References

[1] R. H. T. Bates, "Mode theory approach to arrays," *IEEE Trans. Antennas and Propagation* (*Communications*), vol. AP-13, pp. 321–322, March 1965.

[2] J. L. Allen, "On surface-wave coupling between elements of large arrays," *IEEE Trans. Antennas and Propagation* (*Communications*), vol. AP-13, pp. 638–639, July 1965.

[3] L. W. Lechtreck, "Cumulative coupling in antenna arrays," *Proc. 1965 Internat'l Symp. IEEE G-AP*, pp. 144–149.

[4] H. Querido, J. Frank, and T. C. Cheston, "Wide band phase shifters," *IEEE Trans. Antennas and Propagation* (*Communications*), vol. AP-15, p. 300, March 1967.

[5] P. W. Hannan and M. A. Balfour, "Simulation of a phased-array antenna in waveguide," *IEEE Trans. Antennas and Propagation*, vol. AP-13, pp. 342–353, May 1965.

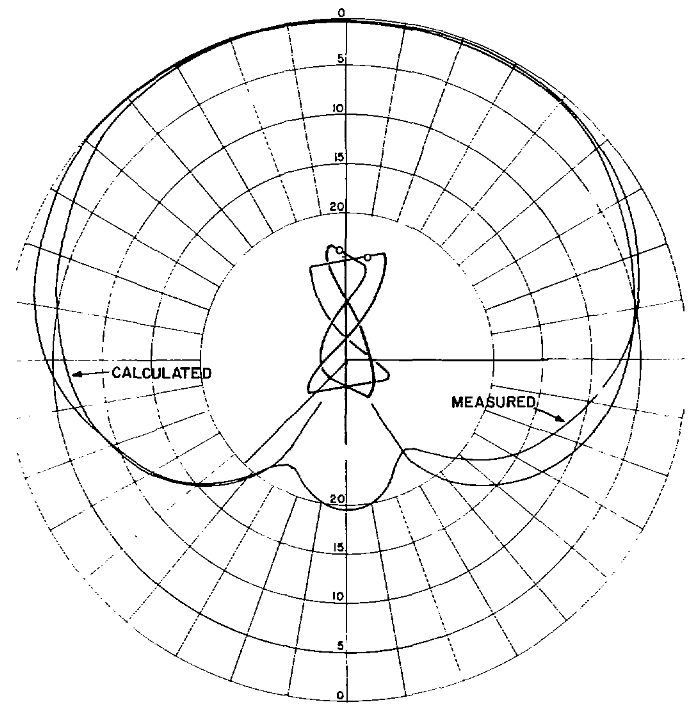

Fig. 1.

Multielement, Fractional Turn Helices

Abstract—**Two new resonant antennas are discussed: 1) A bifilar helix with antiphase feed that radiates a sin θ shaped, circularly polarized pattern with the null perpendicular to the helical axis; and 2) a quadrafilar helix with feed currents in quadrature that radiates a cardiod shaped, circularly polarized pattern.**

Introduction

The properties of short helices with several parallel resonant elements are well known.[1] Circular polarization obtains when

$$A = pL \tag{1}$$

where A = turn area, p = pitch, $L = 1/2\pi$ wavelength. For a helix with $\frac{1}{2}$ turn elements the radius and axial length are about 0.1 λ for optimum circular polarization. The pattern varies as sin θ, the null lies along the helical axis.

Tests at this laboratory indicate that feeding pairs of resonant (0.5 λ long), $\frac{1}{2}$ turn helical elements in antiphase rather than in parallel produces interesting characteristics. If the axial length = 0.27 λ and the radius = 0.09 λ, the bifilar helix radiates a circularly polarized pattern with sin θ shape, the null is now perpendicular to the helix axis.

Two bifilar helices with orthogonal radials (a quadrafilar helix) fed in phase quadrature produce a cardiod pattern shape (130° 3-dB beamwidth, 180° 6-dB beamwidth) with less than 3-dB axial ratio over the hemisphere (Fig. 1). Reversing the phase between bifilar helices causes the beam to cover the opposite hemisphere with no change in polarization.

Z in $\cong 50\ \Omega$ at resonance for each pair of thin elements.

Analysis

Half-Turn Bifilar Helix

An insight is afforded by the model of Fig. 2(a). The helical sides are approximated by linear and semi-circular pieces. The current distribution assumed is sinusoidal with the phase velocity of light along the wires. The dotted current I_D is the vector sum of the currents in the circle. If the antenna is removed from the diagram (Fig. 2(b)), the current distribution is seen to be similar to that on the loop-diople antenna.[2]

The radiation field expression for the small loop-dipole is:

$$\begin{aligned} E_\theta &= j\,\frac{60\pi[I_D]\sin\theta}{r}\left[\frac{L}{\lambda}e^{-jkr}\right]; \\ E\phi &= \frac{120\pi^2[I_L]\sin\theta}{r}\left[\frac{A}{\lambda^2}\,e^{-jkr}\right] \end{aligned} \tag{2}$$

$$\begin{aligned} E_\theta &= j\,\frac{K1}{r}\,(\sin\theta)e^{-jkr} \\ E\phi &= \frac{K2}{r}\,(\sin\theta)e^{-jkr} \end{aligned} \tag{3}$$

where I_D = dipole current, I_L = loop current, $k = 2\pi/\lambda$, A = loop area, L = dipole length, λ = wavelength of operation, K_1, K_2 = constants for a given helix. Formulas (3) show the orthogonal components $E\theta$ from the sides and $E\phi$ from the semi-circles to be in phase quadrature. If length/diameter of the helix is adjusted to make $K_1 = K_2$, the radiation will be circularly polarized over the entire sphere.

Experimental measurements indicate axial length/diameter = 1.5 produces optimum polarization. Both calculated and experimental patterns vary as sin θ with a null perpendicular to the axis and the radial wires of the helix, supporting the many approximations made in the analysis.

Quadrafilar Helix

Accepting the equivalence between the bifilar helix and the loop dipole, a model for the quadrafilar helix can be made of two orthogonal loop dipole antennas (Fig. 3). The field equations are

$$\begin{aligned} E_{\theta_1} &= jK\sin\theta[e^{-j(kr+\alpha)}]; \\ E_{\phi_1} &= K\sin\theta[e^{-j(kr+\alpha)}] \\ E_{\theta_2} &= Ke^{-jkr}(\sin\phi + j\cos\phi\cos\theta); \\ E_{\phi_2} &= Ke^{-jkr}(\cos\phi\cos\theta - j\sin\phi). \end{aligned}$$

If the helices are fed in phase quadrature, $\alpha = \pi/2$ and

$$E_{\theta T} = E_{\theta_1} + E_{\theta_2} = \sqrt{\sin^2\theta + \sin^2\phi + \cos^2\phi\cos^2\theta + 2\sin\phi\sin\theta}\ \Big|\ \tan^{-1}\left(\frac{\cos\phi\cos\theta}{\sin\theta + \sin\phi}\right)$$

$$E_{\phi T} = E_{\phi_1} + E_{\phi_2} = \sqrt{\sin^2\theta + \sin^2\phi + \cos^2\phi\cos^2\theta + 2\sin\phi\sin\theta}\ \Big|\ \tan^{-1} - \left(\frac{\sin\theta + \sin\phi}{\cos\theta\cos\phi}\right).$$

The relative phase is 90° and $E_\theta = E_\phi$ for all θ and ϕ. The shape of the pattern is cardiod with a maximum along the axis of the antenna. Fig. 1 compares calculated and measured data.

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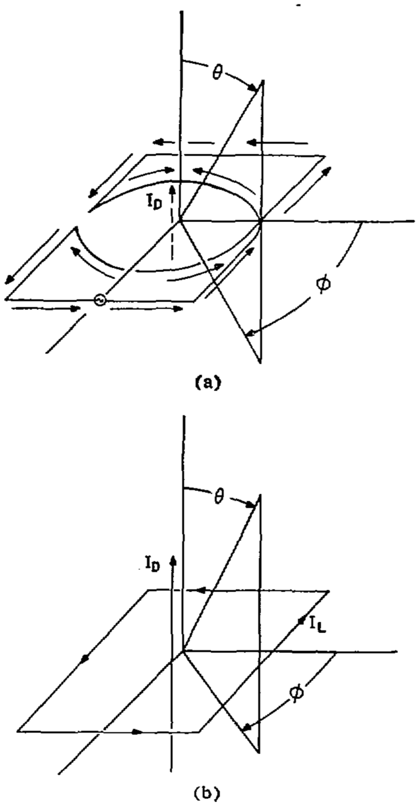

Fig. 2.

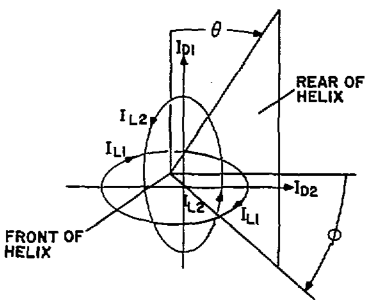

Fig. 3.

Realization

Fig. 4 pictures the quadrafilar helix. The wide beamwidth and excellent axial ratio are ideal for low altitude gravity gradient stabilized satellites radiating to simple linear ground antennas.

The helical elements are 0.040-inch diameter beryllium copper tubing. The antenna is fed with a split sheath balun that also serves as the mechanical support for the elements.

The split sheath balun slot length is varied to produce a real antenna terminal impedance. The Z_0 of a $\lambda/4$ transformer inside the balun is adjusted to match the real impedance to 50 Ω. Phase quadrature between bifilar helices is obtained by adjusting the element lengths about resonance until the input impedances are in phase quadrature. The pattern of Fig. 1 was taken with this antenna.

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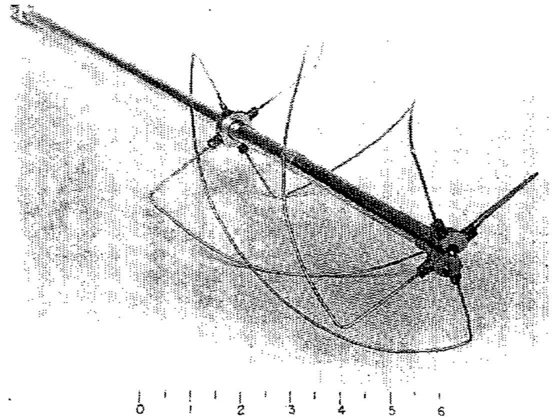

Fig. 4

References

[1] H. A. Wheeler, "A helical antenna for circular polarization," *Proc. IRE*, vol. 35, pp. 1484–1488, December 1947.

[2] G. H. Brown and O. M. Woodward, Jr., "Circularly-polarized omni-directional antenna," *RCA Rev.*, vol. 8, pp. 259–260, June 1947.

On the Distributional Formulation of Maxwell's Equations

In a recently published textbook,[1] a distributional formulation of Maxwell's equations is given. The author has defined, for example, the curl of a vector distribution $\overline{T}$ as

$$\int \overline{F}\cdot\nabla \times \overline{T}dv = \int \overline{T}\cdot\nabla \times \overline{F}dV \quad (1)$$

where $\overline{F}$ is a test vector. By using definition (1), among other relationships, Maxwell's equations were rewritten[2] as

$$\int_0^T dt \int \left(\overline{H}\cdot\nabla\times\overline{F}-\overline{F}\cdot\overline{J}+\epsilon\overline{E}\cdot\frac{\partial \overline{F}}{\partial t}\right) dV = \int \epsilon\overline{E}\cdot\overline{F}\Big]_0^T dV \quad (2a)$$

$$\int_0^T dt \int \left(\overline{E}\cdot\nabla\times\overline{F}-\overline{F}\cdot\overline{J}_m-\mu\overline{H}\cdot\frac{\partial \overline{F}}{\partial t}\right) dV = -\int \mu\overline{H}\cdot\overline{F}\Big]_0^T dV \quad (2b)$$

where $\overline{F}$ is a test vector.

The definition (1) and therefore the distributional form of Maxwell's equations given by (2a) and (2b) are ambiguous. To see this, we first recall that a scalar functional denoted by $V(x)$, on a set of test functions, is a rule

$$\langle V, \phi\rangle = \int_{-\infty}^{\infty} V(x)\phi(x)dx \quad (3)$$

that assigns a complex *number* (the weighted average of V with respect to the weighting function ϕ) for every member $\phi(x)$ of the set.

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[1] J. Van Bladel, *Electromagnetic Fields*. New York: McGraw-Hill, 1964, p. 544.

[2] *Ibid.*, eq. (A6.36).

The definition (1), however, yields the *sum* of three members without any sense of direction, and, in fact, no interpretation of this sum is offered in that textbook.

One way to overcome this ambiguity is to define a vector distribution with the aid of a test dyadic as follows.

Definition 1: A test dyadic is a diagonal dyadic whose elements are test functions (not necessarily the same), i.e.,

$$\bar{\bar{\Phi}}(\bar{r}) = \phi_1(\bar{r})\overline{U}_x\overline{U}_x + \phi_2(\bar{r})\overline{U}_y\overline{U}_y + \phi_3(\bar{r})\overline{U}_z\overline{U}_z$$

where $\overline{U}_x$, $\overline{U}_y$, and $\overline{U}_z$ are, respectively, unit vectors in the direction of the x, y, and z coordinates.

Definition 2: For any vector field $\overline{V}(\bar{r})$, a vector functional is a rule

$$\langle \overline{V}, \bar{\bar{\Phi}}\rangle = \int \overline{V}\cdot\bar{\bar{\Phi}}dV \quad (4)$$

that assigns to each test dyadic three complex numbers (a vector)

$$\langle V_1, \phi_1\rangle, \quad \langle V_2, \phi_2\rangle, \quad \text{and} \quad \langle V_3, \phi_3\rangle.$$

It follows that the definitions of the gradient, divergence, and curl for distributions could be given, respectively, by

$$\int \bar{\bar{\Phi}}\cdot\nabla f dV = -\int f\nabla\cdot\bar{\bar{\Phi}}dV \quad (5a)$$

$$\int \bar{\bar{\Phi}}(\nabla\cdot\overline{F})dV = -\int (\overline{F}\cdot\nabla)\bar{\bar{\Phi}}dV \quad (5b)$$

$$\int \bar{\bar{\Phi}}\cdot(\nabla\times\overline{F})dV = \int \overline{F}\cdot(\nabla\times\bar{\bar{\Phi}})dV \quad (5c)$$

where the introduction of the concept of a test dyadic allows us to examine the action of each component of a vector distribution on a test function.

With the definitions of the gradient, divergence, and curl of vector distributions given by (5), Maxwell's equations for distributions can be written, unambiguously, as

$$\int_{t_0}^T dt \int \left(\overline{H}\cdot\nabla\times\bar{\bar{\Phi}}-\bar{\bar{\Phi}}\cdot\overline{J}+\overline{D}\cdot\frac{\partial \bar{\bar{\Phi}}}{\partial t}\right) dV = \int \bar{\bar{\Phi}}\cdot\overline{D}\Big]_{t_0}^T dV \quad (6a)$$

$$\int_{t_0}^T dt \int \left(\overline{E}\cdot\nabla\times\bar{\bar{\Phi}}-\overline{B}\cdot\frac{\partial \bar{\bar{\Phi}}}{\partial t}\right) dV = -\int \bar{\bar{\Phi}}\cdot\overline{B}\Big]_{t_0}^T dV \quad (6b)$$