}{\varepsilon}} = \frac{Z_0}{\sqrt{\varepsilon_r}}$

$$\vec{E}_{1+} = \vec{1}_x A_{1+} e^{-jk_0 z} \qquad \vec{H}_{1+} = \frac{j}{\omega\mu_0} rot \vec{E}_{1+} = \frac{j}{\omega\mu_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_{1+}e^{-jk_0 z} & 0 & 0 \end{vmatrix} = \vec{1}_y \frac{A_{1+}}{Z_0} e^{-jk_0 z}$$

$$\vec{E}_{1-} = \vec{1}_x A_{1-} e^{jk_0 z} \qquad \vec{H}_{1-} = -\vec{1}_y \frac{A_{1-}}{Z_0} e^{jk_0 z} \qquad \vec{E}_{2+} = \vec{1}_x A_{2+} e^{-jk_0 z}$$

$$\vec{H}_{2+} = \vec{1}_y \frac{A_{2+}}{Z} e^{-jk_0 z} \quad \rightarrow \vec{1}_{E_{1-}} \times \vec{1}_{H_{1-}} = -\vec{1}_z \quad \textit{odbiti val}$$

pri $z = 0: \ \vec{E}_{1+} + \vec{E}_{1-} = \vec{E}_{2+} \rightarrow A_{1+} + A_{1-} = A_{2+}$

$$\vec{H}_{1+} + \vec{H}_{1-} = \vec{H}_{2+} \rightarrow \frac{A_{1+}}{Z_0} - \frac{A_{1-}}{Z_0} = \frac{A_{2+}}{Z} \rightarrow A_{2+} = \frac{Z}{Z_0}(A_{1+} - A_{1-})$$

$$A_{1+} + A_{1-} = \frac{Z}{Z_0}(A_{1+} - A_{1-}) \rightarrow A_{1-} = \frac{Z - Z_0}{Z + Z_0} A_{1+} = -\frac{\sqrt{\varepsilon_r} - 1}{\sqrt{\varepsilon_r} + 1} A_{1+}$$

$$\Gamma = \frac{A_{1-}}{A_{1+}} = -\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = -0,172 \qquad A_{2+} = A_{1+} + A_{1-} = (1 + \Gamma) A_{1+}$$

$$\vec{S}_{2+} = \vec{S}_2 = \frac{1}{2}\vec{E}_{2+} \times \vec{H}_{2+}^{\ *} = \frac{1}{2}\vec{1}_x (1+\Gamma) A_{1+} e^{-jkz} \times \vec{1}_y \frac{(1+\Gamma) A_{1+}^{\ *}}{Z} e^{jkz} = \vec{1}_z \frac{(1+\Gamma)^2 A_{1+}^{\ 2}}{2Z} =$$

$$\underline{\underline{\vec{1}_z 0,515 \frac{W}{m^2}}}$$

$$\vec{S}_1 = \frac{1}{2}(\vec{E}_{1+} + \vec{E}_{1-}) \times (\vec{H}_{1+} + \vec{H}_{1-})^* = \frac{1}{2}\vec{1}_x (A_{1+} e^{jk_0 z} + \Gamma A_{1+} e^{jk_0 z}) \times \vec{1}_y \left(\frac{A_{1+}}{Z_0} e^{jk_0 z} - \frac{\Gamma A_{1+}}{Z_0} e^{-jk_0 z}\right) =$$

$$\vec{1}_z \frac{A_{1+}^{\ 2}}{2Z_0}(e^{-jk_0 z} + \Gamma e^{jk_0 z})(e^{jk_0 z} + \Gamma e^{-jk_0 z}) = \vec{1}_z \frac{\left|A_{1+}^{\ 2}\right|}{Z_0}(1 - |\Gamma|^2 + j2\Gamma \sin 2k_0 z) =$$

$$\underline{\underline{\vec{1}_z 0,531 \frac{W}{m^2}(0,971 + j0,343 \sin 2k_0 z)}}$$

Rešitev pisnega izpita iz ELEKTROMAGNETIKE (UNI) – 5/3/1999

① $(\eta,\psi,\phi) \to x = a\,\mathrm{ch}\eta \sin\psi\cos\phi\ ;\ y = a\,\mathrm{ch}\eta\sin\psi\sin\phi\ ;\ z = a\,\mathrm{sh}\eta\cos\psi$

$h_\eta = h_\psi = a\sqrt{\mathrm{sh}^2\eta + \cos^2\psi}\ ;\ h_\phi = a\,\mathrm{ch}\eta\sin\psi\ ;$ Geometrija naloge: $\frac{\partial}{\partial\psi} = \frac{\partial}{\partial\phi} = 0$

$$0 = \Delta V = \frac{1}{h_\eta h_\psi h_\phi}\frac{\partial}{\partial\eta}\left(h_\psi h_\phi \frac{1}{h_\eta}\frac{\partial V}{\partial\eta}\right) \to 0 = \frac{\partial}{\partial\eta}\left(\mathrm{ch}\eta\frac{\partial V}{\partial\eta}\right) \to \underline{\frac{\partial V}{\partial\eta} = \frac{C}{\mathrm{ch}\eta}}$$

$$\vec{D} = \varepsilon\vec{E} = -\varepsilon\,\mathrm{grad}\,V = -\vec{1}_\eta\varepsilon\frac{1}{h_\eta}\frac{\partial V}{\partial\eta} = \underline{\vec{1}_\eta\frac{-\varepsilon C}{h_\eta\,\mathrm{ch}\eta}}\ ;\ z=0 \to \eta=0;\ \mathrm{ch}\eta = 1;\ h_\eta = \sqrt{a^2-x^2-y^2};\ \vec{1}_\eta = \pm\vec{1}_z$$

$$\sigma = \vec{1}_n\cdot\vec{D} = \underline{\frac{-2\varepsilon C}{\sqrt{a^2-x^2-y^2}}}\ ;\ Q = \int_0^{2\pi}\int_0^a \sigma(\rho,\varphi)\,\rho\,d\rho\,d\varphi = -4\pi\varepsilon C\int_0^a\frac{\rho\,d\rho}{\sqrt{a^2-\rho^2}} = -4\pi\varepsilon C a \to \underline{C = \frac{-Q}{4\pi\varepsilon a}}$$

$$\underline{\underline{\sigma = \frac{Q}{2\pi a\sqrt{a^2-x^2-y^2}}}}$$

② $\Delta V = 0 \to V(\rho,\varphi,z) = \sum C_n \rho^{\pm n}\cos n\left(\varphi - \frac{\pi}{2}\right)$; SAMO SODI n!

$h \ll a \to n = 2$ zadošča

$$\int_{\pi/4}^{3\pi/4}\sum C_n \underbrace{\rho^{\pm n}}_{a^{+n}}\cos n\left(\varphi-\tfrac{\pi}{2}\right)\cos 2\left(\varphi-\tfrac{\pi}{2}\right)d\varphi = \int_{\pi/4}^{3\pi/4} V_0\cos 2\left(\varphi-\tfrac{\pi}{2}\right)d\varphi = \quad V_0 = \frac{\pi}{4}C_2 a^{+2}$$

$$C_2 = \frac{4V_0}{\pi a^2}\ ;\ V\left(\rho=h;\varphi=\tfrac{\pi}{2};z\right) = C_2\rho^{+2}\cos 2\left(\varphi-\tfrac{\pi}{2}\right) = \frac{4V_0}{\pi a^2}h^{+2}\cdot 1 = \underline{\underline{\frac{4V_0h^2}{\pi a^2} = 1.91\,\mathrm{mV}}}$$

③ $\vec{A} = \vec{1}_{\phi_x}\frac{\mu_0 I a^2}{4\pi r^2}\sin\Theta_x\ ;$ Koordinate (r,Θ_x,ϕ_x) $\quad \sin\Theta_x = \sqrt{1-\sin^2\Theta\cos^2\phi}$

$$\vec{1}_{\phi_x} = -\vec{1}_\Theta\frac{\sin\phi}{\sqrt{1-\sin^2\Theta\cos^2\phi}} - \vec{1}_\phi\frac{\cos\Theta\cos\phi}{\sqrt{1-\sin^2\Theta\cos^2\phi}}$$

$$\underline{\underline{\vec{A} = \frac{\mu_0 I a^2}{4\pi r^2}\left(-\vec{1}_\Theta\sin\phi - \vec{1}_\phi\cos\Theta\cos\phi\right)}}$$

④ $\mathrm{rot}\vec{E} = -j\omega\mu\vec{H} \to \vec{H} = \frac{j}{\omega\mu}\mathrm{rot}\vec{E} = \underline{\underline{-\vec{1}_x\frac{\beta}{\omega\mu}\cos\alpha x\, e^{-j\beta z} - \vec{1}_z\frac{j\alpha}{\omega\mu}\sin\alpha x\, e^{-j\beta z}}}$

$$\vec{S} = \tfrac{1}{2}\vec{E}\times\vec{H}^* = \underline{\underline{\vec{1}_x\frac{j\alpha}{2\omega\mu}\cos\alpha x\sin\alpha x + \vec{1}_z\frac{\beta}{2\omega\mu}\cos^2\alpha x}} \qquad \rho = \varepsilon\,\mathrm{div}\vec{E} = \underline{\underline{0}}$$

$$\vec{J} = \mathrm{rot}\vec{H} - j\omega\varepsilon\vec{E} = \vec{1}_y\left(\frac{j\beta^2}{\omega\mu}\cos\alpha x\,e^{-j\beta z} + \frac{j\alpha^2}{\omega\mu}\cos\alpha x\,e^{-j\beta z}\right) - j\omega\varepsilon\vec{1}_y\cos\alpha x\,e^{-j\beta z} = \underline{\underline{0}}$$

⑤ $Z_K = \frac{d}{w}Z_0\ ;\ dR = \frac{2dz}{w\,\delta\,\gamma} = \frac{2dz}{w\gamma\sqrt{\frac{2}{\omega\mu\gamma}}} = \frac{2}{w}\sqrt{\frac{\omega\mu}{2\gamma}}\,dz$

$$\frac{dP}{P} = -\frac{dR}{Z_K} = -\frac{2}{wZ_K}\sqrt{\frac{\omega\mu}{2\gamma}}\,dz \longrightarrow \ln P = -\frac{2l}{wZ_K}\sqrt{\frac{\omega\mu}{2\gamma}} + C$$

$$A\,[\mathrm{dB}] = 10\log_{10}\frac{P(0)}{P(l)} = \frac{10}{\ln 10}\frac{2l}{wZ_K}\sqrt{\frac{\omega\mu}{2\gamma}} \quad ;\ \mu = \mu_0 \text{ za baker}$$

$$A/l = \frac{10}{\ln 10}\frac{2}{wZ_K}\sqrt{\frac{\omega\mu_0}{2\gamma}} = \underline{\underline{\frac{10}{\ln 10}\frac{2}{dZ_0}\sqrt{\frac{\omega\mu_0}{2\gamma}} = 0{,}0061\,\mathrm{dB/m} = 6.1\,\mathrm{dB/km}}}$$

Rešitev pisnega izpita iz ELEKTROMAGNETIKE – 9/7/1999

①

A) V_0 $\square$ 0 $\longrightarrow V_A = \sum_{k=0}^{\infty} \frac{4V_0}{(2k+1)\pi} \sin\frac{(2k+1)\pi}{a}x \times \frac{\operatorname{sh}\frac{(2k+1)\pi}{a}y}{\operatorname{sh}(2k+1)\pi}$

B) V_0 $\square$ 0 $\longrightarrow V_B = \sum_{k=0}^{\infty} \frac{4V_0}{(2k+1)\pi} \sin\frac{(2k+1)\pi}{a}y \, \frac{\operatorname{sh}\frac{(2k+1)\pi}{a}(a-x)}{\operatorname{sh}(2k+1)\pi}$

V_0 $\square$ 0 $\longrightarrow$

$$\underline{\underline{V = V_A + V_B = \sum_{k=0}^{\infty} \frac{4V_0}{(2k+1)\pi \operatorname{sh}(2k+1)\pi}\left(\sin\frac{(2k+1)\pi}{a}x \operatorname{sh}\frac{(2k+1)\pi}{a}y + \sin\frac{(2k+1)\pi}{a}y \operatorname{sh}\frac{(2k+1)\pi}{a}(a-x)\right)}}$$

---

② (u, v, z) $\quad \frac{\partial}{\partial v} = \frac{\partial}{\partial z} = 0 \rightarrow V = Au + B$

$x = a \operatorname{ch} u \cos v \quad \Delta V = 0$

$y = a \operatorname{sh} u \sin v \quad V(u=0) = 0 \rightarrow B = 0$

$z = z$

$A = -\frac{q}{2\pi\varepsilon_0} \rightarrow V = -\frac{q}{2\pi\varepsilon_0} u$

PLOSKEV: $u = -\frac{2\pi\varepsilon_0 V}{q}$

$\cos v = \frac{x}{\alpha} = \sqrt{1-\sin^2 v} = \sqrt{1-\left(\frac{y}{\beta}\right)^2}$

$x = a \operatorname{ch}\left(-\frac{2\pi\varepsilon_0 V}{q}\right)\cos v = \alpha \cos v \qquad \left(\frac{x}{\alpha}\right)^2 + \left(\frac{y}{\beta}\right)^2 = 1$

$y = a \operatorname{sh}\left(-\frac{2\pi\varepsilon_0 V}{q}\right)\sin v = \beta \sin v$

ENAČBA ELIPSE (cev eliptičnega prereza)

$$\underline{\underline{\left(\frac{x}{a \operatorname{ch}\left(-\frac{2\pi\varepsilon_0 V}{q}\right)}\right)^2 + \left(\frac{y}{a \operatorname{sh}\left(-\frac{2\pi\varepsilon_0 V}{q}\right)}\right)^2 = 1}}$$

---

③ $\Delta V = 0$; $\vec{E}_0 = -\vec{1}_z E_0 e^{j\omega t} = \left[-\vec{1}_r \cos\theta + \vec{1}_\theta \sin\theta\right] E_0 e^{j\omega t}$ $\qquad \operatorname{div}\vec{J} + j\omega\rho = 0 \longrightarrow \operatorname{div}\vec{K} + j\omega\sigma = 0$

$\vec{E} = \left[-\vec{1}_r\left(1+\frac{2a^3}{r^3}\right)\cos\theta + \vec{1}_\theta\left(1-\frac{a^3}{r^3}\right)\sin\theta\right] E_0 e^{j\omega t}$ $\qquad \vec{K} = \vec{1}_\theta K(\theta)$; $\operatorname{div}\vec{K} = \frac{1}{r^2 \sin\theta}\frac{d}{d\theta}\left(r\sin\theta K(\theta)\right) = \frac{1}{a\sin\theta}\frac{d}{d\theta}\left(\sin\theta K(\theta)\right)$

$\sigma = \vec{1}_r \cdot \vec{D}\Big|_{r=a} = -\left(1+\frac{2a^3}{a^3}\right)\cos\theta\,\varepsilon_0 E_0 e^{j\omega t} = \underline{-3\cos\theta\,\varepsilon_0 E_0 e^{j\omega t}}$ $\qquad \underline{\frac{d}{d\theta}\left(\sin\theta K(\theta)\right) = 3\cos\theta\sin\theta\, j\omega a\varepsilon_0 E_0 e^{j\omega t}}$

$\cos\theta\sin\theta = \frac{1}{2}\sin 2\theta$; $\int \frac{1}{2}\sin 2\theta\, d\theta = -\frac{1}{4}\cos 2\theta + C$; $\vec{K} = -\vec{1}_\theta \frac{3}{4} j\omega a\varepsilon_0 E_0 e^{j\omega t} \frac{\cos 2\theta - 1}{\sin\theta}$ $\longleftarrow$ $C = \frac{1}{4}$ daje rezultat omejen $\theta = 0$ $\theta = \pi$

$\cos 2\theta - 1 = \cos^2\theta - \sin^2\theta - 1 = -2\sin^2\theta$; $\quad \underline{\underline{\vec{K} = \vec{1}_\theta \frac{3}{2} j\omega a\,\varepsilon_0 E_0 e^{j\omega t} \sin\theta}}$

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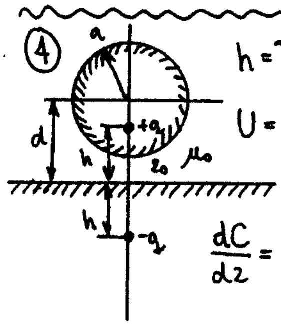
④



$h = \sqrt{d^2 - a^2}$

$U = \frac{q}{2\pi\varepsilon_0} \ln\frac{h+d-a}{h-(d-a)}$

$\frac{dC}{dz} = \frac{q}{U} = \frac{2\pi\varepsilon_0}{\ln\frac{h+d-a}{h-(d-a)}}$

$Z_K = \sqrt{\frac{\frac{dL}{dz}}{\frac{dC}{dz}}}$; TEM: $c_0 = \frac{1}{\sqrt{\frac{dL}{dz}\cdot\frac{dC}{dz}}} \rightarrow \frac{dL}{dz} = \frac{1}{c_0^2 \frac{dC}{dz}} \rightarrow Z_K = \frac{1}{c_0 \frac{dC}{dz}}$

$$Z_K = \frac{\ln\frac{h+d-a}{h-(d-a)}}{2\pi\varepsilon_0 c_0} = \underline{\underline{\frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}} \ln\frac{\sqrt{d^2-a^2}+(d-a)}{\sqrt{d^2-a^2}-(d-a)}}}$$

---

⑤ $\vec{E} = \vec{E}_v + \vec{E}_0 = \vec{1}_E E_v\left(e^{-jkz} - e^{+jkz}\right)$; $\vec{S}_v = \frac{1}{2}\vec{E}_v \times \vec{H}_v = \vec{1}_z \frac{|E_v|^2}{2Z_0} \longrightarrow E_v = \sqrt{2Z_0 S_v}$

$\vec{H} = \vec{1}_H \frac{E_v}{Z_0}\left(e^{-jkz} + e^{+jkz}\right)$ $\qquad |\vec{K}| = \frac{2}{Z_0}\sqrt{2Z_0 S_v} = \sqrt{\frac{8S_v}{Z_0}} = \sqrt{\frac{8\cdot 10^3\,\mathrm{W/m^2}}{120\pi\,\mathrm{V/A}}} = \underline{\underline{4.607\,\frac{\mathrm{A}}{\mathrm{m}}}}$

$\vec{H}(z=0) = \vec{1}_H \frac{2E_v}{Z_0}$

$\vec{K} = \vec{1}_n \times \vec{H}(z=0) = \vec{1}_E \frac{2E_v}{Z_0}$ $\qquad \vec{F} = I\,d\vec{x} \times \vec{B}_v = \vec{1}_z |\vec{K}|\cdot A\cdot\mu_0|\vec{H}_v| = \vec{1}_z \frac{|\vec{K}|^2 A\mu_0}{4} = \underline{\underline{6.667\cdot 10^{-7}\,\mathrm{N}}}$

REŠITEV 1. kolokvija iz elektromagnetike – 8.12.1999

① $V(x,y)=\sum_n C_n \sin\left(\frac{n\pi}{a}x\right)\mathrm{sh}\left(\frac{n\pi}{a}y\right)$; $V(x,b)=\sum_n C_n \sin\left(\frac{n\pi}{a}x\right)\mathrm{sh}\left(\frac{n\pi}{a}b\right)=\begin{cases}\frac{2V_0}{a}x\ ; & 0\le x<\frac{a}{2}\\ \frac{2V_0}{a}(x-a)\ ; & \frac{a}{2}<x\le\frac{a}{2}\end{cases}\Bigg/\cdot\int_0^a \sin\left(\frac{m\pi}{a}x\right)dx$

#1 $\int_0^a\left(\sum_n C_n \sin\left(\frac{n\pi}{a}x\right)\mathrm{sh}\left(\frac{n\pi}{a}b\right)\right)\sin\left(\frac{m\pi}{a}x\right)dx=\int_0^a C_m\sin^2\left(\frac{m\pi}{a}x\right)\mathrm{sh}\left(\frac{m\pi}{a}b\right)dx=C_m\frac{a}{2}\mathrm{sh}\left(\frac{m\pi}{a}b\right)$

#2 $\int_0^{a/2}\frac{2V_0}{a}x\sin\left(\frac{m\pi}{a}x\right)dx+\int_{a/2}^a\frac{2V_0}{a}(x-a)\sin\left(\frac{m\pi}{a}x\right)dx=-\frac{2V_0a}{m\pi}\cos\left(\frac{m\pi}{2}\right)$

#1 = #2 $\Rightarrow C_m=\frac{-4V_0}{m\pi\,\mathrm{sh}\left(\frac{m\pi}{a}b\right)}\cos\left(\frac{m\pi}{2}\right)\Rightarrow V(x,y)=\sum_{m=1}^{\infty}\frac{-4V_0}{m\pi\,\mathrm{sh}\left(\frac{m\pi}{a}b\right)}\cos\left(\frac{m\pi}{2}\right)\sin\left(\frac{m\pi}{a}x\right)$

② $V_1=V_{16}=-V_4=-V_{13}$; $V_2=V_5=V_{12}=V_{15}=-V_3=-V_8=-V_9=-V_{14}$; $V_6=V_{11}=-V_7=-V_{10}$

$$\left.\begin{aligned}4V_1&=10+V_2+V_5+10\\4V_2&=10+V_3+V_6+V_1\\4V_6&=V_2+V_7+V_{10}+V_5\end{aligned}\right\}\quad\left.\begin{aligned}4V_1&=10+V_2+V_2+10\\4V_2&=10-V_2+V_6+V_1\\4V_6&=V_2-V_6-V_6+V_2\end{aligned}\right\}\quad\left.\begin{aligned}4V_1&=20+2V_2\ /\cdot3\\5V_2&=10+V_1+V_6\ /\cdot12\\4V_6&=2V_2-2V_6\ /\cdot2\end{aligned}\right\}\quad\begin{aligned}12V_1&=60+6V_2\\60V_2&=120+12V_1+12V_6\\12V_6&=4V_2\end{aligned}\qquad\begin{aligned}V_2&=\frac{18}{5}V\\V_1&=\frac{34}{5}V\\V_6&=\frac{6}{5}V\end{aligned}$$

③ $\Delta V=0$ $\quad\vec{E}=-\mathrm{grad}V=\left(-\vec{1}_rE_0\left(1+2\frac{a^3}{r^3}\right)\cos\Theta+\vec{1}_\Theta E_0\left(1-\frac{a^3}{r^3}\right)\sin\Theta\right)e^{j\omega t}$ $\quad\vec{1}_n=\vec{1}_r$

$\sigma=\vec{1}_n\cdot\varepsilon_0\vec{E}=-3\varepsilon_0E_0\cos\Theta\,e^{j\omega t}$ $\quad\mathrm{div}\vec{J}+j\omega\rho=0\Rightarrow\mathrm{div}\vec{K}+j\omega\sigma=0$ $\quad\vec{K}=\vec{1}_\Theta K(\Theta)$ $\quad\frac{\partial}{\partial\phi}=0$ $r=a$

$\frac{1}{a\sin\Theta}\frac{\partial}{\partial\Theta}(\sin\Theta K_\Theta)=j\omega3\varepsilon_0E_0\cos\Theta\,e^{j\omega t}\Rightarrow K_\Theta=j\frac{3}{2}\omega\varepsilon_0E_0a\sin\Theta\,e^{j\omega t}+\frac{C}{\sin\Theta}$

$\vec{K}=\vec{1}_\Theta\left(j\frac{3}{2}\omega\varepsilon_0E_0a\sin\Theta\,e^{j\omega t}+\frac{C}{\sin\Theta}\right)$

④ $h_{q_i}=\sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2+\left(\frac{\partial y}{\partial q_i}\right)^2+\left(\frac{\partial z}{\partial q_i}\right)^2}$ $\quad h_\eta=a\sqrt{\mathrm{sh}^2\eta+\cos^2\psi}$ $\quad h_\psi=a\sqrt{\mathrm{sh}^2\eta+\cos^2\psi}$ $\quad h_\phi=a\,\mathrm{ch}\eta\sin\psi$

$$\mathrm{rot}\vec{F}=\frac{1}{a^3(\mathrm{sh}^2\eta+\cos^2\psi)\mathrm{ch}\eta\sin\psi}\begin{vmatrix}\vec{1}_\eta a\sqrt{\mathrm{sh}^2\eta+\cos^2\psi} & \vec{1}_\psi a\sqrt{\mathrm{sh}^2\eta+\cos^2\psi} & \vec{1}_\phi a\,\mathrm{ch}\eta\sin\psi\\ \frac{\partial}{\partial\eta} & \frac{\partial}{\partial\psi} & \frac{\partial}{\partial\phi}\\ a\sqrt{\mathrm{sh}^2\eta+\cos^2\psi}\cdot F_\eta & a\sqrt{\mathrm{sh}^2\eta+\cos^2\psi}\cdot F_\psi & a\,\mathrm{ch}\eta\sin\psi\cdot F_\phi\end{vmatrix}$$

⑤ $\vec{H}=\frac{1}{\mu_0}\mathrm{rot}\vec{A}=\vec{1}_\rho\frac{C}{2\mu_0}\frac{\ln\rho}{\rho}\cos\frac{\varphi}{2}-\vec{1}_\varphi\frac{C}{\mu_0\rho}\sin\frac{\varphi}{2}$ $\quad\vec{J}=\mathrm{rot}\vec{H}=\vec{1}_z\frac{C}{4\mu_0}\frac{\ln\rho}{\rho^2}\sin\frac{\varphi}{2}$

$I=\oint\vec{H}\,d\vec{s}=\frac{C}{\mu_0}\int_0^{2\pi}\left(\vec{1}_\rho\frac{\ln\rho}{2\rho}\cos\frac{\varphi}{2}-\vec{1}_\varphi\frac{1}{\rho}\sin\frac{\varphi}{2}\right)\vec{1}_\varphi\rho\,d\varphi=-\frac{C}{\mu_0}\int_0^{2\pi}\sin\frac{\varphi}{2}\,d\varphi=\frac{-4C}{\mu_0}$

Rešitev 2. kolokvija iz ELEKTROMAGNETIKE – 21.1.2000

① $\vec{E}_+=(\vec{1}_x+j\vec{1}_y)Ae^{-jk_0z}$; $\vec{E}_+(z=0)+\vec{E}_-(z=0)=0 \Rightarrow \vec{E}_-=-(\vec{1}_x+j\vec{1}_y)Ae^{jk_0z}$

$\vec{E}=\vec{E}_++\vec{E}_-=(\vec{1}_x+j\vec{1}_y)A(e^{-jk_0z}-e^{jk_0z})$; $\vec{H}=\frac{j}{\omega\mu}\mathrm{rot}\,\vec{E}=\frac{A}{Z_0}(-j\vec{1}_x+\vec{1}_y)(e^{-jk_0z}+e^{jk_0z})$

$\vec{S}=\frac12\vec{E}\times\vec{H}^*=\frac12A(\vec{1}_x+j\vec{1}_y)(e^{-jk_0z}-e^{jk_0z})\times\frac{A^*}{Z_0}(j\vec{1}_x+\vec{1}_y)(e^{jk_0z}+e^{-jk_0z})=-\vec{1}_z\frac{j2AA^*}{Z_0}\sin 2k_0z$

② $Q=\omega_0\frac{W}{P}$; ω_0: $\vec{E}=\vec{1}_\rho\frac{A}{\rho}\sin\frac{3\pi}{l}z$; $\vec{H}=\frac{j}{\omega\mu_0}\mathrm{rot}\,\vec{E}=\vec{1}_\varphi\frac{j3\pi A}{\omega\mu_0\rho l}\cos\frac{3\pi}{l}z$

$\vec{J}(\omega=\omega_0)=0=\mathrm{rot}\,\vec{H}\big|_{\omega=\omega_0}-j\omega_0\varepsilon\vec{E}$; $\mathrm{rot}\,\vec{H}=\vec{1}_\rho\frac{j9\pi^2A}{\omega_0\mu_0l^2\rho}\sin\frac{3\pi}{l}z=\vec{1}_\rho\frac{j\omega_0\varepsilon A}{\rho}\sin\frac{3\pi}{l}z \Rightarrow$

$\omega=\frac{3\pi}{l}\frac{1}{\sqrt{\mu_0\varepsilon}}=\frac{3\pi c_0}{l}\frac{1}{\sqrt{\varepsilon_r}}$; $W=\frac12\int_v\varepsilon|\vec{E}_{max}|^2dv=\frac12\int_0^l\int_0^{2\pi}\int_{r_n}^{r_z}\varepsilon\frac{|A|^2}{\rho^2}\sin^2\frac{3\pi}{l}z\,\rho\,d\rho\,d\varphi\,dz=$

$=\frac{\varepsilon|A|^2\pi l}{2}\ln\frac{r_z}{r_n}$; $P=\frac12\int_v\gamma|\vec{E}|^2dv=\ldots=\frac{\gamma|A|^2\pi l}{2}\ln\frac{r_z}{r_n}$; $Q=\omega_0\frac{W}{P}=\frac{3\pi}{l\gamma}\sqrt{\frac{\varepsilon}{\mu_0}}$

③ $M=\frac1I\oint\vec{V}_m\,d\vec{s}$; žica: $\vec{V}_m=-\vec{1}_z\left(\frac{\mu_0I}{2\pi}\ln\rho+C\right)$

$M=\frac1I\int_0^{2\pi}-\vec{1}_z\left(\frac{\mu_0I}{2\pi}\ln a+C\right)\cdot\vec{1}_\varphi\cdot a\cdot d\varphi=0$ ker je $\vec{1}_z\cdot\vec{1}_\varphi=0$

④ $\vec{E}=\vec{1}_yE_0\sin\frac{3\pi}{a}x\,e^{-j\beta z}$; $\vec{H}=\frac{j}{\omega\mu_0}\mathrm{rot}\,\vec{E}=-\vec{1}_x\frac{E_0\beta}{\omega\mu_0}\sin\frac{3\pi}{a}x\,e^{-j\beta z}+\vec{1}_z\frac{j3\pi E_0}{\omega\mu_0a}\cos\frac{3\pi}{a}x\,e^{-j\beta z}$

$\vec{S}=\frac12\vec{E}\times\vec{H}^*=\frac12\left(\vec{1}_yE_0\sin\frac{3\pi}{a}x\,e^{-j\beta z}\right)\times\left(-\vec{1}_x\frac{E_0^*\beta}{\omega\mu_0}\sin\frac{3\pi}{a}x\,e^{j\beta z}-\vec{1}_z\frac{j3\pi E_0^*}{\omega\mu a}\cos\frac{3\pi}{a}x\,e^{j\beta z}\right)=$

$=\vec{1}_z\frac{E_0E_0^*\beta}{2\omega\mu_0}\sin^2\frac{3\pi}{a}x-\vec{1}_x\frac{j3\pi E_0E_0^*}{4\omega\mu_0a}\sin\frac{6\pi}{a}x$

$P=\int_A\vec{S}\,d\vec{A}=\int_0^a\int_0^b\left(\vec{1}_z\frac{|E_0|^2\beta}{2\omega\mu_0}\sin^2\frac{3\pi}{a}x-\vec{1}_x\frac{j3\pi|E_0|^2}{4\omega\mu_0a}\sin\frac{6\pi}{a}x\right)\cdot\vec{1}_z\,dx\,dy=\frac{|E_0|^2\beta ab}{4\omega\mu_0}$

$x=0$: $\vec{K}=\vec{1}_x\times\vec{H}=-\vec{1}_y\frac{j3\pi E_0}{\omega\mu a}e^{-j\beta z}$; $y=0$: $\vec{K}=\vec{1}_y\times\vec{H}=\vec{1}_z\frac{E_0\beta}{\omega\mu_0}\sin\frac{3\pi}{a}x\,e^{-j\beta z}+\vec{1}_x\frac{j3\pi E_0}{\omega\mu a}\cos\frac{3\pi}{a}x\,e^{-j\beta z}$

$x=a$: $\vec{K}=-\vec{1}_x\times\vec{H}=-\vec{1}_y\frac{j3\pi E_0}{\omega\mu a}e^{-j\beta z}$; $y=b$: $\vec{K}=-\vec{1}_y\times\vec{H}=-\vec{1}_z\frac{E_0\beta}{\omega\mu_0}\sin\frac{3\pi}{a}x\,e^{-j\beta z}-\vec{1}_x\frac{j3\pi E_0}{\omega\mu a}\cos\frac{3\pi}{a}x\,e^{-j\beta z}$

⑤ $f=\frac{1}{2\sqrt{\mu_0\varepsilon_0\varepsilon_r}}\sqrt{\left(\frac{l}{a}\right)^2+\left(\frac{m}{b}\right)^2+\left(\frac{n}{c}\right)^2}$; $l,m,n\in\mathbb{Z}$, vsaj dva morata biti različna od nič!

Najnižja resonančna frekvenca v primeru, da bi bil $\varepsilon_r=1$ je za $m=1$, $n=1$ ($b,c>a$!) enaka $f(\varepsilon_r=1)=4.48\,\mathrm{GHz}$. Zahtevana resonančna frekvenca je nižja $\Rightarrow \varepsilon_r>1$

$f=\frac{f(\varepsilon_r=1)}{\sqrt{\varepsilon_r}} \Rightarrow \varepsilon_r=\left(\frac{f(\varepsilon_r=1)}{f}\right)^2=20.1$

Rešitve izpita ELEKTROMAGNETIKA 12/7/2000

①

$$\mathrm{rot}(\vec{1}_x \Delta A) + \mathrm{rot}(\vec{1}_y \Delta B) + \mathrm{rot}(\vec{1}_z \Delta C) + \mathrm{rot}(\mathrm{rot}(\mathrm{rot}(\vec{1}_x A + \vec{1}_y B + \vec{1}_z C))) = ?$$

$A(x,y,z)$
$B(x,y,z)$
$C(x,y,z)$

$$\vec{F} = \vec{1}_x A + \vec{1}_y B + \vec{1}_z C$$

$$\mathrm{rot}(\Delta\vec{F}) + \mathrm{rot}(\mathrm{rot}(\mathrm{rot}\vec{F})) =$$

$$= \mathrm{rot}(\mathrm{grad}(\mathrm{div}\vec{F}) - \mathrm{rot}(\mathrm{rot}\vec{F})) + \mathrm{rot}(\mathrm{rot}(\mathrm{rot}\vec{F})) =$$

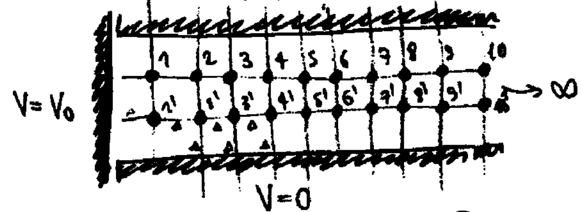
$$= \mathrm{rot}(\mathrm{grad}(\mathrm{div}\vec{F})) =$$

$$V = \mathrm{div}\vec{F}$$

$$= \mathrm{rot}(\mathrm{grad}\, V) = 0$$

③

$V_n = ?$



$$4V_n = V_{n-1} + V_n + V_{n+1} + 0$$

$$3V_n = V_{n-1} + V_{n+1} \qquad V_{n+1} = \alpha V_n \quad V_n = \alpha V_{n-1}$$

$$3 = \alpha^{-1} + \alpha$$

$$\alpha^2 - 3\alpha + 1 = 0$$

$$\alpha = \frac{3 \pm \sqrt{9-4}}{2}$$

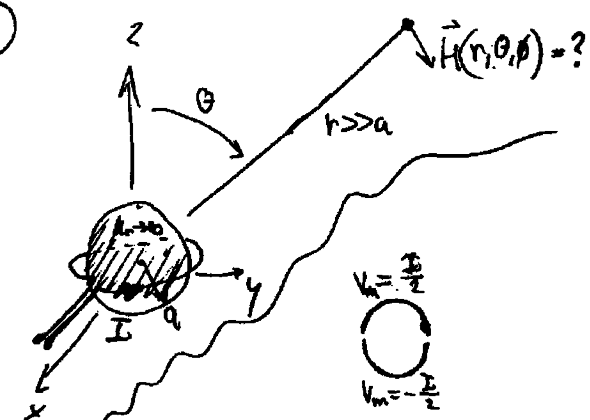
$$\alpha = \frac{3 \pm \sqrt{5}}{2}$$

$$\alpha = \frac{3-\sqrt{5}}{2}$$

$$V_n = \left(\frac{3-\sqrt{5}}{2}\right) V_{n-1}$$

$$\underline{\underline{V_n = \left(\frac{3-\sqrt{5}}{2}\right)^n V_0}}$$

④



$$V_m = \sum \frac{C_m}{r^{m+1}} P_m(\cos\theta) \approx \frac{C_1}{r^2}\cos\theta$$

$$\int_{-1}^{1} \frac{C_1}{a^2}\cos\theta \cos\theta\, d\cos\theta = \int_{-1}^{1} V_m(a)\cos\theta\, d\cos\theta$$

$$\frac{C_1}{a^2}\frac{2}{3} = \frac{I}{2}\cdot\frac{1}{2} + \frac{I}{2}\cdot\frac{1}{2} = \frac{I}{2} \rightarrow \boxed{C_1 = \frac{3}{4}a^2 I}$$

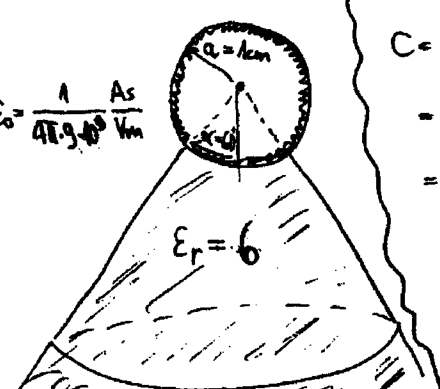
$$V_m \approx \frac{3}{4}a^2 I \frac{\cos\theta}{r^2} = \frac{3\pi a^2 I}{4\pi r^2}\cos\theta$$

$$\underline{\underline{\vec{H} = -\mathrm{grad}\, V_m = \frac{3\pi a^2 I}{4\pi r^3}\left(\vec{1}_r 2\cos\theta + \vec{1}_\theta \sin\theta\right)}}$$

②

$C = ?$

$$\varepsilon_0 = \frac{1}{4\pi \cdot 9 \cdot 10^9}\frac{As}{Vm}$$



$$\Omega = 2\pi(1-\cos\alpha) = \pi$$

$$C = \Omega \varepsilon_r \varepsilon_0 a + (4\pi - \Omega)\varepsilon_0 a =$$

$$= \pi \varepsilon_r \varepsilon_0 a + 3\pi\varepsilon_0 a = \pi(\varepsilon_r + 3)\varepsilon_0 a =$$

$$= \pi(6+3)\frac{1}{4\pi \cdot 9 \cdot 10^9}\frac{As}{Vm}\cdot 0{,}01\,m =$$

$$= \frac{9}{36}\cdot 10^{-11}\,F = \underline{\underline{2{,}5\,pF}}$$

⑤

$\varepsilon_0, \mu_0, \rho = 0, \vec{J} = 0$

$$\vec{E} = \vec{E}_0 e^{-j\beta x}\cos\alpha(y+z)$$

$\alpha = ?$

smer $\vec{E}_0 = ?$

$\vec{H} = ?$

$$\mathrm{div}\vec{E} = 0 = -j\beta E_{0x} e^{-j\beta x}\cos\alpha(y+z) - \alpha E_{0y} e^{-j\beta x}\sin\alpha(y+z) - \alpha E_{0z} e^{-j\beta x}\sin\alpha(y+z)$$

$\rho = 0$

$$\underline{\underline{E_{0x} = 0}} \qquad \underline{\underline{E_{0y} = -E_{0z}}}$$

$$\underline{\underline{\vec{E}_0 = \frac{\vec{1}_y - \vec{1}_z}{\sqrt{2}} E_0}}$$

$$\vec{H} = -\frac{1}{j\omega\mu}\mathrm{rot}\vec{E} = -\frac{1}{j\omega\mu}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & E_z \end{vmatrix} = -\frac{1}{j\omega\mu}\left(\vec{1}_x \frac{2E_{0y}}{\sqrt{2}}\alpha e^{-j\beta x}\sin\alpha(y+z) + \vec{1}_y \frac{E_{0y}}{\sqrt{2}} j\beta e^{-j\beta x}\cos\alpha(y+z) - \vec{1}_z \frac{E_{0y}}{\sqrt{2}} j\beta e^{-j\beta x}\cos\alpha(y+z)\right)$$

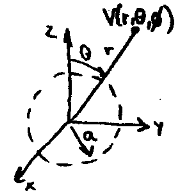
$$\mathrm{rot}\vec{H} = \vec{1}_x 0 + \vec{1}_y\left(\sqrt{2}E_{0y}\alpha^2 e^{-j\beta x}\cos\alpha(y+z) + \frac{E_{0y}}{\sqrt{2}}\beta^2 e^{-j\beta x}\cos\alpha(y+z)\right) + \vec{1}_z\left(-\frac{E_{0y}}{\sqrt{2}}\beta^2 e^{-j\beta x}\cos\alpha(y+z) - \sqrt{2}E_{0y}\alpha^2 e^{-j\beta x}\cos\alpha(y+z)\right)$$

Pogoj: $\beta^2 + 2\alpha^2 = k^2 = \omega^2\mu_0\varepsilon_0 \rightarrow \underline{\underline{\alpha = \frac{\sqrt{k^2 - \beta^2}}{\sqrt{2}}}}$

Rešitev ELEKTROMAGNETIKA 04.10.2000

1

$V = \begin{cases} V_0 ; & r < a \\ V_0 e^{-\alpha r} ; & r \geq a \end{cases}$



$\varepsilon = \varepsilon_0 \; ; \; \omega = 0$

$\rho = ? \; ; \; \sigma = ? \; ; \; Q = ?$

$\vec{E} = -\mathrm{grad}\, V \; ; \; \vec{E}_n = 0 \; ; \; \vec{E}_z = -\vec{1}_r V_0 (-\alpha) e^{-\alpha r} = \vec{1}_r \alpha V_0 e^{-\alpha r}$

$\rho = \mathrm{div}(\varepsilon \vec{E}) \; ; \; \underline{\rho_n = 0} \; ; \; \rho_z = \frac{\alpha V_0}{r^2} \frac{\partial}{\partial r}(r^2 e^{-\alpha r}) = \underline{\alpha V_0 \left(\frac{2}{r} - \alpha\right) e^{-\alpha r}}$

$\sigma(r=a) = \vec{1}_r (\vec{D}_z - \vec{D}_n) = \underline{\alpha \varepsilon V_0 e^{-\alpha a}}$

$Q = \oint \vec{D}_n d\vec{A} = \underline{0}$

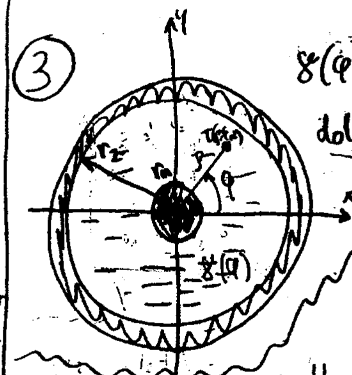
meja okoli izhodišča

3

$\gamma(\varphi) = \gamma_0 \left(\frac{1}{2} - \frac{1}{2}\sin\varphi\right)^2$

dolžina kabla l

$R = ?$



$\gamma(\varphi) \to \vec{E} = \vec{1}_\rho \frac{C}{\rho}$

$U = C \ln \frac{r_2}{r_1}$

$\vec{E} = \vec{1}_\rho \frac{U}{\ln \frac{r_2}{r_1}} \frac{1}{\rho}$

$\vec{J} = \gamma \vec{E} = \gamma_0 \left(\frac{1}{2} - \frac{1}{2}\sin\varphi\right)^2 \frac{U}{\ln\frac{r_2}{r_1}} \frac{1}{\rho}$

$I = \int \vec{J} d\vec{A} = l \int_0^{2\pi} J \rho \, d\varphi = \frac{3}{8} \cdot 2\pi l \gamma_0 \frac{U}{\ln \frac{r_2}{r_1}}$

$R = \frac{U}{I} = \underline{\underline{\frac{4 \ln \frac{r_2}{r_1}}{3\pi l \gamma_0}}}$

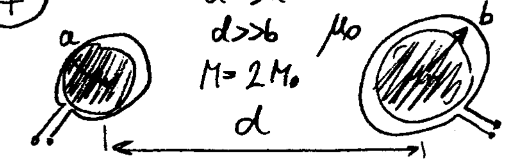
4

$d \gg a$

$d \gg b$

$M = 2 M_0$

μ_0



$\mu_r = ?$

Krogla μ_r v homogenem polju: $\vec{B}_N = \vec{B}_0 \frac{3\mu_r}{2+\mu_r}$

1. zanka, 2. zanka (recipročnost!)

$M = \left(\frac{3\mu_r}{2+\mu_r}\right)\left(\frac{3\mu_r}{2+\mu_r}\right) M_0$

$2 = \left(\frac{3\mu_r}{2+\mu_r}\right)^2$

$\frac{3\mu_r}{2+\mu_r} = \sqrt{2}$

$3\mu_r = 2\sqrt{2} + \mu_r \sqrt{2}$

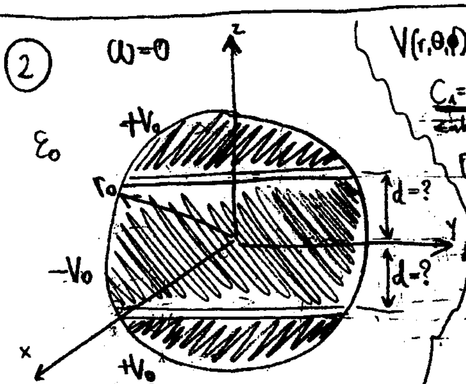
$\mu_r(3 - \sqrt{2}) = 2\sqrt{2}$

$\mu_r = \frac{2\sqrt{2}}{3 - \sqrt{2}} = \underline{\underline{1{,}784}}$

2

$\omega = 0$

ε_0



$\vec{E}(r \gg a) = \min$

$V(r,\theta,\phi) = \sum_n C_n r^{-(n+1)} P_n(\cos\theta)$

$C_n = 0$ zaradi simetrije

zahtevamo $C_0 = 0 \; ; \; P_0(\cos\theta) = 1$

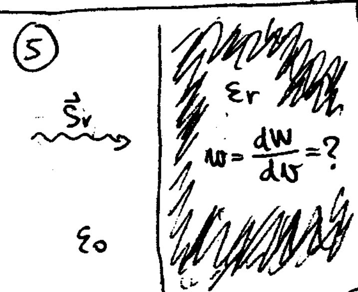
$0 = \int V(r = a, \theta, \phi)\, d\cos\theta =$

$= \int_{-1}^{-d/r} V_0 \, d\cos\theta + \int_{-d/r}^{d/r} -V_0 \, d\cos\theta + \int_{d/r}^{1} V_0 \, d\cos\theta =$

$= 2V_0(1 - d/r) - 2V_0 \, d/r =$

$0 = 2V_0(1 - 2d/r) \Rightarrow \underline{\underline{d = \frac{r}{2}}}$

5



$\Gamma = \frac{1-n}{1+n} \qquad \vec{S} = \vec{S}_v(1 - \Gamma^2) = \vec{S}_v \frac{4n}{(1+n)^2}$

$w = \frac{dW}{dV} = \frac{S}{c} = \frac{S n}{c_0} = \frac{S_v}{c_0} \frac{4n^2}{(1+n)^2}$

$n = \sqrt{\varepsilon_r}$

$w = \underline{\underline{\frac{S_v}{c_0} \frac{4\varepsilon_r}{(1+\sqrt{\varepsilon_r})^2}}}$

Elektromagnetika, kolokvij 12.12.2000, rešitve

1) $\rho(r,\theta,\phi) = \ ? \quad \frac{\partial}{\partial\phi} = 0$

$V(r,\theta,\phi) = V_0\, r \sin 3\theta$

$\vec{E} = -\,grad\, V = \underline{-\,\vec{1}_r\ V_0 \sin 3\theta - \vec{1}_\theta\ V_0\, 3\cos 3\theta}$

$$\rho = div\,\vec{D} = div\,\varepsilon_0\,\vec{E} = \varepsilon_0 \frac{1}{r^2 \sin\theta}\left[-\frac{\partial}{\partial r}\left(r^2 \sin\theta\ V_0 \sin 3\theta\right) - \frac{\partial}{\partial\theta}\left(r\sin\theta\ V_0\, 3\cos 3\theta\right)\right] =$$

$$= \varepsilon_0 \frac{1}{r^2 \sin\theta}\left[-\,V_0\, 2r\ \sin\theta \sin 3\theta - 3V_0\, r\,(\cos\theta\cos 3\theta + \sin\theta\,(-3\sin 3\theta))\right] =$$

$$= \varepsilon_0 \frac{V_0}{r\sin\theta}\left[-2\sin\theta\sin 3\theta - 3\cos\theta\cos 3\theta + 9\sin\theta\sin 3\theta\right]$$

$$= \underline{\underline{\varepsilon_0 \frac{V_0}{r}\left(7\sin 3\theta - 3\,\mathrm{ctg}\,\theta\cos 3\theta\right)}}$$

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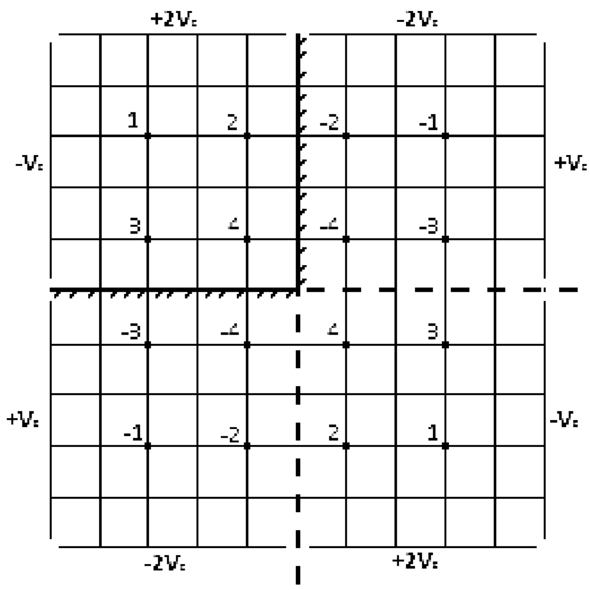
2)



$4V_1 = -V_0 + 2V_0 + V_2 + V_3 \ \rightarrow\ 4V_1 = V_0 + V_2 + V_3$

$4V_2 = 2V_0 + V_1 - V_2 + V_4 \ \rightarrow\ 5V_2 = 2V_0 + V_1 + V_4$

$4V_3 = -V_0 + V_1 - V_3 + V_4 \ \rightarrow\ 5V_3 = -V_0 + V_1 + V_4$

$4V_4 = V_2 + V_3 - V_4 - V_4 \ \rightarrow\ 6V_4 = V_2 + V_3 \ \rightarrow\ V_4 = \frac{V_2 + V_3}{6}$

$19V_2 = 9V_0 + V_3 + 4V_4 \ \rightarrow\ 55V_2 = 27V_0 + 5V_3$

$19V_3 = -3V_0 + V_2 + 4V_4 \ \rightarrow\ 55V_3 = -9V_0 + 5V_2$

$\underline{\underline{V_2 = \frac{12}{25}V_0}}$

$\underline{\underline{V_3 = -\frac{3}{25}V_0}}$

$\underline{\underline{V_4 = \frac{3}{50}V_0}}$

$\underline{\underline{V_1 = \frac{17}{50}V_0}}$

---

3) $V\left(x,y,z=\frac{c}{2}\right) = V_0 \sin\left(\frac{\pi}{a}x\right) \qquad V\left(x,y,z=-\frac{c}{2}\right) = -V_0 \sin\left(\frac{\pi}{a}x\right)$

$\frac{\partial}{\partial x} \neq 0 \quad \frac{\partial}{\partial y} \neq 0 \quad \frac{\partial}{\partial z} \neq 0 \qquad k_x = \frac{\pi}{a}, \quad k_y = \frac{\pi}{b}, \quad k_z = \sqrt{{k_x}^2 + {k_y}^2} = \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2},$

Nastavek: $V\left(x,y,z=\frac{c}{2}\right) = \sum_m \sum_n C_{mn} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \sinh\left(\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\ \frac{c}{2}\right) = V_0 \sin\frac{\pi}{a}$

$sinh\left(k_z\left(-\frac{c}{2}\right)\right) = -\, sinh\left(k_z\,\frac{c}{2}\right)$ → rešitev pri $z = -\frac{c}{2}$ glede na vzbujanje $(-\,V_0\, sin\,\frac{\pi}{a}x)$ predstavlja zrcalno sliko glede na rešitev pri $z = \frac{c}{2}$, zato je nesmiselna.

Glede na vsiljen potencial $(sin\,\frac{\pi}{a}x)$ na pokrovih sklepamo: $m = 1$;

$$V_0 = \sum_n C_n\, sin\left(\frac{n\pi}{b}y\right) sinh\left(\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\;\frac{c}{2}\right)$$

Koeficiente C_n določimo s Fourierjevo analizo, tako da čimbolje zadostimo pogoju $V = V_0$ na pokrovu:

$$\int_0^b V_0\left(sin\left(\frac{l\pi}{b}y\right)\right)dy = \int_0^b C_n\, sin\left(\frac{n\pi}{b}y\right) sinh\left(\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\;\frac{c}{2}\right) sin\left(\frac{l\pi}{b}y\right)dy\ ; \qquad n \to l$$

$$\frac{V_0\, b}{l\pi}\,(1-\cos(l\pi)) = C_l\, sinh\left(\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{l\pi}{b}\right)^2}\;\frac{c}{2}\right)\left(\frac{b}{2}\right)$$

$$C_l = \frac{2\,V_0}{l\pi\, sinh\left(\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{l\pi}{b}\right)^2}\;\frac{c}{2}\right)}(1-\cos(l\pi))$$

$$\underline{\underline{V(x,y,z) = \sum_{l=1}^{\infty}\frac{2\,V_0}{l\pi}(1-\cos(l\pi))\, sin\left(\frac{\pi}{a}x\right) sin\left(\frac{l\pi}{b}y\right)\frac{sinh\left(\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{l\pi}{b}\right)^2}\; z\right)}{sinh\left(\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{l\pi}{b}\right)^2}\;\frac{c}{2}\right)}}}$$

---

4) $\frac{\partial}{\partial r} \neq 0 \quad \frac{\partial}{\partial \theta} \neq 0 \quad \frac{\partial}{\partial \varphi} = 0 \qquad \vec{J}_0 = -\vec{1}_z J_0 = -\vec{1}_z \gamma_0 E_0$

$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0 \qquad \vec{E} = -grad\,V = -\vec{1}_z\frac{\partial V}{\partial z} \quad \to \quad \frac{\partial V_0}{\partial z} = E_0 \;\int dz$

V prostoru "γ_0": Potencial: $V_0\,(r,\theta) = (A_0 r + B_0 r^{-2})\, cos\theta$

Polje: $\vec{E}_0 = -grad\,V = -\vec{1}_r\,(A_0 - \frac{2B_0}{r^3})\, cos\theta + \vec{1}_\theta\,(A_0 + \frac{B_0}{r^3})\, sin\theta$

V prostoru "γ_N": Potencial: $V_N\,(r,\theta) = (A_N r + B_N r^{-2})\, cos\theta$

Polje: $\vec{E}_N = -grad\,V = -\vec{1}_r\,(A_N - \frac{2B_N}{r^3})\, cos\theta + \vec{1}_\theta\,(A_N + \frac{B_N}{r^3})\, sin\theta$

1. Pri $r \gg b$ je polje nespremenjeno:

   $\vec{E}_0 = -\vec{1}_r\,(A_0 - \frac{2B_0}{r^3})\, cos\theta + \vec{1}_\theta\,(A_0 + \frac{B_0}{r^3})\, sin\theta \approx -\vec{1}_r\, A_0\, cos\theta + \vec{1}_\theta\, A_0\, sin\theta = -\vec{1}_z\, A_0 = -\vec{1}_z\, E_0 \;\to\; \underline{A_0 = E_0}$

2. Pri $r = a$ (na površini krogle) je: $\vec{E}_t = \vec{E}_\theta = 0 \quad ; \quad \vec{1}_\theta E_N(r=a) = 0 = (A_N + \frac{B_N}{r^3})\, sin\theta \to B_N = -A_N a^3$

3. Pri $r = b$ je: $\vec{E}_{tN} = \vec{E}_{t0} \quad ; \quad \vec{J}_{nN} = \vec{J}_{n0}$

   $A_N + \frac{B_N}{b^3} = E_0 + \frac{B_0}{b^3} \quad ; \quad \gamma_N(A_N - \frac{2B_N}{b^3}) = \gamma_0(E_0 - \frac{2B_0}{b^3})$

   $A_N(1 - \frac{a^3}{b^3}) = E_0 + \frac{B_0}{b^3} \quad ; \quad A_N\gamma_N(1 + \frac{2a^3}{b^3}) = E_0\gamma_0 - \frac{2B_0}{b^3}\gamma_0$

   $A_N(1 - \frac{a^3}{b^3}) = E_0 + \frac{B_0}{b^3} \quad ; \quad E_0\gamma_N b^3\left(1 + \frac{2a^3}{b^3}\right) + B_0\gamma_N\left(1 + \frac{2a^3}{b^3}\right) = E_0\gamma_0 b^3\left(1 - \frac{a^3}{b^3}\right) - 2B_0\gamma_0\left(1 - \frac{a^3}{b^3}\right)$

$$A_N = \frac{E_0 b^3 + B_0}{b^3\left(1-\frac{a^3}{b^3}\right)} \qquad ; \qquad \underline{\underline{B_0 = \frac{E_0 b^3\left(\gamma_0\left(1-\frac{a^3}{b^3}\right)-\gamma_N\left(1+\frac{2a^3}{b^3}\right)\right)}{2\gamma_0\left(1-\frac{a^3}{b^3}\right)+\gamma_N\left(1+\frac{2a^3}{b^3}\right)}}}$$

$$\underline{\underline{A_N = \frac{3E_0\gamma_0}{2\gamma_0\left(1-\frac{a^3}{b^3}\right)+\gamma_N\left(1+\frac{2a^3}{b^3}\right)}}} \qquad ; \qquad \underline{\underline{B_N = -\frac{3E_0\gamma_0 a^3}{2\gamma_0\left(1-\frac{a^3}{b^3}\right)+\gamma_N\left(1+\frac{2a^3}{b^3}\right)}}}$$

$$\underline{\underline{\vec{E}_0 = -\vec{1}_r\, E_0\left(1-\frac{b^3\left(\gamma_0\left(1-\frac{a^3}{b^3}\right)-\gamma_N\left(1+\frac{2a^3}{b^3}\right)\right)}{r^3\gamma_0\left(1-\frac{a^3}{b^3}\right)+\gamma_N\left(1+\frac{2a^3}{b^3}\right)}\right)cos\theta + \vec{1}_\theta\, E_0\left(1+\frac{b^3\left(\gamma_0\left(1-\frac{a^3}{b^3}\right)-\gamma_N\left(1+\frac{2a^3}{b^3}\right)\right)}{r^3 2\gamma_0\left(1-\frac{a^3}{b^3}\right)+\gamma_N\left(1+\frac{2a^3}{b^3}\right)}\right)\, sin\theta}}$$

$$\underline{\underline{\vec{E}_N = -\vec{1}_r\,\frac{3E_0\gamma_0}{2\gamma_0\left(1-\frac{a^3}{b^3}\right)+\gamma_N\left(1+\frac{2a^3}{b^3}\right)}\left(1-\frac{2a^3}{r^3}\right)cos\theta + \vec{1}_\theta\,\frac{3E_0\gamma_0}{2\gamma_0\left(1-\frac{a^3}{b^3}\right)+\gamma_N\left(1+\frac{2a^3}{b^3}\right)}\left(1+\frac{a^3}{r^3}\right)\, sin\theta}}$$

---

5) $\vec{F} = \vec{1}_\rho + \vec{1}_\varphi\,\rho + \vec{1}_z\,\rho\cos\varphi$

$$\oint \vec{F}\cdot d\vec{s} = \int_2^4 \vec{F}\ \vec{1}_z\, dz + \int_\pi^{-\frac{\pi}{2}} \vec{F}\left(-\vec{1}_\varphi\right)\rho(-d\varphi) + \int_2^1 \vec{F}\ \left(-\vec{1}_\rho\right)(-d\rho) + \int_4^2 \vec{F}\ \left(-\vec{1}_z\right)(-dz) + \int_1^2 \vec{F}\ \vec{1}_\rho\, d\rho +$$

$$+\int_{-\frac{\pi}{2}}^{\pi} \vec{F}\ \vec{1}_\varphi\,\rho\, d\varphi =$$

$$= \Big|_{\substack{\rho=2\\ \varphi=\pi}} \rho\cos\varphi\ \Big[_2^4\, z + \Big|_{\substack{\rho=2\\ z=4}} \rho^2\ \Big[_\pi^{-\frac{\pi}{2}}\, \varphi + \Big|_{\substack{\varphi=-\pi/2\\ z=4}} \Big[_2^1\, \rho + \Big|_{\substack{\rho=1\\ \varphi=-\pi/2}} \rho\cos\varphi\ \Big[_4^2\, z + \Big|_{\substack{\varphi=-\pi/2\\ z=2}} \Big[_1^2\, \rho + \Big|_{\substack{\rho=2\\ z=2}} \rho^2\ \Big[_{-\frac{\pi}{2}}^{\pi}\, \varphi =$$

$$= 2\,(-1)\,2 + 4\left(-\frac{\pi}{2}-\pi\right) + (1-2) + 0 + (2-1) + 4\left(\pi+\frac{\pi}{2}\right) = \underline{\underline{-4}}$$

Elektromagnetika, kolokvij 15.02.2001, rešitve

1) $\vec{J} = \vec{1}_y J_0 \quad [A/m], \qquad r \gg a$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{v'} \vec{J}\,(\vec{r}')\,\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dv' = \frac{\mu_0 J_0}{4\pi}\int_{v'} \vec{1}_y\,\frac{e^{-jkr}}{r}\,dv' = \underline{\frac{\mu_0 a^3}{4\pi} J_0 \frac{e^{-jkr}}{r}\,(\vec{1}_r \sin\theta\sin\phi + \vec{1}_\theta\cos\theta\sin\phi + \vec{1}_\phi\cos\phi)}$$

$$\vec{H} = \frac{1}{\mu}\,rot\,\vec{A} = \frac{a^3}{4\pi} J_0 \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\,\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \sin\theta\sin\phi\,\frac{e^{-jkr}}{r} & r\cos\theta\sin\phi\,\frac{e^{-jkr}}{r} & r\sin\theta\cos\phi\,\frac{e^{-jkr}}{r}\end{vmatrix} =$$

$$= \frac{a^3}{4\pi} J_0 \frac{1}{r^2\sin\theta}\,[\vec{1}_r\,(\cos\theta\cos\phi\,e^{-jkr} - \cos\theta\cos\phi\,e^{-jkr}) + r\,\vec{1}_\theta\,(\sin\theta\cos\phi\,\frac{e^{-jkr}}{r} + (jk)\sin\theta\cos\phi\,e^{-jkr}) +$$

$$+ r\sin\theta\;\vec{1}_\phi\,((-jk)\cos\theta\sin\phi\,e^{-jkr} - \cos\theta\sin\phi\,\frac{e^{-jkr}}{r})]$$

$$\underline{\underline{\vec{H} = \frac{a^3}{4\pi} J_0 \frac{e^{-jkr}}{r}\left[\left(\frac{1}{r} + jk\right)\left(\vec{1}_\theta\cos\phi - \vec{1}_\phi\cos\theta\sin\phi\right)\right]}}$$

---

2) $\vec{K}(z=0) = \vec{1}_\rho K_0\,J_0\,(\alpha\rho) \qquad \sigma = ?$

$$\sigma = \frac{j}{\omega}\,div\,\vec{K} = \frac{j}{\omega}\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}\,\rho\;K_0\,J_0\,(\alpha\rho)\right) = \underline{\underline{\frac{j}{\omega}\frac{K_0}{\rho}\,(J_0\,(\alpha\rho) + \rho\alpha\,J_0'(\alpha\rho))}}$$

---

3) $a = 4\ cm, \quad b = 3\ cm, \quad c = 9\ cm, \qquad \varepsilon_r = ? \qquad f_{l,m,n} = \frac{c_0}{2}\sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$ m,n,p → cela števila

$f_{011} - f_{101} = 288{,}8\ MHz \qquad \frac{c_0}{2\sqrt{\varepsilon_r}}\left(\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2} - \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}\right) = 288{,}8\ MHz$

$$\underline{\underline{\varepsilon_r = \left(\frac{c_0}{2\ x\ 288{,}8\ MHz}\left(\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2} - \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}\right)\right)^2 = 16{,}32}}$$

---

4) $rot\,\vec{H} = \gamma\,\vec{E} + j\omega\varepsilon\,\vec{E} = j\omega\varepsilon_0\underbrace{\left(\frac{\gamma}{j\omega\varepsilon_0} + 1\right)}_{\varepsilon_r}\vec{E}$

$k = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r}$ V praznem prostoru

$k = k_0\sqrt{1 + \frac{\gamma}{j\omega\varepsilon_0}} \approx k_0\left(1 + \frac{\gamma}{2j\omega\varepsilon_0}\right) = \underline{k_0\left(1 - j\frac{\gamma}{2\omega\varepsilon_0}\right) = \beta_0 - j\alpha} \quad \rightarrow \quad \underline{\alpha = \frac{\gamma}{2\omega\varepsilon_0}}$

$\underline{l' = l\,\frac{k}{\beta} = \frac{\sqrt{(\beta)^2 + \left(\frac{\pi}{b}\right)^2}}{\beta}} \quad \rightarrow \quad \underline{\underline{\alpha' = \alpha\,\frac{\sqrt{(\beta)^2 + \left(\frac{\pi}{b}\right)^2}}{\beta}}} \qquad \underline{\underline{\alpha_{dB/m} = \alpha'_{Np/m}\,\frac{20}{\ln 10}}}$

5) $\underline{\vec{1}_{kv} = \vec{1}_z}$ $\underline{\vec{1}_{ko} = -\vec{1}_z}$

$$\vec{E} = \vec{E}_v + \vec{E}_o = \vec{1}_x A e^{-jk_0 z} + j\,\vec{1}_y A e^{-jk_0 z} + \vec{1}_x \Gamma A e^{+jk_0 z} + j\,\vec{1}_y \Gamma A e^{+jk_0 z}$$

$$\vec{H} = \vec{H}_v + \vec{H}_o = \left(\vec{1}_{kv} \times \frac{\vec{E}_v}{Z}\right) + \left(\vec{1}_{ko} \times \frac{\vec{E}_o}{Z}\right) = \underline{\vec{1}_y \frac{A}{Z} e^{-jk_0 z} - j\,\vec{1}_x \frac{A}{Z} e^{-jk_0 z} - \vec{1}_y \frac{\Gamma A}{Z} e^{+jk_0 z} + j\,\vec{1}_x \frac{\Gamma A}{Z} e^{+jk_0 z}}$$

$$S = \frac{1}{2}\left|\vec{E} \times \vec{H}^*\right| = \frac{1}{2}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ A e^{-jk_0 z} + \Gamma A e^{+jk_0 z} & j\,A e^{-jk_0 z} + j\,\Gamma A e^{+jk_0 z} & 0 \\ j\,\frac{A^*}{Z} e^{+jk_0 z} - j\,\frac{\Gamma^* A^*}{Z} e^{-jk_0 z} & \frac{A^*}{Z} e^{+jk_0 z} - \frac{\Gamma^* A^*}{Z} e^{-jk_0 z} & 0 \end{vmatrix} =$$

$$= \vec{1}_z \left(\frac{AA^*}{Z} - \frac{AA^*\Gamma^*}{Z} e^{-2jk_0 z} + \frac{AA^*\Gamma}{Z} e^{+2jk_0 z} - \frac{AA^*\Gamma\Gamma^*}{Z}\right) = \vec{1}_z \frac{AA^*}{Z}\left(1 - \Gamma^* e^{-2jk_0 z} + \Gamma e^{+2jk_0 z} - \Gamma\Gamma^*\right) =$$

$$= \underline{\underline{\vec{1}_z \frac{|A|^2}{Z}\left(1 - \Gamma 2j sin 2k_0 z - |\Gamma|^2\right)}}$$

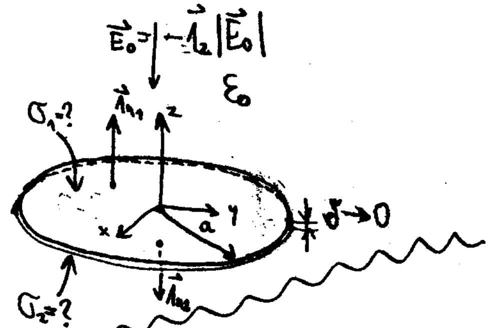
Odbojnost:

$$E_v + E_o = E_p \;;\; -\frac{1}{Z_0}\left[E_v - E_o\right] = -\frac{1}{Z} E_p \;;\; \frac{E_o}{E_v} = \frac{Z - Z_0}{Z + Z_0} = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} = \Gamma \;;\; \Gamma^2 = |\Gamma|^2$$

$$\underline{\underline{\Gamma = \Gamma^* = -|\Gamma| = \frac{\Gamma A e^{+jk_0 z}}{A e^{-jk_0 z}} = \Gamma e^{+2jk_0 z}}}$$

Rešitev ELEKTROMAGNETIKA 23.02.2001

1



Plošča je pravokotna na polje in nemoti polja!

$\sigma_1 = \vec{1}_{n_1} \cdot \varepsilon_0 \vec{E}_0 = \underline{\underline{-\varepsilon_0 |\vec{E}_0|}}$ $\quad \vec{1}_{n_1} = \vec{1}_z$

$\sigma_2 = \vec{1}_{n_2} \cdot \varepsilon_0 \vec{E}_0 = \underline{\underline{+\varepsilon_0 |\vec{E}_0|}}$ $\quad \vec{1}_{n_2} = -\vec{1}_z$

3

$\mu_r(W_{max}) = ?$ NOTRANJA MAGNETNA ENERGIJA V KROGLI

$W_{max} = ?$

$H = H_0 \frac{3}{2+\mu_r}$; $B = B_0 \frac{3\mu_r}{2+\mu_r}$; $W = \frac{1}{2} HBdv$.

$W = \frac{1}{2} H_0 B_0 \frac{9\mu_r}{(2+\mu_r)^2} v$ $\quad \frac{dW}{d\mu_r} = 0 = \frac{1}{2} H_0 B_0 \frac{9(2+\mu_r)^2 - 9\mu_r 2(2+\mu_r)}{(2+\mu_r)^4}$

$(2+\mu_r)^2 - 2\mu_r(2+\mu_r) = 0 \to 2+\mu_r - 2\mu_r = 0 \to \underline{\mu_r = 2}$

$W_{max} = \frac{1}{2} H_0 B_0 \frac{9 \cdot 2}{(2+2)^2} v = \frac{9}{8} W_0$; $v = \frac{4\pi r_0^3}{3}$

$$\underline{\underline{W_{max} = \frac{3\pi}{4} \mu_0 H_0^2 r_0^3}}$$

4

$\vec{E} = \vec{1}_x E_0 \sin(\alpha y) e^{-j\beta z}$; $\alpha^2 + \beta^2 = k^2$

$\vec{H} = ?$; $\vec{S} = ?$; $\rho = ?$; $\vec{J} = ?$

$$\vec{H} = \frac{j}{\omega\mu_0} \operatorname{rot} \vec{E} = \frac{j}{\omega\mu_0} \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \underline{\underline{\vec{1}_y \frac{\beta}{\omega\mu_0} E_0 \sin\alpha y \, e^{-j\beta z} - \vec{1}_z \frac{j\alpha}{\omega\mu_0} E_0 \cos\alpha y \, e^{-j\beta z}}}$$

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \underline{\underline{\vec{1}_y \frac{j\alpha}{2\omega\mu_0} |E_0|^2 \sin\alpha y \cos\alpha y + \vec{1}_z \frac{\beta}{2\omega\mu_0} |E_0|^2 \sin^2 \alpha y}}$$

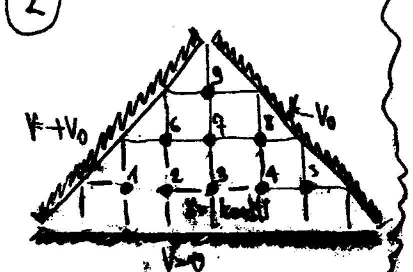
$\rho = \operatorname{div}(\varepsilon_0 \vec{E}) = \underline{0}$

$$\vec{J} = \operatorname{rot} \vec{H} - j\omega\varepsilon_0 \vec{E} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y & H_z \end{vmatrix} - \vec{1}_x j\omega\varepsilon_0 E_0 \sin\alpha y \, e^{-j\beta z} =$$

$$= \vec{1}_x \left[\frac{j\alpha^2}{\omega\mu_0} E_0 \sin\alpha y \, e^{-j\beta z} + \frac{j\beta^2}{\omega\mu_0} E_0 \sin\alpha y \, e^{-j\beta z} \right] - \vec{1}_x j\omega\varepsilon_0 E_0 \sin\alpha y \, e^{-j\beta z} =$$

$\underline{\underline{\vec{J} = 0}}$.

2



$V_{1-9} = ?$

Simetrija: $\underline{\underline{V_3 = V_7 = V_9 = 0}}$

$$\left.\begin{array}{l} 4V_1 = 2V_0 + V_2 \\ 4V_2 = V_1 + V_6 \\ 4V_6 = 2V_0 + V_2 \end{array}\right\} 16V_2 = 4V_1 + 2V_0 + V_2 \Big\} \begin{array}{l} 16V_2 = 2V_0 + V_2 + 2V_0 + V_2 \\ 14V_2 = 4V_0 \end{array}$$

$\underline{\underline{V_2 = \frac{2}{7} V_0}}$

$V_1 = \frac{1}{4}(2V_0 + V_2) = \underline{\underline{\frac{4}{7} V_0}}$

$V_6 = \frac{1}{4}(2V_0 + V_2) = \underline{\underline{\frac{4}{7} V_0}}$

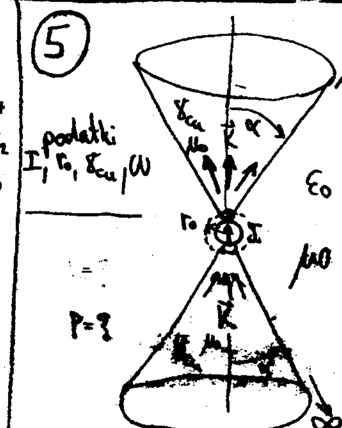
$V_5 = -V_1 = \underline{\underline{-\frac{4}{7} V_0}}$ $\quad V_8 = -V_6 = \underline{\underline{-\frac{4}{7} V_0}}$

$V_4 = -V_2 = \underline{\underline{-\frac{2}{7} V_0}}$

5

podatki $I, r_0, \gamma_{Cu}, \omega$

$P = ?$



$\delta = \sqrt{\frac{2}{\omega\mu_0\gamma}}$; $dR = \frac{dr}{\gamma\delta \, 2\pi r \sin\alpha}$

$R = 2\int_{r_0}^{\infty} dR$; $P = \frac{1}{2}|I|^2 R$

$$P = \frac{1}{2}|I|^2 2 \int_{r_0}^{\infty} \frac{dr}{\gamma\sqrt{\frac{2}{\omega\mu_0\gamma}} \, 2\pi r \sin\alpha}$$

$$P = \frac{|I|^2}{2\pi \sin\alpha} \sqrt{\frac{\omega\mu_0}{2\gamma}} \ln r \Big|_{r_0}^{\infty} = \underline{\underline{\infty}}$$

Pri resnični anteni je dolžina stožcev omejena in so izgube zelo majhne.

Rešitev ELEKTROMAGNETIKA 04.07.2001

① $\oint_A (\mathrm{grad}U \times \mathrm{grad}V)\cdot \vec{1}_r\, dA = ?$ (r,θ,ϕ)

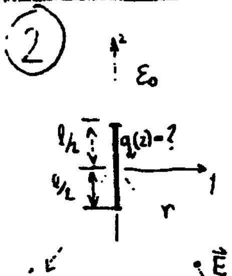
A = krogla s polmerom R v koordinatnem izhodišču

$U = r^3 \sin\theta\cos\phi$; $V = r^3\sin\theta\sin\phi$

$\oint_A (\mathrm{grad}U \times \mathrm{grad}V)\cdot \vec{1}_r\, dA = \int_v \mathrm{div}(\mathrm{grad}U \times \mathrm{grad}V)\, dv =$

$= \int_v (\mathrm{grad}V \cdot \mathrm{rot}(\mathrm{grad}U) - \mathrm{grad}U \cdot \mathrm{rot}(\mathrm{grad}V))\, dv = \underline{\underline{0}}$

②



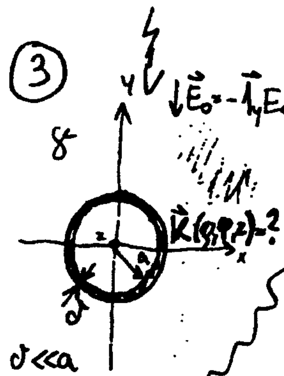
$\vec{E}(r \gg l) = \vec{1}_r \frac{C}{r^2}$

$\vec{E}(r \gg l) \approx \vec{1}_r \frac{Q}{4\pi\varepsilon_0 r^2}$

$Q = 4\pi\varepsilon_0 r^2 \cdot \frac{C}{r^2}$

$q(z) = \frac{Q}{l} = \underline{\underline{\frac{4\pi\varepsilon_0 C}{l}}}$

③



$\gamma \ll a$

$V = (A\varrho + \frac{B}{\varrho})\sin\varphi$

$\vec{E} = -\mathrm{grad}V =$

$\vec{E} = -\vec{1}_\varrho (A - \frac{B}{\varrho^2})\sin\varphi - \vec{1}_\varphi (A + \frac{B}{\varrho^2})\cos\varphi$

$\varrho \gg a \rightarrow \vec{E} \approx \vec{E}_0 \rightarrow A = E_0$

$V = (E_0\varrho + \frac{B}{\varrho})\sin\varphi$

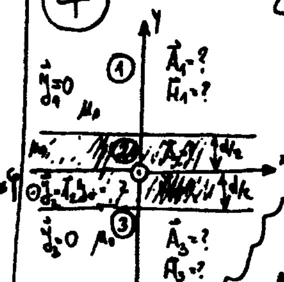
$V(\varrho = a) = 0 \rightarrow B = -E_0 a^2$

$\vec{J} = \gamma\vec{E}$; $\vec{J}(\varrho = a) = -\vec{1}_\varrho 2E_0 a \sin\varphi$

$\vec{K}(\varphi) = \vec{1}_\varphi \int_\varphi^{\pi/2} J_\varrho(\varphi)\, a\, d\varphi = \vec{1}_\varphi \int_\varphi^{\pi/2} (-2E_0)\sin\varphi\, a\, d\varphi$

$\underline{\underline{\vec{K}(\varphi) = -\vec{1}_\varphi 2E_0 a\cos\varphi}}$

④



$\vec{A} = \vec{1}_z A(y)$ iz geometrije

$\vec{H} = \frac{1}{\mu}\mathrm{rot}\vec{A} = \frac{1}{\mu}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A \end{vmatrix} = \vec{1}_x \frac{1}{\mu}\frac{\partial A}{\partial y}$

$\vec{J} = \mathrm{rot}\vec{H} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{1}{\mu}\frac{\partial A}{\partial y} & 0 & 0 \end{vmatrix} = -\vec{1}_z \frac{1}{\mu}\frac{\partial^2 A}{\partial y^2}$

$\frac{\partial^2 A}{\partial y^2} = -\mu_0 J$

$\frac{\partial A}{\partial y} = -\mu_0 J y + C_A$

$A = -\frac{\mu_0 J}{2} y^2 + C_A y + C_B$

② $\vec{A}_2 = \vec{1}_z \left(-\frac{\mu_0 J}{2} y^2 + C_{B2}\right)$; $\vec{H}_2 = -\vec{1}_x J y$; $C_{A2} = 0$ zaradi simetrije

① $y = \frac{d}{2} \rightarrow -\frac{\mu_0 J}{2}\left(\frac{d}{2}\right)^2 + C_{B2} = C_{A1}\frac{d}{2} + C_{B1}$; $-J\frac{d}{2} = \frac{C_{A1}}{\mu_0} \rightarrow C_{A1} = -\mu_0 J \frac{d}{2}$

$C_{B1} = \frac{\mu_0 J}{2}\left(\frac{d}{2}\right)^2 + C_{B2} \rightarrow \vec{A}_1 = \vec{1}_z\left(-\mu_0 J \frac{d}{2} y + \frac{\mu_0 J}{2}\left(\frac{d}{2}\right)^2 + C_{B2}\right)$; $\vec{H}_1 = -\vec{1}_x J \frac{d}{2}$

③ $y = -\frac{d}{2} \rightarrow C_{A3} = \mu_0 J \frac{d}{2}$; $C_{B3} = -\frac{\mu_0 J}{2}\left(\frac{d}{2}\right)^2 + C_{B2}$

$\vec{A}_3 = \vec{1}_z\left(\mu_0 J \frac{d}{2} y - \frac{\mu_0 J}{2}\left(\frac{d}{2}\right)^2 + C_{B2}\right)$; $\vec{H}_3 = +\vec{1}_x J \frac{d}{2}$

⑤ $\mu_0, \varepsilon_0, \gamma = 0$

$\vec{E} = [\vec{1}_x(1+j) + \vec{1}_y(1-j)]Ce^{-jkz}$

$\vec{H} = ?$; $\vec{S} = ?$; $\vec{J} = ?$; $\varrho = ?$

$\vec{H} = \frac{j}{\omega\mu_0}\mathrm{rot}\vec{E} = \frac{j}{\omega\mu_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & 0 \end{vmatrix} = \frac{k}{\omega\mu_0}[\vec{1}_x(-(1-j))C + \vec{1}_y(1+j)C]e^{-jkz}$

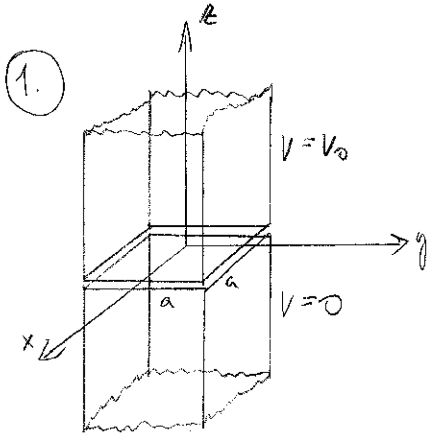
$\underline{\underline{\vec{H} = [\vec{1}_x(j-1) + \vec{1}_y(j+1)]\frac{C}{Z_0}e^{-jkz}}}$

$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}[\vec{1}_x(1+j) + \vec{1}_y(1-j)]Ce^{-jkz} \times [\vec{1}_x(-j-1) + \vec{1}_y(-j+1)]\frac{C^*}{Z_0}e^{jkz} = \underline{\underline{\vec{1}_z\frac{2CC^*}{Z_0}}}$

$\vec{J} = \mathrm{rot}\vec{H} - j\omega\varepsilon_0\vec{E} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & 0 \end{vmatrix} - j\omega\varepsilon_0\vec{E} = -jk[\vec{1}_x(-(j+1)) + \vec{1}_y(j-1)]\frac{C}{Z_0}e^{-jkz} - j\omega\varepsilon_0[\vec{1}_x(1+j) + \vec{1}_y(1-j)]Ce^{-jkz} = \underline{\underline{0}}$

$\varrho = \mathrm{div}(\varepsilon_0\vec{E}) = \underline{\underline{0}}$

1.



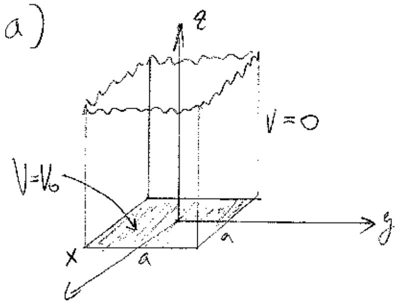
Postopek:

a) izračunamo porazdelitev potenciala v polneskončni cevi kvadratnega preseka s potencialom V_0 na osnovni ploskvi in potencialom 0 na stenah

b) izraz apliciramo na naš primer

c) rezultat

a)



osnovna enačba

$$V(x,y,z) = X(x)Y(y)Z(z)$$

laplasova enačba

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\underbrace{\frac{1}{X}\frac{\partial^2 X}{\partial x^2}}_{-k^2} + \underbrace{\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}}_{-l^2} + \underbrace{\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}}_{k^2+l^2} = 0$$

mejni pogoji

1. $V=0$, ko $x=0$
2. $V=0$, ko $x=a$
3. $V=0$, ko $y=0$
4. $V=0$, ko $y=a$
5. $V=V_0(x,y)$, ko $z=0$
6. $V \to 0$, ko $z \to \infty$

nastavki

$-k^2 \to \quad X(x) = A\sin kx + \cancel{B\cos kx}^{=0}$ (1)

$-l^2 \to \quad Y(y) = C\sin ly + \cancel{D\cos ly}^{=0}$ (3)

$k^2+l^2 \to \quad Z(z) = \cancel{E\, e^{\sqrt{k^2+l^2}\,z}}^{=0} + F e^{-\sqrt{k^2+l^2}\,z}$ (6)

točki (2) in (4) povedita:

$\sin ka = 0 \;\to\; k = \frac{n\pi}{a} \quad n = 1,2,3\ldots$

$\sin la = 0 \;\to\; l = \frac{m\pi}{a} \quad m = 1,2,3\ldots$

nadaljevanje (1.) naloge

rešitev

$$V(x,y,z) = ACF\, e^{-\pi\sqrt{(\frac{n}{a})^2+(\frac{m}{a})^2}\cdot z} \cdot \sin\left(\frac{n\pi}{a}x\right)\cdot\sin\left(\frac{m\pi}{a}y\right)$$

$$V(x,y,z) = \sum_{m,n=1}^{\infty} C_{m,n}\, e^{-\pi\sqrt{(\frac{n}{a})^2+(\frac{m}{a})^2}\cdot z}\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{a}y\right)$$

iščemo $C_{m,n}$ (ortogonalnost, mejni pogoji)

$$V(x,y,0) = \sum_{m,n=1}^{\infty} C_{m,n}\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{a}y\right) = V_0(x,y) \quad \text{v točki } z=0$$

$$\sum_{m,n=1}^{\infty} C_{m,n}\int_0^a \sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{n'\pi}{a}x\right)dx\int_0^a \sin\left(\frac{m\pi}{a}y\right)\sin\left(\frac{m'\pi}{a}y\right)dy =$$

$$= \int_0^a\!\!\int_0^a V_0(x,y)\sin\left(\frac{n'\pi}{a}x\right)\sin\left(\frac{m'\pi}{a}y\right)dx\,dy$$

$$C_{m,n} = \frac{4}{a^2}\int_0^a\!\!\int_0^a V_0(x,y)\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{a}y\right)dx\,dy$$

$$C_{m,n} = \begin{cases} \frac{4V_0}{a^2}\,\frac{2a}{n\pi}\,\frac{2a}{m\pi} = \frac{16V_0}{mn\pi^2} & \text{; ko } m,n = \text{liha števila} \\ 0 & \text{; ko } m,n = \text{soda števila} \end{cases}$$

končna porazdelitev

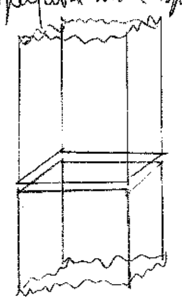
$$V(x,y,z) = \frac{16V_0}{\pi^2}\sum_{m,n=1,3,5\ldots}^{\infty}\frac{1}{mn}\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{a}y\right)e^{-\pi\sqrt{(\frac{m}{a})^2+(\frac{n}{a})^2}\cdot z}$$

b) vpeljava nove spremenljivke

$U = \frac{V_0}{2}$

$U = 0$

$U = -\frac{V_0}{2}$



$V = \frac{V_0}{2} + \frac{V_0}{2} = V_0$

$V = \frac{V_0}{2}$

$V = 0$

$$U = V - \frac{V_0}{2}$$

$$V = U + \frac{V_0}{2}$$

$$V = \begin{cases} V_0 - V(x,y,z) & ; z>0 \\ +\frac{V_0}{2} & , z=0 \\ 0 + V(x,y,-z) & ; z<0 \end{cases}$$

nadaljevanje 1. naloge

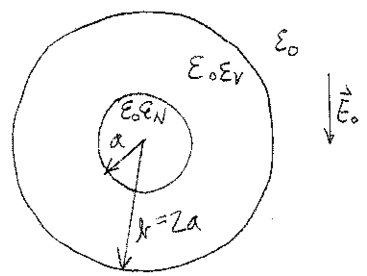
c) rezultat

$$V = \begin{cases} V_0 - \dfrac{16V_0}{\pi^2}\displaystyle\sum_{m,n=1,3,5\ldots}^{\infty} \frac{1}{mn}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right) e^{-\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{a}\right)^2}\cdot z} & ; z>0 \\ +\dfrac{V_0}{2} & ; z=0 \\ \dfrac{16V_0}{\pi^2}\displaystyle\sum_{m,n=1,3,5\ldots}^{\infty} \frac{1}{mn}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right) e^{+\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{a}\right)^2}\cdot z} & ; z<0 \end{cases}$$

2. podano: $\varepsilon_N = 4$, $b = 2a$, $|E_N| = \frac{1}{2}|E_0|$

dielektrična krogla z dielektrično oblogo v el. polju

$\varepsilon_V = ?$



$$\Delta V = 0 \text{ pri } \frac{\partial}{\partial\phi} = 0 \; ; \; \vec{E} = -\operatorname{grad} V = -\left(\vec{1}_r\frac{\partial V}{\partial r} + \vec{1}_\theta\frac{1}{r}\frac{\partial V}{\partial\theta}\right)$$

znotraj krogle: $V_N = A_N r\cos\theta$

$\vec{E}_N = -A_N(\vec{1}_r\cos\theta - \vec{1}_\theta\sin\theta)$

v oblogi: $V_V = (A_V r + B_V r^{-2})\cos\theta$

$\vec{E}_V = -\left(\vec{1}_r\left(A_V - \frac{2B_V}{r^3}\right)\cos\theta - \vec{1}_\theta\left(A_V + \frac{B_V}{r^3}\right)\sin\theta\right)$

zunaj: $V_Z = (A_Z r + B_Z r^{-2})\cos\theta$

$\vec{E}_Z = -\left(\vec{1}_r\left(A_Z - \frac{2B_Z}{r^3}\right)\cos\theta - \vec{1}_\theta\left(A_Z + \frac{B_Z}{r^3}\right)\sin\theta\right)$

	pogoji za tangencialni $\vec{E}$	pogoji za normalni $\vec{D}$
meja krogla-obloga	(1) $A_N = A_V + \frac{B_V}{a^3}$	(3) $\varepsilon_N A_N = \varepsilon_V\left(A_V - \frac{2B_V}{a^3}\right)$
meja obloga-zunaj	(2) $A_V + \frac{B_V}{b^3} = A_Z + \frac{B_Z}{b^3}$	(4) $\varepsilon_V\left(A_V - \frac{2B_V}{b^3}\right) = A_Z - \frac{2B_Z}{b^3}$

2×(2)+(4): $A_V(2+\varepsilon_V) + \frac{2B_V}{b^3}(1-\varepsilon_V) = 3A_Z$ (5) izločimo B_Z

2×ε_V×(1)+(3): $A_V = \frac{2\varepsilon_V+\varepsilon_N}{3\varepsilon_V}\cdot A_N$ (6) izrazimo A_V z A_N

ε_V×(1)−(3): $B_V = \frac{\varepsilon_V-\varepsilon_N}{3\varepsilon_V}a^3 A_N$ (7) izrazimo B_V z A_N

(6) in (7) vstavimo v (5) in dobimo:

$$A_N\left(\frac{(2\varepsilon_V+\varepsilon_N)(2+\varepsilon_V)}{3\varepsilon_V} - \frac{2a^3}{b^3}\frac{(\varepsilon_V-\varepsilon_N)^2}{3\varepsilon_V}\right) = 3A_Z$$

modeliranje (2.) naloge

podano: $\frac{|E_0|}{|E_N|} = 2$

$\varepsilon_N = 4$

$b = 2a$

$$\frac{A_z}{A_N} = \frac{E_0}{E_N} = \frac{(2\varepsilon_v + \varepsilon_N)(2+\varepsilon_v) - \frac{2a^3}{b^3}(\varepsilon_v - \varepsilon_N)^2}{9\varepsilon_v} = 2$$

$$\frac{2\varepsilon_v^2 + 8\varepsilon_v + 8 - \frac{\varepsilon_v^2}{4} + 2\varepsilon_v - 4}{9\varepsilon_v} = 2$$

$$\frac{7}{4}\varepsilon_v^2 - 8\varepsilon_v + 4 = 0$$

$$\varepsilon_{v_{1,2}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{36}}{2\,\frac{7}{4}}$$

$\varepsilon_1 = \frac{4}{7}$ //

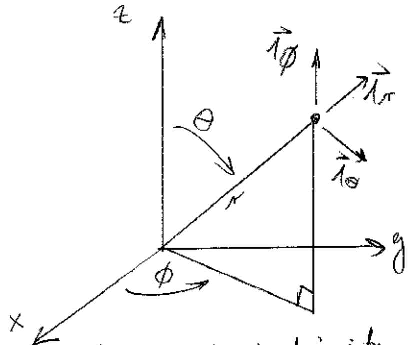
$\varepsilon_2 = 4$ ✓ $\qquad \underline{\underline{\varepsilon_v = 4}}$

(3.)

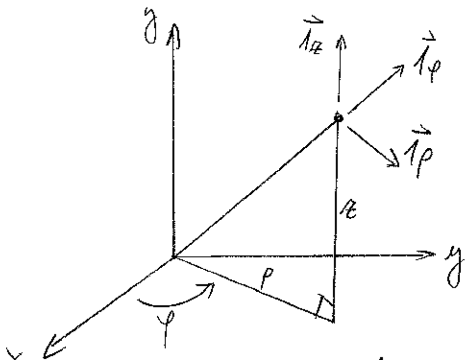
$$\vec{1}_\rho = \vec{1}_r \cdot \sin\Theta + \vec{1}_\Theta \cos\Theta$$

$$\vec{1}_\varphi = \vec{1}_\phi$$

$$\vec{1}_z = \vec{1}_r \cos\Theta - \vec{1}_\Theta \sin\Theta$$

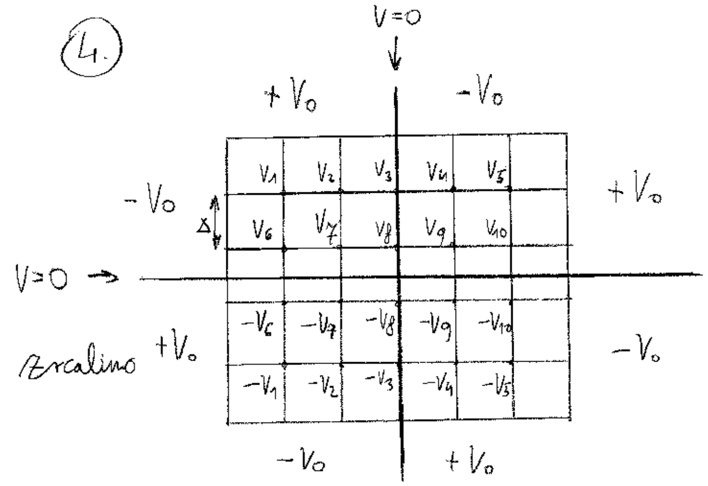


kroglni koordinatni sistem



valjni koordinatni sistem

(4.)



$V_3 = V_8 = 0$

$V_4 = -V_2$

$V_9 = -V_7$

$V_5 = -V_1$

$V_{10} = -V_6$

osnovne enačbe

(1) $4V_1 = V_2 + V_6 + V_0 - V_0 = V_2 + V_6$

(2) $4V_2 = V_0 + V_1 + V_7$

(3) $4V_6 = -V_0 + V_1 - V_6 + V_7$

$5V_6 = -V_0 + V_1 + V_7$

(4) $4V_7 = V_2 + V_6 - V_7$

$5V_7 = V_2 + V_6$

(1) in (4)

$5V_7 = V_2 + V_6$

$4V_1 = V_2 + V_6$

$5V_7 = 4V_1$

$V_1 = \frac{5}{4}V_7$

(2) in (3)

$5V_6 = -V_0 + V_1 + V_7$

$4V_2 = V_0 + V_1 + V_7$

$5V_6 - 4V_2 = -2V_0$

$V_2 = \frac{5V_6 + 2V_0}{4}$

(1) in (2), izničimo V_2

$4V_2 = V_0 + V_1 + V_7$

$-4 \cdot / \; 4V_1 = V_2 + V_6 \Rightarrow 4V_2 = 16V_1 - 4V_6$

$0 = 15V_1 - 4V_6 - V_0 - V_7$

$\xrightarrow{\frac{5}{4}V_7}$ $0 = \frac{71}{4}V_7 - 4V_6 - V_0$

$4V_6 = \frac{71}{4}V_7 - V_0$

$V_6 = \frac{71}{16}V_7 - \frac{V_0}{4}$

(3) in (4), izničimo V_6

$5V_6 = -V_0 + V_1 + V_7$

$-5 \cdot / \; 5V_7 = V_2 + V_6 \Rightarrow 5V_6 = 25V_7 - 5V_2$

$0 = 24V_7 - 5V_2 + V_0 - V_1 \leftarrow \frac{5}{4}V_7$

$0 = \frac{91}{4}V_7 - 5V_2 + V_0$

$5V_2 = \frac{91}{4}V_7 + V_0$

$V_2 = \frac{91}{20}V_7 + \frac{V_0}{5}$

↳

nadaljevanje (4) naloge

enačimo V_2

$$\frac{\frac{91}{4}V_7 + V_0}{5} = \frac{5V_6 + 2V_0}{4}$$

$$V_6 = \frac{\left(\frac{91}{4}V_7 + V_0\right)\cdot 4 - 10V_0}{25}$$

$$V_6 = \frac{91V_7 + 4V_0}{25} - \frac{2V_0}{5}$$

enačimo V_6

$$\frac{71}{16}V_7 - \frac{V_0}{4} = \frac{91V_7 + 4V_0}{25} - \frac{2V_0}{5}$$

$$\frac{71}{16}V_7 - \frac{V_0}{4} = \frac{91}{25}V_7 + \frac{4}{25}V_0 - \frac{10}{25}V_0$$

$$\left(\frac{71}{16} - \frac{91}{25}\right)V_7 = \left(\frac{1}{4} - \frac{6}{25}\right)V_0$$

$$\frac{319}{400}V_7 = \frac{1}{100}V_0$$

$$V_7 = \underline{\underline{\frac{4}{319}V_0}}$$

vstavljanje

$$V_1 = \frac{5}{4}V_7 = \underline{\underline{\frac{5}{319}V_0}}$$

$$V_2 = \frac{91}{20}V_7 + \frac{V_0}{5} = \frac{91}{20}\frac{4V_0}{319} + \frac{V_0}{5}$$

$$V_2 = \left(\frac{364}{6380} + \frac{1}{5}\right)V_0 = \frac{410}{1595}V_0 = \underline{\underline{\frac{82}{319}V_0}}$$

$$V_6 = \frac{71}{16}V_7 - \frac{V_0}{4} = \frac{71}{1276}V_0 - \frac{V_0}{4}$$

$$V_6 = \frac{71 - 319}{1276}V_0 = \underline{\underline{-\frac{62}{319}V_0}}$$

končni rezultati

$$V_1 = \frac{5}{319}V_0$$

$$V_2 = \frac{82}{319}V_0$$

$$V_3 = 0$$

$$V_4 = -V_2 = -\frac{82}{319}V_0$$

$$V_5 = -V_1 = -\frac{5}{319}V_0$$

$$V_6 = -\frac{62}{319}V_0$$

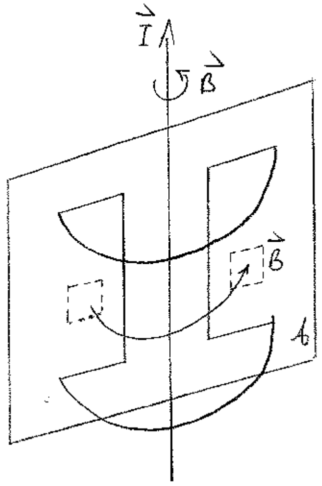
$$V_7 = \frac{4}{319}V_0$$

$$V_8 = 0$$

$$V_9 = -V_7 = -\frac{4}{319}V_0$$

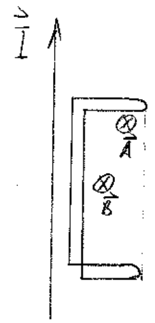
$$V_{10} = -V_6 = \frac{62}{319}V_0$$

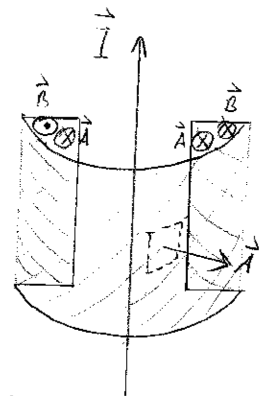
(5)



$$\oint_L \vec{B}\cdot d\vec{l} = 0$$

polje skozi isto ploskev vstopi in izstopi, zato je enako 0.





$$\Phi = \int_A \vec{B}\cdot d\vec{A} + \int_A B\cdot(-d\vec{A}) = 0$$

sledi : $\underline{\underline{M = \frac{\Phi}{I} = 0}}$

(1.) iščemo:
- energijo resonatorja
- kvaliteto resonatorja

$$\vec{E} = \vec{1}_z E_0 J_0(\alpha_1 \rho)\ ;\ \alpha_1 = \frac{2{,}405\ldots}{a}$$

$$Q = \omega \frac{W}{P}$$

$$W = \frac{1}{2}\varepsilon \int_V |E_{max}|^2 dV$$

energija resonatorja

$$W = \frac{1}{2}\varepsilon \int_V |E_{max}|^2 dV = \frac{1}{2}\varepsilon E_0^2 \int_0^a\int_0^{2\pi}\int_0^b J_0^2(\alpha_1\rho)\rho\, d\rho\, d\varphi\, dz$$

nova spremenljivka

$$\alpha_1 \rho = k$$

$$\alpha_1 d\rho = dk$$

$$d\rho = dk/\alpha_1$$

$$W = \frac{\varepsilon E_0^2}{2}\int_0^{\alpha_1 a}\int_0^{2\pi}\int_0^b J_0^2(k)\ \frac{k\cdot dk}{\alpha_1^2}\cdot d\varphi \cdot dz$$

$$W = \frac{\varepsilon E_0^2 \cdot 2\pi b}{2}\ \frac{1}{\alpha_1^2\cdot 2}\ (\alpha_1 a J_0'(\alpha_1 a))^2$$

$$W = \frac{\varepsilon \pi b E_0^2}{2\alpha_1^2}\cdot J_1^2(\alpha_1 a)\cdot \alpha_1^2\cdot a^2$$

$$\underline{W = \frac{1}{2}\pi \varepsilon E_0^2 a^2 b\, J_1^2(\alpha_1 a)}$$

V nalogi je podano:

$$J_n'(x) = -\frac{n}{x}J_n(x) + J_{n-1}(x)$$

$$J_0'(\alpha a) = -\frac{0}{\alpha a}J_0(\alpha a) + J_{-1}(\alpha a)$$

$$J_{-n}(x) = (-1)^n J_n(x)$$

$$J_{-1}(\alpha a) = (-1)^1 J_1(\alpha a)$$

posledično:

$$J_0'(\alpha_1 a) = -J_1(\alpha_1 a)$$

$$J_0'(x) = -J_1(x)$$

moč resonatorja

$$P = \frac{1}{2}\int_V |\vec{E}|^2 \gamma\, dV$$

$$P = \frac{\gamma}{2}\int_0^a\int_0^{2\pi}\int_0^b E_0^2 J_0^2(\alpha_1\rho)\rho\, d\rho\, d\varphi\, dz$$

$$P = \frac{\gamma E_0^2 2\pi b}{2}\int_0^{\alpha_1 a}\frac{k}{\alpha_1}J_0^2(k)\frac{dk}{\alpha_1}$$

$$P = \frac{\gamma E_0^2 \pi b}{2\alpha_1^2}(\alpha_1 a J_0'(\alpha_1 a))^2$$

$$P = \frac{\gamma E_0^2 \pi a^2 b}{2} J_0'^2(\alpha_1 a)$$

nadaljevanje (1.) naloge

magnetno polje

$$\vec{H} = \frac{j}{\omega\mu}\operatorname{rot}\vec{E} = \frac{j}{\omega\mu\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ 0 & 0 & E_0 J_0(\alpha_1\rho)\end{vmatrix}$$

$$\vec{H} = \frac{j}{\omega\mu\rho}(-\rho\cdot\vec{1}_\varphi)E_0 J_0'(\alpha_1\rho)\alpha_1$$

$$\vec{H} = -\vec{1}_\varphi \frac{j\alpha_1}{\omega\mu}E_0 J_0'(\alpha_1\rho)$$

resonančna frekvenca

$$\operatorname{rot}\vec{H} = j\omega\varepsilon\vec{E} = \frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \partial/\partial\rho & \partial/\partial\rho & \partial/\partial z \\ 0 & \frac{-j\alpha_1}{\rho\omega\mu}E_0 J_0'(\alpha_1\rho) & 0\end{vmatrix} =$$

$$= \vec{1}_z \frac{-1}{\rho}\left(j\frac{\alpha_1}{\omega\mu}E_0\alpha_1 J_0''(\alpha_1\rho)\rho + \frac{j\alpha_1}{\omega\mu}E_0 J_0'(\alpha_1\rho)\right) =$$

$$= \vec{1}_z \frac{j\alpha_1 E_0}{\omega\mu\rho}(\alpha_1\rho J_0''(\alpha_1\rho) + J_0'(\alpha_1\rho)) = \vec{1}_z \frac{j\alpha_1 E_0}{\omega\mu\rho}\alpha_1\rho J_0(\alpha_1\rho)$$

enačimo:

$$\operatorname{rot}\vec{H} = j\omega\varepsilon\vec{E}$$

$$\vec{1}_z \frac{j\alpha_1 E_0}{\omega\mu}\alpha_1 J_0(\alpha_1\rho) = j\omega\varepsilon \underbrace{\vec{1}_z E_0 J_0(\alpha_1\rho)}_{=\vec{E}\ \text{podani}}$$

$$\omega\varepsilon = \frac{\alpha_1^2}{\omega\mu}$$

$$\omega^2 = \frac{\alpha_1^2}{\mu\varepsilon}$$

$$\omega = \frac{\alpha_1}{\sqrt{\mu\varepsilon}}$$

kvaliteta resonatorja

$$Q = \omega\frac{W}{P}$$

$$Q = \frac{\alpha_1 \pi\varepsilon E_0^2 a^2 b J_0'^2(\alpha_1 a)\cdot 2}{\gamma\sqrt{\mu\varepsilon}\, E_0^2 \pi a^2 b\, J_0'^2(\alpha_1 a)\cdot 2} = \frac{\alpha_1\varepsilon}{\gamma\sqrt{\mu\varepsilon}}\ ;\ Z = \sqrt{\frac{\mu}{\varepsilon}}\ ;\ \underline{\underline{Q = \frac{\alpha_1}{Z\gamma}}}$$

(2) votlinski resonator – stojni val

$$f_{lmn} = \frac{c_0}{2}\sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$$ resonančna frekvenca

$a > b > c$

ΔX vpliva na a

$a = 5$ cm

$b = 3{,}5$ cm

$c = 3$ cm

$$f_1 = \frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 5{,}229 \text{ GHz}$$

$$f_2 = \frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2} = 5{,}828 \text{ GHz}$$

$$f_3 = \frac{c_0}{2}\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{b}\right)^2} = 6{,}058 \text{ GHz}$$

$$f_4 = \frac{c_0}{2}\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2} = 6{,}583 \text{ GHz}$$

$$f_5 = \frac{c_0}{2}\sqrt{\left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2} = 7{,}070 \text{ GHz}$$

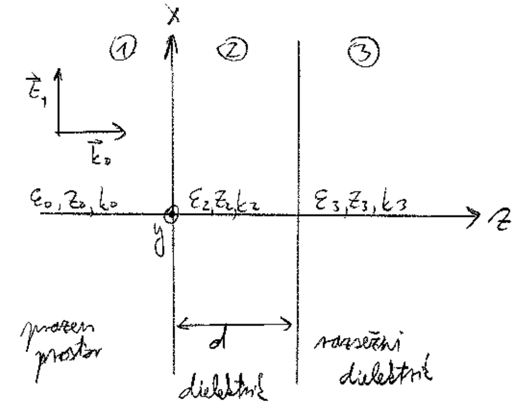
spreminjamo dimenzije a, maksimum je $a \leftrightarrow c$.

$$f_5 - f_2 = 1{,}242 \text{ GHz}$$

$$\underline{\underline{f_4 - f_1 = 1{,}354 \text{ GHz} = \Delta f_{max}}}$$

(3.) iščemo odbojnost pri $z=0$

$$\Gamma = \frac{\text{odbiti val}}{\text{vpadni val}}$$



prazen prostor

$$\vec{E}_1 = \vec{1}_x\left(E_{1+}e^{-jk_0z} + E_{1-}e^{jk_0z}\right)$$

$$\vec{H}_1 = \vec{1}_y\left(\frac{E_{1+}}{Z_0}e^{-jk_0z} - \frac{E_{1-}}{Z_0}e^{jk_0z}\right)$$

dielektrik

$$\vec{E}_2 = \vec{1}_x\left(E_{2+}e^{-jkz} + E_{2-}e^{jkz}\right)$$

$$\vec{H}_2 = \vec{1}_y\left(\frac{E_{2+}}{Z}e^{-jkz} - \frac{E_{2-}}{Z}e^{jkz}\right)$$

meja 1 / 2

$z = 0, \vec{E}_1 = \vec{E}_2, \vec{H}_1 = \vec{H}_2$

$$E_{1+} + E_{1-} = E_{2+} + E_{2-}$$

$$\frac{E_{1+}}{Z_0} - \frac{E_{1-}}{Z_0} = \frac{E_{2+}}{Z_2} - \frac{E_{2-}}{Z_2}$$

$$\Gamma = \frac{E_{1-}}{E_{1+}}$$

meja 2 / 3

$z = d, \vec{E}_2 = \vec{E}_3, \vec{H}_2 = \vec{H}_3$

v ③ ni odbitega vala, temveč le napredujoči

$$E_{2+}e^{-jk_2d} + E_{2-}e^{jk_2d} = E_3e^{-jk_3d}$$

$$\frac{E_{2+}}{Z_2}e^{-jk_2d} - \frac{E_{2-}}{Z_2}e^{jk_2d} = \frac{E_3}{Z_3}e^{-jk_3d}$$

izločimo E_3:

$$\frac{E_{2-}}{E_{2+}} = \frac{Z_3 - Z_2}{Z_3 + Z_2}e^{-2jk_2d}$$

$\hookrightarrow$

nadaljevanje (3) naloge

$$E_{1+}(1+\Gamma) = E_{2+}\left(1+\frac{E_{2-}}{E_{2+}}\right)$$

$$\frac{E_{1+}}{Z_0}(1-\Gamma) = \frac{E_{2+}}{Z_2}\left(1-\frac{E_{2-}}{E_{2+}}\right)$$

$$Z_0\frac{1+\Gamma}{1-\Gamma} = Z_2\frac{1+\frac{E_{2-}}{E_{2+}}}{1-\frac{E_{2-}}{E_{2+}}}$$

odbojnost

$$\Gamma = \frac{\frac{Z_2}{Z_0}\frac{1+\frac{E_{2-}}{E_{2+}}}{1-\frac{E_{2-}}{E_{2+}}}-1}{\frac{Z_2}{Z_0}\frac{1+\frac{E_{2-}}{E_{2+}}}{1-\frac{E_{2-}}{E_{2+}}}+1}$$

vstavimo $\frac{E_{2-}}{E_{2+}} = \frac{Z_3-Z_2}{Z_3+Z_2}e^{-2jk_2d}$

$$\Gamma = \frac{\frac{Z_2}{Z_0}\frac{1+\frac{Z_3-Z_2}{Z_3+Z_2}e^{-2jk_2d}}{1-\frac{Z_3-Z_2}{Z_3+Z_2}e^{-2jk_2d}}-1}{\frac{Z_2}{Z_0}\frac{1+\frac{Z_3-Z_2}{Z_3+Z_2}e^{-2jk_2d}}{1-\frac{Z_3-Z_2}{Z_3+Z_2}e^{-2jk_2d}}+1}$$

(4) $\vec{E} = \vec{1}_x E_0 \sin\left(\frac{\pi}{b}y\right)e^{-j\beta z}$

$$P = \oint_A \vec{S}\cdot d\vec{A}\ ,\ \vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*\ ,\ P = \frac{1}{2}\int_A |K|^2 R_p\, dA$$

magnetno polje

$$H = \frac{j}{\omega\mu}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ E_0\sin(\frac{\pi}{b}y)e^{-j\beta z} & 0 & 0\end{vmatrix} = \frac{j}{\omega\mu}\Big(-\vec{1}_y E_0 \sin(\tfrac{\pi}{b}y)j\beta e^{-j\beta z} - $$
$$-\vec{1}_z E_0\frac{\pi}{b}\cos(\tfrac{\pi}{b}y)e^{-j\beta z}\Big)$$

pretok moči

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{1}_z\frac{E_0^2\beta}{2\omega\mu}\sin^2(\tfrac{\pi}{b}y) - \vec{1}_y\, j\frac{E_0^2\pi}{2b\omega\mu}\sin(\tfrac{\pi}{b}y)\cos(\tfrac{\pi}{b}y)$$

pretočna moč

$$P_p = \oint_A \vec{S}\cdot d\vec{A} = \int_0^a\!\!\int_0^b \vec{1}_z\cdot\vec{S}\,dx\,dy = \int_0^a\!\!\int_0^b \frac{E_0^2\beta}{2\omega\mu}\sin^2(\tfrac{\pi}{b}y)\,dx\,dy = \frac{E_0^2\beta\cdot a\, b}{4\,\omega\,\mu}$$

ploskovni tokovi

$$\vec{K} = \vec{1}_n\times\vec{H}$$

$$x=0:\ \vec{K}_1 = \vec{1}_z\frac{E_0\beta}{\omega\mu}\sin(\tfrac{\pi}{b}y)e^{-j\beta z} + \vec{1}_y\, j\frac{E_0\pi}{\omega\mu b}\cos(\tfrac{\pi}{b}y)e^{-j\beta z}$$

$$x=a:\ \vec{K}_2 = -\vec{1}_z\frac{E_0\beta}{\omega\mu}\sin(\tfrac{\pi}{b}y)e^{-j\beta z} - \vec{1}_y\, j\frac{E_0\pi}{\omega\mu b}\cos(\tfrac{\pi}{b}y)e^{-j\beta z}$$

$$y=0:\ \vec{K}_3 = \vec{1}_x\, j\frac{E_0\pi}{\omega\mu b}e^{-j\beta z}$$

$$y=b:\ \vec{K}_4 = -\vec{1}_x\, j\frac{E_0\pi}{\omega\mu b}e^{-j\beta z}$$

↳

nadaljevanje ④ naloge

izgubna moč

$$P_i = \frac{1}{2}\int_A |K|^2 R_p \, dA \quad ; \quad R_p = \sqrt{\frac{\omega\mu}{2\gamma}}$$

$$P_i = P(x=0) + P(x=a) + P(y=0) + P(y=b)$$

$$P(x=0) = P(x=a) = \frac{1}{2}\int_0^b\int_0^l \frac{E_0}{\omega^2\mu^2}\left(\beta^2 \sin^2\left(\frac{\pi}{b}y\right) + \left(\frac{\pi}{b}\right)^2 \cos^2\left(\frac{\pi}{b}y\right)\right) R_p \, dy \, dz =$$

$$= \frac{E_0^2}{2\omega^2\mu^2}\left(\beta^2 \frac{b}{2} l + \left(\frac{\pi}{b}\right)^2 \frac{b}{2} l\right) \cdot R_p = \frac{E_0^2 R_p \cdot b \cdot l}{4\omega^2\mu^2}\left(\beta^2 + \left(\frac{\pi}{b}\right)^2\right)$$

$$P(y=0) = P(y=b) = \frac{1}{2}\int_0^a\int_0^l \frac{E_0^2 \pi^2}{\omega^2\mu^2 b^2} R_p \, dx \, dz = \frac{E_0^2 R_p \cdot \pi^2 a l}{2\omega^2\mu^2 b^2}$$

$$P_i = 2P(x=0) + 2P(y=0)$$

$$P_i = \frac{E^2 R_p l}{2\omega^2\mu^2}\left(\frac{2a\pi^2}{b^2} + b\beta^2 + \frac{b\pi^2}{b^2}\right) = \frac{E_0^2 R_p l}{2\omega^2\mu^2}\left(b\beta^2 + \frac{\pi^2}{b^2}(b+2a)\right)$$

izračun slabljenja

$$\frac{dP_i}{dl} = -\frac{E_0^2 R_p}{2\omega^2\mu^2}\left(b\beta^2 + \frac{\pi^2}{b^2}(b+2a)\right)$$

$$\frac{dP_i}{P_p} = -\frac{E_0^2 R_p \, 4\omega\mu}{2\omega^2\mu^2 E_0^2 \beta a b}\left(b\beta^2 + \frac{\pi^2}{b^2}(b+2a)\right) \cdot dl$$

$$\ln\frac{P(0)}{P(l)} = \frac{2R_p}{\omega\mu a b \beta}\left(b\beta^2 + \frac{\pi^2}{b^2}(b+2a)\right) \cdot l$$

$$A = 20\log\frac{P(0)}{P(l)} = 20\log e \, \underbrace{\sqrt{\frac{\omega\mu}{2\gamma}}}_{R_p} \frac{2}{\omega\mu a b\beta}\left(b\beta^2 + \frac{\pi^2}{b^2}(b+2a)\right) \cdot l$$

slabljenje na enoto dolžine

$$A/l = 20\log e \sqrt{\frac{2}{\gamma\omega\mu}} \, \frac{b\beta^2 + \frac{\pi}{b^2}(b+2a)}{a b \beta} \quad [dB/m]$$

⑤

$$\vec{K}(r=r_0) = \vec{1}_\phi K_0 \cos\Theta$$

$$\operatorname{div}\vec{J} = -j\omega\rho \Rightarrow \operatorname{div}\vec{K} = -j\omega\sigma$$

$$\sigma = -\frac{1}{j\omega}\operatorname{div}\vec{K} = \frac{j}{\omega}\operatorname{div}\vec{K}$$

$$\operatorname{div}\vec{K} = \frac{1}{r^2\sin\Theta}\left(\frac{\partial(r^2\sin\Theta K_r)}{\partial r} + \frac{\partial(r\sin\Theta K_\Theta)}{\partial\Theta} + \frac{\partial(r K_\phi)}{\partial\phi}\right)$$

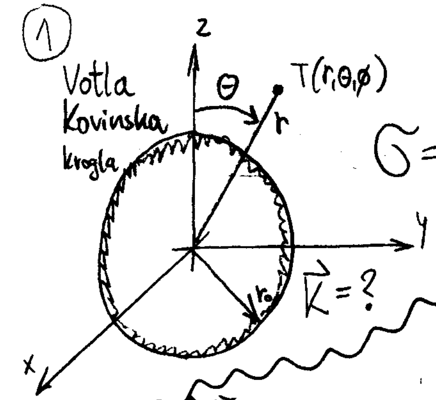
$$\operatorname{div}\vec{K} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 K_r) + \frac{1}{r\sin\Theta}\frac{\partial}{\partial\Theta}(\sin\Theta K_\Theta) + \underbrace{\frac{1}{r\cdot\sin\Theta}\frac{\partial K_\phi}{\partial\phi}}_{\text{ker imamo smer } \vec{1}_\phi}$$

$$\operatorname{div}\vec{K} = \frac{1}{r\cdot\sin\Theta}\cdot\frac{\partial}{\partial\phi}\cdot K_0\cos\Theta = 0$$

$$\sigma = \frac{j}{\omega}\operatorname{div}\vec{K} = 0$$

Rešitev 1. kolokvija iz ELEKTROMAGNETIKE 04.12.2002

1



$\vec{K} = ?$

$\sigma = \sigma_0 \cos\Theta' e^{j\omega t}$

$I = \frac{dQ}{dt} = j\omega Q$

$I(\Theta) = j\omega \int_0^{\Theta}\int_0^{2\pi} \sigma\, r_0^2 \sin\Theta' d\Theta' d\phi' = j\omega 2\pi r_0^2 \int_0^{\Theta} \sigma_0 \cos\Theta' \sin\Theta' d\Theta'$

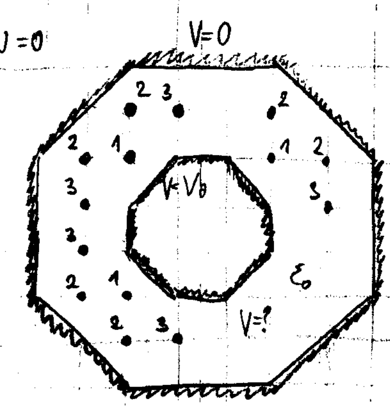
$I(\Theta) = j\omega 2\pi r_0^2 \sigma_0 \int_1^{\cos\Theta} -u\,du = j\omega\pi r_0^2 (1-\cos^2\Theta)$

$\underline{I(\Theta) = j\omega\pi r_0^2 \sin^2\Theta\, \sigma_0}$

$$\vec{K} = -\vec{1}_\Theta \frac{I}{2\pi r_0 \sin\Theta} = \underline{\underline{-\vec{1}_\Theta \frac{j\omega r_0 \sin\Theta}{2}\sigma_0}}$$

3

$\omega = 0$



$4V_1 = 2V_0 + 2V_2$

$4V_2 = V_1 + V_3$

$4V_3 = V_2 + V_3 + V_0$

$8V_1 = 4V_0 + V_1 + V_3$

$V_3 = 7V_1 - 4V_0$

$16V_3 = V_1 + 5V_3 + 4V_0$

$11V_3 = V_1 + 4V_0$

$77V_1 - 44V_0 = V_1 + 4V_0$

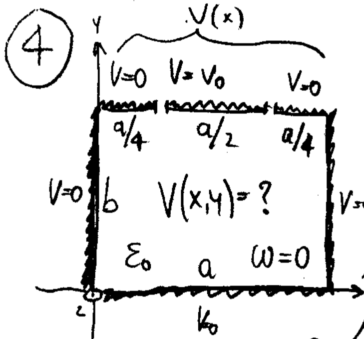
$76V_1 = 48V_0$

$$\underline{\underline{V_1 = \frac{12}{19}V_0}}$$

$$V_3 = 7\cdot\frac{12}{19}V_0 - 4V_0 = \underline{\underline{\frac{8}{19}V_0}}$$

$$V_2 = \frac{V_1+V_3}{4} = \underline{\underline{\frac{5}{19}V_0}}$$

4



$\Delta V = 0$

$V = \sum_n C_n \sin\left(n\frac{\pi}{a}x\right) \mathrm{sh}\left(n\frac{\pi}{a}y\right)$

$V(x) = \begin{cases} 0 ; & 0 < x < a/4 \\ V_0 ; & a/4 < x < 3a/4 \\ 0 ; & 3a/4 < x < a \end{cases}$

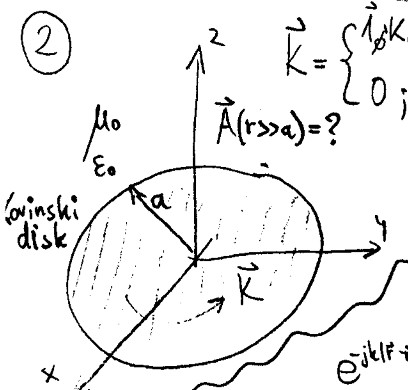
$\int_0^a \sum_n C_n \sin\left(n\frac{\pi}{a}x\right) \mathrm{sh}\left(n\frac{\pi}{a}b\right) \sin\left(m\frac{\pi}{a}x\right) dx = \int_{a/4}^{3a/4} V_0 \sin\left(m\frac{\pi}{a}x\right) dx$

$C_m \mathrm{sh}\left(m\frac{\pi}{a}b\right)\frac{a}{2} = \frac{V_0 a}{m\pi}\left(\cos\left(m\frac{\pi}{4}\right) - \cos\left(m\frac{3\pi}{4}\right)\right)$

$C_m = \frac{2V_0}{\pi}\left(\cos\left(m\frac{\pi}{4}\right) - \cos\left(m\frac{3\pi}{4}\right)\right)\frac{1}{\mathrm{sh}\, m\frac{\pi}{a}b}$

$$\underline{\underline{V = \frac{2V_0}{\pi}\sum_{n=1}^{\infty} \frac{\cos\left(n\frac{\pi}{4}\right) - \cos\left(n\frac{3\pi}{4}\right)}{n\, \mathrm{sh}\left(n\frac{\pi}{a}b\right)} \sin\left(n\frac{\pi}{a}x\right) \mathrm{sh}\left(n\frac{\pi}{a}y\right)}}$$

2



$\vec{A}(r \gg a) = ?$

$\vec{K} = \begin{cases} \vec{1}_\phi K_0 ; & r' \le a \\ 0 ; & r' > a \end{cases}$

$\vec{A} = \frac{\mu_0}{4\pi}\int_{v} \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} e^{-jk|\vec{r}-\vec{r}'|} dv' =$

$\vec{A} = \frac{\mu_0}{4\pi}\int_0^{a}\int_0^{2\pi} \frac{\vec{1}_\phi' K_0}{|\vec{r}-\vec{r}'|} e^{-jk|\vec{r}-\vec{r}'|} r' dr' d\phi'$

$|\vec{r}-\vec{r}'| = \sqrt{r^2 + r'^2 - 2rr'\sin\Theta\cos(\phi-\phi')}$

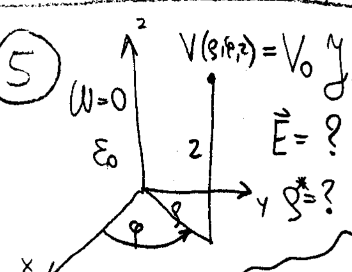
$\frac{1}{|\vec{r}-\vec{r}'|} \approx \frac{1}{r}\left(1 + \frac{r'}{r}\sin\Theta\cos(\phi-\phi')\right)$

$e^{-jk|\vec{r}-\vec{r}'|} \approx e^{-jkr}\left(1 + jkr'\sin\Theta\cos(\phi-\phi')\right)$

$\vec{A} = \frac{\mu_0 K_0}{4\pi}\cdot\frac{e^{-jkr}}{r}\left(\frac{1}{r} + jk\right)\sin\Theta \underbrace{\int_0^a r'^2 dr'}_{a^3/3} \underbrace{\int_0^{2\pi} \vec{1}_\phi' \cos(\phi-\phi') d\phi'}_{\vec{1}_\phi \cdot \pi}$

$$\underline{\underline{\vec{A} = \vec{1}_\phi \frac{\mu_0 K_0 a^3}{12}\frac{e^{-jkr}}{r}\left(\frac{1}{r} + jk\right)\sin\Theta}}$$

5



$V(\rho,\varphi,z) = V_0 J_1(l\rho)\cos\varphi\, \mathrm{sh}\, lz$

$\vec{E} = ?$

$\rho^* = ?$

$\vec{E} = -\mathrm{grad}\, V$

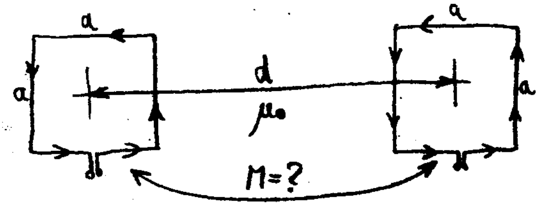
$$\underline{\vec{E} = -\vec{1}_\rho V_0 l J_1'(l\rho)\cos\varphi\, \mathrm{sh}\, lz + \vec{1}_\varphi \frac{V_0}{\rho} J_1(l\rho)\sin\varphi\, \mathrm{sh}\, lz}$$
$$\underline{-\vec{1}_z V_0 l J_1(l\rho)\cos\varphi\, \mathrm{ch}\, lz}$$

$\rho^* = \varepsilon_0 \mathrm{div}\vec{E} = \frac{\varepsilon_0}{\rho}\left[\frac{\partial}{\partial\rho}\left(-\rho V_0 l J_1'(l\rho)\cos\varphi\, \mathrm{sh}\, lz\right) + \frac{\partial}{\partial\varphi}\left(\frac{V_0}{\rho}J_1(l\rho)\sin\varphi\, \mathrm{sh}\, lz\right) - \frac{\partial}{\partial z}\left(V_0 l J_1(l\rho)\cos\varphi\, \mathrm{ch}\, lz\right)\right]$

$\rho^* = \varepsilon_0 V_0 \cos\varphi\, \mathrm{sh}\, lz\left[-\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(l\rho J_1'(l\rho)\right) + \left(\frac{1}{\rho^2} - l^2\right)J_1(l\rho)\right] = \underline{\underline{0}}$

$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial J_m(l\rho)}{\partial\rho}\right) + \left(l^2 - \frac{m^2}{\rho^2}\right)J_m(l\rho) = 0$

1) Kvadratni zanki v isti ravnini, $d \gg a$



$$M = \frac{\mu_0}{4\pi} \iint \frac{d\vec{s}_1 \cdot d\vec{s}_2}{|\vec{r}_2 - \vec{r}_1|} = \frac{\mu_0}{4\pi}\left(\frac{a^2}{d} - \frac{a^2}{d+a} - \frac{a^2}{d-a} + \frac{a^2}{d} + 2\frac{a^2}{d} - \frac{S_1 S_2}{\sqrt{d^2+a^2}}\right) =$$

$$M \approx \frac{\mu_0 a^4}{4\pi d^3}$$

3) $\vec{E} = (\vec{1}_\theta + j\vec{1}_\phi)\, C \frac{e^{-jkr}}{r} \sin\theta \qquad \mu = \mu_0 \quad \varepsilon = \varepsilon_0$

$r \gg \lambda$ približek za velike razdalje!

$\vec{H} = ? \quad \vec{S} = ? \quad \vec{k} = ?$

$r \gg \lambda$: $\frac{\partial}{\partial r} \approx -jk$, $\frac{\partial}{\partial \theta} \approx \frac{\partial}{\partial \phi} \approx 0$

$$\vec{H} = \frac{j}{\omega\mu} \operatorname{rot} \vec{E} = \frac{j}{\omega\mu} \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta \vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & rE_\theta & r\sin\theta E_\phi \end{vmatrix}$$

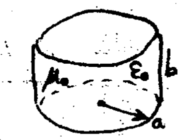
$$\vec{k} = \vec{1}_r \frac{\omega}{c_0}$$

$$\vec{H} = \frac{k}{\omega\mu}\left(-\vec{1}_\theta E_\phi + \vec{1}_\phi E_\theta\right) = \left(-j\vec{1}_\theta + \vec{1}_\phi\right) \frac{C}{Z_0} \frac{e^{-jkr}}{r} \sin\theta$$

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \vec{1}_r \frac{|C|^2}{Z_0} \frac{\sin^2\theta}{r^2}$$

4) Votlinski rezonator TM_{110} $\quad J_1(3.83) = 0$

$\vec{E} = \vec{1}_z E_0 J_1(k\rho) \cos\varphi \qquad a = 1\,\text{cm}$



$H = ? \quad f = ? \quad$ + preveri ME!

$$ka = 3.83 \rightarrow f = \frac{3.83 \cdot c_0}{2\pi a} = 18.3\,\text{GHz}$$

$$\vec{H} = \frac{j}{\omega\mu} \operatorname{rot} \vec{E} = \frac{j}{\omega\mu\rho} \begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & \rho 0 & E_z \end{vmatrix} = \frac{j}{\omega\mu}\left(-\vec{1}_\rho \frac{E_0}{\rho} J_1(k\rho)\sin\varphi - \vec{1}_\varphi E_0 k J_1'(k\rho)\cos\varphi\right)$$

$$\operatorname{div}(\varepsilon_0 \vec{E}) = 0$$

$$\vec{J} = \operatorname{rot}\vec{H} - j\omega\varepsilon\vec{E} = \frac{jE_0}{\omega\mu\rho} \begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ -\frac{J_1(k\rho)}{\rho}\sin\varphi & -\rho J_1'(k\rho)k\cos\varphi & 0 \end{vmatrix} - j\omega\varepsilon_0 \vec{1}_z E_0 J_1(k\rho)\cos\varphi =$$

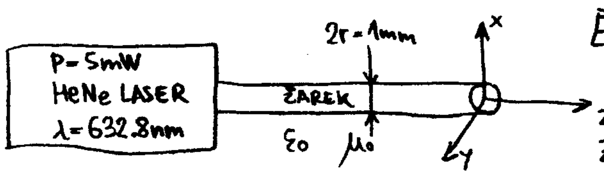
$$= \frac{jE_0}{\omega\mu\rho} \vec{1}_z \left(-\frac{\partial}{\partial \rho}\left(k\rho J_1'(k\rho)\right)\cos\varphi + \frac{J_1(k\rho)}{\rho}\cos\varphi\right) - j\omega\varepsilon_0 E \vec{1}_z J_1(k\rho)\cos\varphi =$$

$$= -\vec{1}_z \frac{jE_0}{\omega\mu_0}\underbrace{\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\left(J_1(k\rho)\right)\right) - \frac{J_1(k\rho)}{\rho^2} + \omega^2\mu_0\varepsilon_0 J_1(k\rho)\right]}_{=0}\cos\varphi = 0$$

Definicija $J_1(k\rho)$:

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\left(J_1(k\rho)\right)\right) - \frac{J_1(k\rho)}{\rho^2} + k^2 J_1(k\rho) = 0$$

2)



$\vec{E} = \vec{1}_x E$

$\vec{k} = \vec{1}_z \cdot k$

$Z_0 = 120\pi\,\Omega$

$\vec{S} = ? \quad \vec{E} = ? \quad \vec{H} = ? \quad \vec{k} = ?$

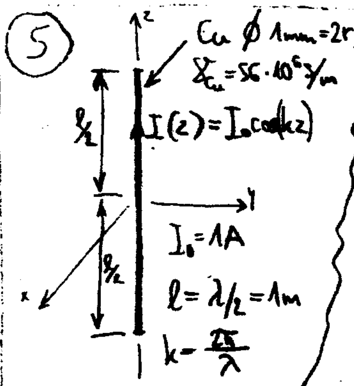
$$\vec{S} = \vec{1}_z S = \vec{1}_z \frac{P}{\pi r^2} = \vec{1}_z 6.37\,\text{kW/m}^2$$

$$S = \frac{|E|^2}{2Z_0} \rightarrow \vec{E} = \vec{1}_x \sqrt{2Z_0 S}\, e^{-jkz+\varphi} = \vec{1}_x \cdot 1.55\,\text{kV/m}\, e^{-jkz+\varphi}$$

$$\vec{H} = \vec{1}_y \frac{E}{Z_0} = \vec{1}_y \cdot 4.11\,\text{A/m}\, e^{-jkz+\varphi}$$

$$\vec{k} = \vec{1}_z k = \vec{1}_z \frac{2\pi}{\lambda} = \vec{1}_z\, 9.93 \cdot 10^6\,\text{m}^{-1}$$

5)



$P_{izgub} = ?$

(toplota = ?)

$$\delta = \sqrt{\frac{2}{\omega\mu\gamma}} \qquad A = 2\pi r \delta \qquad f = \frac{c}{\lambda} = 150\,\text{MHz}$$

$$dP = \frac{1}{2}|I|^2 \frac{dz}{\gamma A} \qquad P = \int_{-1/2}^{+1/2} \frac{1}{2}|I|^2 \frac{dz}{\gamma A}$$

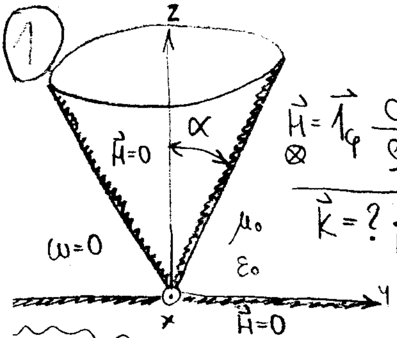
$$P = \frac{|I_0|^2}{4\pi\gamma r\delta} \int_{-\frac{l}{2}}^{+\frac{l}{2}} \cos^2(kz)\,dz = \frac{|I_0|^2\, l}{8\pi\gamma r\delta} = \frac{|I_0|^2\, l}{8\pi r}\sqrt{\frac{\omega\mu_0}{2\gamma}}$$

$$P = \frac{1\,\text{A}^2 \cdot 1\,\text{m}}{8\pi \cdot 0.5 \cdot 10^{-3}\,\text{m}} \sqrt{\frac{2\pi \cdot 150 \cdot 10^6\, 4\pi \cdot 10^{-7}\,\text{Vs}\,\text{Vm}}{\text{s Am}\ 2 \cdot 56 \cdot 10^6\,\text{A}}} = 0.259\,\text{W}$$

Rešitev 2. kolokvija iz ELEKTROMAGNETIKE 24.01.2003

Rešitev ELEKTROMAGNETIKA 07.03.2003

1



$\vec H = \vec 1_\varphi \frac{C}{\rho}$ $\otimes$

$\vec K = ?\ ;\ \vec J = ?$

$\rho = r\sin\Theta$ k.s. (r,Θ,ϕ)

$\vec H = \vec 1_\phi \frac{C}{r\sin\Theta}$ $\qquad \vec K = \vec 1_n \times \vec H$

stožec: $\vec 1_n = \vec 1_\Theta\ ;\ \Theta = \alpha$

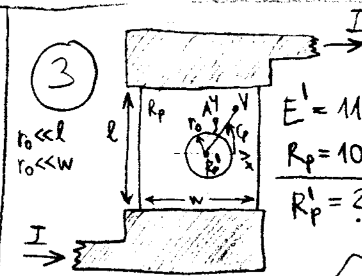
$\vec K = \vec 1_\Theta \times \vec 1_\phi \frac{C}{r\sin\alpha} = \underline{\underline{\vec 1_r \frac{C}{r\sin\alpha}}}$

ravnina xy: $\vec 1_n = -\vec 1_\Theta\ ;\ \Theta = \frac{\pi}{2}$

$\vec K = -\vec 1_\Theta \times \vec 1_\phi \frac{C}{r\sin\frac{\pi}{2}} = \underline{\underline{-\vec 1_r \frac{C}{r}}}$

esni stor: $\vec J = \mathrm{rot}\,\vec H = \underline{\underline{0}}$

3



$E' = 110\% E_0$

$R_p = 100\Omega$

$R_p' = ?$

$\vec K_0 = \vec 1_y \frac{I}{w}\ ;\ \vec E_0 = \vec K_0 R_p = \vec 1_y \frac{IR_p}{w}$

$V = \left(-E_0\rho + \frac{B}{\rho}\right)\sin\varphi\ ;\ V' = A\rho\sin\varphi$

$\vec E = \vec 1_\rho\left(E_0 + \frac{B}{\rho^2}\right)\sin\varphi + \vec 1_\varphi\left(E_0 - \frac{B}{\rho^2}\right)\cos\varphi$

$\vec E' = -\vec 1_\rho A\sin\varphi - \vec 1_\varphi A\cos\varphi = -\vec 1_y A = \vec 1_y E'$

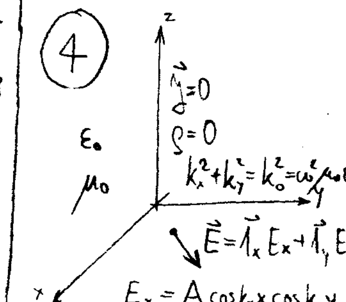
$\vec K = \frac{\vec E}{R_p} \qquad \vec K' = \frac{\vec E'}{R_p'}$

Prestopni pogoji: $\rho = r_0 \begin{cases} E_\varphi = E_\varphi' \to \left(E_0 - \frac{B}{r_0^2}\right)\cos\varphi = -A\cos\varphi \longrightarrow E_0 - \frac{B}{r_0^2} = -A \\ K_\rho = K_\rho' \to \left(E_0 + \frac{B}{r_0^2}\right)\sin\varphi \frac{1}{R_p} = -\frac{A}{R_p'}\sin\varphi \to E_0 + \frac{B}{r_0^2} = -A\frac{R_p}{R_p'} \end{cases}$

$2E_0 = -A\left(1 + \frac{R_p}{R_p'}\right) = E'\left(1 + \frac{R_p}{R_p'}\right)$

$\frac{2E_0}{E'} = 1 + \frac{R_p}{R_p'} \to R_p' = R_p \frac{1}{\frac{2E_0}{E'} - 1} = 100\Omega \frac{1}{\frac{2}{1.1} - 1} = \underline{\underline{122.2\Omega}}$

4



$\vec J = 0$

$\rho = 0$

$k_x^2 + k_y^2 = k_0^2 = \omega^2\mu_0\varepsilon_0$

$\vec E = \vec 1_x E_x + \vec 1_y E_y$

$E_x = A\cos k_x x \cos k_y y$

$E_y = ?\quad \vec H = ?\quad \vec S = ?$

$0 = \mathrm{div}\,\varepsilon_0 \vec E = \varepsilon_0\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right)$

$0 = -Ak_x \sin k_x x \cos k_y y + \frac{\partial E_y}{\partial y}$

$\frac{\partial E_y}{\partial y} = Ak_x \sin k_x x \cos k_y y \ /\int dy$

$E_y = A\frac{k_x}{k_y}\sin k_x x \sin k_y y + C$

$\Delta\vec E + k^2\vec E = 0 \Rightarrow C = 0$

$\underline{\underline{E_y = A\frac{k_x}{k_y}\sin k_x x \sin k_y y}}$

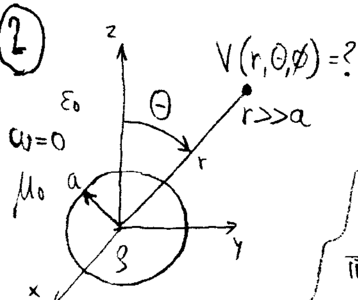
$\vec H = \frac{j}{\omega\mu_0}\mathrm{rot}\,\vec E = \frac{j}{\omega\mu_0}\begin{vmatrix} \vec 1_x & \vec 1_y & \vec 1_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A\cos k_x x\cos k_y y & A\frac{k_x}{k_y}\sin k_x x \sin k_y y & 0 \end{vmatrix} = \vec 1_z \frac{jA}{\omega\mu_0}\left(\frac{k_x^2}{k_y}\cos k_x x \sin k_y y + k_y \cos k_x x \sin k_y y\right) =$

$= \vec 1_z \frac{jAk_0^2}{\omega\mu_0 k_y}\cos k_x x \sin k_y y = \underline{\underline{\vec 1_z j\frac{Ak_0}{Z_0 k_y}\cos k_x x \sin k_y y}}$

$\vec S = \frac{1}{2}\vec E \times \vec H^* = \frac{1}{2}\left(\vec 1_x A\cos k_x x\cos k_y y + \vec 1_y A\frac{k_x}{k_y}\sin k_x x \sin k_y y\right) \times \vec 1_z (-j)\frac{A^* k_0}{Z_0 k_y}\cos k_x x \sin k_y y$

$\underline{\underline{\vec S = j\frac{|A|^2 k_0}{2Z_0 k_y}\left(-\vec 1_x \frac{k_x}{k_y}\sin k_x x \cos k_x x \sin^2 k_y y + \vec 1_y \cos^2 k_x x \sin k_y y \cos k_y y\right)}}$

2



$V(r,\Theta,\phi) = ?$

$\rho(\vec r') = \begin{cases} A\cos\Theta'\ ;\ r' < a \\ 0\ ;\ r' > a \end{cases}$

$V(\vec r) = \frac{1}{4\pi\varepsilon_0}\int_{v'} \frac{\rho(\vec r')}{|\vec r - \vec r'|}\,dv'$ $\qquad dv' = r'^2\sin\Theta'\,dr'\,d\Theta'\,d\phi'$ $\qquad r \gg a > r'$

$|\vec r - \vec r'| = \sqrt{(r\sin\Theta\cos\phi - r'\sin\Theta'\cos\phi')^2 + (r\sin\Theta\sin\phi - r'\sin\Theta'\sin\phi')^2 + (r\cos\Theta - r'\cos\Theta')^2} =$

$= \sqrt{r^2 + r'^2 - 2rr'\sin\Theta\sin\Theta'\cos(\phi-\phi') - 2rr'\cos\Theta\cos\Theta'} \approx r - r'(\sin\Theta\sin\Theta'\cos(\phi-\phi') + \cos\Theta\cos\Theta')$

$\frac{1}{|\vec r - \vec r'|} \approx \frac{1}{r}\left(1 + \frac{r'}{r}(\sin\Theta\sin\Theta'\cos(\phi-\phi') + \cos\Theta\cos\Theta')\right)$

$V(\vec r) \approx \frac{1}{4\pi\varepsilon_0}\int_0^a\int_0^\pi\int_0^{2\pi} A\cos\Theta' \frac{1}{r}\left(1 + \frac{r'}{r}(\sin\Theta\sin\Theta'\cos(\phi-\phi') + \cos\Theta\cos\Theta')\right) r'^2\sin\Theta'\,dr'\,d\Theta'\,d\phi' =$

$= \frac{A}{4\pi\varepsilon_0}\int_0^\pi\int_0^{2\pi} \cos\Theta' \frac{1}{r}\left(\frac{a^3}{3} + \frac{a^4}{4r}(\sin\Theta\sin\Theta'\cos(\phi-\phi') + \cos\Theta\cos\Theta')\right)\sin\Theta'\,d\Theta'\,d\phi' =$

$= \frac{A}{4\pi\varepsilon_0 r}\int_0^\pi \cos\Theta'\left(\frac{a^3}{3} + \frac{a^4}{4r}\cos\Theta\cos\Theta'\right)\sin\Theta'\,d\Theta' = \frac{Aa^3}{4\pi\varepsilon_0 r}\int_{-1}^{1}\left(\frac{u}{3} + \frac{a\cos\Theta}{4r}u^2\right)du =$

$= \frac{Aa^4\cos\Theta}{16\pi\varepsilon_0 r^2}\int_{-1}^{1} u^2\,du = \frac{Aa^4}{24\pi\varepsilon_0}\frac{\cos\Theta}{r^2}$

5

$\varepsilon_r = 80\ ;\ \gamma = 5\,S/m$

$f = 10\,kHz\ ;\ a = 60\,dB$

$h = ?$

$\frac{\omega\varepsilon_0\varepsilon_r}{\gamma} = \frac{2\pi\cdot 10^4/s\ As\cdot 80}{5\,S/m\ 4\pi\cdot 9\cdot 10^9\,Vm} = \underline{2.3\cdot 10^{-6} \ll 1}$

$\delta = \sqrt{\frac{2}{\omega\mu_0\gamma}} = \sqrt{\frac{2\,Am}{2\pi\cdot 10^4 s^{-1}\ 4\pi\cdot 10^{-7}\,Vs\ 5\,S/m}} = \underline{2.25\,m}$

$a = 20\log_{10}\left(\frac{E(0)}{E(h)}\right)\ ;\ \frac{E(h)}{E(0)} = e^{-\frac{h}{\delta}}\ ;\ a = 20\frac{\ln\left(\frac{E(0)}{E(h)}\right)}{\ln 10} = \frac{20h}{\delta\ln 10}$

$\to h = \frac{a\delta\ln 10}{20} = \underline{\underline{15.5\,m}}$

Rešitev 1. kolokvija iz ELEKTROMAGNETIKE 28.11.2003

① $\operatorname{div}\vec{F} = ?$ v elipsoidnih koordinatah (η, ψ, ϕ)

$[x = a\,\mathrm{sh}\eta\sin\psi\cos\phi,\ y = a\,\mathrm{sh}\eta\sin\psi\sin\phi,\ z = a\,\mathrm{ch}\eta\cos\psi]$

$$h_\eta = \sqrt{a^2\mathrm{ch}^2\eta\sin^2\psi\cos^2\phi + a^2\mathrm{ch}^2\eta\sin^2\psi\sin^2\phi + a^2\mathrm{sh}^2\eta\cos^2\psi} = a\sqrt{\mathrm{sh}^2\eta + \sin^2\psi}$$

$$h_\psi = \sqrt{a^2\mathrm{sh}^2\eta\cos^2\psi\cos^2\phi + a^2\mathrm{sh}^2\eta\cos^2\psi\sin^2\phi + a^2\mathrm{ch}^2\eta\sin^2\psi} = a\sqrt{\mathrm{sh}^2\eta + \sin^2\psi}$$

$$h_\phi = \sqrt{a^2\mathrm{sh}^2\eta\sin^2\psi\sin^2\phi + a^2\mathrm{sh}^2\eta\sin^2\psi\cos^2\phi + 0} = a\,\mathrm{sh}\eta\sin\psi$$

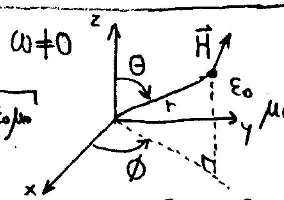
$$\operatorname{div}\vec{F} = \frac{1}{h_\eta h_\psi h_\phi}\left[\frac{\partial}{\partial\eta}(h_\psi h_\phi F_\eta) + \frac{\partial}{\partial\psi}(h_\eta h_\phi F_\psi) + \frac{\partial}{\partial\phi}(h_\eta h_\psi F_\phi)\right] =$$

$$= \frac{1}{a(\mathrm{sh}^2\eta + \sin^2\psi)\mathrm{sh}\eta\sin\psi}\left[\frac{\partial}{\partial\eta}\left(\mathrm{sh}\eta\sin\psi\sqrt{\mathrm{sh}^2\eta+\sin^2\psi}\,F_\eta\right) + \frac{\partial}{\partial\psi}\left(\mathrm{sh}\eta\sin\psi\sqrt{\mathrm{sh}^2\eta+\sin^2\psi}\,F_\psi\right)\right] + \frac{1}{a\,\mathrm{sh}\eta\sin\psi}\frac{\partial F_\phi}{\partial\phi}$$

③ Prazen prostor $\quad \omega \neq 0$

$$\vec{H} = \vec{1}_\phi C\frac{e^{-jkr}}{r\sin\theta};\ k = \omega\sqrt{\varepsilon_0\mu_0}$$

$\vec{S} = ?$, Izvori = ?



$$\vec{E} = \frac{1}{j\omega\varepsilon_0}(\operatorname{rot}\vec{H} - \vec{J}) = \frac{1}{j\omega\varepsilon_0}\left[\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & r0 & r\sin\theta\, C\frac{e^{-jkr}}{r\sin\theta}\end{vmatrix} - \vec{J}\right] =$$

$$= \frac{1}{j\omega\varepsilon_0}\left[\vec{1}_\theta jkC\frac{e^{-jkr}}{r\sin\theta} - \vec{J}\right] = \vec{1}_\theta\frac{kC}{\omega\varepsilon_0}\frac{e^{-jkr}}{r\sin\theta} - \frac{\vec{J}}{j\omega\varepsilon_0} \quad \vec{J} = 0$$

$$\vec{H} = \frac{j}{\omega\mu_0}\operatorname{rot}\vec{E} = \frac{j}{\omega\mu_0}\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & \frac{kC}{\omega\varepsilon_0}\frac{e^{-jkr}}{\sin\theta} & 0\end{vmatrix} - \frac{1}{k^2}\operatorname{rot}\vec{J} = \vec{H} - \frac{1}{k^2}\operatorname{rot}\vec{J}$$

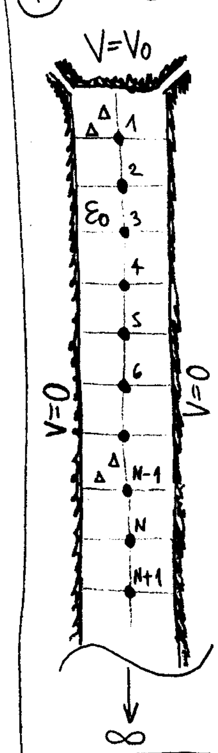
$$\rho = \operatorname{div}(\varepsilon_0\vec{E}) = \frac{\varepsilon_0}{r^2 s}\frac{\partial}{\partial\theta}\left[r\sin\theta\frac{kC}{\omega\varepsilon_0}\frac{e^{-jkr}}{r\sin\theta}\right] = \underline{0}$$

Singularnost na osi z: $I = \oint\vec{H}\cdot d\vec{s} = \int_0^{2\pi}\vec{H}\cdot\vec{1}_\phi r\sin\theta\,d\phi = \underline{Ce^{-jkr}}$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\vec{1}_\theta\frac{kC}{\omega\varepsilon_0}\frac{e^{-jkr}}{r\sin\theta}\times\vec{1}_\phi C^*\frac{e^{+jkr}}{r\sin\theta} =$$

$$= \vec{1}_r\frac{k}{2\omega\varepsilon_0}|C|^2\frac{1}{r^2\sin^2\theta} = \vec{1}_r\frac{z_0|C|^2}{2r^2\sin^2\theta}$$

④ $\omega = 0$



$V_N = ?$

$4V_1 = V_0 + V_2$
$4V_2 = V_1 + V_3$
$4V_3 = V_2 + V_4$
$4V_4 = V_3 + V_5$
$\vdots$
$4V_N = V_{N-1} + V_{N+1}$
$\vdots$
∞

Rešitev v obliki: $V_N = C\alpha^N$

$$V_0 = C\alpha^0 \longrightarrow \underline{C = V_0}$$

$$4C\alpha^N = C\alpha^{N-1} + C\alpha^{N+1}$$

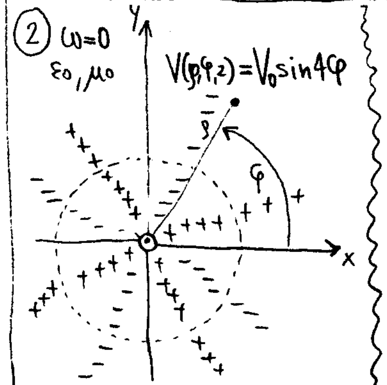
$$0 = \alpha^2 - 4\alpha + 1$$

$$\alpha = \frac{4 \pm \sqrt{16-4}}{2} = \underline{2 \pm \sqrt{3}}$$

Potencial upada → izberemo $\alpha < 1$

$$\underline{\underline{V_N = V_0\left(2 - \sqrt{3}\right)^N}}$$

② $\omega = 0$, ε_0, μ_0



$\vec{E} = ?$, $\rho_e = ?$ električna, Singularnost $\rho=0$? v osi "z"

$$\vec{E} = -\operatorname{grad}V = -\vec{1}_\rho\frac{\partial V}{\partial\rho} - \vec{1}_\varphi\frac{1}{\rho}\frac{\partial V}{\partial\varphi} - \vec{1}_z\frac{\partial V}{\partial z} =$$

$$= -\vec{1}_\rho 0 - \vec{1}_\varphi\frac{4V_0}{\rho}\cos 4\varphi - \vec{1}_z 0 = \underline{\underline{-\vec{1}_\varphi\frac{4V_0}{\rho}\cos 4\varphi}}$$

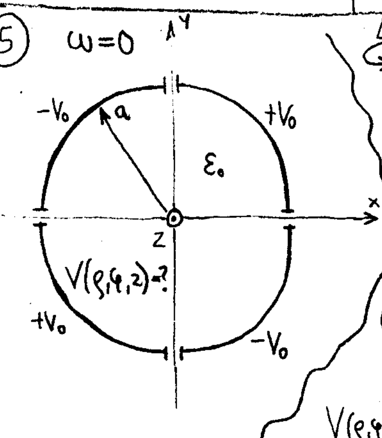
$$\rho_e = \operatorname{div}(\varepsilon_0\vec{E}) = \frac{\varepsilon_0}{\rho}\left[\frac{\partial}{\partial\varphi}\left(-\frac{4V_0}{\rho}\cos 4\varphi\right)\right] =$$

$$\underline{\underline{\rho_e = \frac{16V_0\varepsilon_0}{\rho^2}\sin 4\varphi}}$$

$$\left.\begin{array}{l}\Delta V = -\frac{\rho_e}{\varepsilon_0}\\ \rho \to 0\end{array}\right\} V' = V + A\frac{\sin 4\varphi}{\rho^4} \to \underline{\underline{A = 0}}$$

ni ρ-pola v osi z

⑤ $\omega = 0$



$$\overset{\Delta V = 0}{\curvearrowright} V(\rho,\varphi,z) = \sum_n C_n\rho^n\sin n\varphi \quad \Big/ \int_0^{2\pi}\sin m\varphi\,d\varphi\ @\ \rho = a$$

$$\int_0^{2\pi}\sum_n C_n a^n\sin n\varphi\,\sin m\varphi\,d\varphi = +V_0\int_0^{\pi/2}\sin m\varphi\,d\varphi - V_0\int_{\pi/2}^{\pi}\sin m\varphi\,d\varphi + V_0\int_{\pi}^{3\pi/2}\sin m\varphi\,d\varphi - V_0\int_{3\pi/2}^{2\pi}\sin m\varphi\,d\varphi$$

$$C_m a^m\pi = \frac{V_0}{m}\left[\cos 0 - \cos m\frac{\pi}{2} - \cos m\frac{\pi}{2} + \cos m\pi + \cos m\pi - \cos m\frac{3\pi}{2} - \cos m\frac{3\pi}{2} + \cos m2\pi\right]$$

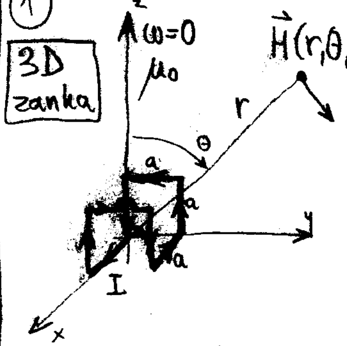
$$C_m = \frac{2V_0}{m\pi a^m}\underbrace{\left[1 - \cos m\frac{\pi}{2} + \cos m\pi - \cos m\frac{3\pi}{2}\right]}_{= 4 \neq 0\ @\ m = 4k+2;\ k = 0,1,2,3,\ldots}$$

$$\underline{\underline{V(\rho,\varphi,z) = \sum_{k=0}^{\infty}\frac{8V_0}{(4k+2)\pi}\left(\frac{\rho}{a}\right)^{4k+2}\sin(4k+2)\varphi}}$$

II. kolokvij iz ELEKTROMAGNETIKE
12/01/2004

1

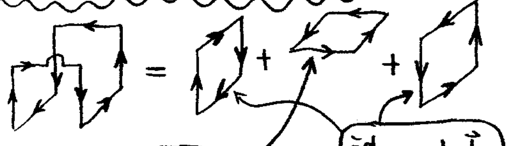
3D zanka



$\omega=0$, μ_0

$\vec{H}(r,\theta,\phi)=?$

$r \gg a$



dipol $\vec{1}_z$

četveropol $\vec{1}_y$ zanemarljiv glede na polje dipola!

$A=a^2$

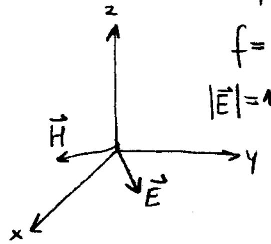
$$\vec{A}=\vec{1}_\phi \frac{\mu_0 I A}{4\pi r^2}\sin\theta = \vec{1}_\phi \frac{\mu_0 I a^2}{4\pi r^2}\sin\theta$$

$$\vec{H}=\frac{1}{\mu_0}\,\mathrm{rot}\vec{A}=\underline{\underline{\frac{Ia^2}{4\pi r^3}\left(\vec{1}_r 2\cos\theta+\vec{1}_\theta\sin\theta\right)}}$$

3

$\mu_0,\ \varepsilon_0,\ \vec{J}=0,\ \rho=0$

PLANI VAL V PRAZNEM PROSTORU



$f=1\,\mathrm{GHz}$

$|\vec{E}|=100\,\frac{V_{eff}}{m}$

$$\vec{E}=\left(\vec{1}_x+\vec{1}_y\right)f(x,y,z)$$

$$\vec{H}=\left(\vec{1}_x-\vec{1}_y\right)\frac{f(x,y,z)}{Z_0}$$

$\vec{k}=?\ ;\ \vec{S}=?$

$$\vec{k}=\vec{1}_k\cdot k \qquad k=\frac{\omega}{c_0}=\frac{2\pi\cdot 10^9\,\mathrm{rd/s}}{3\cdot 10^8\,\mathrm{m/s}}=\underline{20.9\,\mathrm{rd/m}}$$

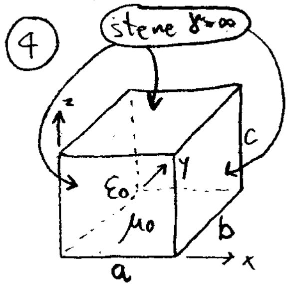
$$\vec{1}_k=\vec{1}_s=\frac{\vec{E}\times\vec{H}^*}{|\vec{E}\times\vec{H}^*|}=\frac{(\vec{1}_x+\vec{1}_y)\times(\vec{1}_x-\vec{1}_y)}{|(\vec{1}_x+\vec{1}_y)\times(\vec{1}_x-\vec{1}_y)|}=\frac{-\vec{1}_z\cdot 2}{2}=\underline{-\vec{1}_z}$$

$$\underline{\underline{\vec{k}=-\vec{1}_z\cdot 20.9\,\mathrm{rd/m}}}$$

$$\vec{S}=\vec{E}_{eff}\times\vec{H}^*_{eff}=-\vec{1}_z\cdot 2\cdot\frac{|\vec{E}|^2}{Z_0}=\underline{\underline{-\vec{1}_z\cdot 53\,\frac{W}{m^2}}}$$

4

stene $\gamma\to\infty$



VOTLINSKI REZONATOR

$$\vec{E}=\vec{1}_z\,\frac{10V}{m}\sin\frac{\pi}{a}x\,\sin\frac{\pi}{b}y$$

$a=2c\ ;\ b=3c\ ;\ f=1\,\mathrm{GHz}$

$a=?,\ b=?,\ c=?,\ W=?$

$$k=\sqrt{k_x^2+k_y^2}=\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2}=\sqrt{\left(\frac{\pi}{2c}\right)^2+\left(\frac{\pi}{3c}\right)^2}=\frac{\pi}{c}\sqrt{\frac{1}{4}+\frac{1}{9}}$$

$$k=\frac{\omega}{c_0}=\frac{2\pi f}{c_0}\ \rightarrow c=\frac{c_0}{2f}\sqrt{\frac{1}{4}+\frac{1}{9}}=\underline{\underline{90.14\,\mathrm{mm}}}$$

$$a=2c=\underline{\underline{180.3\,\mathrm{mm}}},\quad b=3c=\underline{\underline{270.4\,\mathrm{mm}}}$$

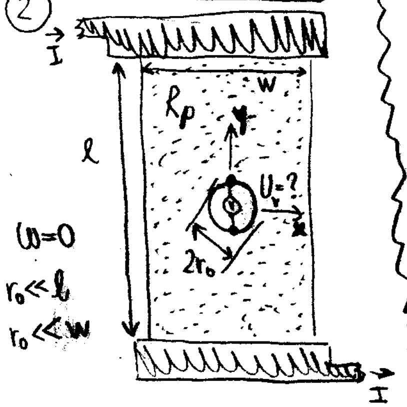
$$W=W_e+W_m=W_{emax}=\frac{1}{2}\int_V\varepsilon_0|E|^2\,dv=$$

$$=\frac{\varepsilon_0}{2}\int_0^a\int_0^b\int_0^c\left(\frac{10V}{m}\sin\frac{\pi}{a}x\sin\frac{\pi}{b}y\right)^2 dx\,dy\,dz=\frac{\varepsilon_0}{2}\left(\frac{10V}{m}\right)^2\frac{a}{2}\,\frac{b}{2}\,c=$$

$$=\frac{1\,\mathrm{As}}{2\cdot 4\pi\cdot 9\cdot 10^9\,\mathrm{Vm}}\,\frac{100\,V^2}{m^2}\cdot 0.09\,\mathrm{m}\cdot 0.135\,\mathrm{m}\cdot 0.09\,\mathrm{m}=\underline{\underline{4.857\cdot 10^{-13}\,\mathrm{J}}}$$

2

TANKOPLASTNI UPOR



$\omega=0$

$r_0\ll l$

$r_0\ll w$

$$\vec{K}_0=-\vec{1}_y\frac{I}{w}\ ;\ \vec{E}_0=R_p\vec{K}_0=-\vec{1}_y\frac{IR_p}{w}\ ;\ E_0=\frac{IR_p}{w}$$

$$\Delta V=0\rightarrow V=\left(E_0\rho+C\rho^{-1}\right)\sin\varphi=E_0\left(\rho+\frac{r_0^2}{\rho}\right)\sin\varphi$$

$$\rho=r_0\rightarrow K_\rho=0, E_\rho=0\ \rightarrow C=E_0r_0^2$$

$$\vec{E}=-\mathrm{grad}V=-\vec{1}_\rho\left(E_0-\frac{C}{\rho^2}\right)\sin\varphi-\vec{1}_\varphi\left(E_0+\frac{C}{\rho^2}\right)\cos\varphi$$

$$U_v=V\left(r_0,\frac{\pi}{2}\right)-V\left(r_0,\frac{3\pi}{2}\right)=2E_0r_0-(-2E_0r_0)=$$

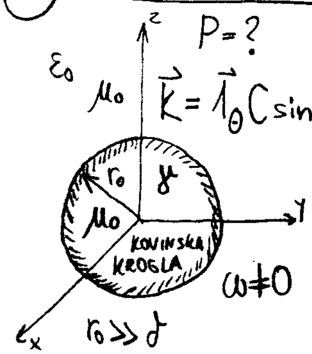
$$=4E_0r_0=\underline{\underline{4\,\frac{IR_p}{w}\,r_0}}$$

5

MOČ PLOSKOVNEGA TOKA

$P=?$

$\varepsilon_0\quad\mu_0\quad\vec{K}=\vec{1}_\theta C\sin\theta$



$\omega\neq 0$

$r_0\gg\delta$

$$\delta=\sqrt{\frac{2}{\omega\mu_0\gamma}}\qquad \frac{dP}{dA}=\frac{1}{2}|\vec{K}|^2\frac{1}{\delta\gamma}\qquad \cos\theta=u,\quad \sin\theta\,d\theta=-du$$

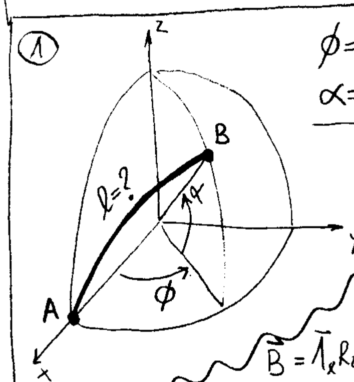
$$P=\int_0^\pi\int_0^{2\pi}\frac{1}{2}|\vec{K}|^2\frac{1}{\delta\gamma}r_0^2\sin\theta\,d\theta\,d\phi=$$

$$=\pi\frac{1}{\delta\gamma}r_0^2\int_0^\pi|C|^2\sin^2\theta\sin\theta\,d\theta=\frac{\pi r_0^2|C|^2}{\delta\gamma}\int_0^\pi(1-\cos^2\theta)\sin\theta\,d\theta=$$

$$=\frac{\pi r_0^2|C|^2}{\delta\gamma}\int_{-1}^{1}(1-u^2)\,du=\frac{4\pi r_0^2|C|^2}{3\delta\gamma}=\underline{\underline{\frac{4\pi r_0^2|C|^2}{3}\sqrt{\frac{\omega\mu_0}{2\gamma}}}}$$

ELEKTROMAGNETIKA 20/02/2004

1



$\phi = 14^\circ$ Zemlja = krogla

$\alpha = 46^\circ$ $R_z = 6378\,km$

$l = ?$ (najkrajša)

$\vec{A} = \vec{1}_x R_z$

$\vec{B} = \vec{1}_x R_z \cos\alpha\cos\phi + \vec{1}_y R_z \cos\alpha\sin\phi + \vec{1}_z R_z \sin\alpha$

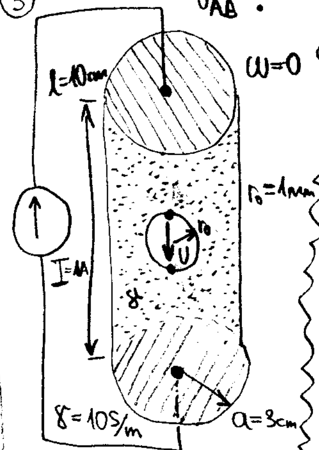
$\vec{A}\cdot\vec{B} = R_z^2 \cos\alpha \cos\phi$

$l = R_z \arccos\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right) = R_z \arccos(\cos\alpha\cos\phi)$

$l = 6378\,km \arccos(\cos 46^\circ \cos 14^\circ)$

$\underline{\underline{l = 5301\,km}}$

3



$U_{AB} = ?$

$\omega = 0$

$\vec{E}_0 = -\vec{1}_z \frac{I}{\pi a^2 \gamma}$; $E_0 = \frac{I}{\pi a^2 \gamma}$

$\Delta V = 0 \rightarrow V = (Ar + Br^{-2})\cos\theta$

$V(\infty) = V_0 \rightarrow A = E_0$

$\vec{J} = -\gamma\,\mathrm{grad}\,V = \vec{1}_r \gamma\left(-A + \frac{2B}{r^3}\right)\cos\theta + \vec{1}_\theta \gamma\left(A + \frac{B}{r^3}\right)\sin\theta$

$\vec{J}(r_0)\cdot\vec{1}_r = 0 \rightarrow -A + \frac{2B}{r_0^3} = 0$

$B = \frac{A r_0^3}{2}$

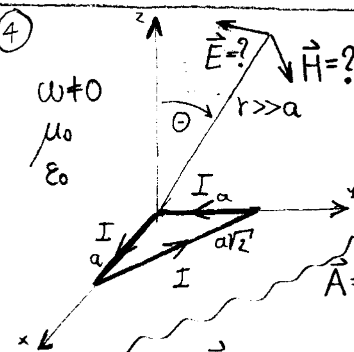
$V = E_0\left(r + \frac{r_0^3}{2r^2}\right)\cos\theta$

$U = V(r=r_0, \theta=0) - V(r=r_0, \theta=\pi) = 2E_0\left(r_0 + \frac{r_0^3}{2r_0^2}\right) =$

$= 2\frac{I}{\pi a^2 \gamma}\,\frac{3}{2} r_0 = \frac{3 I r_0}{\pi a^2 \gamma} = \frac{3 \cdot 1A \cdot 10^{-3} m}{\pi \cdot 9 \cdot 10^{-4} m^2 \cdot 10 S/m} =$

$\underline{\underline{= 0.1061\,V = 106.1\,mV}}$

4



$\omega \neq 0$

μ_0

ε_0

$A = \frac{a^2}{2}$

$\rho = 0 \rightarrow V = 0$

$\vec{A} = \vec{1}_\phi \frac{\mu_0}{4\pi} I A \frac{e^{-jkr}}{r}\left(jk + \frac{1}{r}\right)\sin\theta$

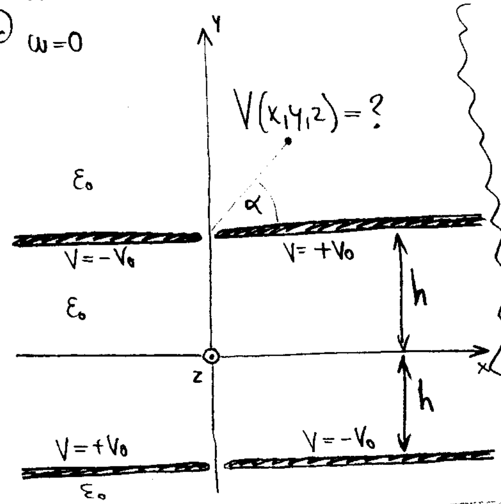
$\vec{A} = \vec{1}_\phi \frac{\mu_0}{4\pi} I \frac{a^2}{2} \frac{e^{-jkr}}{r}\left(jk + \frac{1}{r}\right)\sin\theta$

$$\vec{H} = \frac{1}{\mu_0}\mathrm{rot}\vec{A} = \frac{1}{4\pi} I \frac{a^2}{2} \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & r0 & \sin^2\theta e^{-jkr}\left(jk + \frac{1}{r}\right) \end{vmatrix} =$$

$$\underline{\underline{= \frac{1}{4\pi} I \frac{a^2}{2} e^{-jkr}\left[\vec{1}_r\left(\frac{jk}{r^2} + \frac{1}{r^3}\right)2\cos\theta + \vec{1}_\theta\left(-\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3}\right)\sin\theta\right]}}$$

$\underline{\underline{\vec{E} = -j\omega\vec{A} = -\vec{1}_\phi \frac{j\omega\mu_0}{4\pi} I \frac{a^2}{2} \frac{e^{-jkr}}{r}\left(jk + \frac{1}{r}\right)\sin\theta}}$ $\rho = 0$, $V = 0$, $\mathrm{grad}\,V = 0$

2



$\omega = 0$

$V(x,y,z) = ?$

$\Delta V = 0 \rightarrow V = C_1\alpha + C_2$

Problem kvadranta

$\alpha = \mathrm{arctg}\frac{y-h}{x} = \arccos\frac{x}{\sqrt{x^2 + (y-h)^2}}$

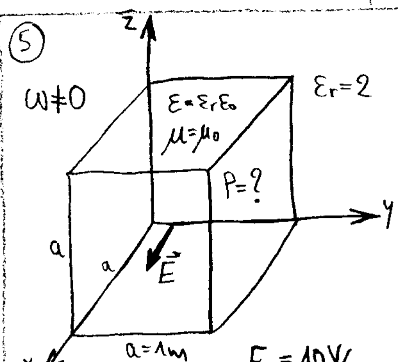
$\alpha = 0 \rightarrow C_2 = +V_0$

$\alpha = \pi \rightarrow C_1 = -\frac{2V_0}{\pi}$

$\underline{\underline{V = -\frac{2V_0}{\pi}\arccos\frac{x}{\sqrt{x^2+(y-h)^2}} + V_0}}$

Spodnji dve elektrodi nimata vpliva!

5



$\omega \neq 0$

$\varepsilon_r = 2$

$\vec{E} = \vec{1}_x E_0 e^{-jky}$

$k = \omega\sqrt{\mu\varepsilon}$

$E_0 = 10\,V/m$

$f = 1\,GHz$

$P = ?$

$\vec{H} = \frac{j}{\omega\mu}\mathrm{rot}\vec{E} = -\vec{1}_z \frac{k}{\omega\mu} E_0 e^{-jky}$

$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{1}_y \frac{k}{2\omega\mu}|E_0|^2 = \vec{1}_y \frac{|E_0|^2}{2\sqrt{\frac{\mu}{\varepsilon}}}$

$P = \oint_{kocka} \vec{S}\cdot d\vec{A} = \underline{\underline{0\,W}}$ (ploskev y=0 in ploskev y=a se uničita, ostale imajo skalarni produkt 0)

ELEKTROMAGNETIKA 9/7/2004

1 Zemljepisni koordinatni sistem (λ, φ, h)

λ = zem. dolžina, φ = zem. širina, h = nadm. višina

$R_Z = 6378\,km$

$h_\lambda, h_\varphi, h_h = ?$

$r = h + R_Z$

$x = r\cos\lambda\cos\varphi = \cos\lambda\cos\varphi\,(h+R_Z)$

$y = r\sin\lambda\cos\varphi = \sin\lambda\cos\varphi\,(h+R_Z)$

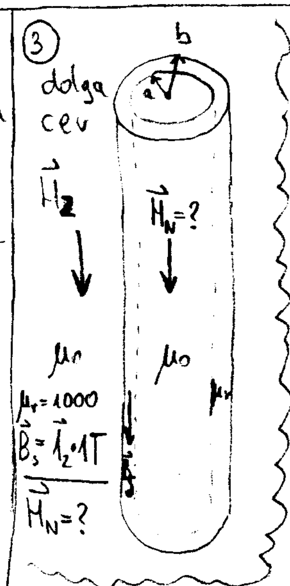
$z = r\sin\varphi \quad = \sin\varphi\,(h+R_Z)$

$$h_\lambda = \sqrt{\left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2 + \left(\frac{\partial z}{\partial \lambda}\right)^2} = \cos\varphi\,(h+R_Z)$$

$$h_\varphi = \sqrt{\left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2 + \left(\frac{\partial z}{\partial \varphi}\right)^2} = (h+R_Z)$$

$$h_h = \sqrt{\left(\frac{\partial x}{\partial h}\right)^2 + \left(\frac{\partial y}{\partial h}\right)^2 + \left(\frac{\partial z}{\partial h}\right)^2} = 1$$

3 dolga cev



$\mu_r = 1000$

$\vec{B}_s = \vec{1}_z \cdot 1T$

$\vec{H}_N = ?$

$\mu_0 = 4\pi \cdot 10^{-7}\,Vs/Am$

$$\vec{H}_s = \frac{\vec{B}_s}{\mu_r \mu_0} = \frac{\vec{1}_z 1T}{10^3 \cdot 4\pi \cdot 10^{-7}\,\frac{Vs}{Am}}$$

$\vec{H}_s = \vec{1}_z 796\,A/m$

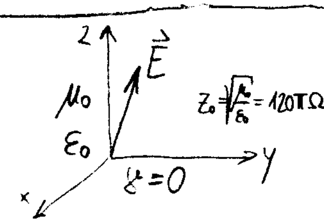
$\vec{K} = 0$

$\vec{H}_N = \vec{H}_s = \vec{1}_z \cdot 796\,A/m$

4 $\vec{E} = (\vec{1}_y + j\vec{1}_z) E_0 e^{-jkx}$

$E_0 = 100\,V/m$; $k = \omega\sqrt{\mu_0 \varepsilon_0} = \frac{\omega}{c_0}$

$\vec{E}(t) = ?$ $\vec{H}(t) = ?$ $\vec{S}(t) = ?$



$$\vec{E}(t) = Re\left[(\vec{1}_y + j\vec{1}_z) E_0 e^{j(\omega t - kx)}\right] = \vec{1}_y E_0 \cos(\omega t - kx) - \vec{1}_z E_0 \sin(\omega t - kx) =$$

$$= \vec{1}_y\, 100\,V/m \cos\left[\omega\left(t - \frac{x}{c_0}\right)\right] - \vec{1}_z\, 100\,V/m \sin\left[\omega\left(t - \frac{x}{c_0}\right)\right]$$

$$\vec{H} = \frac{j}{\omega\mu_0} \operatorname{rot} \vec{E} = \frac{j}{\omega\mu_0} \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 e^{-jkx} & jE_0 e^{-jkx} \end{vmatrix} = (-j\vec{1}_y + \vec{1}_z)\frac{E_0}{Z_0} e^{-jkx}$$

$$\vec{H}(t) = Re\left[(-j\vec{1}_y + \vec{1}_z)\frac{E_0}{Z_0} e^{-jkx}\right] = \vec{1}_y\, 0.265\,A/m \sin\left[\omega\left(t - \frac{x}{c_0}\right)\right] + \vec{1}_z\, 0.265\,A/m \cos\left[\omega\left(t - \frac{x}{c_0}\right)\right]$$

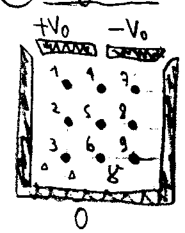
$$\vec{S}(t) = \vec{E}(t) \times \vec{H}(t) = \left(\vec{1}_y\, 100\,V/m \cos\left[\omega\left(t - \frac{x}{c_0}\right)\right] - \vec{1}_z\, 100\,V/m \sin\left[\omega\left(t - \frac{x}{c_0}\right)\right]\right) \times$$

$$\times \left(\vec{1}_y\, 0.265\,A/m \sin\left[\omega\left(t - \frac{x}{c_0}\right)\right] + \vec{1}_z\, 0.265\,A/m \cos\left[\omega\left(t - \frac{x}{c_0}\right)\right]\right) =$$

$$= \vec{1}_x\, 26.5\,W/m^2 \cos^2\left[\omega\left(t - \frac{x}{c_0}\right)\right] + \vec{1}_x\, 26.5\,W/m^2 \sin^2\left[\omega\left(t - \frac{x}{c_0}\right)\right] =$$

$$= \vec{1}_x\, 26.5\,W/m^2$$

2 dolg žleb



$V_{1\ldots9} = ?$

Simetrija: $V_4 = V_5 = V_6 = 0$

$V_1 = -V_7$

$V_2 = -V_8$

$V_3 = -V_9$

$4V_1 = V_0 + V_2$

$4V_2 = V_1 + V_3$

$4V_3 = V_2$

$16V_2 = 4V_1 + V_2$

$15V_2 = 4V_1 = V_0 + V_2 \rightarrow V_2 = \frac{1}{14}V_0$, $V_8 = -\frac{1}{14}V_0$

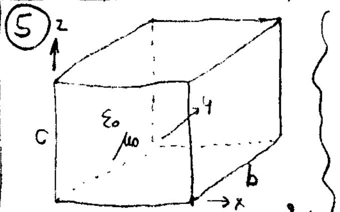
$V_1 = \frac{V_0 + V_2}{4} = \frac{15}{56}V_0$

$V_7 = -\frac{15}{56}V_0$

$V_3 = \frac{V_2}{4} = \frac{1}{56}V_0$

$V_9 = -\frac{1}{56}V_0$

5



$c_0 = 3 \cdot 10^8\,m/s$

Votlinski rezonator = kvader

$a = 2cm$; $b = 3cm$; $c < a < b$

$\Delta f = 1MHz$ za najnižji rezonanci

$c = ?$

$$f_{lmn} = \frac{c_0}{2}\sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$$

$$f_{110} = \frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 9013.878\,MHz$$

$a < b \rightarrow f_{011} < f_{101}$

$f_{011} = f_{110} + \Delta f = 9014.878\,MHz$

$$f_{011} = \frac{c_0}{2}\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2} \rightarrow c = \frac{1}{\sqrt{\left(\frac{2f_{011}}{c_0}\right)^2 - \left(\frac{1}{b}\right)^2}} = 1.99968\,cm$$

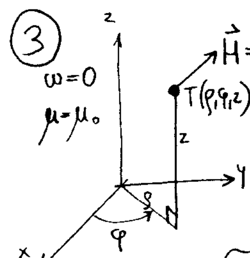
IZPIT ELEKTROMAGNETIKA 22.10.2004

① $\operatorname{div}(\operatorname{grad}U \times \operatorname{grad}V) = ?$ $v(x,y,z)$

$\vec{A} = \operatorname{grad}U$; $\vec{B} = \operatorname{grad}V$

$$\operatorname{div}(\operatorname{grad}U \times \operatorname{grad}V) = \operatorname{div}(\vec{A}\times\vec{B}) =$$
$$= \vec{B}\cdot \operatorname{rot}\vec{A} - \vec{A}\cdot\operatorname{rot}\vec{B} =$$
$$= \operatorname{grad}V \cdot \operatorname{rot}(\operatorname{grad}U) - \operatorname{grad}U\cdot\operatorname{rot}(\operatorname{grad}V) =$$
$$= \operatorname{grad}V\cdot 0 - \operatorname{grad}U\cdot 0 = \underline{\underline{0}}$$

③ $\omega = 0$, $\mu = \mu_0$



$\vec{H} = \vec{1}_\varphi \frac{A}{\rho} e^{-\frac{\rho^2}{a^2}}$

$\vec{J} = ?$ $\vec{K} = ?$

$I = ?$

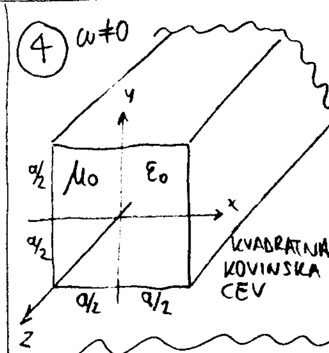
$$\vec{J} = \operatorname{rot}\vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \rho H_\varphi(\rho) & 0 \end{vmatrix} = \vec{1}_z \frac{1}{\rho} A\, e^{-\frac{\rho^2}{a^2}}\left(-2\frac{\rho}{a^2}\right) =$$
$$= \underline{\underline{-\vec{1}_z \frac{2A}{a^2} e^{-\frac{\rho^2}{a^2}}}}$$

$\underline{\underline{\vec{K} = 0}}$

V osi z:

$$I = \lim_{\rho\to 0} \oint \vec{H}\,d\vec{s} = \lim_{\rho\to 0}\int_0^{2\pi} \vec{1}_\varphi \frac{A}{\rho} e^{-\frac{\rho^2}{a^2}} \cdot \vec{1}_\varphi \rho\, d\varphi = \underline{\underline{2\pi A}}$$

④ $\omega \neq 0$



$$\vec{E} = \left(\vec{1}_x \cos\frac{\pi}{a}y + j\vec{1}_y\cos\frac{\pi}{a}x\right)\cdot E_0 \cdot e^{-j\beta_z z}$$

$$k^2 = \omega^2\mu_0\varepsilon_0 = (\pi/a)^2 + \beta_z^2$$

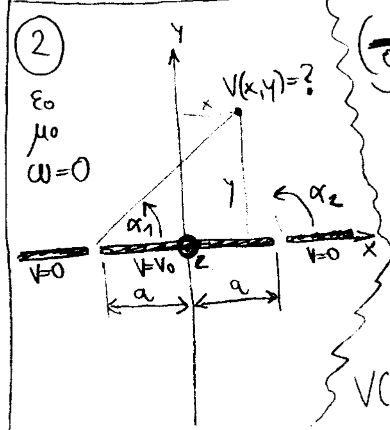
$P = ?$ (v cevi)

$$\vec{H} = \frac{j}{\omega\mu_0}\operatorname{rot}\vec{E} = \frac{j}{\omega\mu_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & 0\end{vmatrix} = -j\vec{1}_x \frac{\beta_z}{\omega\mu_0}\cos\frac{\pi}{a}x\, e^{-j\beta_z z}E_0 +$$
$$+ \vec{1}_y \frac{\beta_z}{\omega\mu_0}\cos\frac{\pi}{a}y\, e^{-j\beta_z z}E_0 + \vec{1}_z\frac{\pi/a}{\omega\mu_0}\left(\sin\frac{\pi}{a}y - j\sin\frac{\pi}{a}x\right)e^{-j\beta_z z}E_0$$

$$\vec{1}_z\cdot\vec{S} = \vec{1}_z\cdot\left(\frac{1}{2}\vec{E}\times\vec{H}^*\right) = \frac{1}{2}\left(E_x H_y^* - E_y H_x^*\right) = \underline{\frac{\beta_z|E_0|^2}{2\omega\mu_0}\left(\cos^2\frac{\pi}{a}y + \cos^2\frac{\pi}{a}x\right)}$$

$$P = \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \vec{S}\cdot\vec{1}_z\, dA = \underline{\underline{\frac{\beta_z|E_0|^2}{2\omega\mu_0}a^2}}$$

② ε_0, μ_0, $\omega = 0$



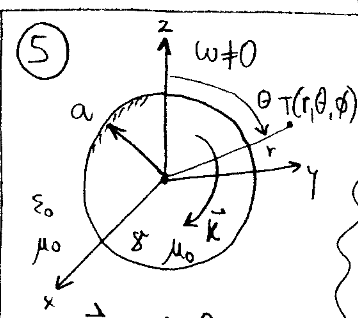
$$\underbrace{\left(\underset{0}{\text{—}}\ \underset{V_0}{\text{—}}\ \underset{0}{\text{—}}\right)}_{V(x,y)} = \underbrace{\left(\underset{-\frac{V_0}{2}}{\text{—}}\ \underset{+\frac{V_0}{2}}{\text{—}}\ \underset{+\frac{V_0}{2}}{\text{—}}\right)}_{V_1(x,y)} + \underbrace{\left(\underset{+\frac{V_0}{2}}{\text{—}}\ \underset{+\frac{V_0}{2}}{\text{—}}\ \underset{-\frac{V_0}{2}}{\text{—}}\right)}_{V_2(x,y)}$$

$$V_1(x,y) = +\frac{V_0}{2} - \frac{V_0}{\pi}\alpha_1 = \frac{V_0}{2} - \frac{V_0}{\pi}\operatorname{arctg}\frac{y}{x+a}$$
$$V_2(x,y) = -\frac{V_0}{2} + \frac{V_0}{\pi}\alpha_2 = -\frac{V_0}{2} + \frac{V_0}{\pi}\operatorname{arctg}\frac{y}{x-a}$$

$$\underline{\underline{V(x,y) = \frac{V_0}{\pi}\left[\operatorname{arctg}\frac{y}{x-a} - \operatorname{arctg}\frac{y}{x+a}\right]}}$$

PAZI KVADRANT arctg !

⑤



ε_0, μ_0

$\vec{K} = \vec{1}_\theta K_0 \sin\theta$

$P = ?$

$$\delta = \sqrt{\frac{2}{\omega\mu_0\gamma}} \qquad R_p = \frac{1}{\gamma\delta} = \sqrt{\frac{\omega\mu_0}{2\gamma}}$$

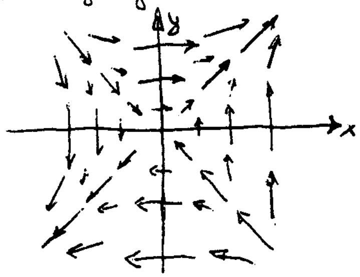
$$\frac{dP}{dA} = \frac{1}{2}|\vec{K}|^2 R_p$$

$$P = \int_0^{\pi}\int_0^{2\pi}\frac{dP}{dA}\,a^2\sin\theta\, d\theta\, d\phi = \int_0^{\pi}\int_0^{2\pi}\frac{1}{2}|K_0|^2\sin^2\theta R_p a^2\sin\theta\, d\theta\, d\phi$$

$$P = \pi|K_0|^2 a^2 R_p\int_0^{\pi}\sin^3\theta\, d\theta = \underline{\underline{\frac{4}{3}\pi|K_0|^2 a^2\sqrt{\frac{\omega\mu_0}{2\gamma}}}}$$

1\. kolokvij iz EM 2.12.2004

(1) $\vec{F} = \vec{1}_x y + \vec{1}_y x$



$\operatorname{div} \vec{F} = \frac{\partial y}{\partial x} + \frac{\partial x}{\partial y} = 0 \Rightarrow$ NI IZVOROV

$$\operatorname{rot} \vec{F} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & 0 \end{vmatrix} = \vec{1}_x 0 + \vec{1}_y \cdot 0 + \vec{1}_z (1-1) = 0$$

NI VRTINCEV

(2) element ploskve $dA = r^2 \sin\theta \, d\theta \, d\phi$

površina izseka $A = \int_0^{2\pi} \int_\delta^\beta a^2 \sin\theta \, d\theta \, d\phi = 2\pi a^2 (\cos\delta - \cos\beta)$

Ko je $\delta = 0$ in $\beta = \pi$ je $A = 4\pi a^2$ celotna krogla

$$\phi = \frac{A}{4\pi a^2} Q = \frac{Q}{2} (\cos\delta - \cos\beta)$$

Ko je $\delta = 0$ in $\beta = \frac{\pi}{2}$ imamo polkroglo ploskev katero je pretok $\phi_{1/2} = \frac{Q}{2}$

(3)

$$\vec{H} = \frac{1}{\mu_0} \operatorname{rot} \vec{A} = \frac{1}{\mu_0} \frac{1}{\varrho} \begin{vmatrix} \vec{1}_\varrho & \varrho \vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial \varrho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & 0 & A_0 \frac{\cos\varphi}{\varrho} \end{vmatrix} =$$

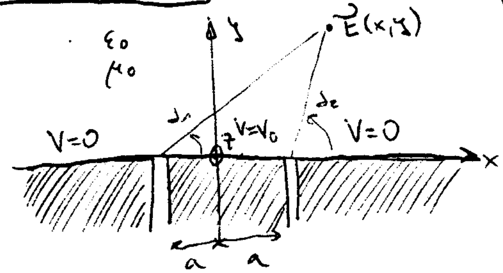
$$= \frac{1}{\mu_0 \varrho} \left(-\vec{1}_\varrho A_0 \frac{\sin\varphi}{\varrho} + \varrho \vec{1}_\varphi A_0 \frac{\cos\varphi}{\varrho^2} \right) =$$

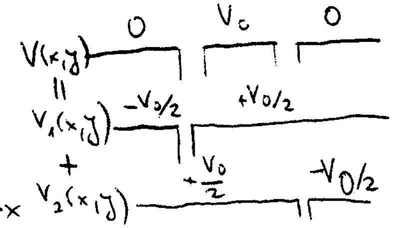
$$= \frac{A_0}{\mu_0 \varrho} \left(-\vec{1}_\varrho \frac{\sin\varphi}{\varrho} + \vec{1}_\varphi \frac{\cos\varphi}{\varrho} \right)$$

$$\vec{J} = \operatorname{rot} \vec{H} = \frac{1}{\varrho} \begin{vmatrix} \vec{1}_\varrho & \varrho \vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial \varrho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ -\frac{A_0 \sin\varphi}{\mu_0 \varrho^2} & \frac{A_0 \cos\varphi}{\mu_0 \varrho} & 0 \end{vmatrix} = 0$$

$$I = \oint \vec{H} \, ds = \frac{A_0}{\mu_0} \int_0^{2\pi} \left(-\vec{1}_\varrho \frac{\sin\varphi}{\varrho^2} + \vec{1}_\varphi \frac{\cos\varphi}{\varrho} \right) \vec{1}_\varphi \varrho \, d\varphi =$$

$$= \frac{A_0}{\mu_0} \int_0^{2\pi} \cos\varphi \, d\varphi = \frac{A_0}{\mu_0} \left[\sin\varphi \right]_0^{2\pi} = 0$$





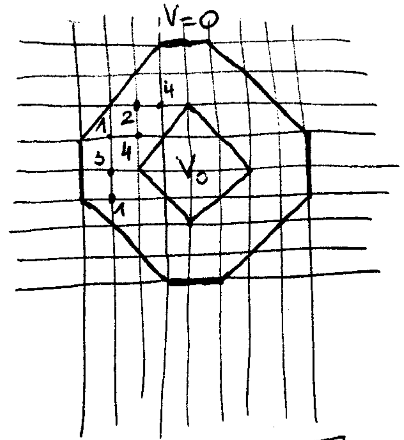
$$V_1(x,y) = +\frac{V_0}{2} - \frac{V_0}{\pi} \delta_1 = \frac{V_0}{2} - \frac{V_0}{\pi} \operatorname{arctg} \frac{y}{x+a}$$

$$V_2(x,y) = -\frac{V_0}{2} + \frac{V_0}{\pi} \delta_1 = -\frac{V_0}{2} + \frac{V_0}{\pi} \operatorname{arctg} \frac{y}{x-a}$$

$$V(x,y) = \frac{V_0}{\pi} \left[\operatorname{arctg} \frac{y}{x-a} - \operatorname{arctg} \frac{y}{x+a} \right]$$

Pazi na kvadrant!

$\vec{E} = -\operatorname{grad} V = \ldots$



$\underline{V_4 = V_5}$

$4V_1 = 0 + 0 + V_4 + V_3$

$4V_2 = 0 + 0 + V_4 + V_4$

$4V_3 = 0 + V_1 + V_0 + V_1$

$4V_4 = V_1 + V_2 + V_0 + V_0$

$4V_1 = V_4 + V_3$

$4V_2 = 2V_4 \Longrightarrow V_4 = 2V_2$

$4V_3 = 2V_1 + V_0$

$4V_4 = V_1 + V_2 + 2V_0$

$4V_1 = 2V_2 + V_3 \Rightarrow V_3 = 4V_1 - 2V_2$

$4V_3 = 2V_1 + V_0$

$8V_2 = V_1 + V_2 + 2V_0$

$16V_1 - 8V_2 = 2V_1 + V_0$

$8V_2 = V_1 + V_2 + 2V_0$

$14V_1 = 8V_2 + V_0$

$7V_2 = V_1 + 2V_0 \Rightarrow V_1 = 7V_2 - 2V_0$

$14(7V_2 - 2V_0) = 8V_2 + V_0$

$98V_2 - 28V_0 = 8V_2 + V_0$

$90V_2 = 29V_0$

$\boxed{V_2 = \frac{29}{90} V_0}$

$V_1 = 7V_2 - 2V_0 = \boxed{\frac{23}{90} V_0 = V_1}$

$V_3 = 4V_1 - 2V_2$

$\boxed{V_3 = \frac{34}{90} V_0}$

$\boxed{V_4 = \frac{58}{90} V_0}$

2. kolokvij EM 17.1.2005

(1) $A'_{\square} = a^2$

$A'_{\circ} = \pi a^2$

$$\vec{A} = \vec{1}_\phi \frac{\mu_0 I A'}{4\pi r^2} \sin\theta$$

$$\vec{H} = \frac{I A'}{4\pi r^3}\left(\vec{1}_r 2\cos\theta + \vec{1}_\theta \sin\theta\right) \text{ polje zanke}$$

$$\vec{H} = \frac{IA'}{4\pi}\left(\vec{1}_r\left(\frac{1}{r_0^3}+\frac{2}{r^3}\right)\cos\theta + \vec{1}_\theta\left(-\frac{1}{r_0^3}+\frac{1}{r^3}\right)\sin\theta\right)$$

(2) $M_{00} = \dfrac{\pi \mu_0 a^4}{2d^3}$

$$M_{0\square} = \frac{1}{I}\oint \vec{1}_\phi \frac{\mu I x^2}{4\pi d^2}\,\frac{a}{d}\,\vec{1}_\phi\, a\, d\varphi$$

$$M_{0\square} = \frac{\mu_0 x^2 a^2}{2d^3}$$

$M_{00} = M_{0\square}$

$$\frac{\pi\mu_0 a^4}{2d^3} = \frac{\mu_0 x^2 a^2}{2d^3}$$

$\pi a^2 = x^2$ Površina mora ostati ista

$x = a\sqrt{\pi}$

(3)

$$\vec{H} = \frac{j}{\omega\mu_0}\,\mathrm{rot}\,\vec{E} = \frac{j}{\omega\mu_0}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & \cancel{r\sin\theta}\, E_0 \frac{e^{-j\beta r}}{\cancel{r\sin\theta}} \end{vmatrix} = -\vec{1}_\theta \frac{E_0}{Z_0}\frac{e^{-j\beta r}}{r\sin\theta}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\begin{vmatrix} \vec{1}_r & \vec{1}_\theta & \vec{1}_\phi \\ 0 & 0 & E_0\frac{e^{-j\beta r}}{r\sin\theta} \\ 0 & -\frac{E_0^*}{Z_0}\frac{e^{+j\beta r}}{r\sin\theta} & 0 \end{vmatrix} = \frac{1}{2}\vec{1}_r \frac{E_0 E_0^*}{Z_0}\frac{1}{r^2\sin^2\theta}$$

$$\vec{J} = \mathrm{rot}\,\vec{H} - j\omega\varepsilon\vec{E} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & -\cancel{r}\frac{E_0}{Z_0}\frac{e^{-j\beta r}}{\cancel{r}\sin\theta} & 0 \end{vmatrix} - j\omega\varepsilon\,\vec{1}_\phi E_0 \frac{e^{-j\beta r}}{r\sin\theta} = 0$$

(4) $I = \oint_\Delta \vec{H}\,d\vec{s} = \int_0^{2\pi} \vec{1}_\varphi H\,\vec{1}_\varphi\,\varrho\,d\varphi = H\varrho 2\pi$ $\qquad \vec{H} = \vec{1}_\varphi \dfrac{I}{2\pi\varrho}e^{-j\beta z}$

$$\mathrm{rot}\,\vec{H} = \underset{=0}{\vec{J}} + j\omega\varepsilon\vec{E} \qquad \vec{E} = \frac{1}{j\omega\varepsilon}\,\mathrm{rot}\,\vec{H} = \frac{1}{j\omega\varepsilon}\frac{1}{\varrho}\begin{vmatrix} \vec{1}_\varrho & \varrho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\varrho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \varrho\frac{I}{2\pi\varrho}e^{-j\beta z} & 0 \end{vmatrix} = \frac{1}{j\omega\varepsilon}\frac{1}{\varrho}\left[\underset{=0}{\vec{1}_z\frac{\partial}{\partial\varrho}\left(\frac{I}{2\pi}e^{-j\beta z}\right)} - \vec{1}_\varrho\frac{\partial}{\partial z}\left(\frac{I}{2\pi}e^{-j\beta z}\right)\right]$$

$$\vec{E} = \vec{1}_\varrho \frac{I}{2\pi\varrho\sqrt{\varepsilon_r}}Z_0 e^{-j\beta z} = \vec{1}_\varrho I Z_0 e^{-j\beta z}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\begin{vmatrix} \vec{1}_\varrho & \vec{1}_\varphi & \vec{1}_z \\ IZ_0 e^{-j\beta z} & 0 & 0 \\ 0 & \frac{I^*}{2\pi\varrho}e^{-j\beta z} & 0 \end{vmatrix} = \frac{1}{2}\vec{1}_z \frac{II^* Z_0}{2\pi\varrho}$$

(5)

$$f = \frac{1}{2\sqrt{\varepsilon_0\varepsilon_r\mu_0}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{c}\right)^2}$$

m, n, p so cela števila

vsaj dva sta različna od 0!

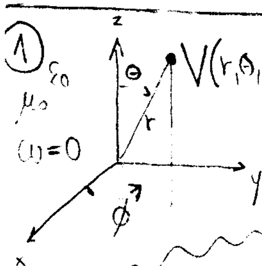
$m = 1$, $n = 0$, $p = 1$

$$f^2 = \frac{1}{4\varepsilon_0\varepsilon_r\mu_0}\left[\left(\frac{1}{a}\right)^2+\left(\frac{1}{c}\right)^2\right] \Rightarrow \varepsilon_r = \frac{c_0^2}{4f^2}\left(\frac{1}{a^2}+\frac{1}{c^2}\right) = 3{,}25$$

ELEKTROMAGNETIKA 7/7/2005

1

ε_0, μ_0, $\omega=0$



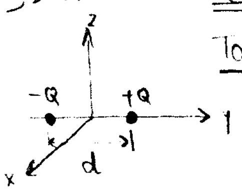
$$V(r,\theta,\phi) = \frac{C}{r^2}\sin\theta\sin\phi$$

Izvori polja = ?

$\sin\theta\sin\phi = \cos\theta_y$

$$V = \frac{C}{r^2}\cos\theta_y \Rightarrow \Delta V = 0 \;;\; \underline{\underline{\rho = 0}}$$

$$\oint \vec{D}\cdot d\vec{A} = 0 \Rightarrow \underline{\underline{Q=0,\ q=0}}$$

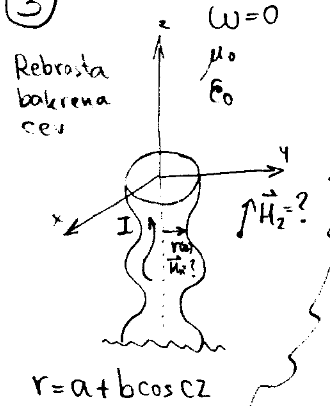


Točkasti dipol Qd v smeri y:

$$V = \frac{Qd}{4\pi\varepsilon_0}\frac{\cos\theta_y}{r^2}$$

$$\underline{\underline{Qd = 4\pi\varepsilon_0 C}}$$

3



$$I = \oint \vec{H}\cdot d\vec{s}$$

$$\underline{\underline{\vec{H}_n = 0}}$$

$$\underline{\underline{\vec{H}_z = \vec{1}_\varphi \frac{I}{2\pi\rho}}}$$

4

$\vec{k}_1 = (\vec{1}_x + \vec{1}_y + \vec{1}_z)$ rd/m

$\vec{k}_2 = (\vec{1}_x - \vec{1}_y + \vec{1}_z)$ rd/m

$\vec{E}(0,0,0) = 0$; μ_0, ε_0

$f = ?$; $\vec{1}_E = ?$

$$k = \frac{\omega}{c_0} = \frac{2\pi f}{c_0} \rightarrow f = \frac{c_0}{2\pi}|\vec{k}_1| = \underline{\underline{82.7\ \mathrm{MHz}}}$$

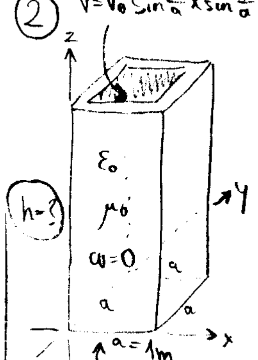
$$\vec{E}_1 = \vec{E}_{10}e^{-j\vec{k}_1\cdot\vec{r}} \qquad \vec{E} = \vec{E}_1 + \vec{E}_2 = \vec{E}_{10}e^{-j\vec{k}_1\cdot\vec{r}} + \vec{E}_{20}e^{-j\vec{k}_2\cdot\vec{r}}$$

$$\vec{E}_2 = \vec{E}_{20}e^{-j\vec{k}_2\cdot\vec{r}} \qquad \vec{E}(0,0,0) = \vec{E}_{10} + \vec{E}_{20} \rightarrow \vec{E}_{10} = -\vec{E}_{10}$$

$$\left.\begin{array}{l}\vec{E}_{10}\cdot\vec{k}_1 = 0\\ \vec{E}_{10}\cdot\vec{k}_2 = 0\end{array}\right\} \vec{1}_E = \pm\frac{\vec{k}_1\times\vec{k}_2}{|\vec{k}_1\times\vec{k}_2|} = \pm\frac{2(\vec{1}_x - \vec{1}_z)}{|2(\vec{1}_x - \vec{1}_z)|} = \underline{\underline{\pm\frac{\vec{1}_x - \vec{1}_z}{\sqrt{2}}}}$$

2

$V = V_0\sin\frac{\pi}{a}x\sin\frac{\pi}{a}y$



(h=?)

$|Q_{dno}| = 1\%|Q|$

$$\underline{V = C\sin\frac{\pi}{a}x\sin\frac{\pi}{a}y\,\mathrm{sh}\frac{\pi\sqrt{2}}{a}z} \;;\; C = \frac{V_0}{\mathrm{sh}\frac{\pi\sqrt{2}}{a}h}$$

$$\vec{E} = -\mathrm{grad}\,V \;;\; Q_{dno} = \int_{A_{dno}}\vec{E}\cdot\vec{1}_z\,dA \;;\; Q = \int_{A_{pokrov}}\vec{E}\cdot(-\vec{1}_z)\,dA$$

$$\underline{E_z = -C\sin\frac{\pi}{a}x\sin\frac{\pi}{a}y\frac{\pi\sqrt{2}}{a}\mathrm{ch}\frac{\pi\sqrt{2}}{a}z}$$

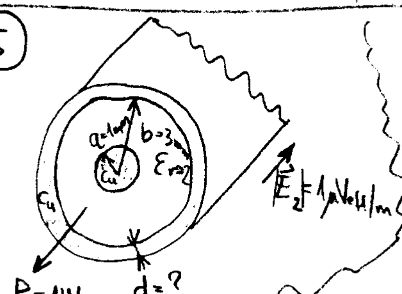
$$\left|\frac{Q}{Q_{dno}}\right| = 100 = \mathrm{ch}\frac{\pi\sqrt{2}}{a}h \rightarrow h = \frac{a}{\pi\sqrt{2}}\,\mathrm{arch}\,100$$

$\mathrm{ch}\,u = \frac{1}{2}(e^u + e^{-u})$ $\qquad u = \ln(\mathrm{ch}\,u + \sqrt{\mathrm{ch}^2u - 1}) = \underline{5.298}$

$(e^u)^2 - 2\,\mathrm{ch}\,u\,e^u + 1 = 0$ $\qquad \underline{\underline{h = 1.193\ \mathrm{m}}}$

$$e^u = \frac{2\,\mathrm{ch}\,u \pm\sqrt{4\,\mathrm{ch}^2u - 4}}{2}$$

5



$P = 1\ \mathrm{W}$

$\gamma_{Cu} = 56\cdot10^6\ \mathrm{S/m}$

$f = 10\ \mathrm{MHz}$

$Z_0 = 120\pi\ \Omega$

$\mu_0 = 4\pi\cdot10^{-7}\ \frac{\mathrm{Vs}}{\mathrm{Am}}$

$$Z_k = \frac{Z_0}{2\pi\sqrt{\varepsilon_r}}\ln\frac{b}{a} = \underline{46.6\ \Omega}$$

$$\text{Plašč: } K_{eff} = \frac{I_{eff}}{2\pi b} = \frac{1}{2\pi b}\sqrt{\frac{P}{Z_k}} = \underline{\underline{7.77\ \mathrm{A/m}}}$$

$$R_p = \frac{1}{\gamma_{Cu}\delta} = \underline{0.84\ \mathrm{m}\Omega} \qquad \delta = \sqrt{\frac{2}{\omega\mu_0\gamma_{Cu}}} = \underline{21.27\ \mu\mathrm{m}}$$

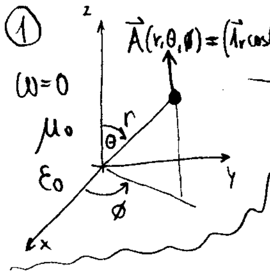
$$E_n = K_{eff}\cdot R_p = \underline{6.52\ \mathrm{mV_{eff}/m}}$$

$$\frac{E_n}{E_z} = e^{\frac{d}{\delta}} \rightarrow d = \delta\ln\frac{E_n}{E_z} = \underline{\underline{187\ \mu\mathrm{m}}}$$

Pisni izpit iz ELEKTROMAGNETIKE

29.09.2005

① $\omega=0$, μ_0, ε_0



$$\vec{A}(r,\theta,\phi)=(\vec{1}_r\cos\theta-\vec{1}_\theta\sin\theta)\,C\ln(r\sin\theta)$$

$\vec{J}=?$, $\vec{K}=?$, $I=?$

$$\vec{1}_r\cos\theta-\vec{1}_\theta\sin\theta=\vec{1}_z$$

$$r\sin\theta=\rho$$

$$\vec{A}=\vec{1}_z C\ln\rho$$

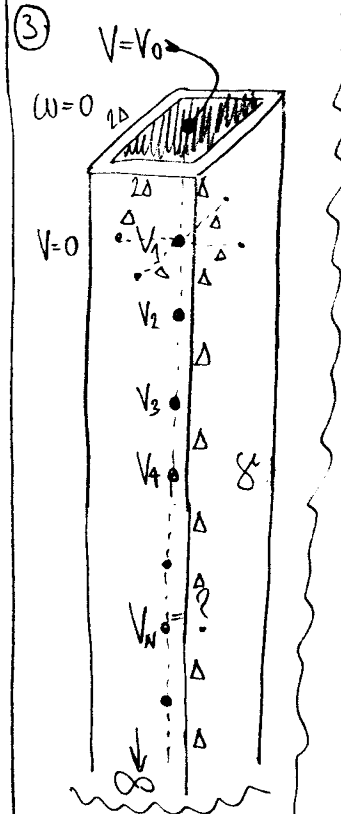
$$\vec{H}=\frac{1}{\mu_0}\operatorname{rot}\vec{A}=\frac{1}{\rho}\begin{vmatrix}\vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z}\\ 0 & \rho 0 & C\ln\rho\end{vmatrix}=-\vec{1}_\varphi\frac{C}{\mu_0\rho}$$

$$\vec{J}=\operatorname{rot}\vec{H}=\frac{1}{\rho}\begin{vmatrix}\vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z}\\ 0 & -\rho\frac{C}{\mu_0\rho} & 0\end{vmatrix}=\underline{\underline{0}}$$

V osi z:

$$I=\oint\vec{H}\cdot d\vec{s}=\int_0^{2\pi}-\vec{1}_\varphi\frac{C}{\mu_0\rho}\cdot\vec{1}_\varphi\rho\,d\varphi=\underline{\underline{-2\pi C/\mu_0}}$$

③ $V=V_0$, $\omega=0$, $V=0$



$$6V_N=V_{N-1}+V_{N+1}+0+0+0+0$$

Iščemo rešitev v obliki:

$$V_N=V_0a^N\ ;\ 0<a<1$$

$$6V_0a^N=V_0a^{N-1}+V_0a^{N+1}$$

$$0=a^2-6a+1$$

$$a=\frac{6\pm\sqrt{36-4}}{2}=3\pm2\sqrt{2}$$

$$a=3-2\sqrt{2}=\underline{0.1716}$$

$$\underline{\underline{V_N=V_0\ 0.1716^N}}$$

④ $\varepsilon_r=80$, $f=100\,\mathrm{MHz}$, $\mu=\mu_0$, $\varepsilon_0=\frac{1}{4\pi\cdot9\cdot10^9}\frac{As}{Vm}$

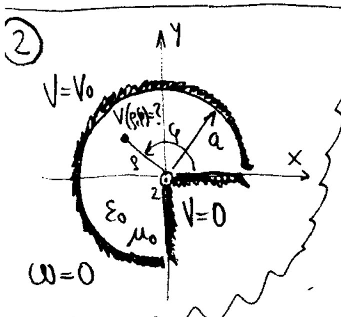
$\rho=?$ ($|I_{polni}|=|I_{prevodni}|$)

$$\operatorname{rot}\vec{H}=\vec{J}+j\omega\varepsilon\vec{E}=(\gamma+j\omega\varepsilon_0\varepsilon_r)\vec{E}$$

$$\gamma=\omega\varepsilon_0\varepsilon_r$$

$$\rho=\frac{1}{2\pi f\varepsilon_0\varepsilon_r}=\frac{4\pi\cdot9\cdot10^9\,Vm}{2\pi\cdot10^8\,s^{-1}As\ 80}=\underline{\underline{2{,}25\,\Omega m}}$$

② $V=V_0$, $\omega=0$



$$V(\rho,\varphi)=\sum_m C_m\rho^m\sin m\varphi$$

$$m=\frac{2}{3},\frac{4}{3},\frac{6}{3},\frac{8}{3}\ldots$$

$$V(\rho,\varphi)=\sum_{k=1}^{\infty}C_k\rho^{\frac{2}{3}k}\sin\tfrac{2}{3}k\varphi\ ;\ k=1,2,3,4\ldots\ ;\ V(a,\varphi)=V_0$$

$$\int_0^{3\pi/2}\sum_{k=1}^{\infty}C_k a^{\frac{2}{3}k}\sin\tfrac{2}{3}k\varphi\sin\tfrac{2}{3}l\varphi\,d\varphi=\int_0^{3\pi/2}V_0\sin\tfrac{2}{3}l\varphi\,d\varphi$$

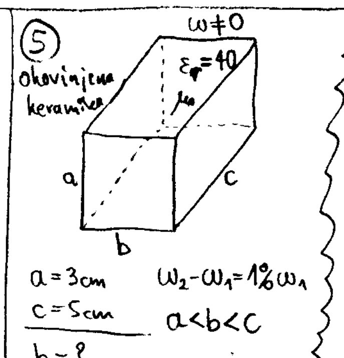
$$\frac{1}{2}C_l a^{\frac{2}{3}l}\frac{3\pi}{2}=V_0\frac{3}{2l}\left(-\cos\tfrac{2}{3}l\varphi\right)\Big|_0^{3\pi/2}=\frac{3V_0}{2l}(1-\cos\pi l)$$

$$C_l=\frac{2V_0}{\pi l}a^{-\frac{2}{3}l}(1-\cos\pi l)$$

sodi l → $C_l=0$

$$\underline{\underline{V(\rho,\varphi)=\sum_{l=1}^{\infty}\frac{2V_0(1-\cos\pi l)}{\pi l}\left(\frac{\rho}{a}\right)^{\frac{2}{3}l}\sin\tfrac{2}{3}l\varphi}}$$

⑤ $\omega\neq0$



$a=3\,cm$, $c=5\,cm$, $b=?$

$\omega_2-\omega_1=1\%\,\omega_1$, $a<b<c$

$$\omega=\frac{\pi C_0}{\sqrt{\varepsilon_r}}\sqrt{\left(\frac{l}{a}\right)^2+\left(\frac{m}{b}\right)^2+\left(\frac{n}{c}\right)^2}$$

$$\omega_1=\frac{\pi C_0}{\sqrt{\varepsilon_r}}\sqrt{\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}$$

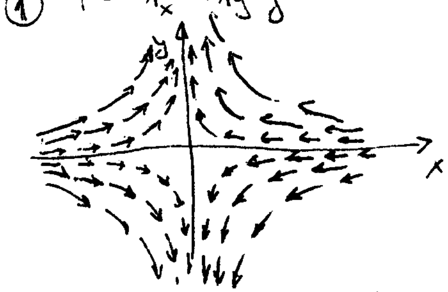
$$\omega_2=\frac{\pi C_0}{\sqrt{\varepsilon_r}}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{c}\right)^2}=1.01\,\omega_1$$

$$\left(\frac{1}{a}\right)^2+\left(\frac{1}{c}\right)^2=1.01^2\left[\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2\right]$$

$$b=\frac{1}{\sqrt{\frac{(1/a)^2+(1/c)^2}{1.01^2}-\left(\frac{1}{c}\right)^2}}=\underline{\underline{3.041\,cm}}$$

1. kolokvij iz EM - 24. 11. 2005

1 $\vec{F} = -\vec{1}_x x + \vec{1}_y y$



$\operatorname{div} \vec{F} = \frac{\partial}{\partial x}(-x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(0) = 0$ NI IZVOROV!

$\operatorname{rot} \vec{F} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x & y & 0 \end{vmatrix} = \vec{1}_x \cdot 0 + \vec{1}_y \cdot 0 + \vec{1}_z \cdot 0$ NI VRTINCEV!

2 $\vec{E} = \vec{1}_x \frac{U}{d}$

$\vec{H} = \vec{1}_z \frac{I}{a}$

$\vec{S} = \vec{E} \times \vec{H} = \left(\vec{1}_x \frac{U}{d}\right) \times \left(\vec{1}_z \frac{I}{a}\right) = -\vec{1}_y \frac{UI}{ad}$

$W = \frac{1}{2} \int_V \vec{E} \cdot \vec{D}\, dv = \frac{1}{2} \varepsilon_0 \int_V \vec{E} \cdot \vec{E}\, dv =$

$= \frac{1}{2} \varepsilon_0 \left(\frac{U}{d}\right)^2 abd = \frac{1}{2} \varepsilon_0 \frac{U^2}{d} ab$

3 $\vec{A} = (\vec{1}_\rho \sin\varphi + \vec{1}_\varphi \cos\varphi)\, \rho^2 \cos^2\varphi$

$\vec{A} = \vec{1}_y x^2$

$\vec{H} = \frac{1}{\mu} \operatorname{rot} \vec{A} = \frac{1}{\mu} \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x^2 & 0 \end{vmatrix} = \vec{1}_z 2x \frac{1}{\mu}$

$\vec{J} = \operatorname{rot} \vec{H} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 2x\frac{1}{\mu} \end{vmatrix} = -\vec{1}_y 2 \frac{1}{\mu}$

5 $\operatorname{rot}(\operatorname{rot} \vec{A}) = -\vec{1}_z \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial A_z}{\partial \rho}\right) = \mu \vec{J} = \mu \left(-\vec{1}_z \frac{K}{\rho}\right)$

I $\rho < b$

$0 = -\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial A_{z1}}{\partial \rho}\right) \Big| \int d\rho$

$C_1 = \rho \frac{\partial A_{z1}}{\partial \rho}$

$\frac{C_1}{\rho} = \frac{\partial A_{z1}}{\partial \rho} \Big| \int d\rho$

$C_1 \ln\rho + C_2 = A_{z1}$

II $b < \rho < a$

$\mu \frac{K}{\cancel{\rho}} = \frac{1}{\cancel{\rho}} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial A_{z2}}{\partial \rho}\right) \Big| \int d\rho$

$\mu K \rho + C_3 = \rho \frac{\partial A_{z2}}{\partial \rho}$

$\mu K + \frac{C_3}{\rho} = \frac{\partial A_{z2}}{\partial \rho} \Big| \int d\rho$

$\mu K + C_3 \ln\rho + C_4 = A_{z2}$

III $a < \rho$

$C_5 \ln\rho + C_6 = A_{z3}$

izberemo $C_4 = 0$

$C_1 = 0$ ker v osi ni prevega toka

meja $\rho = b$

$C_2 = A_{z1} = \mu K b + C_3 \ln b = A_{z2}$

$C_2 = \mu K b - \mu K b \ln b$

$\frac{\partial A_{z1}}{\partial \rho}\Big|_{\rho=b} = \frac{\partial A_{z2}}{\partial \rho}\Big|_{\rho=b}$

$C_3 = -\mu K b$

meja $\rho = a$ $\quad A_{z2}|_{\rho=a} = A_{z3}|_{\rho=a}$

$\mu K a + C_3 \ln a = C_5 \ln a + C_6$

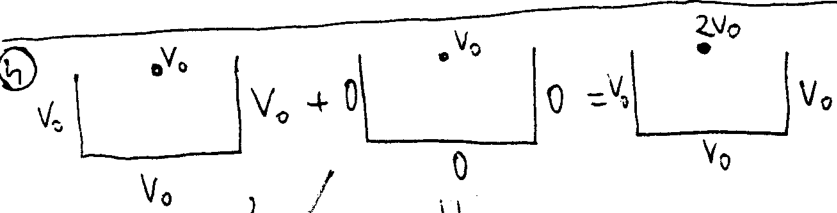
$C_6 = \mu K a (1 - \ln a)$

$\frac{\partial A_{z2}}{\partial \rho}\Big|_{\rho=a} = \frac{\partial A_{z3}}{\partial \rho}\Big|_{\rho=a}$

$\mu K + \frac{C_3}{a} = \frac{C_5}{a}$

$C_5 = 0$

4



$V(x,y) = V_0$

$V(x,y) = \sum_{n=1}^{\infty} B_n \sin\left(n \frac{\pi}{a} x\right) \operatorname{sh}\left(n \frac{\pi}{a} y\right)$

$\int_0^a V(x, y=b) \sin\left(\frac{n\pi}{a} x\right) dx = \int_0^a V(x) \sin\left(\frac{n\pi}{a} x\right) dx$

$\int_0^a \left[B_n \sin\left(n \frac{\pi}{a} x\right) \operatorname{sh}\left(n \frac{\pi}{a} b\right) \sin\left(\frac{n\pi}{a} x\right) dx = 2 \int_0^{a/2} \frac{V_0}{a/2} x \sin\left(\frac{n\pi}{a} x\right) dx\right.$

$B_n \operatorname{sh}\left(n \frac{\pi}{a} b\right) \left[\frac{x}{2} - \frac{a}{4n\pi} \sin\left(2 \frac{n\pi}{a} x\right)\right]_0^a = 2 \frac{V_0}{a/2} \left[-\frac{a}{n\pi} \cos\left(\frac{n\pi}{a} x\right) + \frac{a^2}{(n\pi)^2} \sin\left(\frac{n\pi}{a} x\right)\right]_0^{a/2}$

$B_n \operatorname{sh}\left(n \frac{\pi}{a} b\right) \frac{a}{2} = \frac{V_0}{a} \frac{a}{n\pi} \left[-\frac{a}{2} \underbrace{\cos\left(\frac{n\pi}{a} \frac{a}{2}\right)}_{0} + \frac{a}{n\pi} \sin\left(\frac{n\pi}{a} \frac{a}{2}\right)\right]$

$B_n = \frac{\frac{V_0}{a}\left(\frac{a}{n\pi}\right)^2}{\frac{a}{2} \operatorname{sh}\left(n \frac{\pi}{a} b\right)} = \frac{2V_0}{(n\pi)^2 \operatorname{sh}\left(n \frac{\pi}{a} b\right)}$

$V(x,y) = V_0 + \sum_{n=1}^{\infty} \frac{2V_0}{(n\pi)^2 \operatorname{sh}\left(n \frac{\pi}{a} b\right)} \sin\left(n \frac{\pi}{a} x\right) \operatorname{sh}\left(n \frac{\pi}{a} y\right)$

2. kolokvij EM 17. 1. 2006 (rešitve)

(1) $V_3 = V_7 = V_9 = 0$

$$\begin{aligned} 4V_1 &= 0 + V_0 + V_2 + 0 \\ 4V_2 &= V_1 + V_0 + 0 + V_6 \\ 4V_6 &= 0 + V_2 + 0 + 0 \end{aligned} \quad \left| \quad \begin{aligned} 4V_1 &= V_0 + V_2 \\ 4V_2 &= V_1 + V_0 + V_6 \\ V_2 &= 4V_6 \end{aligned} \right\} \quad \begin{aligned} 4V_2 &= \frac{V_0}{4} + \frac{V_2}{4} + V_2 + \frac{V_2}{4} \\ 4V_2 &= \frac{5}{4}V_0 + \frac{2}{4}V_2 \end{aligned}$$

$$16V_2 = 5V_0 + 2V_2$$

$$\boxed{V_2 = \frac{5}{14}V_0 = -V_4}$$

$$\boxed{V_6 = \frac{1}{4}\frac{5}{14}V_0 = \frac{5}{56}V_0 = -V_8}$$

$$V_1 = \frac{V_0}{4} + \frac{V_2}{4} = \frac{V_0}{4} + \frac{5}{56}V_0$$

$$\boxed{V_1 = \frac{19}{56}V_0 = -V_5}$$

(2) $\vec{A} = \vec{1}_\phi \frac{\mu_0 I_1 a^2}{4} \frac{\sin\theta}{r^2}$ $\qquad b \gg a$

$$\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1$$

$$M = \frac{1}{I_a}\oint_{\partial\Pi} \vec{A}\cdot d\vec{s} = \frac{I}{I_a}\int_0^{2\pi} \vec{1}_\phi \frac{\mu_0 I_1 a^2}{4}\frac{1}{b^2}\cdot\vec{1}_\phi\, b\, d\phi$$

$$\boxed{M = \frac{\pi\mu_0 a^2}{2b}}$$

(3) $$\vec{H} = \frac{j}{\omega\mu_0}\operatorname{rot}\vec{E} = \frac{j}{\omega\mu_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0\sin\left(\frac{\pi}{a}x\right)e^{-jkz} & 0 \end{vmatrix} =$$

$$\vec{H} = -\vec{1}_x\frac{E_0 k}{\omega\mu}\sin\left(\frac{\pi}{a}x\right)e^{-jkz} + \vec{1}_z\frac{jE_0}{\omega\mu a}\cos\left(\frac{\pi}{a}x\right)e^{-jkz} = \vec{1}_x H_x + \vec{1}_z H_z$$

LEVA STENA $x=0$

$$\vec{K}_{leva} = \vec{1}_x \times \vec{H}\Big|_{x=0} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 1 & 0 & 0 \\ \frac{-E_0 k}{\omega\mu}\underbrace{\sin\left(\frac{\pi}{a}x\right)}_{0}e^{-jkz} & 0 & \frac{jE_0\pi}{\omega\mu_0 a}\underbrace{\cos\left(\frac{\pi}{a}x\right)}_{1}e^{-jkz} \end{vmatrix}$$

$$\vec{K}_{leva} = -\vec{1}_y\frac{jE_0\pi}{\omega\mu a}e^{-jkz}$$

DESNA STENA $x=a$

$$\vec{K}_{desna} = -\vec{1}_x \times \vec{H}\Big|_{x=a} = -\vec{1}_y\frac{jE_0\pi}{\omega\mu a}e^{-jkz}$$

ZGORNJA STENA $y=b$

$$\vec{K}_{zgornja} = -\vec{1}_y \times \vec{H}\Big|_{y=b} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 0 & -1 & 0 \\ H_x & 0 & H_z \end{vmatrix}$$

$$\vec{K}_{zgornja} = \vec{1}_x\frac{jE_0\pi}{\omega\mu a}\cos\left(\frac{\pi}{a}x\right)e^{-jkz} + \vec{1}_z\frac{jE_0 k}{\omega\mu}\sin\left(\frac{\pi}{a}x\right)e^{-jkz}$$

SPODNJA STENA $y=-b$

$$\vec{K}_{spodnja} = -\vec{K}_{zgornja}$$

(4) pravokotni resonator

$$f_\square = \frac{c_0}{2}\sqrt{\frac{2}{a^2}}$$

valjni resonator

$$f_0 = \frac{2{,}405\, c_0}{2\pi r}$$

$$f_\square = f_0 \Rightarrow r = \frac{2{,}405 a}{\pi\sqrt{2}} = 5{,}4\text{ cm}$$

(5) $$\Gamma = \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}} = -\frac{1}{3}$$

$$\vec{E}_1 = \vec{1}_x E_v e^{-jk_0 z} + \vec{1}_x E_v \Gamma e^{+jk_0 z}$$

$$\vec{H}_1 = \vec{1}_y\frac{E_v}{Z_0}e^{-jk_0 z} - \vec{1}_y\frac{E_v}{Z_0}\Gamma e^{+jk_0 z}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ E_x & 0 & 0 \\ 0 & H_y^* & 0 \end{vmatrix} = \vec{1}_z\frac{1}{2}\left(E_x\cdot H_y^*\right)$$

$$= \vec{1}_z\frac{1}{2}\left(\frac{E_v E_v^*}{Z_0}\underbrace{e^{-jk_0 t}\cdot e^{+jk_0 t}}_{1} - \frac{E_v E_v^*}{Z_0}e^{-2jk_0 t}\Gamma + \frac{E_v E_v^*}{Z_0}\Gamma e^{2jk_0 z} - \frac{E_v E_v^*}{Z_0}\Gamma^2\underbrace{e^{+jk_0 z}\cdot e^{-jk_0 z}}_{1}\right)$$

$$= \vec{1}_z\frac{1}{2}\frac{E_v E_v^*}{Z_0}\left(1 + \Gamma\, 2j\sin(k_0 z) - \Gamma^2\right)$$

ELEKTROMAGNETIKA 17/2/2006

① $V(u,v,z) = C \,\mathrm{sh}\,u \cos v$

$x = f \,\mathrm{ch}\,u \cos v$ $\quad \mu_0, \varepsilon_0$

$y = f \,\mathrm{sh}\,u \sin v$ $\quad \omega = 0$

$z = z$

$h_z = 1$

$h_u = h_v = f\sqrt{\mathrm{sh}^2 u + \sin^2 v}$

$\rho = ?,\ \sigma = ?,\ q = ?,\ Q = ?$

$$\rho = \varepsilon_0 \Delta V = \varepsilon_0 \left[\frac{\frac{\partial^2 V}{\partial u^2} + \frac{\partial^2 V}{\partial v^2}}{f^2(\mathrm{sh}^2 u + \sin^2 v)} + \frac{\partial^2 V}{\partial z^2} \right] = \underline{\underline{0}}$$

$$\vec{D} = -\varepsilon_0 \,\mathrm{grad}\, V = -\vec{1}_u \frac{\varepsilon_0}{h_u} \mathrm{ch}\,u \cos v + \vec{1}_v \frac{\varepsilon_0}{h_v} \mathrm{sh}\,u \sin v$$

Na traku širine 2f $(-f<x<f, y=0) \rightarrow u=0$

$x = f\cos v$

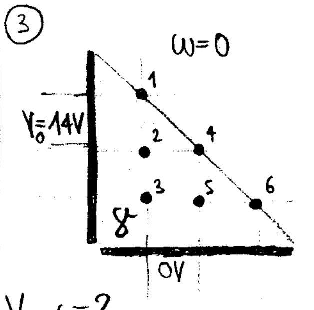
$\sin v = \sqrt{1-(x/f)^2}$

$$\sigma(-f<x<f, y=0) = 2\,\vec{1}_n \cdot \vec{D} =$$

$$= -2\frac{\varepsilon_0}{h_u}\cdot\frac{x}{f} = -2\frac{x\varepsilon_0}{f\sin v} = \underline{\underline{\frac{-2\varepsilon_0 x}{\sqrt{f^2-x^2}}}}$$

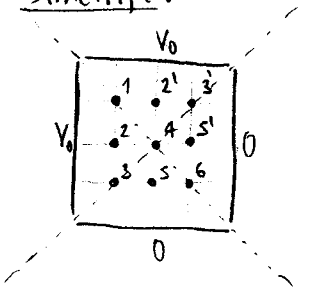
$\left.\begin{array}{l} \underline{\underline{q=0}} \\ \underline{\underline{Q=0}} \end{array}\right\}$ ni drugih singularnosti

③



$V_{1\ldots6} = ?$

Simetrija :



$$V_3 = V_4 = \frac{V_0}{2} = \underline{\underline{7V}}$$

$4V_1 = V_0 + V_0 + V_2 + V_2 = 2V_0 + 2V_2$

$4V_2 = V_0 + V_1 + V_4 + V_3 = 2V_0 + V_1$

$4V_5 = V_3 + V_4 + V_6 + 0 = V_0 + V_6$

$4V_6 = V_5 + V_5 + 0 + 0 = 2V_5$

$2V_1 = V_0 + V_2$ $\qquad 4V_5 = V_0 + V_6$

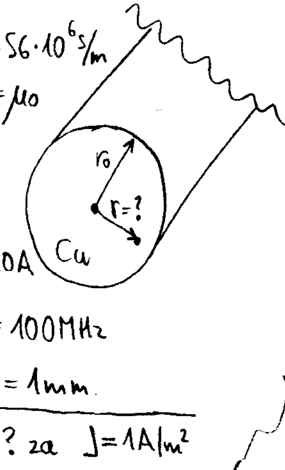
$4V_2 = 2V_0 + V_1$ $\qquad 2V_6 = V_5$

$V_2 = \frac{5}{7}V_0 = \underline{\underline{10V}};\ \underline{\underline{V_1 = 12V}};\quad V_6 = \frac{V_0}{7} = \underline{\underline{2V}};\ \underline{\underline{V_5 = 4V}}$

④

$\gamma = 56\cdot10^6\,\mathrm{S/m}$

$\mu = \mu_0$



$I = 10A$

$f = 100MHz$

$r_0 = 1mm$

$r = ?$ za $J = 1A/m^2$

$$\delta = \sqrt{\frac{2}{\omega\mu\gamma}}$$

$$\underline{\delta = 6.73\cdot10^{-6}\,m}$$

$$J_0 = \frac{I}{2\pi r_0 \delta} = \underline{2.37\cdot10^8\,A/m^2}$$

$$J = J_0 e^{-\frac{r_0 - r}{\delta}}$$

$$-\frac{r_0 - r}{\delta} = \ln\frac{J}{J_0} \qquad r_0 - r = \delta \ln\frac{J_0}{J}$$

$$r = r_0 - \delta \ln\frac{J_0}{J} = 1mm - 6.73\mu m \cdot 19.28 =$$

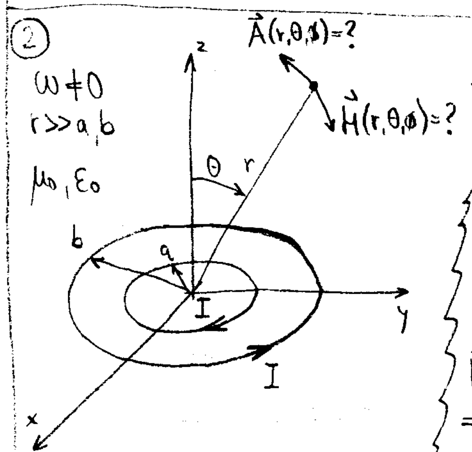
$$= 1mm - 0{,}1297mm = \underline{\underline{0{,}8703mm}}$$

②

$\omega \neq 0$

$r \gg a, b$

μ_0, ε_0



$$\vec{A}_1 = \vec{1}_\phi \frac{\mu_0 I}{4\pi}(-\pi a^2)\frac{e^{-jkr}}{r}\left(jk + \frac{1}{r}\right)\sin\theta$$

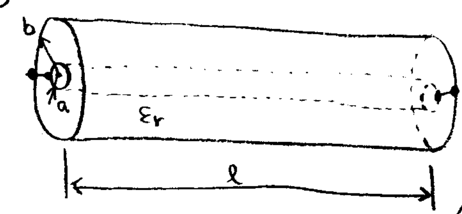
$$\vec{A}_2 = \vec{1}_\phi \frac{\mu_0 I}{4\pi}(\pi b^2)\frac{e^{-jkr}}{r}\left(jk + \frac{1}{r}\right)\sin\theta$$

$$\underline{\underline{\vec{A} = \vec{A}_1 + \vec{A}_2 = \vec{1}_\phi \frac{\mu_0 I}{4}(b^2 - a^2)\frac{e^{-jkr}}{r}\left(jk + \frac{1}{r}\right)\sin\theta}}$$

$$\vec{H} = \frac{1}{\mu_0}\mathrm{rot}\vec{A}$$

$$\underline{\underline{\vec{H} = \frac{I}{4}(b^2 - a^2)\frac{e^{-jkr}}{r}\left[\vec{1}_r\left(\frac{jk}{r} + r^2\right)2\cos\theta + \vec{1}_\theta\left(-k^2 + \frac{jk}{r} + \frac{1}{r^2}\right)\sin\theta\right]}}$$

⑤



$a = 1mm$ $\quad \varepsilon_r = 2{,}6$

$b = 3mm$ $\quad$ TEM valovi

$l = 1m$

$f_{res} = ?$ (najniža)

$$\vec{E} = \vec{1}_\rho \frac{C}{\rho}\sin kz \qquad \vec{E}(z=0) = 0 \qquad \vec{E}(z=l) = 0$$

$$\sin kl = 0 \rightarrow kl = m\pi\ ;\ m = 1,2,3,\ldots$$

$$k = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} \qquad \omega = \frac{m\pi}{l\sqrt{\mu_0\varepsilon_0\varepsilon_r}}$$

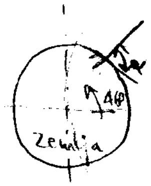
$$f_{res} = \frac{\omega}{2\pi} = \frac{m c_0}{2l\sqrt{\varepsilon_r}} = \underline{m\cdot 93.03MHz}$$

najniža $\underline{\underline{f_{res} = 93.03MHz}}$

ELEKTROMAGNETIKA

10.7.2006

① $\vec{H} = C\left(\vec{1}_r \frac{2\cos\theta}{r^3} + \vec{1}_\theta \frac{\sin\theta}{r^3}\right)$

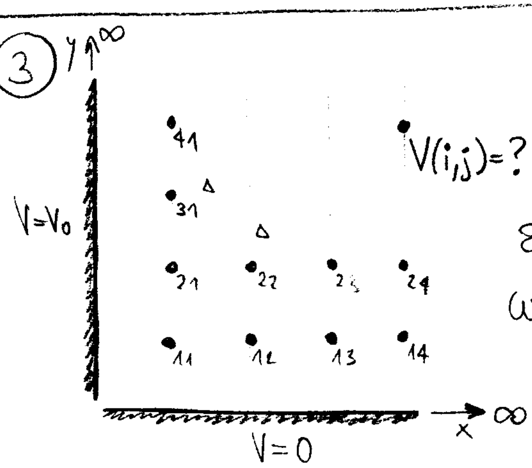


$\alpha = \operatorname{arc\,tg} \frac{H_r}{H_\theta} =$ $\qquad \theta = \frac{\pi}{2} - 46^\circ = 44^\circ$

$= \operatorname{arc\,tg} \frac{2\cos\theta}{\sin\theta} = \operatorname{arc\,tg}\left(\frac{2}{\operatorname{tg}\theta}\right)$

$\underline{\underline{\alpha = 64.227^\circ = 1.121\,\text{rd}}}$

③



ε_0

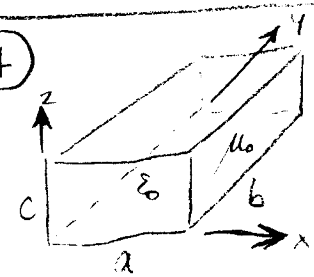
$\omega = 0$

$V = V_0 \frac{2}{\pi}\varphi \qquad \varphi = \operatorname{arc\,tg}\frac{y}{x} \qquad x = j\Delta$

$\varphi = \operatorname{arc\,tg}\frac{i}{j} \qquad y = i\Delta$

$\underline{\underline{V(i,j) = V_0 \frac{2}{\pi} \operatorname{arc\,tg}(i/j)}}$

④



$a = 10\,\text{cm}$

$\vec{E} = C \sin\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$

$f(m=1, n=3) = f(m=2, n=1)$

$b = ? \qquad f = ?$

$f = \frac{c_0}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{3}{b}\right)^2} = \frac{c_0}{2}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$

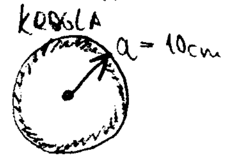
$\frac{1}{a^2} + \frac{9}{b^2} = \frac{4}{a^2} + \frac{1}{b^2} \rightarrow \frac{8}{b^2} = \frac{3}{a^2}$

$b = \sqrt{\frac{8}{3}}\,a = \underline{\underline{16.33\,\text{cm}}}$

$f = \frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{3}{b}\right)^2} = \underline{\underline{3.138\,\text{GHz}}}$

②

KOVINSKA KROGLA



ε_0

$\omega = 0$

ENAKA KAPACITIVNOST

KOVINSKA KROŽNA PLOŠČA $\quad b = ?$

$C_{krogle} = 4\pi\varepsilon_0 a$

$C_{plošče} = 8\varepsilon_0 b$

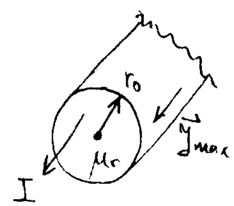
$b = \frac{4\pi\varepsilon_0}{8\varepsilon_0}a = \frac{\pi}{2}a = \underline{\underline{15.7\,\text{cm}}}$

⑤ Jeklena žica $\quad f = 100\,\text{MHz}$

$r_0 = 0.5\,\text{mm} \quad \gamma = 3\cdot 10^6\,\text{S/m}$

$I = 1\,\text{A} \quad J_{max} = 10^8\,\text{A/m}^2$

$\mu_r = ? \qquad \delta \ll r_0$



$J_{max} = \frac{I}{2\pi r_0 \delta} \qquad \delta = \sqrt{\frac{2}{\omega\mu_0\mu_r\gamma}} = \sqrt{\frac{1}{\pi f \mu_0\mu_r\gamma}}$

$\left(\frac{I}{2\pi r_0 J_{max}}\right)^2 = \frac{1}{\pi f \mu_0 \mu_r \gamma} \qquad \underline{\mu_0 = 4\pi\cdot 10^{-7}\,\frac{\text{Vs}}{\text{Am}}}$

$\mu_r = \frac{4\pi r_0^2 J_{max}^2}{I^2 f \mu_0 \gamma} = \underline{\underline{83.3}}$

ELEKTROMAGNETIKA

27.9.2006

① dani vektorji $\vec{A}, \vec{B}, \vec{C}$

$\vec{A}\times\vec{B} \neq 0$; $\vec{B}\times\vec{C} \neq 0$; $\vec{C}\times\vec{A} \neq 0$

poiskati $\vec{D} = ?$ da velja $\vec{D}\cdot\vec{A} = \vec{D}\cdot\vec{B} = \vec{D}\cdot\vec{C} = 0$

$$\left.\begin{matrix}\vec{D}\cdot\vec{A}=0\\ \vec{D}\cdot\vec{B}=0\end{matrix}\right\} \rightarrow \vec{D} \parallel \vec{A}\times\vec{B}$$

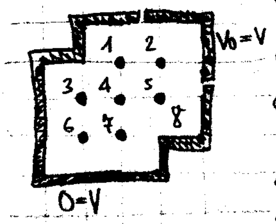
$$\left.\begin{matrix}\vec{D}\cdot\vec{B}=0\\ \vec{D}\cdot\vec{C}=0\end{matrix}\right\} \rightarrow \vec{D} \parallel \vec{B}\times\vec{C}$$

$\vec{D} = 0$ ali $\vec{A} \parallel \vec{C}$

ni možno zaradi $\vec{C}\times\vec{A} \neq 0$

$$\underline{\underline{\vec{D} = 0}}$$

③ $\omega = 0$



$V_1 = V_5$

$V_3 = V_7$

$4V_1 = V_2 + V_4$

$4V_2 = 2V_0 + 2V_1$ — IZLOČIM V_2

$4V_4 = 2V_1 + 2V_3$

$4V_3 = V_4 + V_6$

$4V_6 = 2V_3$ — IZLOČIM V_3, V_6

$8V_3 = 2V_4 + V_3 \rightarrow 7V_3 = 2V_4$

$14V_4 = 7V_1 + 2V_4 \rightarrow 12V_4 = 7V_1$

$8V_1 = V_0 + V_1 + 2V_4$

$7V_1 = V_0 + 2V_4$

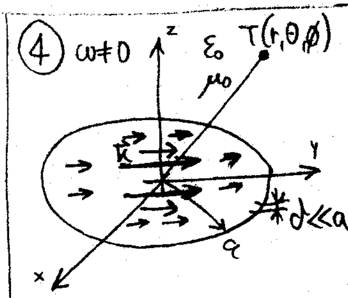
$12V_4 = V_0 + 2V_4 \rightarrow 10V_4 = V_0 \qquad V_1 = \frac{12}{7}V_4$

$\underline{\underline{V_4 = \frac{1}{10}V_0}} \qquad \underline{\underline{V_1 = V_5 = \frac{6}{35}V_0}} \qquad V_2 = \frac{1}{2}(V_0 + V_1) \qquad \underline{\underline{V_2 = \frac{41}{70}V_0}}$

$V_3 = \frac{2}{7}V_4 \qquad V_6 = \frac{1}{2}V_3$

$\underline{\underline{V_3 = V_7 = \frac{1}{35}V_4}} \qquad \underline{\underline{V_6 = \frac{1}{70}V_0}}$

④ $\omega \neq 0$



$d \ll a \rightarrow \vec{K} \approx \vec{J}d$

$\sigma \approx \rho d$

$$\underline{\vec{K} = (\vec{1}_r \sin\phi + \vec{1}_\phi \cos\phi)\, C \cos\frac{\pi r}{2a}}$$

$\vec{1}_r \sin\phi + \vec{1}_\phi \cos\phi = \vec{1}_y$

$r = \sqrt{x^2 + y^2}$

$\sigma = ?$

1.ME $\quad \mathrm{rot}\vec{H} = \vec{J} + j\omega\varepsilon_0\vec{E} \quad /\mathrm{div}$

3.ME $\quad \mathrm{div}(\varepsilon_0\vec{E}) = \rho$

$0 = \mathrm{div}(\mathrm{rot}\vec{H}) = \mathrm{div}\vec{J} + j\omega\,\mathrm{div}(\varepsilon_0\vec{E})$

$0 = \mathrm{div}\vec{J} + j\omega\rho$

$\rho = \frac{-1}{j\omega}\mathrm{div}\vec{J}$

$\sigma = \frac{j}{\omega}\mathrm{div}(\vec{K}d)$

$\sigma = \frac{j}{\omega}\mathrm{div}\vec{K}$

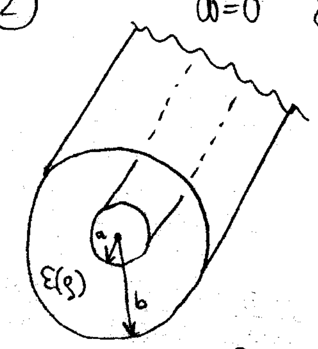
$\mathrm{div}\vec{K} = \frac{\partial}{\partial y}\left(C\cos\frac{\pi r}{2a}\right)$

$\mathrm{div}\vec{K} = -C\sin\left(\frac{\pi r}{2a}\right)\frac{\pi}{2a}\frac{\partial r}{\partial y}$

$\frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} = \frac{y}{r} = \sin\phi$

$$\underline{\underline{\sigma = -\frac{j\pi C}{2a\omega}\sin\left(\frac{\pi r}{2a}\right)\sin\phi}}$$

② $\omega = 0$



$a = 1\,\mathrm{cm} \quad \varepsilon(\rho) = \varepsilon_0\frac{\rho}{a}$

$b = 3\,\mathrm{cm} \quad l = 1\,\mathrm{m} \quad C = ?$

$dU = Q\frac{d\rho}{2\pi\rho l\varepsilon} \qquad \varepsilon_0 = \frac{1}{4\pi\cdot 9\cdot 10^9}\cdot\frac{\mathrm{As}}{\mathrm{Vm}}$

$U = \int_a^b dU = \frac{Q}{2\pi l}\int_a^b \frac{d\rho}{\rho\varepsilon_0\frac{\rho}{a}} = \frac{Qa}{2\pi l\varepsilon_0}\int_a^b\frac{d\rho}{\rho^2}$

$U = \frac{Qa}{2\pi l\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q(b-a)}{2\pi l\varepsilon_0 b}$

$C = \frac{Q}{U} = \frac{2\pi\varepsilon_0 l b}{b-a} = \frac{2\pi\,\mathrm{As}\,1\mathrm{m}\cdot 0.03\mathrm{m}}{4\pi\cdot 9\cdot 10^9\,\mathrm{Vm}\cdot 0.02\mathrm{m}} = \frac{1}{12\cdot 10^9}\,\mathrm{F}$

$$\underline{\underline{C = 83.33\,\mathrm{pF}}}$$

⑤ molekula O_2

$f = 60\,\mathrm{GHz}$

$n \approx 1 \rightarrow \mu = \mu_0, \ \varepsilon \approx \varepsilon_0$

$\alpha = 14\,\mathrm{dB/km}$

$\gamma = ?$

$\alpha = 14\,\mathrm{dB/km} = 0.014\,\mathrm{dB/m}$

1.ME $\quad \mathrm{rot}\vec{H} = \vec{J} + j\omega\varepsilon_0\vec{E} = (\gamma + j\omega\varepsilon_0)\vec{E} = j\omega\varepsilon_0\underbrace{\left(\frac{\gamma}{j\omega\varepsilon_0} + 1\right)}_{\varepsilon_r}\vec{E}$

$\Delta\vec{E} + k^2\vec{E} = 0$

$k^2 = \omega^2\mu_0\varepsilon_0\varepsilon_r = \omega^2\mu_0\varepsilon_0\left(\frac{\gamma}{j\omega\varepsilon_0} + 1\right) \rightarrow k = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{1 - j\frac{\gamma}{\omega\varepsilon_0}}$

$\frac{\gamma}{\omega\varepsilon_0} \ll 1 \rightarrow \sqrt{1 - j\frac{\gamma}{\omega\varepsilon_0}} \approx 1 - \frac{j\gamma}{2\omega\varepsilon_0} \rightarrow k \approx \omega\sqrt{\mu_0\varepsilon_0} - \frac{j\gamma}{2\varepsilon_0}\sqrt{\mu_0\varepsilon_0} =$

$\vec{E} = \vec{E}(0)e^{-jkz} = \vec{E}_0 e^{-jk_0 z}e^{-\frac{\gamma Z_0}{2}z} \qquad \Rightarrow k_0 - j\frac{\gamma Z_0}{2}$

$|\vec{E}| = |\vec{E}(0)|\,e^{-\alpha z} \qquad a = \frac{20}{\ln 10}\alpha = \frac{10}{\ln 10}\gamma Z_0$

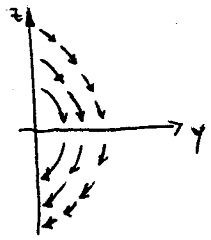
$$\gamma = \frac{a\ln 10}{10 Z_0} = \frac{0.014\,\mathrm{m}^{-1}\ln 10}{10\cdot 120\pi\,\Omega} = \underline{\underline{8.55\cdot 10^{-6}\,\mathrm{S/m} = 8.55\,\mu\mathrm{S/m}}}$$

Rešitve 1. kolokvija EM – 30/11/2006

① $\vec{F}(r,\theta,\phi) = \vec{1}_\theta \frac{1}{r}$

$$\mathrm{div}\,\vec{F} = \frac{1}{r^2\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\frac{1}{r} r\sin\theta\right)\right) = \frac{1}{r^2}\frac{\cos\theta}{\sin\theta}$$ IZVORI SO!

$\phi = \frac{\pi}{2}$



$$\mathrm{rot}\,\vec{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & \vec{1}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & \frac{1}{r}r & 0 \end{vmatrix} = \vec{1}_\phi r\sin\theta \frac{\partial}{\partial r}(1) - \vec{1}_r\frac{\partial}{\partial\phi}(1) = 0$$

$$\oint \vec{F}\,d\vec{l} = \Bigg| \int_{\substack{r=a\\ \phi=0}}^{2\pi} \vec{1}_\theta \frac{1}{r}\,\vec{1}_\phi\, r\sin\theta\, d\phi = 0$$ NI VRTINCEV!

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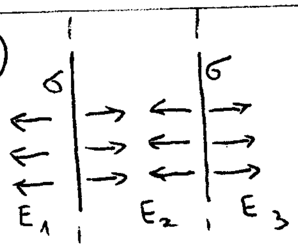
$$\oint \vec{F}\,d\vec{l} = \int_0^\pi \vec{1}_\theta \frac{1}{r}\vec{1}_\theta r^2 \sin\theta\, d\theta + \int_\pi^{2\pi} \vec{1}_\theta\frac{1}{r}\vec{1}_\theta r^2\sin\theta\, d\theta = r\int_0^\pi \sin\theta\, d\theta + r\int_\pi^{2\pi}\sin\theta\, d\theta = r\Big[-\cos\theta\Big]_0^\pi + r\Big[-\cos\theta\Big]_\pi^{2\pi} = 2r - 2r = 0$$

---

②



$(\vec{D}_1 - \vec{D}_2)\cdot\vec{1}_n = \sigma$ $\qquad \vec{1}_n = \vec{1}_x$

$(\vec{E}_1 - \vec{E}_2)\cdot\vec{1}_x = \frac{\sigma}{\varepsilon_0}$

$$\left.\begin{aligned} E_{1x} - E_{2x} &= \frac{\sigma}{\varepsilon_0} \\ E_{2x} - E_{3x} &= \frac{\sigma}{\varepsilon_0} \end{aligned}\right\} \begin{aligned} &\text{seštejemo } E_{1x} - E_{3x} = \frac{2\sigma}{\varepsilon_0} \\ &\text{odštejemo } E_{1x} - 2E_{2x} + E_{3x} = 0 \end{aligned} \quad\Bigg\}\quad \begin{aligned} &\text{iz simetrije } |E_{1x}| = |E_{3x}| \\ &E_{1x} = -E_{3x} = -\frac{\sigma}{\varepsilon_0} \\ &E_{2x} = 0 \end{aligned}$$

$\vec{E}_1 = -\vec{1}_x \frac{\sigma}{\varepsilon_0}$

$\vec{E}_2 = 0$

$\vec{E}_3 = +\vec{1}_x\frac{\sigma}{\varepsilon_0}$

---

③ $\vec{A}(r,\theta,\phi) = \vec{1}_\phi \frac{\sin\theta}{r}$

$$\vec{H} = \frac{1}{\mu_0}\mathrm{rot}\,\vec{A} = \frac{1}{\mu_0}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & r\sin\theta A_\phi \end{vmatrix} = \vec{1}_r \frac{2\cos\theta}{\mu_0 r^2}$$

$$\vec{J} = \mathrm{rot}\,\vec{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ H_r & 0 & 0 \end{vmatrix} = \frac{1}{r^2\sin\theta}\left(-\vec{1}_\phi r\sin\theta\frac{\partial}{\partial\theta}H_r\right) = \vec{1}_\phi\frac{2\sin\theta}{\mu_0 r^3}$$

---

④ $V = -V_0\sin^2\left(\frac{\pi}{a}x\right) = -V_0\left(\cos\left(\frac{2\pi}{a}x\right) - 1\right) = +V_0 - V_0\cos\left(\frac{2\pi}{a}x\right) = V_1 + V_2$

$V_2(x,y) = AC\cos\left(\frac{2\pi}{a}x\right)\mathrm{ch}\left(\frac{2\pi}{a}y\right) \Rightarrow AC = -\frac{V_0}{\mathrm{ch}\left(\frac{2\pi}{a}b\right)}$

$V_1(x,y) = \sum_{n=1} \frac{2V_0(1-\cos(n\pi))}{n\pi\,\mathrm{sh}\left(n\frac{\pi}{a}b\right)}\sin\left(n\frac{\pi}{a}x\right)\mathrm{sh}\left(n\frac{\pi}{a}y\right)$

---

⑤

$V_1 = V_2 = V_3 = V_4$

$V_5 = V_6 = V_7 = V_8$

$6V_1 = V_5 + V_2 + V_3 + V_0$

$6V_5 = V_1 + V_6 + V_7$

$6V_1 = V_5 + 2V_1 + V_0$

$6V_5 = V_1 + 2V_5$

$4V_1 = V_5 + V_0$

$4V_5 = V_1$

$16V_5 = V_5 + V_0$

$$\boxed{V_5 = \frac{V_0}{15}}$$

$$\boxed{V_1 = \frac{4}{15}V_0}$$

Rešitve 2. kolokvija EM – 11/01/2007

① $\vec{E} = \vec{1}_x E_0 \sin(k_y \cdot y) \cdot e^{-jk_z z}$ $\qquad k_y^2 + k_z^2 = k_0^2 = \omega^2 \mu_0 \varepsilon_0$

$$\vec{H} = \frac{j}{\omega\mu_0} \operatorname{rot} \vec{E} = \frac{j}{\omega\mu_0}\left[\vec{1}_y \frac{\partial}{\partial z}(E_x) - \vec{1}_z \frac{\partial}{\partial y}(E_x)\right] = \vec{1}_y \frac{E_0 k_z}{\omega\mu_0} \sin(k_y y) e^{-jk_z z} - \vec{1}_z \frac{j k_y E_0}{\omega\mu_0} \cos(k_y y) e^{-jk_z z}$$

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{1}{2}\left[\vec{1}_z E_x H_y^* - \vec{1}_y E_x H_z^*\right] = \vec{1}_y \frac{E_0 E_0^* k_y}{2\omega\mu_0} \cos(k_y y) \sin(k_y y) + \vec{1}_z \frac{E_0 E_0^* k_z}{2\omega\mu_0} \sin^2(k_y y)$$

$$\operatorname{rot} \vec{H} = \vec{1}_x \frac{\partial H_z}{\partial y} + \vec{1}_z \underbrace{\frac{\partial H_y}{\partial x}}_{=0} - \vec{1}_y \frac{\partial H_y}{\partial z} - \vec{1}_y \underbrace{\frac{\partial H_z}{\partial x}}_{=0} = \vec{1}_x \frac{j k_y^2 E_0}{\omega\mu_0} \sin(k_y y) e^{-jk_z z} + \vec{1}_x \frac{j k_z^2 E_0}{\omega\mu_0} \sin(k_y y) e^{-jk_z z}$$

$$\vec{J} = \operatorname{rot} \vec{H} - j\omega\varepsilon_0 \vec{E} = \vec{1}_x \frac{j E_0 k_0}{\omega\mu_0} \sin(k_y y) e^{-jk_z z} - j\omega\varepsilon_0 \vec{1}_x \sin(k_y y) \cdot e^{-jk_z z} = \vec{1}_x E_0 \sin(k_y y) e^{-jk_z z}\left[\frac{j\omega^2\mu_0\varepsilon_0}{\omega\mu_0} - j\omega\varepsilon_0\right] = 0$$

$$\varrho = \varepsilon \operatorname{div} \vec{E} = 0$$

② $\vec{E} = \vec{1}_y E_0 \sin\left(\frac{n\pi x}{a}\right)$ $\qquad \vec{H} = \frac{j}{\omega\mu_0} \operatorname{rot} \vec{E} = \frac{j}{\omega\mu_0} \vec{1}_z \frac{\partial E_y}{\partial x} = \vec{1}_z \frac{j E_0 n\pi}{\omega\mu_0 a} \cos\left(\frac{n\pi x}{a}\right)$

$\vec{J} = \operatorname{rot} \vec{H} - j\omega\varepsilon_0 \vec{E} = 0$ $\qquad \operatorname{rot} \vec{H} = -\vec{1}_y \frac{\partial}{\partial x}(H_z) = \vec{1}_y \frac{jE_0}{\omega\mu_0}\left(\frac{n\pi}{a}\right)^2 \sin\left(\frac{n\pi}{a} x\right)$

$$\vec{J} = \vec{1}_y j E_0 \sin\left(\frac{n\pi x}{a}\right)\left[\frac{1}{\omega\mu_0}\left(\frac{n\pi}{a}\right)^2 - \omega\varepsilon_0\right] = 0$$

$$\left(\frac{n\pi}{a}\right)^2 = \omega^2 \varepsilon_0 \mu_0$$

$$\omega = \frac{n\pi}{a} \cdot \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{n\pi}{a} c_0$$

$$n = 1 \quad f_1 = \frac{c_0}{2a} = \frac{3 \cdot 10^8\,\mathrm{m/s}}{2 \cdot 0{,}01\,\mathrm{m}} = 15\ \mathrm{GHz}$$

③ $\vec{E} = \vec{1}_\varrho \frac{I}{2\pi\varrho} \cdot Z_0 e^{-jkz}$

$$\vec{H} = \frac{j}{\omega\mu_0} \operatorname{rot} \vec{E} = \frac{j}{\omega\mu_0} \frac{1}{\varrho} \varrho \vec{1}_\varphi \frac{\partial}{\partial z}\left(\frac{I}{2\pi\varrho} Z_0 e^{-jkz}\right) =$$

$$= \vec{1}_\varphi \frac{I}{2\pi\varrho} \frac{k Z_0}{\omega\mu_0} e^{-jkz} = \vec{1}_\varphi \frac{I}{2\pi\varrho} e^{-jkz}$$

$$\vec{S} = \vec{E} \times \vec{H}^* = \begin{vmatrix} \vec{1}_\varrho & \vec{1}_\varphi & \vec{1}_z \\ E_\varrho & 0 & 0 \\ 0 & H_\varphi & 0 \end{vmatrix} = \vec{1}_z \frac{I I^* Z_0}{(2\pi\varrho)^2}$$

④ $\vec{B} = \vec{1}_\varphi \frac{\mu_0 I}{2\pi\varrho}$ $\qquad \vec{A} = -\vec{1}_z\left(\frac{\mu_0 I_1}{2\pi} \ln\varrho + C\right)$

$$M = \frac{1}{I} \oint_s \vec{A}\, d\vec{s} = 0 \quad \text{ker je } \vec{1}_z \cdot \vec{1}_\varphi = 0$$

ali

$$M = \frac{1}{I} \int_A \vec{B}\, d\vec{A} = 0 \qquad d\vec{A} = \vec{1}_z dA \quad \vec{1}_\varphi \cdot \vec{1}_z = 0$$

⑤ $\vec{E} = -\operatorname{grad} V = -\vec{1}_\varrho E_0 e^{j\omega t}\left(1 + \frac{a^2}{\varrho^2}\right)\sin\varphi - \vec{1}_\varphi E_0 e^{j\omega t}\left(1 - \frac{a^2}{\varrho^2}\right)\cos\varphi$

$$\sigma = \vec{1}_n \cdot \vec{D}(\varrho = a) = \vec{1}_n \cdot \varepsilon_0 \vec{E}\Big|_{\varrho=a} = \vec{1}_\varrho \cdot \varepsilon_0 \vec{E}\Big|_{\varrho=a} = -2\varepsilon_0 E_0 \sin\varphi\, e^{j\omega t}$$

$$\operatorname{div} \vec{K} + j\omega\sigma = 0 \qquad \vec{K} = \vec{1}_\varphi K(\varphi)$$

$$\operatorname{div} \vec{K} = \frac{1}{a} \frac{\partial K(\varphi)}{\partial\varphi} = -j\omega\sigma$$

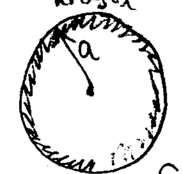
$$\frac{\partial K}{\partial\varphi} = a\, 2j\omega\varepsilon_0 E_0 e^{j\omega t} \sin\varphi \longrightarrow \vec{K} = -\vec{1}_\varphi\left(2j\omega\varepsilon_0 E_0 a e^{j\omega t} \cos\varphi + C\right)$$

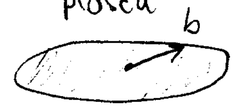
Elektromagnetika

9. 3. 2007

1

kovinska krogla $\omega = 0$ ε_0 krožna kovinska plošča





$a = 10\,cm$ $\quad C_{krogle} = C_{plošče} \quad b = ?$

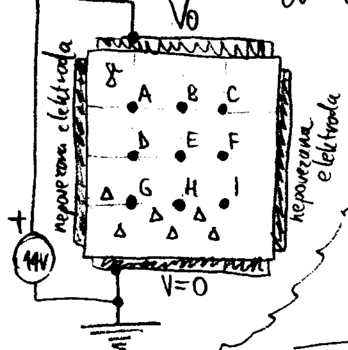
$$C_{krogle} = 4\pi\varepsilon_0 a$$

$$C_{krožne plošče} = 8\varepsilon_0 b$$

$$b = \frac{\pi}{2} a = \underline{\underline{15{,}7\,cm}}$$

3

$\omega = 0$



Simetrija:

$$V_D = V_E = V_F = \frac{V_0}{2} = \underline{\underline{7V}}$$

nepovezana

$$V_A = V_C = V_0 - V_G$$

$$V_G = V_I \qquad V_B = V_0 - V_H$$

$$4V_G = \frac{V_0}{2} + \frac{V_0}{2} + V_H + 0$$

$$4V_H = V_G + \frac{V_0}{2} + V_G + 0$$

$$4V_G = V_0 + V_H$$

$$4V_H = 2V_G + \frac{V_0}{2} \rightarrow 8V_H = 4V_G + V_0$$

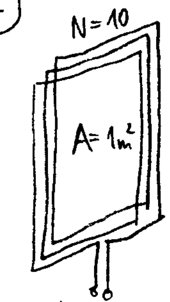
$$8V_H = (V_0 + V_H) + V_0$$

$$7V_H = 2V_0$$

$$V_H = \frac{2}{7}V_0 = \underline{\underline{4V}} \longrightarrow V_B = \underline{\underline{10V}}$$

$$V_G = \frac{V_0 + V_H}{4} = \underline{\underline{4{,}5V}} = V_I \longrightarrow V_A = \underline{\underline{9{,}5V}} = V_C$$

4



ε_0, μ_0, $f = 100\,kHz$, $|\vec{E}| = 10\,mV/m$, $d = 100\,km$

$U_{imax} = ?$

$$\lambda = \frac{c_0}{f} = 3\,km \ll d$$

$$|\vec{H}| = \frac{|\vec{E}|}{Z_0} = \underline{26{,}5\,\mu A/m}$$

$$U_i = -j\omega N|\vec{B}|A$$

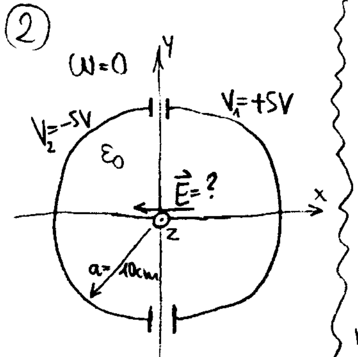
$$U_i = -j\omega N|\vec{H}|\mu_0 A$$

$$U_i = -j\,10^5 \cdot 2\pi\,rd/s \cdot 10 \cdot 26.5 \cdot 10^{-6}\,A/m \cdot 4\pi \cdot 10^{-7}\,Vs/Am \cdot 1m^2$$

$$\underline{\underline{U_i = -j\,209\,\mu V}}$$

2

$\omega = 0$



$$\vec{E}(x=0, y=0) = ?$$

$$V(\varrho,\varphi) = \sum_{n=1}^{\infty} C_n \varrho^n \cos n\varphi$$

$$\vec{E} = -\mathrm{grad}\,V = -\vec{1}_\varrho \sum_{n=1}^{\infty} n C_n \varrho^{n-1}\cos n\varphi + \vec{1}_\varphi \sum_{n=1}^{\infty} n C_n \varrho^{n-1} \sin n\varphi$$

$$\vec{E}(\varrho=0) = -\vec{1}_\varrho C_1 \cos\varphi + \vec{1}_\varphi C_1 \sin\varphi = \underline{-\vec{1}_x C_1}$$

$$\int_0^{2\pi} \sum_{n=1}^{\infty} C_n a^n \cos n\varphi \cos m\varphi \, d\varphi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V_1 \cos m\varphi \, d\varphi + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} V_2 \cos m\varphi \, d\varphi$$

$$m = n = 1 \rightarrow C_1 a\pi = 2V_1 - 2V_2 \longrightarrow C_1 = \frac{2(V_1 - V_2)}{a\pi}$$

$$\vec{E}(\varrho=0) = -\vec{1}_x \frac{2(V_1 - V_2)}{a\pi} = \underline{\underline{-\vec{1}_x \cdot 63.66\,V/m}}$$

5

$$\vec{E} = \vec{1}_x E_0 e^{-\frac{x^2+y^2}{2a^2}} e^{-jkz}$$

$k = \omega\sqrt{\mu_0\varepsilon_0} \qquad Z_0 = 120\pi\,\Omega$

$\lambda = 0{,}633\,\mu m$

$a = 1\,mm \gg \lambda$

$E_0 = 10^3\,V/m$

$P = ?$

$$a \gg \lambda \rightarrow \vec{H} \approx \frac{1}{Z_0} \vec{1}_k \times \vec{E}$$

$$\vec{S} = \frac{1}{2}\vec{E} \times \vec{H}^* \approx \vec{1}_z \frac{|\vec{E}|^2}{2Z_0} = \vec{1}_z \frac{|E_0|^2}{2Z_0} e^{-\frac{x^2+y^2}{a^2}}$$

$$\frac{\varrho^2}{a^2} = u$$

$$P = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \vec{S} \cdot \vec{1}_z \, dx\, dy = \int_0^{\infty}\int_0^{2\pi} \vec{S} \cdot \vec{1}_z \, \varrho \, d\varrho \, d\varphi =$$

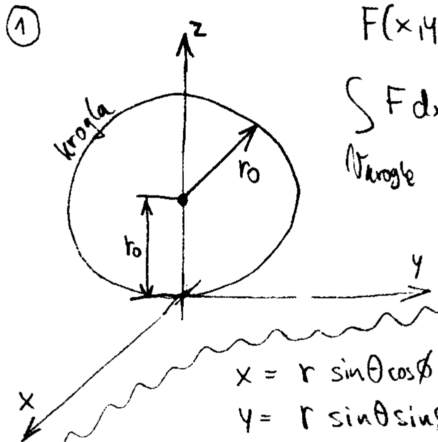
$$= \frac{|E_0|^2}{2Z_0} 2\pi \int_0^{\infty} e^{-\frac{\varrho^2}{a^2}} \varrho \, d\varrho = \frac{|E_0|^2}{2Z_0} 2\pi \frac{a^2}{2} \int_0^{\infty} e^{-u} du = \frac{|E_0|^2 \pi a^2}{2Z_0} =$$

$$= \frac{|10^3\,V/m|^2 \pi (10^{-3}\,m)^2}{2 \cdot 120\pi\,\Omega} = \frac{1W}{240} = \underline{\underline{4{,}167\,mW}}$$

ELEKTROMAGNETIKA

6/9/2007

①



$F(x,y,z) = x^2 z$

$\int_{V_{krogle}} F\,dx\,dy\,dz = ?$

$dV = dx\,dy\,dz = r^2 \sin\theta \,dr\,d\theta\,d\phi$

$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r_0 + r\cos\theta$
premaknjeni krogelni KS

$$\int_{V_{krogle}} F\,dx\,dy\,dz = \int_0^{r_0}\int_0^{\pi}\int_0^{2\pi} r^2\sin^2\theta\cos^2\phi\,(r_0 + r\cos\theta)\, r^2\sin\theta\, dr\,d\theta\,d\phi =$$

$u = \cos\theta;\ du = -\sin\theta\, d\theta;\ 1-u^2 = \sin^2\theta$

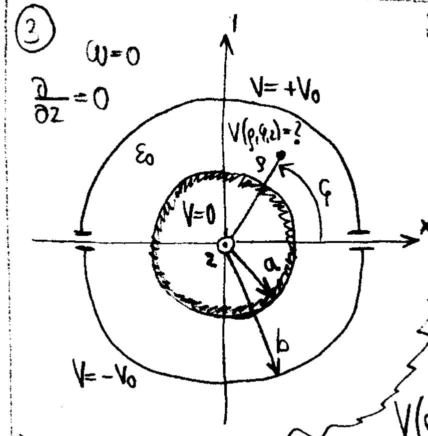
$$= \pi \int_0^{r_0}\int_0^{\pi} r^4 \sin^3\theta (r_0 + r\cos\theta)\,dr\,d\theta = \pi \int_0^{r_0}\int_{-1}^{+1} r^4(1-u^2)(r_0 + ru)\,dr\,du =$$

$$= \pi \int_0^{r_0} r^4\left(2r_0 - \tfrac{2}{3} r_0\right) dr = \pi \int_0^{r_0}\left(\tfrac{4}{3} r_0 r^4\right) dr = \underline{\underline{\pi\,\tfrac{4}{15} r_0^6}}$$

③

$\omega = 0$

$\frac{\partial}{\partial z} = 0$



$\Delta V = 0 \qquad \frac{\partial}{\partial z} = 0$

$V(\rho,\varphi) = (A\rho^n + B\rho^{-n})\cdot$
$\cdot\,(C\cos n\varphi + D\sin n\varphi)$
$C = 0 \leftarrow$ tehnično $\rightarrow D = 1$

$V(\rho,\varphi) = (A\rho^n + B\rho^{-n})\sin n\varphi$

$V(\rho = a, \varphi) = 0 \rightarrow 0 = Aa^n + Ba^{-n}$
$\rightarrow B = -Aa^{2n}$

$$V(\rho,\varphi) = \sum_n A_n\left(\rho^n - \frac{a^{2n}}{\rho^n}\right)\sin n\varphi$$

$$\rho = b \rightarrow \int_0^{2\pi} A_n\left(b^n - \frac{a^{2n}}{b^n}\right)\sin n\varphi \sin m\varphi\, d\varphi = V_0\int_0^{\pi} \sin m\varphi\, d\varphi - V_0\int_{\pi}^{2\pi} \sin m\varphi\, d\varphi$$

$$A_m\left(b^m - \frac{a^{2m}}{b^m}\right)\pi = \frac{2V_0}{m}(1-\cos m\varphi) \rightarrow A_m = \frac{2V_0(1-\cos m\varphi)}{m\pi\left(b^m - \frac{a^{2m}}{b^m}\right)}$$

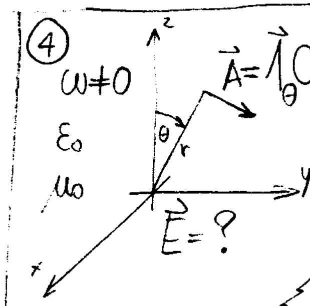
$$\underline{\underline{V(\rho,\varphi) = \sum_{m=0}^{\infty} \frac{2V_0(1-\cos m\varphi)}{m\pi\left(b^m - \frac{a^{2m}}{b^m}\right)}\left(\rho^m - \frac{a^{2m}}{\rho^m}\right)\sin m\varphi}}$$

④

$\omega \neq 0$

ε_0

μ_0



$\vec{A} = \vec{1}_\theta C \frac{e^{-jkr}}{r}$

$\vec{E} = ?$

Lorentz:
$\mathrm{div}\vec{A} = -j\omega\mu_0\varepsilon_0 V$

$$\mathrm{div}\vec{A} = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(C\frac{e^{-jkr}}{r} r\sin\theta\right) = C\frac{e^{-jkr}}{r^2}\frac{\cos\theta}{\sin\theta}$$

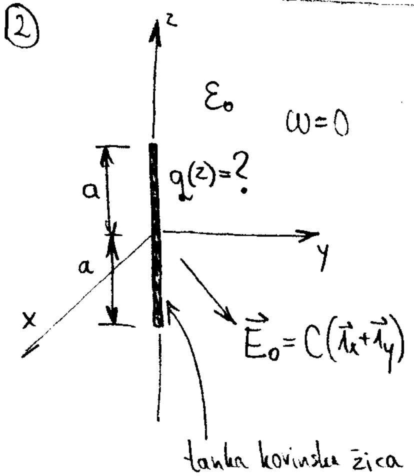
$$V = \frac{j}{\omega\mu_0\varepsilon_0}\mathrm{div}\vec{A} = \frac{jC}{\omega\mu_0\varepsilon_0}\frac{e^{-jkr}}{r^2}\frac{\cos\theta}{\sin\theta}$$

$$\vec{E} = -j\omega\vec{A} - \mathrm{grad}\,V = -\vec{1}_\theta j\omega C\frac{e^{-jkr}}{r} - \frac{jC}{\omega\mu_0\varepsilon_0}\cdot$$

$$\cdot\left[\vec{1}_r\left(\frac{-2}{r^3} - \frac{jk}{r^2}\right)e^{-jkr}\frac{\cos\theta}{\sin\theta} + \vec{1}_\theta\frac{e^{-jkr}}{r^3}\frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta}\right] =$$

$$\underline{\underline{= j\omega C\left[\vec{1}_r\left(\frac{2}{k^2r^3} + \frac{j}{kr^2}\right)e^{-jkr}\frac{\cos\theta}{\sin\theta} + \vec{1}_\theta\left(\frac{1}{k^2r^3\sin^2\theta} - \frac{1}{r}\right)e^{-jkr}\right]}}$$

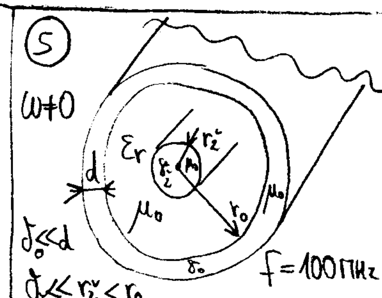
②



$\underline{\underline{q(z) = 0}} \longleftarrow \vec{E}_0 \perp \vec{1}_z$

⑤

$\omega \neq 0$



$\delta_0 \ll d$

$d \ll r_ž < r_0$

$f = 100\,\mathrm{MHz}$

$r_ž = 1\,\mathrm{mm} \quad \gamma_ž = 56\cdot 10^6\,\mathrm{S/m}$

$r_0 = 3\,\mathrm{mm} \quad P_ž = P_0$

$\gamma_0 = ?$

$$P_ž = \frac{1}{2}R_ž|I|^2;\quad R_ž = \frac{l}{\gamma_ž 2\pi r_ž \delta_ž} = \frac{l}{2\pi r_ž}\sqrt{\frac{\omega\mu_0}{2\gamma_ž}}$$

$$P_0 = \frac{1}{2}R_0|I|^2;\quad R_0 = \frac{l}{\gamma_0 2\pi r_0 \delta_0} = \frac{l}{2\pi r_0}\sqrt{\frac{\omega\mu_0}{2\gamma_0}}$$

$$\delta = \sqrt{\frac{2}{\omega\mu_0\gamma}}$$

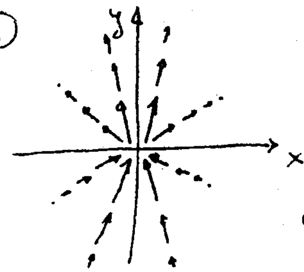
$$P_ž = P_0 \rightarrow R_ž = R_0 \rightarrow \frac{1}{r_ž}\sqrt{\frac{1}{\gamma_ž}} = \frac{1}{r_0}\sqrt{\frac{1}{\gamma_0}}$$

$$\gamma_0 = \gamma_ž\left(\frac{r_ž}{r_0}\right)^2 = 56\cdot 10^6\,\mathrm{S/m}\left(\frac{1\,\mathrm{mm}}{3\,\mathrm{mm}}\right)^2 = \underline{\underline{6{,}22\cdot 10^6\,\mathrm{S/m}}}$$

1. kolokvij EM 4.12.2007

$\vec{F} = \vec{1}_\varrho \frac{1}{\varrho} \sin\varphi$

(1)



$\operatorname{div}\vec{F} = \frac{1}{\varrho}\left(\frac{\partial}{\partial\varrho}\left(\frac{1}{\varrho}\sin\varphi\,\varrho\right) + 0 + 0\right) = 0$

singularnost pri $\varrho = 0$

$\oint_A \vec{F}\,d\vec{A} = \left| \int_{\varrho=a}\int_0^{2\pi}\int_0^{l} \vec{F}\cdot(\vec{1}_\varrho)\,a\,d\varphi\,dz = 0 \right.$

NI IZVOROV!

$\operatorname{rot}\vec{F} = \frac{1}{\varrho}\begin{vmatrix} \vec{1}_\varrho & \vec{1}_\varphi \varrho & \vec{1}_z \\ \frac{\partial}{\partial\varrho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ \frac{1}{\varrho}\sin\varphi & 0 & 0 \end{vmatrix} = -\vec{1}_z \frac{1}{\varrho^2}\cos\varphi$

SO VRTINCI!

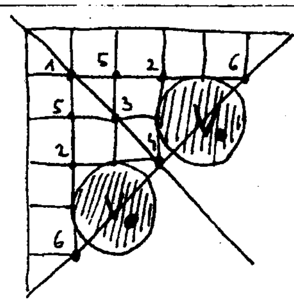
(2)

$\vec{B} = \operatorname{rot}\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & \vec{1}_\theta r & \vec{1}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & r\cdot\frac{r}{\sin\theta} & 0 \end{vmatrix} = \frac{1}{r^2\sin\theta}\left(\vec{1}_\phi\, r\sin\theta\,\frac{1}{\sin\theta}\,2r\right) = \vec{1}_\phi \frac{2}{\sin\theta}$

$\vec{H} = \frac{\vec{B}}{\mu_0}$ $\quad \vec{J} = \operatorname{rot}\vec{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & \vec{1}_\theta r & \vec{1}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & \frac{2}{\mu\sin\theta} r\sin\theta \end{vmatrix} = \frac{1}{r^2\sin\theta}\left(-\vec{1}_\theta r\frac{\partial}{\partial r}\left(\frac{2r}{\mu}\right)\right) = -\vec{1}_\theta \frac{2}{\mu}\frac{1}{r\sin\theta}$

$I = \oint \vec{H}\cdot d\vec{s} = \int_0^{2\pi} \vec{1}_\phi \frac{2}{\mu\sin\theta}\cdot\vec{1}_\phi\, r\sin\theta\, d\phi = \frac{4\pi r}{\mu_0}$

(3)



$4V_1 = 2V_5$

$4V_2 = V_5 + 2V_0$

$4V_3 = 2V_5 + 2V_0$

$4V_4 = 4V_0 \quad \Rightarrow \boxed{V_4 = V_0}$

$4V_5 = V_1 + V_2 + V_3$

$4V_6 = 2V_0 \quad \Rightarrow \boxed{V_6 = \frac{V_0}{2}}$

$4V_5 = \frac{V_5}{2} + \frac{V_5}{4} + \frac{V_0}{2} + \frac{V_5}{2} + \frac{V_0}{2}$

$4V_5 = \frac{5}{4}V_5 + V_0 \quad \Rightarrow \boxed{V_5 = \frac{4}{11}V_0}$

$4V_2 = \frac{4}{11}V_0 + 2V_0 \quad \Rightarrow \frac{26}{11}V_0 \Rightarrow \boxed{V_2 = \frac{13}{22}V_0}$

$4V_1 = \frac{8}{11}V_0 \quad \Rightarrow \boxed{V_1 = \frac{2}{11}V_0}$

$4V_3 = \frac{8}{11}V_0 + 2V_0 = \frac{30}{11}V_0 \quad \Rightarrow \boxed{V_3 = \frac{15}{22}V_0}$

(4)

$\Delta V = 0 \qquad \frac{\partial}{\partial z} = 0$

$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

$V(x,y,z) = X(x)\cdot Y(y)$

$\underbrace{\frac{X''}{X}}_{\pm k_x^2} + \underbrace{\frac{Y''}{Y}}_{\pm k_y^2} = 0$

$V = \left(Ae^{k_x x} + Be^{-k_x x}\right)\left(C\cos k_y y + D\sin k_y y\right)$

Mejni pogoji

$\left.\begin{array}{l} V(y=a) = 0 \\ V(y=0) = 0 \end{array}\right\} \Rightarrow C = 0\ ;\ k_x = k_y = n\frac{\pi}{a}$

$V(x\to\infty) = 0 \Rightarrow A = 0$

$V(x,y) = BDe^{-k_x x}\sin\frac{n\pi}{a}y \qquad D = 1$

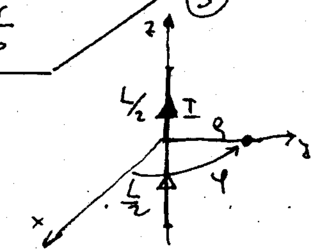
$V(x,y) = \sum_{n=1}^{\infty} B_n e^{-n\frac{\pi}{a}x}\sin\frac{n\pi}{a}y$

$V(x=0) = V_0 = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{a}y \quad \Big/ \int_0^a \sin\frac{m\pi}{a}y\,dy$

$\frac{aV_0}{m\pi}(1-\cos m\pi) = B_m\frac{a}{2}$

$m \to n \qquad B_n = \frac{2V_0}{n\pi}(1-\cos n\pi)$

(5)

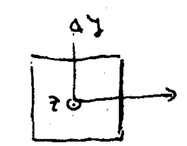


$\vec{B} = \vec{1}_\varphi \frac{\mu_0 I}{4\pi\varrho}\left(\frac{L/2}{\sqrt{\left(\frac{L}{2}\right)^2 + \varrho^2}} - \frac{-L/2}{\sqrt{\left(\frac{L}{2}\right)^2 + \varrho^2}}\right)$

$\varrho = \frac{L}{2}$

$\vec{B} = \vec{1}_\varphi \frac{\mu_0 I}{4\pi\frac{L}{2}}\cdot\frac{2\frac{L}{2}}{\frac{L}{2}\sqrt{2}} = \vec{1}_\varphi \frac{\mu_0 I}{4\pi L}\frac{4}{\sqrt{2}}$

$\vec{1}_\varphi = \vec{1}_z$

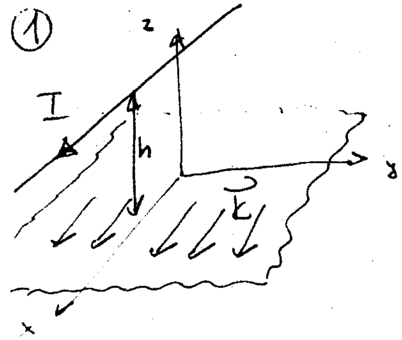


$\vec{B}_\square = 4\vec{B} = 4\,\vec{1}_z \frac{\mu_0 I}{4\pi L}\cdot\frac{4}{\sqrt{2}}$

$\vec{B}_\square = \vec{1}_z \frac{2\sqrt{2}\,\mu_0 I}{\pi L}$

2. kolokvij iz EM 14. 1. 2008

(1)



$$\vec{I} = \oint_{s} \vec{H}\, d\vec{s}$$

$$K \cdot 2a = H \cdot 2a + 0 + H \cdot 2a + 0$$

$$H = \frac{K}{2}$$

$$\vec{H}_{plosnatoga\ toka} = \frac{1}{2}\vec{K} \times \vec{1}_n = \frac{1}{2}\vec{K} \times \vec{1}_z = \frac{1}{2}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 6 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$= -\frac{1}{2}\vec{1}_y\, 6\ A/m = \underline{-3\,\vec{1}_y\ A/m}$$

$$|\vec{H}_{vodiča}| = \frac{I}{2\pi r} = |\vec{H}_{plos.\ toka}| \qquad \vec{1}_\phi = -\vec{1}_k = \vec{1}_x$$

$$I = 2\pi\,(h-z)\cdot|\vec{H}_{p.t.}| = \underline{\underline{47{,}1\ A}}$$

(2)

$$M_1 = \frac{\pi \cdot \mu_0 \cdot N_1 \cdot N_2 \cdot a^2 \cdot a^2}{2 d_1^3} \qquad M_2 = \frac{\pi \cdot \mu_0 \cdot N_1 \cdot N_2 \cdot a^2 \cdot a^2}{2 d_2^3}$$

$$2M_1 = M_2$$

$$\frac{2}{d_1^3} = \frac{1}{d_2^3}$$

$$d_2 = \frac{d_1}{\sqrt[3]{2}} = 39{,}7\ cm$$

Zanki je potrebno zbližati za $\underline{\underline{10{,}3\ cm}}$

(5)

$$f_0 = \frac{c_0}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2}$$

$$a = \frac{c_0}{f\sqrt{2\varepsilon_r}} = 0{,}14\ m$$

$$f_1 = \frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2} = f_0\sqrt{\varepsilon_r} = \underline{\underline{1{,}5\ GHz}}$$

(3)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \vec{1}_x E_0\left(e^{-jk_0 z} + e^{+jk_0 z}\right) =$$

$$= \vec{1}_x E_0\left(\cos k_0 z - j\sin k_0 z + \cos k_0 z + j\sin k_0 z\right) =$$

$$= \vec{1}_x E_0\, 2\cos k_0 z$$

$$\vec{H} = \frac{j}{\omega\mu_0}\operatorname{rot}\vec{E} = \frac{j}{\omega\mu_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_0 2\cos k_0 z & 0 & 0 \end{vmatrix} =$$

$$= \frac{j}{\omega\mu_0}\left(\vec{1}_y\, 2E_0(-\sin k_0 z)\, k_0\right) = -\vec{1}_y \frac{j2E_0 k_0}{\omega\mu_0}\sin k_0 z = -\vec{1}_y \frac{j2E_0}{Z_0}\sin k_0 z$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 2E_0\cos k_0 z & 0 & 0 \\ 0 & \frac{2E_0^*}{Z_0}\sin k_0 z & 0 \end{vmatrix} =$$

$$\vec{S} = \frac{1}{2}\vec{1}_z \frac{4E_0E_0^*\cos k_0 z \sin k_0 z}{Z_0} = \vec{1}_z \frac{E_0E_0^*}{Z_0}\sin 2k_0 z$$

(4)

$$\vec{E} = \frac{1}{j\omega\varepsilon_0}\operatorname{rot}\vec{H}$$

$$\vec{E} = \vec{1}_y \frac{2H_0}{\omega\varepsilon_0}\sin\left(\frac{\pi}{a}x\right)e^{-jk_0 z}$$

$$\operatorname{rot}\vec{H} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{H_0}{k_0}\sin\left(\frac{\pi}{a}x\right)e^{-jk_0 z} & 0 & j\frac{H_0}{\pi}a\cos\left(\frac{\pi}{a}x\right)e^{-jk_0 z} \end{vmatrix} =$$

$$= \vec{1}_y\left(-\frac{H_0}{\cancel{k_0}}\sin\left(\frac{\pi}{a}x\right)e^{-jk_0 z}\cdot(-j\cancel{k_0}) - \vec{1}_y\, j\frac{H_0}{\cancel{\pi}}\cancel{a}\sin\left(\frac{\pi}{a}x\right)e^{-jk_0 z}\,\frac{\cancel{\pi}}{\cancel{a}}\right) =$$

$$= \vec{1}_y\left(j2H_0\sin\left(\frac{\pi}{a}x\right)e^{-jk_0 z}\right)$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 0 & \frac{2H_0}{\omega\varepsilon_0}\sin\left(\frac{\pi}{a}x\right)e^{-jk_0 z} & 0 \\ -\frac{H_0}{k_0}\sin\left(\frac{\pi}{a}x\right)e^{-jk_0 z} & 0 & -\frac{jH_0^* a}{\pi}\cos\left(\frac{\pi}{a}x\right)e^{jk_0 z} \end{vmatrix} =$$

$$= -\vec{1}_x \frac{H_0H_0^* a}{\omega\varepsilon_0\pi}\sin\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{a}x\right)\underbrace{e^{-jk_0 z}e^{+jk_0 z}}_{e^0=1} \underbrace{- \vec{1}_z \frac{H_0H_0^*}{\omega\varepsilon_0 k_0}\sin^2\left(\frac{\pi}{a}x\right)\underbrace{e^{+jk_0 z}e^{-jk_0 z}}_{e^0=1}}_{Re}$$

$$P = \int_A \vec{S}\,d\vec{A} = \int_0^a\int_0^b \frac{|H_0|^2}{\omega\varepsilon_0 k_0}\sin^2\left(\frac{\pi}{a}x\right)dx\,dy = \underline{\underline{\frac{|H_0|^2}{\omega\varepsilon_0 k_0}\, b\,\frac{a}{2}}}$$

ELEKTRO MAGNETIKA

28.03.2008

① $\vec{H}(\rho,\varphi,z) = \begin{cases} \vec{1}_\rho \frac{c}{\rho} \, ; \rho < a \\ 0 \, ; \rho > a \end{cases}$ $\omega = 0$ $\mu = \mu_0$

$\vec{J} = ?$, $\vec{K} = ?$, $I = ?$

$\vec{J}(\rho<a) = \mathrm{rot}\,\vec{H}(\rho<a) = \underline{\underline{0}}$

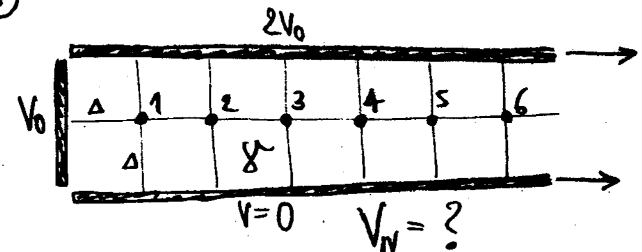
$\vec{J}(\rho>a) = \mathrm{rot}\,\vec{H}(\rho>a) = \underline{\underline{0}}$

$\vec{K}(\rho=a) = \vec{1}_n \times \left(\vec{H}(\rho>a) - \vec{H}(\rho<a)\right) =$
$= \vec{1}_\rho \times \left(0 - \vec{1}_\rho \frac{c}{a}\right) = \underline{\underline{0}}$

$I(\rho=0) = \oint \vec{H}(\rho<a) \cdot \vec{1}_\varphi \rho \, d\varphi = \underline{\underline{0}}$

(takšno polje poganjajo magnetine...)

③ $\omega = 0$ $\gamma \neq 0$



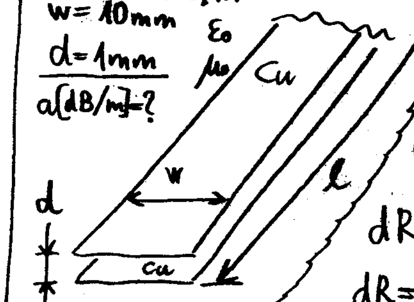
$V_N = ?$

$\underline{\underline{V_N = V_0 \text{ za poljuben } N}}$ (SIMETRIJA!)

④ $f = 1\,GHz$
$\gamma_{Cu} = 56 \cdot 10^6\,S/m$
$w = 10\,mm$
$d = 1\,mm$
$a\,[dB/m] = ?$

stresanje zanemarimo

ε_0 μ_0



$U = Ed$ $Z_0 = 377\,\Omega$

$I = Hw$

$Z_K = \frac{U}{I} = \frac{Ed}{Hw} = \frac{d}{w} Z_0$

$(Z_K = 37.7\,\Omega)$

$dR = \frac{2\,dl}{\delta w \gamma}$ $\delta = \sqrt{\frac{2}{\omega\mu\gamma}} = (2.1\,\mu m)$

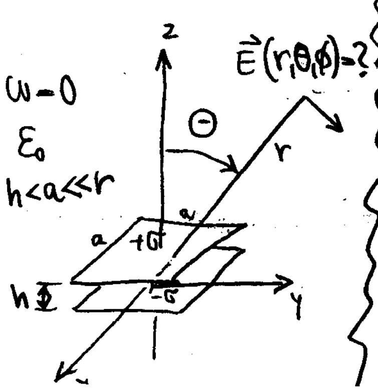
$dR = \frac{2\,dl}{w}\sqrt{\frac{\omega\mu_0}{2\gamma}}$

$\frac{dP}{P} = \frac{dR}{Z_K} = \frac{2\,dl}{d Z_0}\sqrt{\frac{\omega\mu_0}{2\gamma}} \Big/ \int_0^l dl$ $\mu_0 = 4\pi \cdot 10^{-7}\,Vs/Am$

$\ln\frac{P_g}{P_b} = \frac{2l}{d Z_0}\sqrt{\frac{\omega\mu_0}{2\gamma}}$ $a = \frac{10\log P_g/P_b}{l} = \frac{10}{l \ln 10} \ln P_g/P_b$

$a = \frac{10}{\ln 10} \frac{2}{d Z_0}\sqrt{\frac{\omega\mu_0}{2\gamma}} = \underline{\underline{0.193\,dB/m}}$

②



$\vec{E}(r,\theta,\phi) = ?$

$\omega = 0$
ε_0
$h < a \ll r$

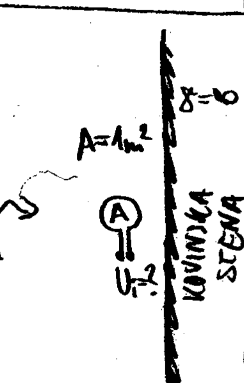
$r \gg h$

$\vec{E} = \frac{Qh}{4\pi\varepsilon_0}\left[\vec{1}_r \frac{2\cos\theta}{r^3} + \vec{1}_\theta \frac{\sin\theta}{r^3}\right]$

$Q = \sigma a^2$

$\underline{\underline{\vec{E} = \frac{\sigma a^2 h}{4\pi\varepsilon_0}\left[\vec{1}_r \frac{2\cos\theta}{r^3} + \vec{1}_\theta \frac{\sin\theta}{r^3}\right]}}$

⑤



$S_V = 1\,W/m^2$ $A = 1\,m^2$

$\lambda = 300\,m$

$U_i = ?$

μ_0
ε_0

$\vec{S}_V = \frac{1}{2}\vec{E}_V \times \vec{H}_V = \vec{1}_s \frac{|E|^2}{2Z_0} = \vec{1}_s \frac{|H|^2 Z_0}{2}$ $|H_V| = \sqrt{\frac{2S_V}{Z_0}} = \underline{0.073\,A/m}$

$U_i = -\frac{d\phi}{dt} = -j\omega\phi = -j\omega BA = -j\omega\mu_0 HA = -j\omega\mu_0 2H_V A$

$|U_i| = \omega\mu_0 2|H_V|A = \frac{2\pi c_0}{\lambda}\mu_0 2|H_V|A = \underline{1.15\,V}$

$(H = H_V + H_O = 2H_V$ ob kovinski steni$)$

ELEKTROMAGNETIKA
10. 07. 2008

1 $\mu=\mu_0$; $\omega=0$

$$\vec{A}(\varrho,\varphi,z)=\begin{cases}\vec{1}_z\, C\ln \varrho/a\ ; & \varrho\le a\\ 0\ ; & \varrho\ge a\end{cases}$$

$\vec{J}=?$; $\vec{K}=?$; $I=?$

$$\vec{H}(\varrho\le a)=\frac{1}{\mu_0}\operatorname{rot}\vec{A}=\frac{1}{\mu_0}\frac{1}{\varrho}\begin{vmatrix}\vec{1}_\varrho & \varrho\vec{1}_\varphi & \vec{1}_z\\ \frac{\partial}{\partial\varrho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z}\\ 0 & \varrho 0 & C\ln\varrho/a\end{vmatrix}=-\vec{1}_\varphi\frac{C}{\mu_0}\frac{1}{\varrho}$$

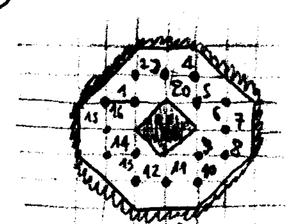
$\vec{H}(\varrho\ge a)=0$

$\vec{J}=\operatorname{rot}\vec{H}$ $\quad \vec{J}(\varrho\le a)=0$ $\quad \vec{J}(\varrho\ge a)=0$

$$\vec{K}(\varrho=a)=\left(\vec{H}(\varrho\le a)-\vec{H}(\varrho\ge a)\right)\times\vec{1}_\varrho=\vec{1}_z\frac{C}{\mu_0 a}$$

$$I(\varrho=0)=\oint\vec{H}(\varrho\le a)\cdot d\vec{s}=\int_0^{2\pi}-\vec{1}_\varphi\frac{C}{\mu_0}\frac{1}{\varrho}\cdot\vec{1}_\varphi\varrho\, d\varphi=-\frac{2\pi C}{\mu_0}$$

3 $\omega=0$



Simetrija $V_1=V_5=V_9=V_{13}$

$V_2=V_4=V_6=V_8=V_{10}=V_{12}=V_{14}=V_{16}$

$V_3=V_7=V_{11}=V_{15}$

$4V_1=2V_2+2V_0\rightarrow 2V_1=V_2+V_0$

$4V_2=V_1+V_3$

$4V_3=2V_2+V_0$

Prva v drugi dve

$8V_2=V_2+V_0+2V_3\rightarrow 7V_2=V_0+2V_3$

$4V_3=V_0+2V_2$

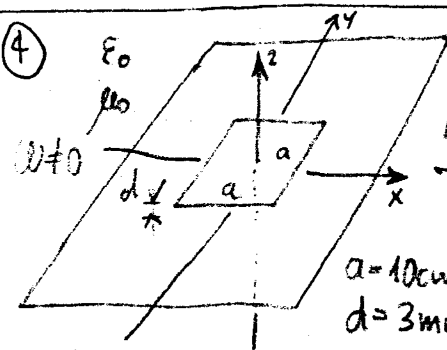
Druga v tretjo

$28V_3=7V_0+2V_0+4V_3\rightarrow 24V_3=9V_0\rightarrow V_3=\frac{3}{8}V_0$

$V_2=\frac{1}{7}(V_0+2V_3)=\frac{2}{8}V_0$

$V_1=\frac{1}{2}(V_0+V_2)=\frac{5}{8}V_0$

4 ε_0 μ_0 $\omega\neq0$



TEM rezonanca

$\vec{E}=\vec{1}_z E(x,y)$

$f_{min}=?$

$a=10\text{cm}$

$d=3\text{mm}\ll a$

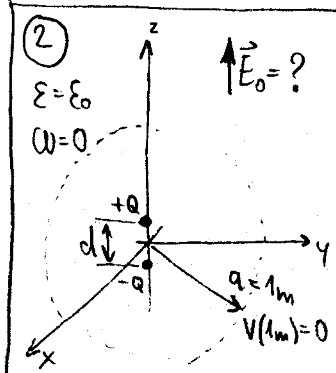
$\vec{E}=\vec{1}_z\left(A\cos k_x x+B\sin k_x x\right)\left(C\cos k_y y+D\sin k_y y\right)$; $k_x^2+k_y^2=k^2$

najnižja rezonanca: $k_x=k=\omega\sqrt{\mu_0\varepsilon_0}$; $k_y=0$

$\vec{E}=\vec{1}_z B\sin kx=\vec{1}_z B\sin\frac{\pi}{a}x\rightarrow k=\frac{\pi}{a}$

$\frac{\pi}{a}=\omega\sqrt{\mu_0\varepsilon_0}=\frac{2\pi f_{min}}{c_0}\rightarrow f_{min}=\frac{c_0}{2a}=1{,}5\,\text{GHz}$

2 $\varepsilon=\varepsilon_0$ $\omega=0$



$\vec{E}_0=?$

$Qd=10^{-9}\,\text{Asm}$

$a=1\text{m}$

$\vec{E}_0=?$

$V(r,\theta,\varphi)=-E_0 z+\frac{Qd}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}$; $z=r\cos\theta$

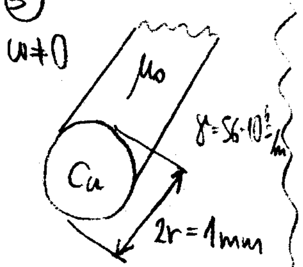
$V(r=a)=0=-E_0 a\cos\theta+\frac{Qd}{4\pi\varepsilon_0}\frac{\cos\theta}{a^2}$

$E_0=\frac{Qd}{4\pi\varepsilon_0 a^3}=\frac{10^{-9}\,\text{Asm}}{4\pi\frac{1}{4\pi\cdot9\cdot10^9}\frac{\text{As}}{\text{Vm}}\cdot1\text{m}^3}$

$E_0=9\,\text{V/m}$

$\vec{E}_0=\vec{1}_z\cdot9\,\text{V/m}$

5 $\omega\neq0$



$R_\sim=10R_=$

$f=?$

$R_==\frac{l}{\gamma A_=}=\frac{l}{\gamma\pi r^2}$ $\qquad \delta=\sqrt{\frac{2}{\omega\mu_0\gamma}}$

$R_\sim=\frac{l}{\gamma A_\sim}=\frac{l}{\gamma\,2\pi r\delta}$

$\frac{R_\sim}{R_=}=10=\frac{r}{2\delta}\rightarrow\delta=\frac{r}{20}=25\,\mu\text{m}$

$f=\frac{\omega}{2\pi}=\frac{1}{\pi\mu_0\gamma\delta^2}=\frac{1\,\text{Am Vm}}{\pi\cdot4\pi\cdot10^{-7}\text{Vs}\cdot56\cdot10^6\text{A}(25\cdot10^{-6}\text{m})^2}$

$f=7{,}24\,\text{MHz}$

ELEKTROMAGNETIKA
25.09.2008

① $\vec{E} = \vec{1}_x C \sin kx \left(e^{j\omega t}\right)$ $\quad \omega \neq 0 \quad k = \omega\sqrt{\mu\varepsilon}$

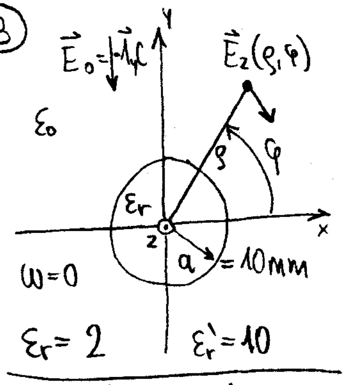
izvori polja $\rho = ?$, $\vec{J} = ?$

$\rho = \operatorname{div}\vec{D} = \operatorname{div}\varepsilon\vec{E} = \underline{\varepsilon C k \cos kx \left(e^{j\omega t}\right)}$

$\vec{J} = \operatorname{rot}\vec{H} - j\omega\varepsilon\vec{E} = \operatorname{rot}\left(\frac{j}{\omega\mu}\operatorname{rot}\vec{E}\right) - j\omega\varepsilon\vec{E} =$

$= \operatorname{rot}\left(\frac{j}{\omega\mu}0\right) - j\omega\varepsilon\vec{E} = \underline{\underline{\vec{1}_x(-j\omega\varepsilon)C\sin kx\left(e^{j\omega t}\right)}}$

③



$\omega = 0$

$\varepsilon_r = 2 \qquad \varepsilon_r' = 10$

$\vec{E}_2 = \vec{E}_2' \longrightarrow a' = ?$

$\vec{E}_0 = -\vec{1}_y C \rightarrow V_0 = C_y = C\rho\sin\varphi$

$V_2 = \left(C\rho + \frac{A}{\rho}\right)\sin\varphi$

$V_N = B\rho\sin\varphi \qquad \vec{E} = -\operatorname{grad}V$

$\vec{E}_2 = -\vec{1}_\rho\left(C - \frac{A}{\rho^2}\right)\sin\varphi - \vec{1}_\varphi\left(C + \frac{A}{\rho^2}\right)\cos\varphi$

$\vec{E}_N = -\vec{1}_\rho B\sin\varphi - \vec{1}_\varphi B\cos\varphi$

$\rho = a: \vec{1}_\rho(\vec{D}_2 - \vec{D}_N) = 0;\ \vec{1}_\varphi(\vec{E}_2 - \vec{E}_N) = 0$

$\left(C - \frac{A}{a^2}\right) = B\varepsilon_r\ ;\ \left(C + \frac{A}{a^2}\right) = B \rightarrow C - \frac{A}{a^2} = \varepsilon_r\left(C + \frac{A}{a^2}\right)$

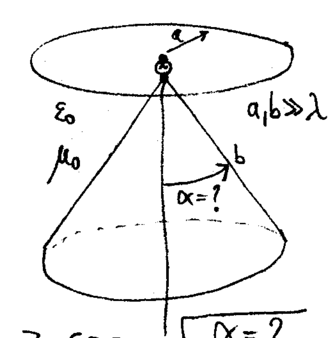
$C(1-\varepsilon_r) = \frac{A}{a^2}(1+\varepsilon_r) \rightarrow A = a^2 C\frac{1-\varepsilon_r}{1+\varepsilon_r}$

$A = A' = a'^2 C\frac{1-\varepsilon_r'}{1+\varepsilon_r'}$

$a' = a\sqrt{\left(\frac{1+\varepsilon_r'}{1-\varepsilon_r'}\right)\left(\frac{1-\varepsilon_r}{1+\varepsilon_r}\right)} = 10\,mm\sqrt{\frac{(1+10)(1-2)}{(1-10)(1+2)}}$

$a' = \sqrt{\frac{11\cdot(-1)}{(-9)\cdot 3}}\cdot 10\,mm = \underline{\underline{6{,}38\,mm}}$

④ Discone antena



$Z = 50\,\Omega \qquad \alpha = ?$

$\vec{E} = \vec{1}_\theta \frac{C}{r\sin\theta}e^{-jkr}$

$\vec{H} = \frac{j}{\omega\mu_0}\operatorname{rot}\vec{E} =$

$= \vec{1}_\phi \frac{j}{\omega\mu_0}(-jk)\frac{C}{r\sin\theta}e^{-jkr} =$

$= \vec{1}_\phi\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{C}{r\sin\theta}e^{-jkr}$

$\underline{I = \oint \vec{H}\cdot d\vec{s} = 2\pi\sqrt{\frac{\varepsilon_0}{\mu_0}}\,C e^{-jkr}}$

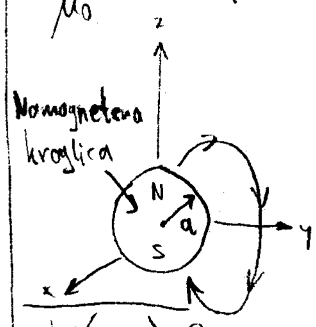
$U = -\int_\alpha^{\pi/2}\vec{E}\cdot d\vec{s} = C\ln(\operatorname{tg}(\theta/2))e^{-jkr}\Big|_\alpha^{\pi/2} = C\ln\frac{\operatorname{tg}\pi/4}{\operatorname{tg}\alpha/2}e^{-jkr}$

$Z = \frac{U}{I} = \frac{Z_0}{2\pi}\ln\frac{\operatorname{tg}\pi/4}{\operatorname{tg}\alpha/2} \rightarrow e^{2\pi Z/Z_0} = \frac{\operatorname{tg}\pi/4}{\operatorname{tg}\alpha/2} = \frac{1}{\operatorname{tg}\alpha/2}$

$\alpha = 2\operatorname{arc}\operatorname{tg} e^{-\frac{2\pi Z}{Z_0}} = \underline{\underline{47^0}}$

② $\omega = 0$

μ_0



$W_m(r>a) = ?$

$\vec{H}(r>a) = \frac{C}{r^3}\left(\vec{1}_r 2\cos\theta + \vec{1}_\theta\sin\theta\right)$

$W_m = \frac{1}{2}\int_V \vec{H}\cdot\vec{B}\,dv = \frac{\mu_0}{2}\int_a^\infty\int_0^\pi\int_0^{2\pi}|\vec{H}|^2 dv$

$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$

$W_m = \frac{\mu_0}{2}\int_a^\infty\int_0^\pi 2\pi|\vec{H}|^2 r^2\sin\theta\,dr\,d\theta =$ $\qquad u = \cos\theta$

$W_m = \mu_0\pi C^2\left(-\frac{1}{3r^3}\right)\Big|_a^\infty\int_0^\pi(4\cos^2\theta + \sin^2\theta)\sin\theta\,d\theta$

$W_m = \frac{\mu_0\pi C^2}{3a^3}\int_{-1}^{+1}(3u^2+1)\,du = \frac{\mu_0\pi C^2}{3a^3}(u^3+u)\Big|_{-1}^{+1}$

$\underline{\underline{W_m = \frac{4\pi\mu_0 C^2}{3a^3}}}$

⑤ Tiskano vezje

$w = 3\,mm$

$d_{cu} = 17{,}5\,\mu m$

$\gamma_{cu} = 56\cdot10^6\,S/m$

$R_\sim = 10\,R_=$

$f = ?$

$\delta = \sqrt{\frac{2}{\omega\mu_0\gamma_{cu}}} = \frac{d_{cu}}{2\cdot10} = \underline{0.875\,\mu m}$

$w \gg d_{cu}$

$\omega = \frac{2}{\delta^2\mu_0\gamma_{cu}}$

$f = \frac{1}{\pi\delta^2\mu_0\gamma_{cu}} = \frac{1\ A\,m\,V\,m}{\pi\cdot(0{,}875\cdot10^{-6}\,m)^2\cdot4\pi\cdot10^{-7}\,Vs\ 56\cdot10^6\,A}$

$\underline{\underline{f = 5{,}9\,GHz}}$

1. kolokvij iz EM 25. 11. 2008

1

$h_\eta = a\sqrt{\cos^2\psi + sh^2\eta}$

$h_\psi = a\sqrt{\cos^2\psi + sh^2\eta}$

$h_\phi = a\, ch\eta \sin\psi$

polje

$\vec{F}(\eta,\psi,\phi) = \vec{1}_\phi \ln\phi$

$$div\,\vec{F} = \frac{1}{h_\eta h_\psi h_\phi}\left(\frac{\partial(F_\phi h_\eta h_\psi)}{\partial\phi}\right) =$$

$$= \frac{1}{a^3(\cos^2\psi + sh^2\eta)\,ch\eta\sin\psi}\,\frac{\partial\left(\ln\phi\, a^2(\cos^2\psi + sh^2\eta)\right)}{\partial\phi} =$$

$$= \frac{1}{a\,ch\eta\sin\psi}\,\frac{\partial(\ln\phi)}{\partial\phi} = \frac{1}{a\phi\, ch\eta\sin\psi}$$

SO VRTINCI

$$rot\,\vec{F} = \frac{1}{h_\eta h_\psi h_\phi}\begin{vmatrix} \vec{1}_\eta h_\eta & \vec{1}_\psi h_\psi & \vec{1}_\phi h_\phi \\ \frac{\partial}{\partial\eta} & \frac{\partial}{\partial\psi} & \frac{\partial}{\partial\phi} \\ \underbrace{h_\eta F_\eta}_{0} & \underbrace{h_\psi F_\psi}_{0} & h_\phi F_\phi \end{vmatrix} =$$

$$= \frac{1}{a^3\left(\sqrt{\cos^2\psi + sh^2\eta}\right)^2 ch\eta\sin\psi}\left[\vec{1}_\eta h_\eta \frac{\partial}{\partial\psi}(h_\phi F_\phi) - \vec{1}_\psi h_\psi \frac{\partial}{\partial\eta}(h_\phi F_\phi)\right] =$$

$$= \frac{1}{a\sqrt{\cos^2\psi + sh^2\eta}\; ch\eta\sin\psi}\left[\vec{1}_\eta \frac{\partial}{\partial\psi}(ch\eta\sin\psi\ln\phi) - \vec{1}_\psi\frac{\partial}{\partial\eta}(ch\eta\sin\psi\ln\phi)\right] =$$

$$= \frac{\ln\phi}{a\sqrt{\cos^2\psi + sh^2\eta}\; ch\eta\sin\psi}\left[\vec{1}_\eta ch\eta\cos\psi - \vec{1}_\psi \sin\psi\, sh\eta\right] =$$

$$= \frac{\ln\phi}{a\sqrt{\cos^2\psi + sh^2\eta}}\left[\vec{1}_\eta\, ctg\psi - \vec{1}_\psi\, tgh\eta\right]$$

2 $\vec{A}(r,\theta,\phi) = \vec{1}_\theta \frac{1}{r}\cdot\sin\theta\cdot tg\phi$

$$\vec{B} = rot\,\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & \vec{1}_\theta r & \vec{1}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & r\frac{1}{r}\sin\theta\, tg\phi & 0 \end{vmatrix} =$$

$$= \frac{1}{r^2\sin\theta}\left[\vec{1}_\phi r\sin\theta\frac{\partial}{\partial r}[\sin\theta\, tg\phi] - \vec{1}_r\frac{\partial}{\partial\phi}[\sin\theta\, tg\phi]\right] =$$

$$= \frac{1}{r^2\sin\theta}\left[-\vec{1}_r\frac{\sin\theta}{\cos^2\phi}\right] = -\vec{1}_r\frac{1}{r^2}\frac{1}{\cos^2\phi}$$

$\vec{B} = \mu_0\vec{H}$

$$\vec{J} = rot\,\vec{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & \vec{1}_\theta r & \vec{1}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ -\frac{1}{\mu_0 r^2\cos^2\phi} & 0 & 0 \end{vmatrix} =$$

$$= \frac{1}{\mu_0 r^2\sin\theta}\left[\vec{1}_\theta r\frac{\partial}{\partial\phi}\left[-\frac{1}{r^2\cos^2\phi}\right]\right] =$$

$$= -\frac{\vec{1}_\theta}{\mu_0 r^3\sin\theta}\left(-\frac{2}{\cos^3\phi}(-\sin\phi)\right) =$$

$$= -\vec{1}_\theta\frac{1}{\mu_0 r^3}\frac{1}{\sin\theta}\frac{2\sin\phi}{\cos^3\phi}$$

3 $\vec{E} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}$ $W = \frac{\varepsilon}{2}\int_v |\vec{E}|^2 dv$

$$W = \frac{\varepsilon}{2}\int_v |\vec{E}|^2 r^2\sin\theta\, dr\, d\theta\, d\phi =$$

$$= \frac{\varepsilon}{2}\int_a^\infty\int_0^\pi\int_0^{2\pi}\left(\frac{Q}{4\pi\varepsilon r^2}\right)^2 r^2\sin\theta\, dr\, d\theta\, d\phi =$$

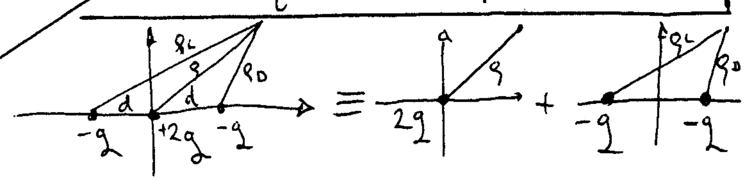
$$= \frac{\varepsilon}{2}\left(\frac{Q}{4\pi\varepsilon}\right)^2 2\pi\int_a^\infty\frac{1}{r^2}\left[-\cos\theta\right]_0^\pi dr =$$

$$= \frac{\varepsilon}{2}\left(\frac{Q}{4\pi\varepsilon}\right)^2 2\pi\cdot 2\int_a^\infty\frac{1}{r^2}dr =$$

$$= \frac{2Q^2}{8\pi\varepsilon}\left[-\frac{1}{r}\right]_a^\infty = \underline{\underline{\frac{Q^2}{8\varepsilon\pi}\frac{1}{a}}}$$

4 ker so preme elektrine neskončno dolge $L\to\infty$ je potencial ene same elektrine $+q$

$$V = -\frac{q}{2\pi\varepsilon_0}\ln r$$



$$V = -\frac{-q}{2\pi\varepsilon}\ln\varrho_L - \frac{+2q}{2\pi\varepsilon}\ln\varrho - \frac{q}{2\pi\varepsilon}\ln\varrho_D = \frac{q}{2\pi\varepsilon}\left(\ln\varrho_L - 2\ln\varrho + \ln\varrho_D\right) = \frac{q}{2\pi\varepsilon}\left(\ln\frac{\varrho_L\varrho_D}{\varrho^2}\right)$$

$$\varrho_D = \sqrt{\varrho^2 + d^2 - 2\varrho d\cos\varphi} \approx \varrho\left(1 - \frac{d}{2\varrho}\cos\varphi\right)$$

$$\varrho_L = \sqrt{\varrho^2 + d^2 - 2\varrho d\cos(180^\circ - \varphi)} = \sqrt{\varrho^2 + d^2 + 2\varrho d\cos\varphi} \approx \varrho\left(1 + \frac{d}{2\varrho}\cos\varphi\right)$$

$$\ln\frac{\varrho_L\varrho_D}{\varrho^2} = \ln\left[\left(1 - \frac{d}{2\varrho}\cos\varphi\right)\left(1 + \frac{d}{2\varrho}\cos\varphi\right)\right] = \ln\left(1 + \left(\frac{d}{2\varrho}\right)^2\cos^2\varphi\right)$$

5 $\boxed{V_5 = V_6 = 0}$ rešujemo z razmikom 2Δ

$\boxed{\begin{matrix} V_1 = V_2 \\ V_3 = V_4 \end{matrix}}$

$4V_1 = 2V_0 + V_3 + V_2$

$4V_3 = -2V_0 + V_1 + V_4$

$3V_1 = 2V_0 + V_3 \Rightarrow V_3 = 3V_1 - 2V_0 \Rightarrow \boxed{V_3 = \frac{1}{2}V_0}$

$3V_3 = -2V_0 + V_1$

$9V_1 - 6V_0 = -2V_0 + V_1 \Rightarrow \boxed{V_1 = \frac{1}{2}V_0}$

2. kolokvij iz EM - 26. 1. 2009

(1) $\vec{A}(\rho,\varphi,z) = -\vec{1}_z z^2\rho^2$

$$\vec{H} = \frac{1}{\mu}\operatorname{rot}\vec{A} = \frac{1}{\mu}\frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & 0 & -z^2\rho^2 \end{vmatrix} =$$

$$= \frac{1}{\mu_0}\frac{1}{\rho}\left[-\rho\vec{1}_\varphi\frac{\partial}{\partial\rho}\left(-z^2\rho^2\right)\right] = \frac{1}{\mu_0}\frac{1}{\rho}\left[z^2\rho\vec{1}_\varphi 2\rho\right]$$

$$\vec{H} = \vec{1}_\varphi\frac{2\rho z^2}{\mu_0}$$

$$\vec{J} = \operatorname{rot}\vec{H} - j\omega\varepsilon_0\vec{E}$$

$$\vec{E} = -j\omega\vec{A}$$

$$\operatorname{rot}\vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \rho\frac{2\rho z^2}{\mu_0} & 0 \end{vmatrix} = \frac{1}{\rho}\left[\vec{1}_z\frac{\partial}{\partial\rho}\left(\frac{2\rho^2 z^2}{\mu_0}\right) - \vec{1}_\rho\frac{\partial}{\partial z}\left(\frac{2\rho^2 z^2}{\mu_0}\right)\right] =$$

$$= \frac{2}{\rho\mu_0}\left(\vec{1}_z 2z^2\rho - \vec{1}_\rho\rho^2 2z\right) = -\vec{1}_\rho\frac{4z\rho}{\mu_0} + \vec{1}_z\frac{4z^2}{\mu_0}$$

$$\vec{J} = -\vec{1}_\rho\frac{4z\rho}{\mu_0} + \vec{1}_z\left[\frac{4z^2}{\mu_0} + \omega^2\varepsilon_0 z^2\rho^2\right]$$

(2)
$$M_1 = \frac{\pi\mu_0 a^2 a^2 N_1 N_2}{2d_1^3} = \frac{\pi\mu_0 a^4 N_1 N_2}{2d_1^2}$$

$$M_2 = \left(\frac{3\mu_r}{2+\mu_r}\right)^2\frac{\pi\mu_0 a^4 N_1 N_2}{2d_2^3}$$

$$M_1 = M_2$$

$$\frac{1}{d_1^3} = \left(\frac{3\mu_r}{2+\mu_r}\right)^2\frac{1}{d_2^3} = \left(\frac{3\cdot 2}{2+2}\right)^2\frac{1}{d_2^3}$$

$$d_2 = \sqrt[3]{9/4}\, d_1$$

$$d_2 = 1{,}31\cdot d_1$$

(3) $\vec{E} = (\vec{1}_x 3 - \vec{1}_y + \vec{1}_z 2)\ \frac{V}{m}$

$$\vec{k} = \vec{1}_k\cdot k$$

$$\vec{1}_k = \pm\frac{\vec{E}\times\vec{1}_z}{|\vec{E}\times\vec{1}_z|} = \pm\frac{(\vec{1}_x 3 - \vec{1}_y + \vec{1}_z 2)\times\vec{1}_z}{\sqrt{1^2+3^2}} = \pm\frac{-\vec{1}_x - \vec{1}_y 3}{\sqrt{10}}$$

$$\vec{H} = \vec{1}_H\cdot H \qquad \vec{1}_H = \vec{1}_k\times\vec{1}_E$$

$$\vec{H} = \vec{1}_k\times\vec{E}\frac{1}{Z_0} = \pm\frac{\vec{1}_x + \vec{1}_y 3}{\sqrt{10}}\times(\vec{1}_x 3 - \vec{1}_y + \vec{1}_z 2)\frac{1}{120\pi}\ \frac{A}{m}$$

$$\vec{H} = \pm\left(\vec{1}_x 6 - \vec{1}_y 2 + \vec{1}_z(-10)\right)\frac{1}{\sqrt{10}\ 120\pi}\ \frac{A}{m}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ 3 & -1 & 2 \\ 6 & -2 & -10 \end{vmatrix}\left(\pm\frac{1}{\sqrt{10}\ 120\pi}\right) =$$

$$\vec{S} = (\vec{1}_x 14 + \vec{1}_y 42)\left(\pm\frac{1}{\sqrt{10}\ 240\pi}\right)$$

$$\vec{S} = \pm(\vec{1}_x + \vec{1}_y 3)\left(\pm\frac{7}{\sqrt{10}\ 120\pi}\right)\ \frac{W}{m^2}$$

(4) $\vec{E} = \vec{1}_x E_0 \sin\left(\frac{\pi}{b}y\right)e^{-jk_0 z}$

$$\vec{H} = \frac{j}{\omega\mu_0}\operatorname{rot}\vec{E} = \frac{j}{\omega\mu_0}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_0\sin\left(\frac{\pi}{b}y\right)e^{-jk_0 z} & 0 & 0 \end{vmatrix} = \frac{j}{\omega\mu_0}\left[\vec{1}_y E_0\sin\left(\frac{\pi}{b}y\right)e^{-jk_0 z}(-jk_0) - \vec{1}_z E_0 e^{-jk_0 z}\cos\left(\frac{\pi}{b}y\right)\frac{\pi}{b}\right]$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \frac{1}{2}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ E_x & 0 & 0 \\ H_x^* & H_y^* & H_z^* \end{vmatrix} = \frac{1}{2}\Bigg[\underbrace{-\vec{1}_y\frac{E_0E_0^* j\pi}{\omega\mu_0 b}\sin\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{b}y\right)}_{Im} + \underbrace{\vec{1}_z\frac{E_0E_0^* k_0}{\omega\mu_0}\sin^2\left(\frac{\pi}{b}y\right)}_{Re}\Bigg]$$

$$P = \int_A \vec{S}\,d\vec{A} = \frac{1}{2}\int_0^a\int_0^b \frac{E_0E_0^* k_0}{\omega\mu_0}\sin^2\left(\frac{\pi}{b}y\right)\,dx\,dy$$

$$P = \frac{|E_0|^2 k_0}{\omega\mu_0}\frac{ab}{4} = \frac{|E_0|^2}{Z_0}\frac{ab}{4}$$

(5) $f = \frac{c_0}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$$f_{1,1} = \frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 1{,}5\cdot 10^9\ Hz$$

$$\frac{1}{a^2} + \frac{1}{b^2} = 100$$

$$f_{1,3} = f_{2,1}$$

$$\frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{3}{b}\right)^2} = \frac{c_0}{2}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$\frac{8}{b^2} = \frac{3}{a^2}$$

$$\frac{1}{a^2} + \frac{3}{8a^2} = 100$$

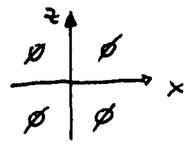
$$a = \sqrt{\frac{11}{800}} = \underline{\underline{11{,}7\ cm}}$$

$$b = \sqrt{\frac{8}{3}a^2} = \sqrt{\frac{11}{300}} = \underline{\underline{19{,}1\ cm}}$$

1. kolokvij iz EM 2. 12. 2009

① $\vec{F}(\varrho,\varphi,z) = \vec{1}_\varphi \frac{1}{\varrho^3}\sin(2\varphi)$ $y=0$ $\varphi=\frac{\pi}{2} \Rightarrow \sin 2\varphi = 0$

$\varphi = \frac{3\pi}{2} \Rightarrow \sin 2\varphi = 0$



$$\operatorname{div}\vec{F}(\varrho,\varphi,z) = \frac{1}{\varrho}\left(\frac{\partial}{\partial\varphi}\left(\frac{1}{\varrho^3}\sin 2\varphi\right)\right) =$$

$$= \frac{1}{\varrho^4} 2\cdot\cos 2\varphi \Rightarrow \text{polje } \vec{F} \text{ ima izvore}$$

$$\operatorname{rot}\vec{F} = \frac{1}{\varrho}\begin{vmatrix} \vec{1}_\varrho & \varrho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\varrho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \varrho\frac{1}{\varrho^3}\sin 2\varphi & 0 \end{vmatrix} = \frac{1}{\varrho}\left(\vec{1}_z\frac{\partial}{\partial\varrho}\left(\frac{1}{\varrho^2}\sin 2\varphi\right)\right) =$$

$$= -\vec{1}_z\frac{2}{\varrho^4}\sin 2\varphi$$

polje $\vec{F}$ ima vrtince

② $\vec{A}(\varrho,\varphi,z) = \vec{1}_z \frac{z^2}{\varrho}\sin\varphi$

$$\vec{B} = \operatorname{rot}\vec{A} = \frac{1}{\varrho}\begin{vmatrix} \vec{1}_\varrho & \varrho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\varrho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \varrho\cdot 0 & \frac{z^2}{\varrho}\sin\varphi \end{vmatrix} =$$

$$= \frac{1}{\varrho}\left[\vec{1}_\varrho\frac{\partial}{\partial\varphi}\left(\frac{z^2}{\varrho}\sin\varphi\right) - \vec{1}_\varphi\cdot\varrho\cdot\frac{\partial}{\partial\varrho}\left(\frac{z^2}{\varrho}\sin\varphi\right)\right] =$$

$$= \frac{1}{\varrho}\left[\vec{1}_\varrho\frac{z^2}{\varrho}\cos\varphi - \vec{1}_\varphi\varrho\left(-\frac{z^2}{\varrho^2}\right)\sin\varphi\right]$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \vec{1}_\varrho\frac{z^2}{\varrho^2\mu_0}\cos\varphi + \vec{1}_\varphi\frac{z^2}{\varrho^2\mu_0}\sin\varphi$$

$\operatorname{rot}\vec{H} = \vec{J} + j\omega\varepsilon_0\vec{E}$ $\omega = 0 \Rightarrow \operatorname{rot}\vec{H} = \vec{J}$

$$\vec{J} = \frac{1}{\varrho}\begin{vmatrix} \vec{1}_\varrho & \varrho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\varrho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ \frac{z^2}{\varrho^2\mu_0}\cos\varphi & \varrho\frac{z^2\sin\varphi}{\varrho^2\mu_0} & 0 \end{vmatrix} =$$

$$= \frac{1}{\varrho}\left[-\vec{1}_\varrho\frac{\partial}{\partial z}\left(\frac{z^2}{\varrho\mu_0}\sin\varphi\right) + \vec{1}_\varphi\varrho\frac{\partial}{\partial z}\left(\frac{z^2}{\varrho^2\mu_0}\cos\varphi\right) + \right.$$

$$\left. + \vec{1}_z\frac{\partial}{\partial\varrho}\left(\frac{z^2}{\varrho\mu_0}\sin\varphi - \frac{\partial}{\partial\varphi}\left(\frac{z^2}{\varrho^2\mu_0}\cos\varphi\right)\right)\right] =$$

$$\vec{J} = -\vec{1}_\varrho\frac{2z}{\varrho^2\mu_0}\sin\varphi + \vec{1}_\varphi\frac{2z}{\varrho^2\mu_0}\cos\varphi$$

③ $4V_1 = V_0 + V_2$ simetrija

$4V_2 = V_0 + V_1 + V_3 + V_4$ $V_1 = -V_6 = \frac{V_0}{3}$

$\left.\begin{array}{l} 4V_3 = \cancel{V_0} + V_2 + V_5 - \cancel{V_0} \\ 4V_4 = V_2 + V_5 \end{array}\right\} V_3 = V_4$ $V_2 = -V_5 = -\frac{V_0}{3}$

$V_3 = V_4 = 0$

$4V_5 = V_3 + V_4 + V_6 - V_0$

$4V_6 = V_5 + V_0$

④ $\Delta V = 0$

$$V = \sum_{m=0}^{\infty}\left(A_n\cos(ny) + B_n\sin(ny)\right)\left(C_n\operatorname{sh}(nx) + D_n\operatorname{ch}(nx)\right)$$

$V(x, y=0) = 0 \Rightarrow A_n = 0$

$V(x, y=a) = 0 \Rightarrow na = k\pi \Rightarrow n = \frac{k}{a}\pi$

$$V = \sum_{k=1}^{\infty}\sin\left(\frac{k}{a}\pi y\right)\left(C_n\operatorname{sh}\left(\frac{k}{a}\pi x\right) + D_n\operatorname{ch}\left(\frac{k}{a}\pi x\right)\right)$$

$$V(x=0, y) = V_1 = \sum_{k=1}^{\infty}\sin\left(\frac{k\pi}{a}x\right)D_n$$

$$D_n = \frac{2}{a}\int_0^a V_1\sin\left(\frac{k\pi}{a}y\right)dy = \frac{2V_1}{a}\left(-\frac{\cos\left(\frac{k\pi}{a}y\right)}{k\pi}a\right)\Big|_0^a = \frac{2V_1}{a}\left(-\frac{(-1)^k}{k\pi}a + \frac{a}{k\pi}\right) = \frac{2V_1}{k\pi}\left(1-(-1)^k\right)$$

$$V(x=a, y) = V_1 = \sum_{k=1}^{\infty}\sin\left(\frac{k\pi}{a}y\right)\left(C_n\operatorname{sh}(k\pi) + B_n\operatorname{ch}(k\pi)\right)$$

$$C_n\operatorname{sh}(k\pi) + B_n\operatorname{ch}(k\pi) = \frac{2}{a}\int_0^a V_1\sin\left(\frac{k\pi}{a}y\right)dy = \frac{2V_1}{k\pi}\left(1-(-1)^k\right)$$

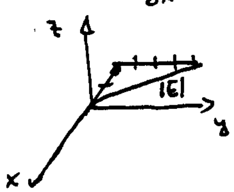
$$C_n = \frac{2V_1}{k\pi}\left(1-(-1)^k\right)\left(1-\operatorname{ch}(k\pi)\right)\frac{1}{\operatorname{sh}(k\pi)}$$

$$V(x,y) = \sum_{k=1}^{\infty}\frac{2V_1\left(1-(-1)^k\right)}{k\pi}\sin\left(\frac{k}{a}\pi y\right)\left(\frac{1-\operatorname{ch}(k\pi)}{\operatorname{sh}(k\pi)}\operatorname{sh}\left(\frac{k\pi}{a}x\right) + \operatorname{ch}\left(\frac{k\pi}{a}x\right)\right)$$

⑤ $V = 2x + 4y$

$\vec{E} = -\operatorname{grad} V =$

$$\vec{E} = -\left(\vec{1}_x\frac{\partial V}{\partial x} + \vec{1}_y\frac{\partial V}{\partial y} + \vec{1}_z\frac{\partial V}{\partial z}\right) = -\left(\vec{1}_x 2 + \vec{1}_y 4 + 0\right)$$



$$|\vec{E}| = \sqrt{2^2+4^2} = \sqrt{20}\ \frac{V}{m}$$

$$W = \frac{1}{2}\int\varepsilon_0|\vec{E}|^2 dv$$

$$W = \frac{1}{2}\ 8{,}85\cdot 10^{-12}\frac{As}{Vm}\cdot 20\frac{V^2}{m^2}\cdot\int dx\,dy\,dz$$

$$W = \frac{1}{2}\cdot 8{,}85\cdot 10^{-12}\cdot 20\cdot 1\frac{VAs}{m^3}\ \cancel{m^3}$$

$$W = 88{,}5\cdot 10^{-12}\ J = \underline{\underline{10\,\varepsilon_0\ J}}$$

Vsi m^3 volumna imajo enako vrednost energije

2. kolokvij EM 21. 2. 2010

① $\vec{H}=\frac{1}{\mu_0}\operatorname{rot}\vec{A}=\frac{1}{\mu_0\rho}\begin{vmatrix}\vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z}\\ 0 & 0 & A_0\frac{e^{-jkz}}{\rho}\end{vmatrix}=\vec{1}_\varphi\frac{A_0e^{-jkz}}{\mu_0\rho^2}$

$\operatorname{rot}\vec{H}=\underset{0}{\vec{J}}+j\omega\varepsilon\vec{E}$

$\vec{E}=\frac{1}{j\omega\varepsilon}\operatorname{rot}\vec{H}=\frac{1}{j\omega\varepsilon}\frac{1}{\rho}\begin{vmatrix}\vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z}\\ 0 & \frac{A_0e^{-jkz}}{\mu_0\rho} & 0\end{vmatrix}$

$\vec{E}=\vec{1}_\rho\frac{A_0k}{\mu\omega\varepsilon}\frac{e^{-jkz}}{\rho^2}+\vec{1}_z\frac{jA_0e^{-jkz}}{\mu\omega\varepsilon\rho^3}$

② $\vec{A}(\vec{r})=\frac{\mu}{4\pi}\int_{V'}\vec{J}(\vec{r}\,')\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dV'\overset{r\gg a}{=}\frac{\mu J_0}{4\pi}\int_{V'}\vec{1}_y\frac{e^{-jkr}}{r}dV'=\frac{\mu a^3}{4\pi}J_0\frac{e^{-jkr}}{r}\left(\vec{1}_r\sin\theta\sin\phi+\vec{1}_\theta\cos\theta\sin\phi+\vec{1}_\phi\cos\phi\right)$

$\vec{H}=\frac{1}{\mu_0}\operatorname{rot}\vec{A}=\frac{a^3}{4\pi}J_0\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\,\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ \sin\theta\sin\phi\frac{e^{-jkr}}{r} & \cos\theta\sin\phi\, e^{-jkr} & \sin\theta\cos\phi\, e^{-jkr}\end{vmatrix}=$

$=\frac{a^3J_0}{4\pi}\frac{1}{r^2\sin\theta}\Big[\vec{1}_r\left(\cos\theta\cos\phi\, e^{-jkr}-\cos\theta\cos\phi\, e^{-jkr}\right)+r\vec{1}_\theta\left(\sin\theta\cos\phi\frac{e^{-jkr}}{r}+jk\sin\theta\cos\phi\, e^{-jkr}\right)+r\sin\theta\,\vec{1}_\phi\Big(-jk\cos\theta\sin\phi\, e^{-jkr}-\cos\theta\sin\phi\frac{e^{-jkr}}{r}\Big)\Big]$

$\vec{H}=\frac{a^3J_0}{4\pi}\frac{e^{-jkr}}{r}\left(\frac{1}{r}+jk\right)\left[\vec{1}_\theta\cos\phi+\vec{1}_\phi\cos\theta\sin\phi\right]$

③ $M_1=\frac{\mu_0 a}{2\pi}\ln\frac{d+\frac{a}{2}}{d-\frac{a}{2}}$

$M_2=\frac{M_1}{3}$

$d\gg a\rightarrow\vec{B}=\text{konst.}$

$M=\frac{1}{I}\int_A\vec{B}\,d\vec{A}$

$A_2=\frac{A_1}{3}$

$a_2^2=\frac{a_1^2}{3}$

$a_2=\frac{a_1}{\sqrt{3}}=2{,}30\text{ mm}$

a se mora zmanjšati za 1,7 mm

④ $\vec{E}=\frac{1}{j\omega\varepsilon_0}\operatorname{rot}\vec{H}=\frac{1}{j\omega\varepsilon_0}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & \frac{H_0}{\omega\mu_0}\beta\sin\left(\frac{\pi}{a}y\right)e^{-j\beta z} & -\frac{jH}{\omega\mu_0}\frac{\pi}{a}\cos\left(\frac{\pi}{a}y\right)e^{-j\beta z}\end{vmatrix}=\vec{1}_x\frac{1}{\underbrace{j\omega\varepsilon_0\,\omega\mu_0}_{k_0^2}}H_0\left[j\left(\frac{\pi}{a}\right)^2\sin\left(\frac{\pi}{a}y\right)e^{-j\beta z}+j\beta^2\sin\left(\frac{\pi}{a}y\right)e^{-j\beta z}\right]$

$\vec{E}=\vec{1}_xH_0\sin\left(\frac{\pi}{a}y\right)e^{-j\beta z}$

$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\frac{1}{2}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ H_0\sin\left(\frac{\pi}{a}y\right)e^{-j\beta z} & 0 & 0\\ 0 & \frac{H_0^*}{\mu\omega}\beta\sin\left(\frac{\pi}{a}y\right)e^{j\beta z} & \frac{jH^*}{\mu\omega}\frac{\pi}{a}\cos\left(\frac{\pi}{a}y\right)e^{j\beta z}\end{vmatrix}=\frac{1}{2}\left[-\vec{1}_y\frac{|H_0|^2j}{\omega\mu_0}\sin\left(\frac{\pi}{a}y\right)\cos\left(\frac{\pi}{a}y\right)+\vec{1}_z\frac{|H_0|^2}{\omega\mu_0}\beta\sin^2\left(\frac{\pi}{a}y\right)\right]$

$P=\oint_A\vec{S}\cdot d\vec{A}=\int_0^a\!\!\int_0^a\vec{S}\cdot\vec{1}_z\,dx\,dy=\int_0^a\!\!\int_0^a\frac{1}{2}\frac{|H_0|^2}{\omega\mu_0}\beta\sin^2\left(\frac{\pi}{a}y\right)\cdot dx\,dy=\frac{H_0^2\beta a^2}{\omega\mu_0 4}$

⑤ $\vec{E}=\vec{1}_\rho\frac{E_0}{\rho}\sin\left(\frac{2\pi}{l}z\right)$

$\vec{H}=\frac{j}{\omega\mu_0}\operatorname{rot}\vec{E}=\frac{j}{\omega\mu_0}\frac{1}{\rho}\begin{vmatrix}\vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z}\\ \frac{E_0}{\rho}\sin\left(\frac{2\pi}{l}z\right) & 0 & 0\end{vmatrix}=\frac{j}{\omega\mu_0}\vec{1}_\varphi\frac{E_0}{\rho}\cos\left(\frac{2\pi}{l}z\right)\frac{2\pi}{l}$

$\operatorname{rot}\vec{H}=\underset{0}{\vec{J}}+j\omega\varepsilon_r\varepsilon_0\vec{E}$

$\operatorname{rot}\vec{H}=\frac{1}{\rho}\begin{vmatrix}\vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z}\\ 0 & \frac{j2\pi}{\omega\mu_0 l}E_\varphi\cos\left(\frac{2\pi}{l}z\right) & 0\end{vmatrix}=-\vec{1}_\rho\frac{j2\pi}{\omega\mu l}\frac{E_0}{\rho}\left(-\sin\left(\frac{2\pi}{l}z\right)\right)\frac{2\pi}{l}=j\omega\varepsilon_r\varepsilon_0\vec{1}_\rho\frac{E_0}{\rho}\sin\left(\frac{2\pi}{l}z\right)$

$\frac{4\pi^2}{\omega\mu l^2}=\omega\varepsilon_0$

$\omega=\sqrt{\frac{1}{\varepsilon_r\varepsilon_0}}\cdot\frac{2\pi}{l}=\frac{c_0 2\pi}{l\sqrt{\varepsilon_r}}\Rightarrow f=\frac{c_0}{l\sqrt{\varepsilon_r}}=\frac{3\cdot10^8\,\text{m/s}}{5\cdot10^{-2}\,\text{m}\sqrt{2}}=4{,}24\text{ GHz}$

ELEKTROMAGNETIKA

02.04.2010

① $\vec{A} = \vec{1}_z C\rho e^{-\alpha\rho} \sin 3\varphi \quad (\omega = 0)$

$\vec{J} = ?\ ,\ \vec{K} = ?\ ,\ I = ?$

$$\vec{H} = \frac{1}{\mu}\,\mathrm{rot}\vec{A} = \frac{1}{\mu\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \rho 0 & C\rho e^{-\alpha\rho}\sin 3\varphi \end{vmatrix} =$$

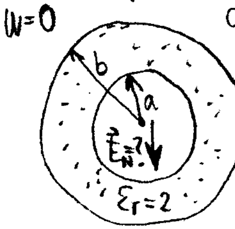
$$= \vec{1}_\rho \frac{C}{\mu} e^{-\alpha\rho}\, 3\cos 3\varphi + \vec{1}_\varphi \frac{C}{\mu}(\alpha\rho - 1) e^{-\alpha\rho} \sin 3\varphi$$

$$\vec{J} = \mathrm{rot}\vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{1}_\rho & \rho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ H_\rho & \rho H_\varphi & 0 \end{vmatrix} = \vec{1}_z \frac{C}{\mu\rho}\left(8 + 2\alpha\rho - \alpha^2\rho^2\right) e^{-\alpha\rho} \sin 3\varphi$$

$\underline{\underline{\vec{K} = 0}}$ ni singularnih ploskev

$I = \oint_{\text{okoli osi } z} \vec{H}\cdot d\vec{s} = \underline{\underline{0}}$ (zaradi $\sin 3\varphi$)

③ $\omega = 0$



$\vec{E}_0 = -\vec{1}_y \cdot 1000\,V/m$

$a = 1cm$

$b = 1{,}5cm$

$\varepsilon_r = 2$

$\vec{E}_N = ?$

$V_Z = A_Z \rho \sin\varphi + B_Z \rho^{-1} \sin\varphi$

$V_\varepsilon = A_\varepsilon \rho \sin\varphi + B_\varepsilon \rho^{-1} \sin\varphi$

$V_N = A_N \rho \sin\varphi$

$A_Z = 1000\,V/m \;\; @\, \rho \to \infty$

$\vec{E} = -\vec{1}_\rho\left(A - \frac{B}{\rho^2}\right)\sin\varphi - \vec{1}_\varphi\left(A + \frac{B}{\rho^2}\right)\cos\varphi$

$\rho = a: \; E_{\varepsilon t} = E_{Nt} \rightarrow A_\varepsilon + \frac{B_\varepsilon}{a^2} = A_N$

$\quad D_{\varepsilon n} = D_{Nn} \rightarrow \left(A_\varepsilon - \frac{B_\varepsilon}{a^2}\right)\varepsilon_r = A_N$

$\Big\} \; A_\varepsilon(1 - \varepsilon_r) + \frac{B_\varepsilon}{a^2}(\varepsilon_r + 1) = 0 \qquad A_\varepsilon = \frac{\varepsilon_r + 1}{\varepsilon_r - 1} B_\varepsilon / a^2$

$\rho = b: \; E_{Zt} = E_{\varepsilon t} \rightarrow A_Z + \frac{B_Z}{b^2} = A_\varepsilon + \frac{B_\varepsilon}{b^2}$

$\quad D_{Zn} = D_{\varepsilon n} \rightarrow A_Z - \frac{B_Z}{b^2} = \left(A_\varepsilon - \frac{B_\varepsilon}{b^2}\right)\varepsilon_r$

$\Big\} \; 2A_Z = A_\varepsilon(\varepsilon_r + 1) + \frac{B_\varepsilon}{b^2}(1 - \varepsilon_r)$

$2A_Z = \frac{(\varepsilon_r+1)^2}{\varepsilon_r - 1}\frac{B_\varepsilon}{a^2} + \frac{B_\varepsilon}{b^2}(1 - \varepsilon_r) \rightarrow B_\varepsilon = \frac{2A_Z(\varepsilon_r - 1)}{(\varepsilon_r+1)^2 a^{-2} - (\varepsilon_r - 1)^2 b^{-2}}$

$B_\varepsilon = \frac{2 \cdot 1000\,V/m \cdot (2-1)}{(2+1)^2 \cdot 10^4 m^{-2} - (2-1)^2\left(\frac{2}{3}\right)^2 \cdot 10^4 m^{-2}} = \underline{0.02338\,Vm}$

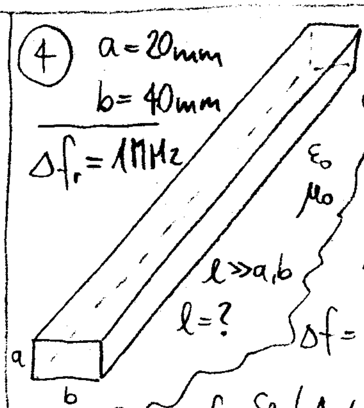
$A_\varepsilon = \frac{2+1}{2-1} B_\varepsilon / a^2 = \underline{701.3\,V/m}$

$A_N = A_\varepsilon + \frac{B_\varepsilon}{a^2} = \underline{\underline{935.1\,V/m}} \qquad \underline{\underline{\vec{E}_N = -\vec{1}_y \cdot 935{,}1\,V/m}}$

④ $a = 20mm$

$b = 40mm$

$\Delta f_r = 1MHz$



ε_0

μ_0

$\ell \gg a, b$

$\ell = ?$

$f_{011} = \frac{c_0}{2}\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{\ell}\right)^2}$

$f_{012} = \frac{c_0}{2}\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{2}{\ell}\right)^2}$

$\Delta f = f_{012} - f_{011}$

$\Delta f = \frac{c_0}{2}\left(\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{2}{\ell}\right)^2} - \sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{\ell}\right)^2}\right) \approx$

$\Delta f \approx \frac{c_0}{2}\left(\frac{1}{b}\left(1 + \frac{1}{2}\left(\frac{2b}{\ell}\right)^2\right) - \frac{1}{b}\left(1 + \frac{1}{2}\left(\frac{b}{\ell}\right)^2\right)\right)$

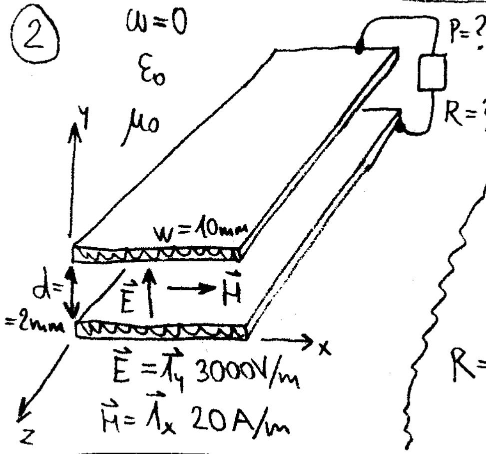
$\frac{2\Delta f}{c_0} \approx \frac{1}{2}\left(\frac{4b}{\ell^2} - \frac{b}{\ell^2}\right) = \frac{3b}{2\ell^2}$

$\ell \approx \sqrt{\frac{3bc_0}{4\Delta f}} = \underline{\underline{3m}}$

② $\omega = 0$

ε_0

μ_0



$P = ?$

$R = ?$

$w = 10mm$

$d = 2mm$

$\vec{E} = \vec{1}_y\, 3000\,V/m$

$\vec{H} = \vec{1}_x\, 20\,A/m$

$\vec{S} = \vec{E} \times \vec{H} = \vec{1}_y\, 3000\,V/m \times \vec{1}_x\, 20\,A/m =$

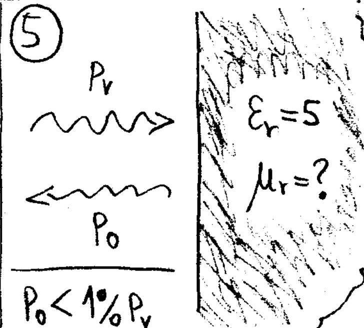
$= \underline{-\vec{1}_z\, 6 \cdot 10^4\, W/m^2}$

$P = \vec{S}\cdot\vec{A} = \vec{S}\cdot(-\vec{1}_z)\,wd =$

$= 6 \cdot 10^4\, W/m^2 \cdot 20 \cdot 10^{-6}\, m^2 = \underline{\underline{1.2W}}$

$R = \frac{U}{I} = \frac{Ed}{Hw} = \frac{3000\,V/m \cdot 2 \cdot 10^{-3} m}{20\,A/m \cdot 10^{-2} m} = \frac{6V}{0.2A} = \underline{\underline{30\Omega}}$

⑤



P_v

P_0

$\varepsilon_r = 5$

$\mu_r = ?$

$P_0 < 1\% P_v$

$\frac{P_0}{P_v} = \frac{|E_0|^2}{|E_v|^2} = |\Gamma|^2 < 1\%$

$E_V + E_0 = E_L$

$H_V + H_0 = H_L$

$H_V = \frac{E_V}{Z_0}$

$H_0 = -\frac{E_0}{Z_0}$

$H_L = \frac{E_L}{Z} \qquad \frac{E_0}{E_V} = \Gamma$

$\frac{E_V}{Z_0} - \frac{E_0}{Z_0} = \frac{E_L}{Z} = \frac{1}{Z}(E_V + E_0)$

$\frac{(1 - \Gamma)}{Z_0} = \frac{(1 + \Gamma)}{Z} \rightarrow Z = Z_0 \frac{1 + \Gamma}{1 - \Gamma}$

$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\ ;\ Z = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} \rightarrow \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{1 + \Gamma}{1 - \Gamma}$

$\mu_r = \varepsilon_r\left(\frac{1 + \Gamma}{1 - \Gamma}\right)^2 \quad |\Gamma|^2 < 0.01 \rightarrow \Gamma_{max} = \pm 0{,}1$

$\underline{\underline{\mu_r = 3.347 \ldots 7.469}}$

ELEKTROMAGNETIKA
01.07.2010

1

$$\rho(r,\theta,\phi) = C e^{-\frac{r}{a}}$$

$Q = 10^{-8}\,As$; $\omega = 0$; $a = 1m$, ε_0, μ_0

$C = ?$

$$Q = \int_0^\infty\int_0^\pi\int_0^{2\pi} \rho r^2 \sin\theta \, dr\, d\theta\, d\phi = 4\pi \int_0^\infty \rho r^2 dr =$$

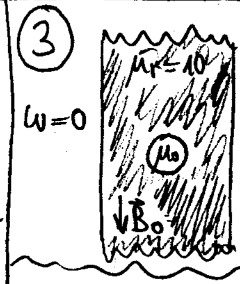
$$= 4\pi C \int_0^\infty e^{-\frac{r}{a}} r^2 dr = 4\pi C \Big[-ar^2 e^{-\frac{r}{a}} \Big|_0^\infty +$$

$$+ \int_0^\infty 2ar\, e^{-\frac{r}{a}} dr \Big] = 4\pi C \Big[0 - 2a^2 r e^{-\frac{r}{a}} \Big|_0^\infty + \int_0^\infty 2a^2 e^{-\frac{r}{a}} dr \Big] =$$

$$= 4\pi C \Big[-0 - 0 - 2a^3 e^{-\frac{r}{a}} \Big|_0^\infty \Big] = \underline{8\pi C a^3}$$

$$C = \frac{Q}{8\pi a^3} = \frac{10^{-8} As}{8\pi (1m)^3} = \underline{\underline{3.98 \cdot 10^{-10} \frac{As}{m^3}}}$$

3



Votlinica polmer $a = 1mm$

$\vec{B}_0 = -\vec{1}_z \cdot 0.1T$

$\vec{B} = ?$ v votlinici

$V_m(r>a) = (Ar + Br^{-2})\cos\Theta$ $\qquad r \gg a: A = \frac{|\vec{B}_0|}{\mu_0 \mu_r}$

$V_m(r<a) = Cr\cos\Theta$

$H_t(r=a): A + Ba^{-3} = C$

$B_n(r=a): (A - 2Ba^{-3})\mu_r = C$

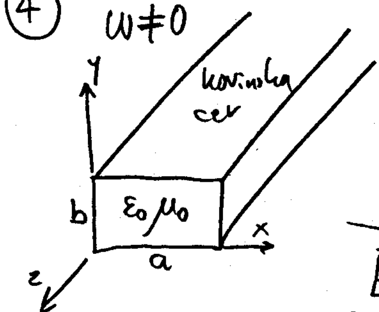
$$2\mu_r A = C(\mu_r + 1)$$

$$C = \frac{2\mu_r A}{\mu_r + 1} = \frac{2|\vec{B}_0|}{(\mu_r+1)\mu_0}$$

$$\vec{B} = -\vec{1}_z \mu_0 C = \underline{\underline{\frac{2\vec{B}_0}{\mu_r + 1} = -\vec{1}_z \cdot 18.2mT}}$$

4

$\omega \neq 0$



$a = 3cm$; $l \to \infty$

$b = 1cm$; $f = 7.5GHz$

$|E_{max}| = 10V/m$

$\vec{E}(x,y,z) = ?$

$$\lambda_0 = \frac{c_0}{f} = 4cm \qquad \underbrace{a > \lambda_0/_2 = 2cm > b}_{\text{rod } TE_{10}}$$

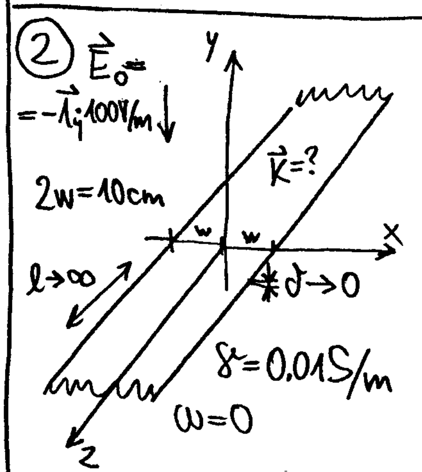
$$\vec{E} = \vec{1}_y C \sin\left(\frac{\pi}{a}x\right) e^{\pm j\beta_z z} \qquad k = \frac{2\pi}{\lambda}$$

$$\underline{\underline{C = 10V/m \cdot e^{j\varphi}}} \text{ ; } \beta_z = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2} = \underline{\underline{1.17rd/cm}}$$

$$\underline{\underline{\vec{E} = \vec{1}_y 10V/m\, e^{j\varphi} \sin(1.047rd/_{cm}\, x)\, e^{\pm j(1.17rd/cm)z}}}$$

2

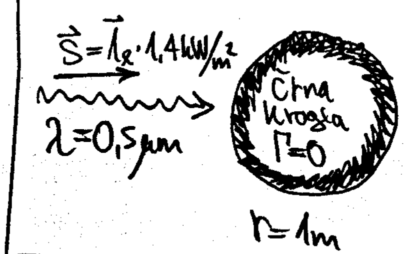
$\vec{E}_0 = -\vec{1}_y 100V/m$



$\underline{\vec{K} = 0 \text{ ker je } \vec{E} \perp \text{ kovinski trak!}}$

5

$\varepsilon_0, \mu_0, \omega \neq 0$



$\vec{F} = ?$

$P = \vec{S} \cdot \vec{A} = |\vec{S}|\pi r^2 = \underline{4398W}$

$\vec{v} = \vec{1}_x c_0 = \underline{\vec{1}_x \cdot 3 \cdot 10^8 m/s}$

$\vec{F}dt = d(m\vec{v}) \to \vec{F} = \vec{v}\frac{dm}{dt} = \vec{v}\frac{P}{c_0^2}$

$W = mc_0^2$; $P = \frac{dW}{dt} = c_0^2 \frac{dm}{dt}$

$$\vec{F} = \vec{v}\frac{P}{c_0^2} = \underline{\underline{\vec{1}_x \cdot 1.466 \cdot 10^{-5} N}}$$

Rešitev 1. kolokvija iz ELEKTROMAGNETIKE 23.11.2010

Rešitve

$$\Delta V = \mathrm{div}(\mathrm{grad}\,V) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial V}{\partial r}r^2\right) = -\frac{\rho}{\varepsilon_0}$$

$\boxed{r > R}$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial V_Z}{\partial r}r^2\right) = 0$$

$$\frac{\partial}{\partial r}\left(\frac{\partial V_Z}{\partial r}r^2\right) = 0 \qquad \Big|\int dr$$

$$\frac{\partial V_Z}{\partial r}r^2 = C_1$$

$$\frac{\partial V_Z}{\partial r} = \frac{C_1}{r^2} \qquad \Big|\int dr$$

$$V_Z = -\frac{C_1}{r} + C_2$$

C_2=0, ker je potencial v neskončnosti nič

$$\vec{E}_Z = -\mathrm{grad}V_Z = -\vec{1}r\frac{C_1}{r^2}$$

$\boxed{r < R}$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial V_N}{\partial r}r^2\right) = -\frac{k\cdot r}{\varepsilon_0}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial V_N}{\partial r}r^2\right) = -\frac{k\cdot r^3}{\varepsilon_0} \qquad \Big|\int dr$$

$$\frac{\partial V_N}{\partial r}r^2 = -\frac{k\cdot r^4}{4\cdot\varepsilon_0} + C_3$$

$$\frac{\partial V_N}{\partial r} = -\frac{k\cdot r^2}{4\cdot\varepsilon_0} + \frac{C_3}{r^2} \qquad \Big|\int dr$$

$$V_N = -\frac{k\cdot r^3}{12\cdot\varepsilon_0} - \frac{C_3}{r} + C_4$$

C_3=0, ker potencial v središču ni neskončen

$$\vec{E}_N = -\mathrm{grad}V_N = \vec{1}r\frac{k\cdot r^2}{4\cdot\varepsilon_0}$$

$\boxed{r = R}$

$$\vec{E}_Z = \vec{E}_N \Rightarrow -\vec{1}r\frac{C_1}{R^2} = \vec{1}r\frac{k\cdot R^2}{4\cdot\varepsilon_0} \Rightarrow C_1 = -\frac{k\cdot R^4}{4\cdot\varepsilon_0}$$

$$V_Z = \frac{k\cdot R^4}{4\cdot\varepsilon_0\cdot r}$$

$$\vec{B} = \nabla\times\vec{A} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & -\frac{\mu_0 I}{4\pi}\ln\left(\frac{x^2+y^2}{\alpha}\right) \end{vmatrix}$$

$$\vec{B} = \vec{1}_x\left[\frac{\partial}{\partial y}\left(-\frac{\mu_0 I}{4\pi}\ln\left(\frac{x^2+y^2}{\alpha}\right)\right)\right] - \vec{1}_y\left[\frac{\partial}{\partial x}\left(-\frac{\mu_0 I}{4\pi}\ln\left(\frac{x^2+y^2}{\alpha}\right)\right)\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[-\vec{1}_x\frac{2y}{x^2+y^2} + \vec{1}_y\frac{2x}{x^2+y^2}\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[-\vec{1}_x\frac{2y}{\rho^2} + \vec{1}_y\frac{2x}{\rho^2}\right]$$

$$\vec{B} = \frac{\mu_0 I}{2\pi\rho^2}\left[-\vec{1}_x y + \vec{1}_y x\right]$$

$$W = \frac{1}{2}\int_v \rho V dv = \frac{1}{2}\int_0^{R_0}\int_0^{\pi}\int_0^{2\pi}\rho V\, r^2\sin\theta\, dr\, d\theta\, d\phi$$

$$V_N = -\frac{\rho}{2\varepsilon_0}\left(R_0^2 - \frac{r^2}{3}\right)$$ smo izračunali na vajah

$$W = \frac{\pi\rho^2}{2\varepsilon_0}\int_0^{R_0}\int_0^{\pi}\left(R_0^2 - \frac{r^2}{3}\right)r^2\sin\theta\, dr\, d\theta$$

$$W = \frac{\pi\rho^2}{2\varepsilon_0}\int_0^{R_0}\left(R_0^2 r^2 - \frac{r^4}{3}\right)dr\,[-\cos\theta]_0^{\pi}$$

$$W = \frac{\pi\rho^2}{\varepsilon_0}\cdot\left[R_0^2\frac{r^3}{3} - \frac{r^5}{3\cdot 5}\right]_0^{R_0} = \frac{\pi\rho^2}{\varepsilon_0}\frac{4}{15}R_0^5$$

$$\rho = \frac{e_0}{\frac{4}{3}\pi\cdot R_0^3}$$

$$W = \frac{\pi}{\varepsilon_0}\left(\frac{e_0}{\frac{4}{3}\pi\cdot R_0^3}\right)^2\frac{4}{15}R_0^5 = \frac{3e_0^2}{20\pi\varepsilon_0 R_0} = m_e c_0^2$$

$$R_0 = \frac{3e_0^2}{20\pi\varepsilon_0 m_e c_0^2} = \frac{3\cdot(1{,}6\cdot 10^{-19}\,\mathrm{As})^2}{20\pi\cdot 8{,}85\cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}\cdot 9{,}1\cdot 10^{-31}\,\mathrm{kg}\cdot(3\cdot 10^8\,\mathrm{m/s})^2} = 1{,}7\cdot 10^{-15}\,\mathrm{m}$$

$$V = \int_{r_1}^{r_2}\vec{E}\cdot d\vec{r}$$

$$V = \int_{r_1}^{r_2}\frac{q}{2\pi\varepsilon_0\varepsilon_r r}dr = \frac{q}{2\pi\varepsilon_0\varepsilon_r}\cdot\ln\frac{r_2}{r_1}$$

$$C = \frac{Q}{V} = \frac{qL}{\frac{q}{2\pi\varepsilon_0\varepsilon_r}\cdot\ln\frac{r_2}{r_1}} = \frac{2\pi\varepsilon_0\varepsilon_r L}{\ln\frac{r_2}{r_1}} = \frac{2\pi\cdot 8{,}85\cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}\cdot 2\cdot 10\,\mathrm{m}}{\ln\frac{13{,}6\,\mathrm{mm}}{5\,\mathrm{mm}}} = 1{,}11\,\mathrm{nF}$$

Vaje iz teorije elektromagnetike, str. 86, vaja 92.

2. kolokvij EM 24.1.2011

① $$\vec{H}=-\frac{1}{j\omega\mu_0}\,\mathrm{rot}\,\vec{E}=-\frac{1}{j\omega\mu_0}\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\cdot\vec{1}_\theta & r\sin\theta\,\vec{1}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & r\cdot 0 & r\sin\theta\, E_0\frac{e^{-jkr}}{r\sin\theta}\end{vmatrix}=-\frac{k}{\omega\mu_0}E_0\vec{1}_\theta\frac{e^{-jkr}}{r\sin\theta}=-\vec{1}_\theta\frac{E_0}{Z_0}\frac{e^{-jkr}}{r\sin\theta}$$

$$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\frac{1}{2}\begin{vmatrix}\vec{1}_r & \vec{1}_\theta & \vec{1}_\phi\\ 0 & 0 & E_0\frac{e^{-jkr}}{r\sin\theta}\\ 0 & -\frac{E_0^*}{Z_0}\frac{e^{jkr}}{r\sin\theta} & 0\end{vmatrix}=\vec{1}_r\frac{E_0E_0^*}{2Z_0(r\sin\theta)^2}\qquad \vec{J}=\mathrm{rot}\,\vec{H}-j\omega\varepsilon_0\vec{E}=\underline{\underline{0}}$$

② $$k=\frac{\omega}{c}=\frac{10^7\,\mathrm{rad/s}}{2\cdot10^8\,\mathrm{m/s}}=0{,}05\,\mathrm{rad/m}$$

$$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*\qquad \vec{1}_k=-\vec{1}_z\qquad Z=\frac{|E|}{|H|}=251{,}33\,\Omega$$

$$\lambda=\frac{2\pi}{k}=125{,}66\,\mathrm{m}\qquad \varepsilon_r=\frac{1}{Zc\varepsilon_0}=2{,}25\qquad \mu_r=\frac{Z}{c\mu_0}=1$$

③ $$\vec{V}_{m_0}=\vec{1}_\phi\frac{\mu_0I_1a^2}{4\pi}\frac{\sin\theta}{r^2}\qquad \theta=\frac{\pi}{2}\Rightarrow\sin\theta=1$$

$$M=\frac{1}{I_1}\oint_{2\pi}\vec{V}_{m_0}\,d\vec{s}_0=\frac{1}{I_1}\int_0^{2\pi}\vec{1}_\phi\frac{\mu_0I_1a^2}{4\pi}\cdot\frac{1}{b^2}\vec{1}_\phi\,b\,d\phi=\frac{\mu_0a^2}{4\pi}\cdot\frac{1}{b}\cdot2\pi=\underline{\underline{\frac{\mu_0a^2}{2b}}}$$

④ $$\vec{H}=-\frac{1}{j\omega\mu_0}\,\mathrm{rot}\,\vec{E}=-\frac{1}{j\omega\mu_0}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & -j\omega\mu_0\frac{a}{\pi}H_0\sin\left(\frac{\pi}{a}x\right)e^{-j\beta z} & 0\end{vmatrix}=$$

$$\vec{H}=-\frac{1}{j\omega\mu_0}\left[\vec{1}_z\left(-j\omega\mu_0\right)\frac{a}{\pi}H_0\frac{\pi}{a}\cos\left(\frac{\pi}{a}x\right)e^{-j\beta z}+\vec{1}_x\,j\omega\mu_0\frac{a}{\pi}H_0\sin\left(\frac{\pi}{a}x\right)(-j\beta)e^{-j\beta z}\right]=$$

$$\vec{H}=\vec{1}_x\,j\beta\frac{a}{\pi}H_0\sin\left(\frac{\pi}{a}x\right)e^{-j\beta z}+\vec{1}_z\,H_0\cos\left(\frac{\pi}{a}x\right)e^{-j\beta z}$$

$$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\frac{1}{2}\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ 0 & -j\omega\mu_0\frac{a}{\pi}H_0\sin\left(\frac{\pi}{a}x\right)e^{-j\beta z} & 0\\ -jH_0^*\beta\frac{a}{\pi}\sin\left(\frac{\pi}{a}x\right)e^{+j\beta z} & 0 & H_0^*\cos\left(\frac{\pi}{a}x\right)e^{+j\beta z}\end{vmatrix}=$$

$$\vec{S}=\underbrace{\frac{1}{2}\vec{1}_x\left[-j\omega\mu_0\frac{a}{\pi}H_0H_0^*\sin\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{a}x\right)\right]}_{Im}+\underbrace{\frac{1}{2}\vec{1}_z\,\omega\mu_0\frac{a}{\pi}H_0H_0^*\beta\frac{a}{\pi}\sin^2\left(\frac{\pi}{a}x\right)}_{Re}$$

$$P=\int_A\vec{S}\cdot d\vec{A}=\int_0^a\int_0^a\frac{1}{2}\omega\mu_0\left(\frac{a}{\pi}\right)^2H_0H_0^*\beta\sin^2\left(\frac{\pi}{a}x\right)dx\,dy$$

$$P=\frac{1}{2}\omega\mu_0\left(\frac{a}{\pi}\right)^2H_0H_0^*\beta\frac{a}{2}a=\frac{H_0H_0^*a^2}{4}\omega\mu_0\left(\frac{a}{\pi}\right)^2\beta$$

$$\boxed{\int\sin^2ax=\frac{x}{2}+\frac{\sin2ax}{4a}\qquad \int_0^a\sin^2\left(\frac{\pi}{a}x\right)dx=\frac{a}{2}}$$

⑤ $$f_{l,m,n}=\frac{c_0}{2}\sqrt{\left(\frac{l}{a}\right)^2+\left(\frac{m}{b}\right)^2+\left(\frac{n}{c}\right)^2}$$

$$f_{101}=\frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{c}\right)^2}=8\,\mathrm{GHz}\qquad \frac{0{,}3}{2}\sqrt{\frac{1}{a^2}+\frac{1}{c^2}}=8$$

$$f_{111}=\frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{2}{a}\right)^2+\left(\frac{1}{c}\right)^2}=10\,\mathrm{GHz}\qquad \frac{0{,}3}{2}\sqrt{\frac{5}{a^2}+\frac{1}{c^2}}=10$$

$$\frac{1}{a^2}+\frac{1}{c^2}=\left(\frac{16}{0{,}3}\right)^2$$

$$\frac{5}{a^2}+\frac{1}{c^2}=\left(\frac{20}{0{,}3}\right)^2$$

$$\frac{5}{a^2}-\frac{1}{a^2}=\left(\frac{20}{0{,}3}\right)^2-\left(\frac{16}{0{,}3}\right)^2$$

$$\underline{\underline{a=5\,\mathrm{cm}}}$$

$$\underline{\underline{c=2\,\mathrm{cm}}}$$

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1 (r,θ,ϕ) $\vec{A}=\vec{1}_\phi r\sin\theta$; $\vec{B}=\vec{1}_\phi C$

$\mathrm{grad}(\vec{A}\cdot\mathrm{rot}\,\vec{B}-\vec{B}\cdot\mathrm{rot}\,\vec{A})=?$

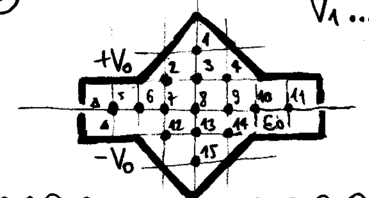
$\vec{A}\cdot\mathrm{rot}\,\vec{B}-\vec{B}\cdot\mathrm{rot}\,\vec{A}=\mathrm{div}(\vec{B}\times\vec{A})$

$\vec{A}\parallel\vec{B}\longrightarrow\vec{A}\times\vec{B}=\underline{0}$

$\mathrm{grad}(\vec{A}\cdot\mathrm{rot}\,\vec{B}-\vec{B}\cdot\mathrm{rot}\,\vec{A})=$

$=\mathrm{grad}(\mathrm{div}(\vec{B}\times\vec{A}))=\mathrm{grad}(\mathrm{div}\,0)=\underline{\underline{0}}$

3



$V_1\dots V_{15}=?$

Simetrija: $\underline{V_5=V_6=V_7=V_8=V_9=V_{10}=V_{11}=0}$ $\quad\underline{V_2=V_4}$

$4V_1=3V_0+V_3=V_0+4V_2=56V_2-32V_0\longrightarrow\underline{\underline{V_2=\frac{33}{52}V_0=V_4}}$

$4V_2=2V_0+V_3\rightarrow V_3=4V_2-2V_0\rightarrow\underline{\underline{V_3=\frac{7}{13}V_0}}$

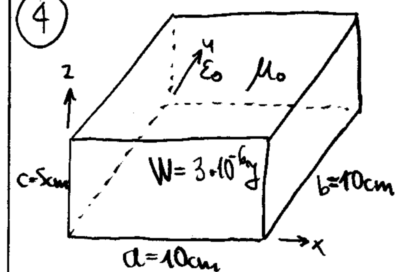
$4V_3=V_1+2V_2=16V_2-8V_0\rightarrow V_1=14V_2-8V_0\rightarrow\underline{\underline{V_1=\frac{23}{26}V_0}}$

spodaj:

$\underline{\underline{V_{12}=V_{14}=-V_2=-\frac{33}{52}V_0}}$

$\underline{\underline{V_{13}=-V_3=-\frac{7}{13}V_0}}$ $\quad\underline{\underline{V_{15}=-V_1=-\frac{23}{26}V_0}}$

4



osnovni rod $\vec{E}_{max}=?$

$f=?$

TE 110

$f=\frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}=\underline{2{,}12\,\mathrm{GHz}}$

$\vec{E}(x,y,z)=\vec{1}_z\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)E_{max}$

$W_e=\frac{1}{2}\int_0^a\int_0^b\int_0^c\varepsilon_0|\vec{E}|^2\,dx\,dy\,dz=\frac{1}{2}\,\frac{a}{2}\,\frac{b}{2}\,c\,\varepsilon_0E_{max}^2$

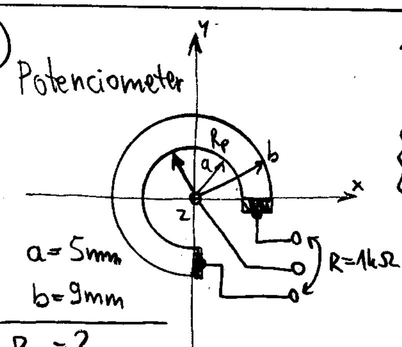
$E_{max}^2=\frac{8W_e}{\varepsilon_0abc}$

$E_{max}=\sqrt{\frac{8W_e}{\varepsilon_0abc}}$

$E_{max}=73{,}7\,\mathrm{kV/m}$

$\underline{\underline{\vec{E}_{max}=\vec{1}_z\,73{,}7\,\mathrm{kV/m}}}$

2 Potenciometer



$a=5\,\mathrm{mm}$

$b=9\,\mathrm{mm}$

$R_p=?$

(ρ,φ,z)

$\vec{E}=\vec{1}_\varphi C/\rho$ $\quad\vec{K}=\frac{\vec{E}}{R_p}$

$U=\int_0^{3\pi/2}\vec{E}\cdot d\vec{s}=\int_0^{3\pi/2}\vec{1}_\varphi\frac{C}{\rho}\cdot\vec{1}_\varphi\,\rho\,d\varphi$

$\underline{U=\frac{3\pi}{2}C}$

$I=\int_a^b\vec{K}\cdot\vec{1}_\varphi\,d\rho=\int_a^b\vec{1}_\varphi\frac{C}{\rho R_p}\cdot\vec{1}_\varphi\,d\rho$

$\underline{I=\frac{C}{R_p}\ln(b/a)}$

$\underline{R=U/I=\frac{3\pi R_p}{2\ln(b/a)}}$

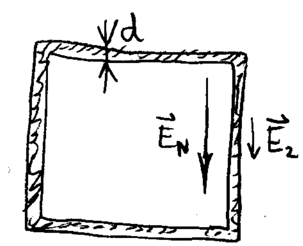
$\underline{\underline{R_p=\frac{2\ln(b/a)}{3\pi}R=125\,\Omega}}$

5 Al oklop $d=1\,\mathrm{mm}$

$\gamma=35\cdot10^6\,\mathrm{S/m}$

$f=100\,\mathrm{kHz}$ $\quad\mu=\mu_0$

$a[\mathrm{dB}]=?$



$\delta=\sqrt{\frac{2}{\omega\mu\gamma}}=\sqrt{\frac{1}{\pi f\mu_0\gamma}}=\underline{269\,\mu\mathrm{m}}$

$\mu_0=4\pi\cdot10^{-7}\,\mathrm{Vs/Am}$

$a_{dB}=20\log_{10}\frac{E_1}{E_2}$ $\quad\frac{E_1}{E_2}=e^{\frac{d}{\delta}}$

$a_{dB}=\frac{20}{\ln10}\ln\left(e^{\frac{d}{\delta}}\right)=\frac{20}{\ln10}\,\frac{d}{\delta}$

$a_{dB}=\frac{20}{2{,}303}\,\frac{1\,\mathrm{mm}}{269\,\mu\mathrm{m}}=\underline{\underline{32{,}3\,\mathrm{dB}}}$

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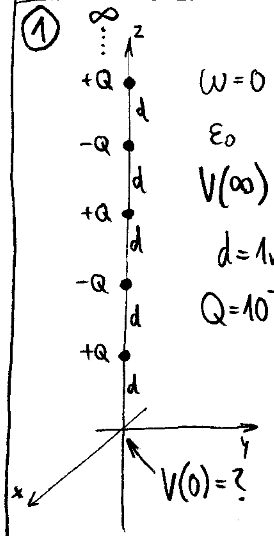
①



$\omega = 0$

ε_0

$V(\infty) = 0$

$d = 1\,m$

$Q = 10^{-9}\,As$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{r_i}$$

$$V = \frac{Q}{4\pi\varepsilon_0 d}\sum_{i=1}^{\infty}\frac{(-1)^{i+1}}{i}$$

$$V = \frac{Q}{4\pi\varepsilon_0 d}\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\dots\right)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$$

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

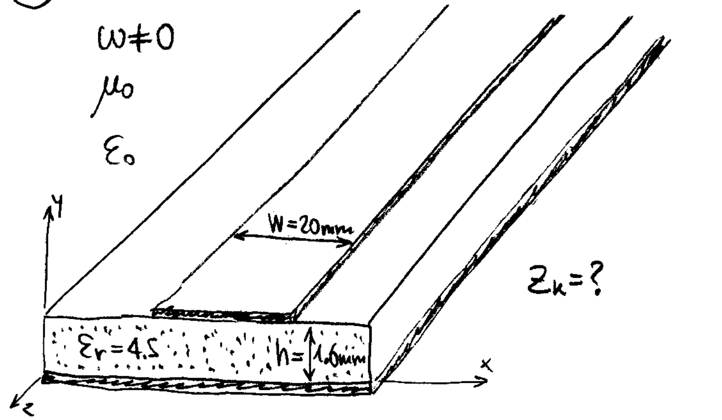
$$V = \frac{Q}{4\pi\varepsilon_0 d}\ln 2 = \frac{10^{-9}As \cdot 0.693}{4\pi \frac{1}{4\pi \cdot 9 \cdot 10^9}\frac{As}{Vm}\cdot 1m} = \underline{\underline{6.24V}}$$

③

$\omega \neq 0$

μ_0

ε_0



$Z_k = ?$

$W = 20mm >> h = 1.6mm \to$ zanemarim stresanje

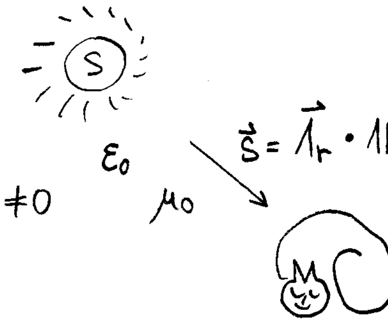
$$C/\ell = \varepsilon_0\varepsilon_r \frac{w}{h}$$

$$L/\ell = \mu_0 \frac{h}{w}$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = Z_0 \approx 120\pi\,\Omega \approx 377\Omega$$

$$Z_k = \sqrt{\frac{L/\ell}{C/\ell}} = \sqrt{\frac{\mu_0\frac{h}{w}}{\varepsilon_0\varepsilon_r\frac{w}{h}}} = \frac{h}{w}\frac{Z_0}{\sqrt{\varepsilon_r}} = \underline{\underline{14.2\,\Omega}}$$

④



$\omega \neq 0$ $\quad \varepsilon_0 \quad \mu_0$

$\vec{S} = \vec{1}_r \cdot 1kW/m^2$

$|\vec{E}_{eff}| = ?$

$|\vec{H}_{eff}| = ?$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{E}_{eff}\times\vec{H}_{eff}^*$$

$$|\vec{E}| = |\vec{H}|Z_0 \qquad |\vec{E}_{eff}| = |\vec{H}_{eff}|Z_0 \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$|\vec{S}| = \frac{|\vec{E}|^2}{2Z_0} = \frac{|\vec{E}_{eff}|^2}{Z_0} = \frac{|\vec{H}|^2 Z_0}{2} = |\vec{H}_{eff}|^2 Z_0$$

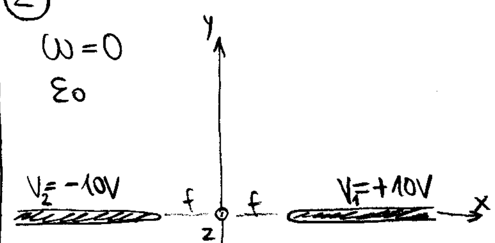
$$|\vec{E}_{eff}| = \sqrt{|\vec{S}|Z_0} = \underline{\underline{614\,V_{eff}/m}}$$

$$|\vec{H}_{eff}| = \sqrt{\frac{|\vec{S}|}{Z_0}} = \underline{\underline{1.63\,A_{eff}/m}}$$

②

$\omega = 0$

ε_0



$$\frac{\partial}{\partial z} = 0$$

$$V(x, y=0) = ?$$

Valjni eliptički KS:

$x = f\,ch\,u\cos v$

$y = f\,sh\,u\sin v$

$z = z$

$$h_u = h_v = f\sqrt{sh^2 u + \sin^2 v}$$

geometrija $\frac{\partial}{\partial u} = 0$

$$\Delta V = \frac{1}{h_u h_v}\frac{\partial}{\partial v}\left(\frac{h_u}{h_v}\frac{\partial V}{\partial v}\right)$$

$$\Delta V = 0 \to \frac{\partial^2 V}{\partial v^2} = 0 \to V = C_1 v + C_2$$

$$0 \le v \le \pi \longrightarrow V = -\frac{20V}{\pi}v + 10V$$

$$u = 0 \to v = \arccos\frac{x}{f}$$

$$V(x, y=0) = \begin{cases} +10V \text{ @ } x > f \\ -\frac{20V}{\pi}\arccos\frac{x}{f} + 10V \text{ @ } -f < x < f \\ -10V \text{ @ } x < f \end{cases}$$

⑤

Morska voda μ_0

$\gamma = 5\,S/m$

$\varepsilon_r = 80 \qquad \omega \neq 0$

polovični $\omega\vec{D} = \vec{J}$ prevodni

$\alpha = ?$

$\beta_z = ?$

$$\Delta\vec{E} + (\omega^2\mu\varepsilon - j\omega\mu\gamma)\vec{E} = 0 \qquad \mu = \mu_0 \quad \varepsilon = \varepsilon_0\varepsilon_r$$

$$\omega\vec{D} = \vec{J} \to \omega = \frac{\gamma}{\varepsilon_0\varepsilon_r} = \frac{5A\;4\pi\cdot 9\cdot 10^9\,Vm}{Vm\;As\cdot 80} = \underline{7.069\cdot 10^9\,rd/s}$$

$$k = \sqrt{\omega^2\mu_0\varepsilon_0\varepsilon_r - j\omega\mu_0\gamma} = \beta_z - j\alpha$$

$$k = \sqrt{(7.069\cdot 10^9 s^{-1})^2 4\pi\cdot 10^{-7}\frac{Vs}{Am}\cdot\frac{1}{4\pi\, 9\cdot 10^9}\frac{As}{Vm}\cdot 80 - j\,7.069\cdot 10^9\cdot 1\cdot 4\pi\cdot 10^{-7}\frac{Vs}{Am}5\frac{S}{m}}$$

$$k = \sqrt{\underbrace{44413\,m^{-2}}_{A} - j\underbrace{44413\,m^{-2}}_{B}} = \sqrt{(\beta_z^2-\alpha^2) - j2\alpha\beta_z} = \sqrt{A - jB}$$

$$\left.\begin{aligned}\beta_z^2 - \alpha^2 &= A\\ 2\alpha\beta_z &= B\end{aligned}\right\} \beta_z^4 - A\beta_z^2 - \frac{B^2}{4} = 0 \to \beta_z = \sqrt{\frac{A \pm\sqrt{A^2+B^2}}{2}} = \underline{\underline{232\,rd/m}} \qquad \alpha = \frac{B}{2\beta_z} = \underline{\underline{96\,Np/m}}$$

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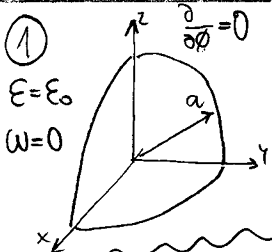
1



$\varepsilon=\varepsilon_0$

$\omega=0$

$$V(r,\theta,\phi)=\begin{cases} C\frac{r}{a}\cos\theta \ @\ r<a \\ C\frac{a^2}{r^2}\cos\theta \ @\ r>a\end{cases}$$

Kje so elektrine : $\rho=?$ $\sigma=?$ $q=?$ $Q=?$

$$\rho=\mathrm{div}(-\varepsilon\,\mathrm{grad}V)=-\varepsilon\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)\right]$$

$\underline{r<a\rightarrow\rho=0}$

$\underline{r>a\rightarrow\rho=0}$

$$D_r=\vec{1}_r\cdot(-\varepsilon\,\mathrm{grad}V)=-\varepsilon\frac{\partial V}{\partial r}$$

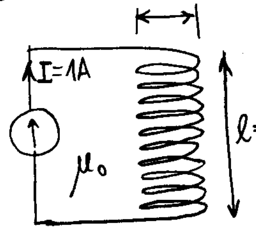
$$\sigma(r=a)=\vec{1}_r(\vec{D}_z-\vec{D}_N)=\left(\frac{2\varepsilon Ca^2}{r^3}\cos\theta+\varepsilon C\frac{1}{a}\cos\theta\right)\Bigg|_{r=a}=\underline{\underline{\frac{3\varepsilon C\cos\theta}{a}}}$$

$$Q(r=0)=\lim_{r\to0}\oint_{4\pi}\vec{D}_N\cdot\vec{1}_r r^2\sin\theta\, d\theta\, d\phi=\underline{\underline{0}}$$

$$q(\theta=0)=\lim_{\theta\to0}\oint_{2\pi}\vec{D}\cdot\vec{1}_\theta\, r\sin\theta\, d\phi=\lim_{\theta\to0}\oint_{2\pi}\frac{1}{r}\frac{\partial V}{\partial\theta}r\sin\theta\, d\theta=\underline{\underline{0}}$$

$$q(\theta=\pi)=\underline{\underline{0}}$$

3



$\omega=0$

N= 200 ovojev

$W_m=?$

$$\oint\vec{H}\cdot d\vec{s}\approx|\vec{H}|l=IN\rightarrow|\vec{H}|=\frac{IN}{l}=\underline{2000\,A/m}$$

$$|\vec{B}|=\mu_0|\vec{H}|=4\pi\cdot10^{-7}\frac{Vs}{Am}\cdot2000\,A/m=\underline{0{,}8\,\pi\,mT}$$

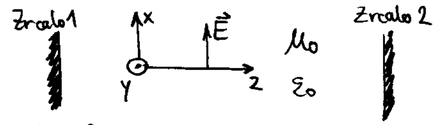
$$W_m=\frac{1}{2}\int_v\vec{H}\cdot\vec{B}\,dv=\frac{1}{2}|\vec{H}||\vec{B}|\,\pi r^2\,l=$$

$$=\frac{1}{2}2000\,A/m\cdot0{,}8\pi\cdot10^{-3}\frac{Vs}{m^2}\cdot\pi\cdot(5\cdot10^{-3}m)^2\cdot0{,}1m=$$

$$=\underline{\underline{1{,}974\cdot10^{-5}\,VAs=19{,}74\,\mu J}}$$

4

HeNe laser $f=474\,THz$ $\omega\neq0$



$\vec{E}=\vec{1}_x C\sin kz$ $C=1000\,V/m$ $k=\omega\sqrt{\mu_0\varepsilon_0}$

$\vec{H}=?$ $\vec{S}=?$ $\vec{J}=?$ $\rho=?$

$$\vec{H}=\frac{j}{\omega\mu_0}\mathrm{rot}\vec{E}=\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ C\sin kz & 0 & 0\end{vmatrix}=\vec{1}_y\frac{jkC}{\omega\mu_0}\cos kz=\vec{1}_y\frac{jC}{Z_0}\cos kz$$

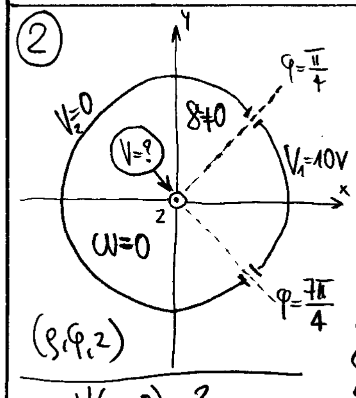
$$\underline{\vec{H}=\vec{1}_y\,j\,2{,}653\,A/m\cos kz}$$

$$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*=\vec{1}_z\frac{1}{2}C\sin kz\,\frac{-jC^*}{Z_0}\cos kz=\underline{\vec{1}_z(-j)\,1{,}33\,kW/m^2\sin kz\cos kz}$$

$$\vec{J}=\mathrm{rot}\vec{H}-j\omega\varepsilon_0\vec{E}=\begin{vmatrix}\vec{1}_x & \vec{1}_y & \vec{1}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & \frac{jC}{Z_0}\cos kz & 0\end{vmatrix}-j\omega\varepsilon_0\vec{1}_x C\sin kz=\underline{\underline{0}}$$

$$\rho=\mathrm{div}(\varepsilon_0\vec{E})=\frac{\partial}{\partial x}(\varepsilon_0 C\sin kz)=\underline{\underline{0}}$$

2



$V(\rho=0)=?$

$$V(\rho,\varphi,z)=\sum_{n=0}^{\infty}C_n\rho^n\cos n\varphi$$ (simetrija $\pm\varphi$ → ni členov $n\sin\varphi$)

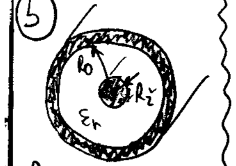
$$\underline{V(\rho=0)=C_0\rho^0\cos0\cdot\varphi=C_0}$$

$$\rho=a\rightarrow\int_0^{2\pi}\cos m\varphi\sum_n C_n\rho^n\cos n\varphi\,d\varphi=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}V_1\cos m\varphi\,d\varphi+\int_{\frac{\pi}{4}}^{\frac{7\pi}{4}}V_2\cos m\varphi\,d\varphi$$

$$m=0\longrightarrow2\pi C_0=\frac{\pi}{2}V_1+\frac{3\pi}{2}V_2$$ (povprečje sosedov $\Delta V=0$)

$$C_0=\frac{1}{4}V_1+\frac{3}{4}V_2=2{,}5V=\underline{\underline{V(\rho=0)}}$$

5



$R_i=0{,}5mm$

$R_0=1{,}8mm$

$\gamma_{Cu}=56\cdot10^6\,S/m$

$\varepsilon_r=1{,}5$ (brez izgub)

$f=1GHz$

$\alpha/l\,[dB/m]=?$

$$\alpha/l\,[dB/m]=\frac{10}{\ln10}\cdot\frac{R/l}{Z_k}=\frac{10}{\ln10}\cdot\frac{\sqrt{\frac{\omega\mu_0}{2\gamma}}\left(\frac{1}{2\pi R_i}+\frac{1}{2\pi R_0}\right)}{\frac{Z_0}{2\pi\sqrt{\varepsilon_r}}\ln R_0/R_i}=\frac{10\sqrt{\frac{\omega\mu_0}{2\gamma}}\left(\frac{1}{R_i}+\frac{1}{R_0}\right)}{\ln10\,\frac{Z_0}{\sqrt{\varepsilon_r}}\ln R_0/R_i}$$

$$\alpha/l\,[dB/m]=\frac{10\sqrt{\frac{2\pi\cdot10^9 s^{-1}\,4\pi\cdot10^{-7}Vs/Am}{2\cdot56\cdot10^6 A/Vm}}\left(\frac{1}{0{,}5\cdot10^{-3}m}+\frac{1}{1{,}8\cdot10^{-3}m}\right)}{\ln10\cdot\frac{377\Omega}{\sqrt{1{,}5}}\ln\frac{1{,}8\cdot10^{-3}m}{0{,}5\cdot10^{-3}m}}=\frac{215\,\Omega/m}{908\,\Omega}$$

$$\underline{\underline{\alpha/l=0{,}236\,dB/m}}$$

ELEKTROMAGNETIKA
18. 9. 2014

1

(r,θ,ϕ) $\quad \omega = 0 \quad \varepsilon = \varepsilon_0$

$$\rho(r) = \begin{cases} 0 \;;\; r<a \\ \frac{C}{r^4} \;;\; r \geq a \end{cases} \qquad a = 1m \qquad C = 10^{-9} Asm$$

$W = ?$

$$\frac{\partial}{\partial\theta} = \frac{\partial}{\partial\phi} = 0 \longrightarrow \vec{D} = \vec{1}_r \frac{\int_a^r \rho(r')\, 4\pi r'^2 dr'}{4\pi r^2}$$

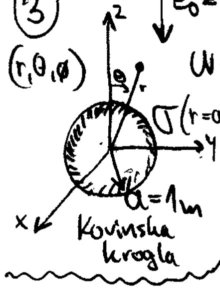
$$\vec{D} = \vec{1}_r \frac{1}{r^2} \int_a^r \frac{C}{r'^4} r'^2 dr' = \vec{1}_r \frac{C}{r^2}\left(-\frac{1}{r'}\Big|_a^r\right) = \vec{1}_r \frac{C}{r^2}\left(\frac{1}{a} - \frac{1}{r}\right)$$

$$W = \frac{1}{2}\int_V \vec{E}\cdot\vec{D}\, dv = \frac{1}{2\varepsilon_0}\int_a^\infty |\vec{D}|^2 4\pi r^2 dr = \frac{C^2}{2\varepsilon_0}\int_a^\infty \frac{1}{r^4}\left(\frac{1}{a}-\frac{1}{r}\right)^2 4\pi r^2 dr$$

$$W = \frac{2\pi C^2}{\varepsilon_0}\int_a^\infty \left(\frac{1}{a^2 r^2} - \frac{2}{a r^3} + \frac{1}{r^4}\right) dr = \frac{2\pi C^2}{\varepsilon_0}\left(\frac{1}{a^3} - \frac{1}{a^3} + \frac{1}{3a^3}\right)$$

$$\underline{\underline{W = \frac{2\pi C^2}{3a^3\varepsilon_0}}} = \frac{2\pi \cdot 10^{-18} A^2s^2m^2 Vm\, 4\pi \cdot 9 \cdot 10^9}{3 \cdot 1m^3\, As} = \underline{\underline{2{,}37 \cdot 10^{-7} J}}$$

3



$\vec{E}_0 = -\vec{1}_z \cdot E_0 \quad E_0 = 1000V/m$

(r,θ,ϕ) $\quad \omega = 0 \quad \varepsilon = \varepsilon_0$

$\sigma(r=a,\theta,\phi) = ?$

$$V(r \geq a, \theta, \phi) = \left(Ar + \frac{B}{r^2}\right)\cos\theta$$

$$V(r \gg a) = V_0 \to A = E_0$$

$$V(r=a) = 0 \to B = -Aa^3 = -E_0 a^3$$

$$V(r \geq a, \theta) = E_0\left(r - \frac{a^3}{r^2}\right)\cos\theta$$

$$\vec{E} = -\text{grad}\, V = -\vec{1}_r E_0\left(1 + \frac{2a^3}{r^3}\right)\cos\theta + \vec{1}_\theta E_0\left(1 - \frac{a^3}{r^3}\right)\sin\theta$$

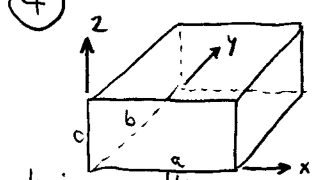
$$\sigma = \vec{1}_r \cdot \vec{D}(r=a) = \vec{1}_r \cdot \varepsilon_0 \vec{E}(r=a) = -\varepsilon_0 E_0 (1+2)\cos\theta$$

$$\underline{\underline{\sigma = -3\varepsilon_0 E_0 \cos\theta}} = -3\, \frac{As \cdot 1000 V/m}{4\pi \cdot 9 \cdot 10^9 Vm}\cos\theta$$

$$\sigma = -2{,}65 \cdot 10^{-8}\frac{As}{m^2}\cos\theta = \underline{\underline{-26{,}5\, \frac{nAs}{m^2}\cos\theta}}$$

$$\varepsilon_0 \approx \frac{1}{4\pi \cdot 9 \cdot 10^9}\frac{As}{Vm}$$

4



okovinjena votlina

$\varepsilon_0 \quad \mu_0, \quad \omega \neq 0$

$a = 10cm \qquad$ v votlini
$b = 7cm \qquad \rho = 0,\ \vec{J} = 0$
$c = 5cm$

$$\vec{E} = \vec{1}_z E_0 \sin\frac{\pi}{a}x \sin\frac{\pi}{b}y$$

$f = ? \qquad \vec{S} = ?$

$$\rho = 0,\ \vec{J} = 0 \to \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 = k_0^2 = \omega^2\mu_0\varepsilon_0 \to f = \frac{c_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

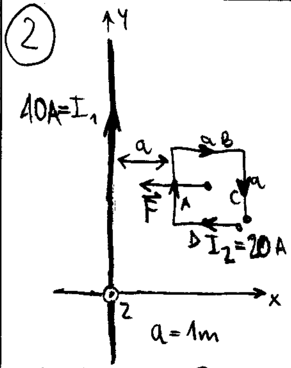
$$\underline{\underline{f = 2{,}616 GHz}}$$

$$\vec{H} = \frac{j}{\omega\mu_0}\text{rot}\vec{E} = \frac{j}{\omega\mu}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z \end{vmatrix} = \frac{jE_0}{\omega\mu_0}\left(\vec{1}_x \frac{\pi}{b}\sin\frac{\pi}{a}x\cos\frac{\pi}{b}y - \vec{1}_y\frac{\pi}{a}\cos\frac{\pi}{a}x\sin\frac{\pi}{b}y\right)$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{1}_z E_0 \sin\frac{\pi}{a}x \sin\frac{\pi}{b}y \times \frac{-jE_0^*}{2\omega\mu_0}\left(\vec{1}_x\frac{\pi}{b}\sin\frac{\pi}{a}x\cos\frac{\pi}{b}y - \vec{1}_y\frac{\pi}{a}\cos\frac{\pi}{a}x\sin\frac{\pi}{b}y\right)$$

$$\underline{\underline{\vec{S} = \frac{j|E_0|^2}{2\omega\mu_0}\left(-\vec{1}_y\frac{\pi}{b}\sin^2\frac{\pi}{a}x\sin\frac{\pi}{b}y\cos\frac{\pi}{b}y - \vec{1}_x\frac{\pi}{a}\sin\frac{\pi}{a}x\cos\frac{\pi}{a}x\sin^2\frac{\pi}{b}y\right)}}$$

2



$\mu = \mu_0 \quad \omega = 0$

$\vec{F} = ?$

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

$$\vec{B}_1(z=0) = -\vec{1}_z \frac{\mu_0 I_1}{2\pi x}$$

$$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C + \vec{F}_D = I_2 \vec{1}_y a \times (-\vec{1}_z)\frac{\mu_0 I_1}{2\pi a} +$$

$$+ \int_a^{2a} I_2 \vec{1}_x dx \times (-\vec{1}_z)\frac{\mu_0 I_1}{2\pi x} + I_2(-\vec{1}_y) a \times (-\vec{1}_z)\frac{\mu_0 I_1}{2\pi \cdot 2a} + \int_a^{2a} I_2(-\vec{1}_x) dx \times (-\vec{1}_z)\frac{\mu_0 I_1}{2\pi x}$$

$$\vec{F}_B = -\vec{F}_D \to \vec{F}_B + \vec{F}_D = 0$$

$$= -\vec{1}_x \frac{\mu_0 I_1 I_2}{2\pi} + \vec{1}_x \frac{\mu_0 I_1 I_2}{2\pi \cdot 2} = \underline{\underline{-\vec{1}_x \frac{\mu_0 I_1 I_2}{4\pi}}}$$

$$\vec{F} = -\vec{1}_x \frac{4\pi \cdot 10^{-7} Vs \cdot 10A \cdot 20A}{Am \cdot 4\pi} = \underline{\underline{-\vec{1}_x \cdot 2 \cdot 10^{-5} N}}$$

$$\mu_0 = 4\pi \cdot 10^{-7} Vs/Am$$

5

$f = 100 kHz$

$\delta = 5m$

Voda $\mu = \mu_0$

$\varepsilon = \varepsilon_r \varepsilon_0 \quad \varepsilon_r = 80$

$\gamma = ?$

pričakujem $\gamma \gg \omega\varepsilon = 2\pi f \varepsilon_0 \varepsilon_r = \underline{4.44 \cdot 10^{-4} S/m}$

$$\delta = \sqrt{\frac{2}{\omega\mu\gamma}} \to \gamma = \frac{2}{\omega\mu\delta^2} = \frac{2}{2\pi f \mu_0 \delta^2}$$

$$\gamma = \frac{2\ Am}{2\pi \cdot 10^5 s^{-1} \cdot 4\pi \cdot 10^{-7} Vs \cdot 25m^2} = \underline{\underline{0{,}101\ S/m}} \gg \omega\varepsilon$$