Unified Presentation of 1/f Noise in Electronic Devices: Fundamental 1/f Noise Sources

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This review represents 1/f noise in electronic devices in terms of the Hooge parameter α_H of the devices. A generalized schematic is given for expressing the noise spectrum $S_I(f)$ in the external circuit in terms of distributed noise sources of the nonuniform devices in terms of α_H; and so one can evaluate α_H from $S_I(f)$. The results can then be compared with Handel's predictions for α_H. Despite the fact that there are several objections to Handel's ***derivation*** *of α_H, it seems that his final result usually agrees with experiment; apparently the results are not sensitive to the details of the (Bremsstrahlung) photon–electron interaction (Appendix I).*

Collision-free devices (pentodes, vacuum photodiodes, secondary emission multiplier stages, etc.) can always be represented by fundamental 1/f noise sources after spurious noise sources have been eliminated or discriminated against. Collision-dominated devices can show fundamental normal collision 1/f noise, Umklapp 1/f noise, intervalley scattering 1/f noise (if there are intervalleys), intervalley + Umklapp 1/f noise and, in long devices, coherent state or Hooge-type 1/f noise. Most of these processes occur, except pure intervalley 1/f noise, which is replaced by intervalley + Umklapp 1/f noise. Such devices include Schottky barrier diodes, n^+-p diodes, p-i-n diodes, n^+-p-n and p^+-n-p BJTs, n-channel and p-channel Si-JFETs, and p-MOS devices operating under strong inversion. The schematic can also be applied to ballistic devices.

I. Introduction

It is the aim of this review paper to present 1/f noise in semiconductors, semiconductor devices, and collision-free devices (like vacuum tubes) from a unified point of view, using an extended version of the Hooge equation [1] as a vehicle. It is then found that the Hooge parameter, introduced by this equation, can be used as a *general measure* of the noisiness of a system or device. It is finally attempted to correlate *measured* values of the Hooge parameter with the values *calculated* from Handel's quantum theory of 1/f noise [2], [3].

The approach is in itself not new: what is new, however, is its generalization to all systems and devices. Also, the program would already give full *practical* benefits if the investigation stopped after the *measurement* of the Hooge parameter. But the *comparison* between the theory and experiment opens up the possibility of refuting or verifying Handel's formulas in a large number of cases, and will result in a *generalized framework* in which all the experimental data can be placed.

Section II-A formulates and generalizes the Hooge equation to all collision-dominated systems involving mobility, diffusion, and cross-section fluctuations. It also applies to collison-free processes involving vacuum tubes, Schottky barrier diodes operating in the thermionic mode and in devices such as p-i-n diodes in which collision processes are not the determining factor. In those cases, the effective number N of carriers is better expressed in terms of the device current.

Section II-B deals with Handel's quantum theories of 1/f noise. The discussion does not imply validity of those equations, but simply states what the theories would *predict*; this is the only way in which any theory, including Handel's, can be verified or refuted by experimental data. Handel's theory is based on the Hooge equation and gives an expression for the Hooge parameter α_H. Theory and experiment thus deal with the same parameter. The main emphasis of the paper is on the generalized framework.

Section III discusses how $S_I(f)$ can be expressed in terms of α_H. To that end, Hooge's equation for $S_I(f)$ is replaced by the spectrum of a *distributed* noise source. In the simplest cases one writes down the Langevin equation involving a random source term $H(x, t)$, linearizes this equation, solves it, and expresses $S_I(f)$ in terms of integrals over the cross spectral intensity $S_H(x, x', f)$ of $H(x, t)$; the latter in turn, is expressed in terms of the Hooge equation. Sometimes this method is inadequate and other methods must be used. These methods are by themselves not new, but are here applied systematically. Some of the applications are new.

Section IV discusses several cases in which the noise does not obey the quantum 1/f noise theory. It is shown that number fluctuation noise gives a 1/f spectrum caused by a distribution of time constants (McWhorter's model). This is the case when there are distributed traps in the surface oxide (MOSFETs, BJTs); it results in a current dependence that is different from what is expected from Hooge's theory.

Section V discusses measurements on many different

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devices. In most cases the predictions made by Handel's theory are verified. This does not necessarily indicate that the *mathematical derivation* of these predictions are correct; this remains open to discussion.

II. General Background of the Problem

A. The Hooge Equation and the Hooge Parameter

1) Collision-Limited Devices: We first turn to the Hooge equation itself. When a constant voltage V is applied to a semiconductor resistor of resistance R, a fluctuating current $I(t)$ is developed. This can only come about because the resistance $R(t)$ of the device fluctuates. Since

$$V = I(t)R(t) = \text{const.}$$

$$\frac{\delta I}{I} = -\frac{\delta R}{R} \qquad \frac{S_I(f)}{I^2} = \frac{S_R(f)}{R^2}. \tag{1}$$

If R and δR are independent of current, $S_I(f)/I^2$ will be independent of current also. This result is true for generation–recombination (g–r) spectra caused by traps; they give Lorentzian spectra of the form const/$(1 + \omega^2\tau^2)$. It is therefore also true for $1/f$ spectra caused by a *superposition* of Lorentzian spectra, as in McWhorter's theory of $1/f$ noise (Section IV). But, as we shall see, the possibility must also be left open that there are true $1/f$ spectra, not caused by such a superposition.

Irrespective of the cause of the $1/f$ noise, $S_I(f)/I^2$ may be written as

$$\frac{S_I(f)}{I^2} = \frac{\text{const}}{f} \tag{1a}$$

and it may be implied that the noise is caused by resistance fluctuations. Clarke and Voss [4], [5] showed the presence of such resistance fluctuations in a beautiful experiment.

The question is now what other parameters enter into the constant introduced by (1a). Hooge suggested that for a rectangular semiconductor the missing parameter was the number N of carriers of the sample and wrote the empirical formula, now known as the *Hooge equation,*

$$\frac{S_I(f)}{I^2} = \frac{S_R(f)}{R^2} = \frac{\alpha_H}{fN}. \tag{2}$$

This equation neither proves anything nor predicts anything, but merely gives an operational definition of the *Hooge parameter* α_H. It is always valid, but is only useful if one can extract useful information out of the value of α_H.

Since a rectangular semiconductor bar of length L and cross-sectional area A has a resistance $R = L^2/(e\mu N)$, where μ is the carrier mobility, N follows from R, and hence α_H from (2). When one does this for a number of different semiconductor resistors of comparable length L, one can characterize the noisiness of the various materials by the parameter α_H. Hooge [1] found in that manner that for many semiconductor samples α_H had a value of about 2×10^{-3}, nearly independent of the material. Hanafi *et al.* [6] found for ten $Hg_{1-x}Cd_xTe$ resistor bars with different doping and (or) different values of x (but all made by similar techniques), that α_H had an average value of 5×10^{-3} with a spread of less than a factor 2. The near constancy of α_H suggests that this $1/f$ noise is due to a *fundamental mechanism* of unknown origin; this is useful information that will be found to be valid in other situations as well.

Later it was found [7], [8] that α_H could be considerably smaller than 2×10^{-3} for sufficiently short resistors ($L < 100\ \mu$m) whereas the Hooge value of 2×10^{-3} was obtained for sufficiently long devices ($L < 500\ \mu$m). A systematic experimental study of the dependence of α_H upon the device length L, which has not been made so far, would be very helpful.

Hooge gave no proof of (2), but it is easily seen that an equation like (2), with constant α_H, could be expected if the $1/f$ noise is generated by N independent carriers. For in that case both I and $S_I(f)$ would be proportional to N so that $S_I(f)/I^2$ would be inversely proportional to N. This would be *fundamental* noise.

Since the resistance R is inversely proportional to the product μN, where μ is the carrier mobility, there can be fluctuations $\delta\mu$ in μ and (or) δN in N, so that, since $\delta\mu$ and δN are independent

$$\frac{\delta R}{R} = -\frac{\delta\mu}{\mu} - \frac{\delta N}{N}$$

or

$$\frac{S_R(f)}{R^2} = \frac{S_\mu(f)}{\mu^2} + \frac{S_N(f)}{N^2}. \tag{3}$$

If the fluctuation in μ predominates

$$\frac{S_I(f)}{I^2} = \frac{S_\mu(f)}{\mu^2} \tag{3a}$$

and the noise is called *mobility fluctuation 1/f noise,* whereas

$$\frac{S_I(f)}{I^2} = \frac{S_N(f)}{N^2} \tag{3b}$$

if the fluctuations in N predominate; the noise is then called *number* fluctuation $1/f$ noise. In principle either relationship can occur, but in practice mobility fluctuation $1/f$ noise predominates in many cases. We come back to that problem in Sections IV and V.

Do (3a) and (3b) result in the Hooge equation? In order that this be the case, $S_\mu(f)$ and $S_N(f)$ must vary as $1/f$ over a wide frequency range and in addition $S_\mu(f)/\mu^2$ and (or) $S_N(f)/N^2$ must vary as $1/N$. We come back to the spectral dependencies in Section IV, but wish to point out here that the latter is the case if $S_N(f)$ is proportional to N.

We shall now show that $S_\mu(f)/\mu^2$ always varies as $1/N$. In addition, if each electron, in and by itself, produces $1/f$ noise, the full Hooge equation (2) results.

The proof is simple, as Hooge [9] and van der Ziel *et al.* [10] have demonstrated. We introduce the short-term mobility μ_i of the individual carriers. If N does not fluctuate, and the μ_i's are independent

$$\mu N = \sum_{i=1}^{N} \mu_i \qquad \mu = \frac{1}{N}\sum_{i=1}^{N} \mu_i \qquad \bar{\mu} = \bar{\mu}_i \tag{4}$$

$$\delta\mu = \frac{1}{N}\sum_{i=1}^{N} \delta\mu_i$$

$$S_\mu(f) = \frac{1}{N^2}\sum_{i=1}^{N} S_{\mu_i}(f) = \frac{1}{N} S_{\mu_i}(f)$$

$$\frac{S_\mu(f)}{(\mu)^2} = \frac{1}{N}\frac{S_{\mu_i}(f)}{(\bar{\mu}_i)^2} \tag{4a}$$

so that $S_\mu(f)/\mu^2$ varies as $1/N$. In addition $S_{\mu_i}(f)/(\mu_i)^2$ is independent of N and was postulated to have a $1/f$ dependence.

We may then write

$$\frac{S_{\mu_i}(f)}{(\mu_i)^2} = \frac{\alpha_H}{f} \qquad \frac{S_I(f)}{(I)^2} = \frac{S_\mu(f)}{(\mu)^2} = \frac{\alpha_H}{fN}. \tag{4b}$$

We thus see that for mobility fluctuations the Hooge equation is always valid and that α_H is defined as f times the relative mobility $1/f$ spectrum $S_{\mu_i}(f)/(\overline{\mu}_i)^2$ of a *single* electron. This would then be *fundamental* $1/f$ noise.

Because of the Einstein relation $eD = kT\mu$, mobility fluctuations correspond to fluctuations in the diffusion constant D. Consequently

$$\frac{\delta D}{D} = \frac{\delta\mu}{\mu} \quad \text{or} \quad \frac{S_D(f)}{D^2} = \frac{S_\mu(f)}{\mu^2}. \tag{5}$$

A Hooge-type equation may therefore also be expected for solid-state devices governed by *diffusion* processes, such as occur in p^+-n and n^+-p junction diodes, p^+-n-p and n^+-p-n BJTs, and Schottky-barrier diodes operating in the *diffusion* mode. Corrections may be needed for degenerate systems.

Since FETs are bias-dependent semiconductor resistors, they should show $1/f$ noise. For devices operating at near-zero drain bias the device is a uniform semiconductor resistor, but for larger bias the resistor becomes nonuniform due to channel pinch-off. For such nonuniform resistors one must replace the Hooge equation by its differential form holding for each section Δx at x

$$S_{\Delta I}(x, f) = \frac{\alpha_H}{fN(x)\,\Delta x} I^2(x) \tag{6}$$

where $N(x)$ is the carrier density per unit length and $I(x)$ the current at x. It is thus possible to treat the Hooge equation as the spectrum of a distributed noise source $H(x, t)$. By evaluating the contributions of individual sections Δx to the spectrum $S_I(f)$ of the total current I, one can express $S_I(f)$ in terms of α_H and other measurable device parameters, so that α_H can be determined for all these devices and the relative noisiness of the various noise mechanisms can be established. The methods for solving such distributed noise problems are discussed in Section III. They work so long as Δx is larger than the free path length of the carriers.

There is one other further noise problem that requires attention. In relatively long n^+-p diodes part of the injected carriers disappears by recombination. In that case the lifetime τ of the individual carriers fluctuates. It is shown in Section III that for $C = 1/\tau$ (C is independent of x)

$$S_C(x, f) = \frac{\alpha_H}{fN(x)\,\Delta x} C^2 \tag{7}$$

so that this problem can be incorporated into the general schematic.

A related problem is the noise due to fluctuations in the contact recombination velocity s_{cn} at an ohmic contact ($s_{cn} \approx 10^7$ cm/s). To that end consider a planar n^+-p diode with a length w_p of the p-region ($w_p \ll L_n$, short diode) where $L_n = (D_n\tau_n)^{1/2}$ is the diffusion length of the electrons in the p-region. Then $I_n = es_{cn}N'(x)$, and, in analogy with (7)

$$S_{s_{cn}}(f) = \frac{\alpha_H}{fN_{\text{eff}}} s_{cn}^2 \tag{7a}$$

where $N_{\text{eff}} = 1/2\,[N(0) + N(w_p)]w_p$ is the effective number of minority carriers in the base region (see below).

A similar effect can occur in the surface recombination velocity s in a junction space-charge region or in the surface recombination velocity in the base region of a BJT. Here s is usually much smaller than s_{cn} ($s_{cn} = 6 \times 10^6$ cm/s in a contact on n-type silicon, and $s < 1$ cm/s for a well-passified surface on n-type silicon).

Many papers have been written about samples of nonrectangular geometry. For references see Hooge *et al.* review paper [11].

2) Collision-Free Devices: Up to here we discussed only semiconductor devices that were *collision limited*, so that α_H was determined by collision processes. We now turn to devices in which collisions either do not exist, as in vacuum tubes and in Schottky-barrier diodes operating in the thermionic mode, or to devices in which collision processes are not the determining factor, as in long p-i-n diodes.

In that case a Hooge type equation of the form [12], [13]

$$\frac{S_I(f)}{(I)^2} = \frac{\alpha_H}{fN} \tag{8}$$

describes the $1/f$ noise. Here α_H may have a different magnitude than in collision-dominated devices, but N again is the number of carriers in the system. This is, e.g., the case for vacuum tubes like space-charge-limited vacuum diodes, triodes, and pentodes, or saturated vacuum photodiodes, and secondary emission multiplication stages, etc. It holds for any system in which the N carriers generate $1/f$ noise independently (*fundamental* $1/f$ noise).

Since the current flow is by *injection*, I/e is the number of carriers injected per second and $N = I\tau/e$ is the number of carriers present in the sample. Consequently, if we substitute for N,

$$S_I(f) = \frac{\alpha_H eI}{f\tau} \tag{8a}$$

where τ is the carrier transit time. For an electron traveling between two parallel electrodes at a distance d_{21}, with negligible charge between them

$$\tau = \frac{2d_{21}}{v_2 + v_1} \tag{8b}$$

where v_2 and v_1 are the carrier velocities at the electrodes 2 and 1, respectively. For space-charge-limited current flow between two parallel electrodes of distance d_1

$$\tau = \frac{3(d_1 - d_m)}{v_1} \tag{8c}$$

where d_m is the distance between the potential minimum and cathode, $v_1 = (2e/m)^{1/2}(V + V_m)^{1/2}$ is the velocity with which the electrons arrive at the anode, V is the anode potential, and V_m the depth of the potential minimum in front of the cathode. For a long p-i-n diode τ is the time constant associated with the generation and recombination of one hole–electron pair. In each case α_H can be determined from $S_I(f)$ if τ is known.

Not all noises in collision-free devices satisfy (8a). Whether or not they do, must be determined by comparing the *measured* value of α_H with the theoretical values predicted in the next section.

B. Handel's Quantum Equations [2], [3]

1) Collision-Free Devices: We first start with a semiclassical consideration of collision-free devices. Since the only physical process present in such devices is *acceleration*, the observed $1/f$ noise must be associated with this acceleration. Now an accelerated electron generates low-frequency Bremsstrahlung; since its energy spectrum is independent of the quantum energy E for small E, and the number spectrum is found by dividing the energy spectrum by hf, it is obvious that this number spectrum varies as $1/f$. The near-field interaction of an electron with its own Bremsstrahlung will therefore give current $1/f$ noise in the external circuit that is described by the Hooge parameter α_H. The effect is semiclassical; to evaluate α_H one needs wave mechanics.

Handel uses a somewhat different model. He splits the electron wave function into a large unperturbed part and a small part that is perturbed by the Bremsstrahlung emission. In the calculation the two parts beat with each other and so give $1/f$ noise back. Handel thus finds the following [14]:

$$\alpha_H = \frac{4\alpha}{3\pi}\frac{\Delta v^2}{c^2} \qquad S_I(f) = \frac{4\alpha}{3\pi}\frac{\Delta v^2}{c^2}\frac{eI}{f\tau}. \tag{9}$$

The first part is known as the *Handel equation*. Here c is the velocity of light, Δv the vectorial change in velocity along the electron path, and α the fine structure constant. For motion between two parallel electrodes of distance d_{12}, with terminal velocities v_1 and v_2, $\Delta v = v_2 - v_1$ and $\tau = 2d_{12}/(v_2 + v_1)$, as mentioned before. The main objection to this approach is against the beat process. For single electrons, in MKS units, where $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$\alpha = \alpha_0 = \frac{\mu_0 ce^2}{2h} = \frac{1}{137} \qquad S_I(f) = \frac{4\alpha_0}{3\pi}\frac{\Delta v^2}{c^2}\frac{eI}{f\tau}. \tag{9a}$$

But in some cases the current flows in charge conglomerates q. Since they are accelerated as a unit, they produce Bremsstrahlung as a unit, and hence generate $1/f$ current noise as a unit; consequently e^2 must be replaced by q^2, or

$$\alpha = \alpha_0\left(\frac{q}{e}\right)^2 \qquad S_I(f) = \frac{4\alpha_0}{3\pi}\left(\frac{q}{e}\right)^2\frac{\Delta v^2}{c^2}\frac{eI}{f\tau}. \tag{9b}$$

As a first example we consider a space-charge-limited vacuum diode. Here the shot noise is space-charge-suppressed by a factor Γ^2 (≈ 0.10 for normal operation) so that $S_I(f)$ may be written

$$S_I(f) = 2eI\Gamma^2 = 2(e\Gamma^2)I = 2qI \tag{10}$$

corresponding to shot noise of charges q, so that the effective charge is $q = e\Gamma^2$. Hence the $1/f$ noise may be written

$$S_I(f) = \frac{4\alpha_0}{3\pi}\Gamma^4\frac{\Delta v^2}{c^2}\frac{eI}{f\tau}. \tag{10a}$$

In vacuum diodes with oxide-coated cathodes the noise is masked by $1/f$ noise generated in the cathode coating, so that (10a) is not verifiable.

In vacuum photodiodes $\Gamma^4 = 1$ (no space-charge suppression), Δv is much larger than in the previous case, and hence $S_I(f)$ becomes

$$S_I(f) = \frac{4\alpha_0}{3\pi}\frac{\Delta v^2}{c^2}\frac{eI}{f\tau} \tag{11}$$

where $\tau = 2d/(v_2 + v_1)$ and $\Delta v = (v_2 - v_1)$. If V_a is the anode voltage and $v_1 \approx 0$, $S_I(f)$ varies as $I\,V_a^{3/2}$; this should be measurable, unless masked by classical $1/f$ noise due to fluctuations in the electron affinity of the photocathode (see Section V).

As a second example we take a secondary emission multiplier stage [15]–[17]. Let I_{pr} be the primary current of the multiplier and δ the secondary multiplication factor, then the output current $I = \delta I_{pr}$, so that the current consists of charge conglomerates of charge $q = \delta e$. Hence

$$S_I(f) = \frac{4\alpha_0}{3\pi}\delta^2\frac{\Delta v^2}{c^2}\frac{eI}{f\tau} \tag{12}$$

when τ is the transit time between the secondary emission electrode (dynode) and the collecting electrode (anode). In secondary emission pentodes this noise is usually masked by the $1/f$ noise, $S_I(f)$, of the primary current. The latter can be suppressed satisfactorily by appropriate cathode feedback [12], [13], [15], [17]; in that case $S_I(f)$ becomes measurable. Again, $S_I(f)$ varies as $I\,V_a^{3/2}$, where V_a is the potential difference between anode and dynode, and this can be verified (see Section V).

In vacuum pentodes cathode $1/f$ noise is distributed between screen grid and anode, whereas partition $1/f$ noise flows from screen grid to anode. The latter is *not* space-charge suppressed, whereas the former is. Nevertheless, the partition $1/f$ noise is *masked* by cathode $1/f$ noise, unless the latter is sufficiently reduced by a feedback resistor R_c in the cathode lead. In that case the partition $1/f$ noise becomes measurable [12], [13], [18] and its possible quantum character can be investigated (Section V).

We have here discussed the predictions made by Handel's quantum $1/f$ noise theory for various vacuum tubes. By comparing the calculated spectra with the experimental data we may then be able to either refute or verify these predictions. The experimental data are independent of Handel's equations (9)–(12), and so can serve as independent checks of those equations.

2) Collision-Dominated Devices: We now turn to quantum $1/f$ noise in semiconductor devices. Here the devices are *collision-dominated* and (9) must be appropriately modified: in (9) Δv^2 must now be averaged over all collisions and be replaced by $\overline{\Delta v^2}$. Bremsstrahlung $1/f$ noise is still considered the initiating process, however. Since the carriers are single electrons or single holes, they always have a charge $\pm e$; hence the fine structure constant α always has the value $\alpha_0 = 1/(137)$. Consequently, for a single scattering process, (9) may be written as

$$\alpha_H = \frac{4\alpha_0}{3\pi}\frac{\overline{\Delta v^2}}{c^2} = 3.09 \times 10^{-3}\frac{\overline{\Delta v^2}}{c^2} \tag{13}$$

where the averaging must be performed in k-space over all scattering angles θ and over the electron-velocity distribution [2], [3], [19].

There are different scattering processes possible, each having its associated mobility μ_i and Hooge parameter α_{Hi}. We then have according to Kousik and van Vliet [20]

$$\frac{1}{\mu} = \sum_i \frac{1}{\mu_i} \qquad \left(\frac{\delta\mu}{\mu^2}\right) = \sum_i \frac{\delta_{\mu_i}}{\mu_i^2}$$
$$\frac{S_{\mu_i}(f)}{\mu_i} = \sum_i \left(\frac{S_{\mu_i}(f)}{\mu_i^4}\right) \tag{13a}$$

Introducing

$$\alpha_H = \frac{S_\mu(f)}{\mu^2}(fN) \qquad \alpha_{H_i} = \frac{S_{\mu_i}(f)}{\mu_i^2}(fN)$$

and multiplying by μ^2 yields

$$\alpha_H = \sum_i \alpha_{H_i}\left(\frac{\mu}{\mu_i}\right)^2. \tag{13b}$$

We now have the following semiclassical processes:

a) Normal Collision Processes (Acoustical Phonon Scattering, Optical Phonon Scattering, Impurity Scattering): Calculating the α_{H_i}'s and μ_i's for each of these processes, Kousik and van Vliet found $\alpha_H = 3.3 \times 10^{-9}$ for n-type Si at $T = 300$ K (mostly acoustical phonon scattering) and $\alpha_H = 1.6 \times 10^{-8}$ for n-type GaAs at $T = 300$ K (mostly optical phonon scattering).

Approximate values can be found by assuming elastic scattering [19]. In that case the change in velocity $\Delta v = 2v \sin\theta$ has a mean square value

$$\overline{\Delta v^2} = 4\overline{v^2}\,\overline{(\sin^2\Theta)} = 2\overline{v^2} = \frac{6kT}{m^*}$$

for a Maxwellian velocity distribution, or

$$\alpha_H = \frac{4\alpha_0}{3\pi}\frac{6kT}{m^*c^2}. \tag{14}$$

For n-type silicon $m^* \simeq m$ and $\alpha_{Hn} = 0.94 \times 10^{-9}$; for p-type silicon $m^*_{\text{eff}} = 0.241m$ and $\alpha_H = 3.9 \times 10^{-9}$ whereas for n-type GaAs $m^* = 0.067m$ and $\alpha_{Hp} = 1.4 \times 10^{-8}$. In all these examples, $T = 300$ K, and m is the mass of the free electron.

p-type silicon has three types of holes with mass $m_1^* = 0.49m$, $m_2^* = 0.24m$, and $m_3^* = 0.16m$, where m is the mass of the free electron. If it is assumed that each hole type occurs with equal probability, then

$$m^*_{\text{eff}} = \frac{3}{1/m_1^* + 1/m_2^* + 1/m_3^*} = 0.241m. \tag{14a}$$

The approximation is often not very reliable, as the examples show; the values for α_H, as obtained by the Kousik–van Vliet method [2], [3], [19], [20] are much more accurate. Combining both considerations yields an estimated value $\alpha_{Hp} \simeq (2\text{–}3)\,\alpha_{Hn}$, or $\alpha_{Hp} \simeq (6\text{–}10) \times 10^{-9}$.

b) Umklapp Processes [2], [3], [19], [20]: In an Umklapp process an electron can give up a momentum h/a to the lattice or accept a momentum h/a from the lattice, while being scattered into the next Brillouin zone; here a is the lattice spacing. Hence $\Delta v = h/(m^*a)$ and

$$\alpha_{Hu} = \frac{4\alpha_0}{3\pi}\left(\frac{h}{m^*ac}\right)^2. \tag{15}$$

This involves an interaction with several acoustical phonons.

For collision processes involving electrons in relatively weakly doped n-type or p-type material (see (13b))

$$\alpha_H = \alpha_{Hu}\left(\frac{\mu}{\mu_u}\right)^2 + \alpha_{H\text{norm}}\left(\frac{\mu}{\mu_{\text{norm}}}\right)^2 \tag{15a}$$

(here μ_u is the Umklapp mobility and μ_{norm} the normal mobility). The term $(\mu/\mu_u)^2$ is very small and hence the first term in (15a) is negligible. However, Umklapp $1/f$ noise might be a significant effect in degenerate semiconductors or metals, in narrow-gap semiconductors, and in strongly inverted MOSFET channels [19]. In that case, Kousik *et al.* find $(\mu/\mu_u) = \exp(-\Theta_D/4T)$, where Θ_D is the Debye temperature.

c) Intervalley Scattering 1/f Noise [2], [3], [20]: In materials such as n-type GaAs the energy E versus the wave vector k has a complex structure. First there is a band having $E(\boldsymbol{k}) = 0$ at $\boldsymbol{k} = 0$, which is the *normal* conduction band, and then there are *six* bands having minima $E_i(\boldsymbol{k})$ at $\boldsymbol{k} = \boldsymbol{k}_i (i = 1\text{–}6)$ that lie higher than the minimum of the conduction band. Transitions are now possible from the bottom of the conduction band ($\boldsymbol{k} = 0$) to each of the intervalleys at $\boldsymbol{k} = \boldsymbol{k}_i$; these are random processes, involving changes in velocity Δv_i and hence giving rise to $1/f$ noise; they are called *intervalley scattering noise*. An equation of the type (15a) results, but with μ_u replaced by the intervalley scattering μ_{int} and μ_{Hu} replaced by the *intervalley scattering noise* parameter $\alpha_{H\text{int}}$. It now turns out for weakly doped n-type silicon that $(\mu/\mu_{\text{int}})^2$ is much larger than $(\mu/\mu_u)^2$, that $\alpha_{H\text{int}}$ is *somewhat* larger than α_{Hu}, and that both are much larger than $\alpha_{H\text{norm}}$. As a consequence, the intervalley scattering noise may predominate. Materials like p-type silicon have no intervalleys and hence no intervalley scattering noise and only the normal collision $1/f$ noise should be present. Materials like n-type Si have no central valley, but six equivalent valleys at $\boldsymbol{k}_i \neq 0$. One can now have Umklapp scattering due to processes $\boldsymbol{k}_i \to -\boldsymbol{k}_i'$ with $\boldsymbol{k}_i' = -\boldsymbol{k}_i$, and valley–valley scattering processes $\boldsymbol{k}_i \to \boldsymbol{k}'_i$, with $\boldsymbol{k}_i' \neq -\boldsymbol{k}_i$.

d) Intervalley Scattering + Umklapp Scattering 1/f Noise (c-process) [21]: Here an intervalley scattering process is followed by an Umklapp transition to an opposite or adjacent intervalley involving a change in momentum $\Delta p = h/a$. This must be considered as a *single* scattering process with $(\Delta v/c)^2 = h^2/(m^*ac)^2$. A calculation shows that μ/μ_c can be approximated by $\exp(-\theta_D/4T)$, where θ_D is the Debye temperature of the material (645 K for Si). Hence [21]

$$\alpha_H = \frac{4\alpha_0}{3\pi}\left\{\left(\frac{h}{m^*ac}\right)^2\exp\left(\frac{-\Theta_D}{2T}\right) + \left(\frac{\Delta v^2}{c^2}\right)_{\text{norm}}\left[1 - \exp\left(-\frac{\Theta_D}{4T}\right)\right]^2\right\} \tag{15b}$$

since $\mu^{-1} = \mu_c^{-1} + \mu_{\text{norm}}^{-1}$. The last term involving $(\Delta v^2/c^2)_{\text{norm}}$ is usually negligible.

The first term of (15b) corresponds to the van der Ziel–Handel [19] heuristic formula for Umklapp $1/f$ noise and the derivation gives it a firmer theoretical basis. It should hold for weakly doped n-type Si, but should be absent for weakly p-type Si because it has no intervalleys. It could also hold for degenerate materials, narrow-bandgap materials, and for strongly inverted MOSFET channels.

e) Coherent State 1/f Noise [22], [23]: In (13) $\Delta v^2/c^2$ is much smaller than unity and hence $\alpha_H \ll 3.09 \times 10^{-3}$. It should therefore be clear that α_H values as large as $(2\text{–}5) \times 10^{-3}$ cannot be explained by Handel's equation. Only in Umklapp processes involving carriers with very low effective masses m^* (as in $Hg_{1-x}Cd_xTe$) can $\Delta v^2/c^2$ be so large that there is a small relativistic correction [24] to (13). Hence the large observed values of α_H for long semiconductor resistors cannot be explained in this manner.

Handel has proposed [22], [23] a different fundamental mechanism, called "coherent state" quantum $1/f$ noise. The term "coherent state" is a wave-mechanical term and the process itself is difficult to explain, and we are not making an attempt here. The result, however, is very simple; Handel predicted

$$\alpha_H = \frac{2\alpha_0}{\pi} = 4.6 \times 10^{-3} \quad (16)$$

in close agreement with some experimental values (2–5) × 10^{-3}. Whether or not this is a coincidence remains to be seen, but at least it explains how Hooge's result might indicate a fundamental process.

Whereas Hooge's result holds for long resistors, it has also been found that α_H can be of the order of 10^{-6}–10^{-9} for short devices [25], [26], [19] like short FETs and BJTs. Hence there should be a transition from "high" 1/f noise (2×10^{-3}) to "low" 1/f noise (10^{-6}–10^{-9}) when going to shorter lengths. Handel [23] has proposed a formula for this transition, but it can only be tested when reliable experimental data have been obtained.

3) Ballistic Devices: We next return to a collision-free device: the n-type Schottky barrier diode operating in the thermionic mode. The electrons with a forward velocity $v_1 > v_0$ can pass the potential barrier and contribute to the forward current. Here

$$v_0 = [e(V_{\text{diff}} - V)/m^*]^{1/2} \quad (17)$$

where $(V_{\text{diff}} - V)$ is the barrier height and m^* the effective mass. The electrons passing the space-charge region are decelerated and hence produce 1/f noise. Trippe gave a computer solution of the problem [27] whereas Luo *et al.* [28] gave a solution in closed form.

To outline their approach we write the second part of (9) in differential form, put $\Delta v = v_1 - v_2$, and $\tau = 2d/(v_1 + v_2)$, where v_1 is the initial velocity (at $x = 0$) and v_2 the final velocity at the barrier (at $x = d$), and d the width of the barrier. This yields

$$dS_I(f) = \frac{4\alpha_0}{3\pi}\frac{(v_1 - v_2)^2}{c^2}\frac{e\,dI_{v1}}{2fd}(v_1 + v_2). \quad (18)$$

Here $dI_{v1} = e\,v_1 dn\,A$ is the differential current, A the cross section of the device, and dn the number of electrons arriving with an initial velocity between v_1 and $v_1 + dv_1$

$$dn = N_d\left(\frac{m^*}{2\pi kT}\right)^{1/2}\exp\left(-\frac{m^* v_1^2}{2kT}\right)dv_1 \quad (18a)$$

where N_d is the donor concentration. Moreover

$$v_2^2 = v_1^2 - 2e\,(V_{\text{dif}} - V)/m^*;\ (v_1 - v_2)^2(v_1 + v_2)$$
$$= (v_1 - v_2)\,[2e(V_{\text{dif}} - V)/m^*] \quad (18b)$$

$$d = \left[2\frac{\epsilon\epsilon_0}{eN_d}(V_{\text{dif}} - V)\right]^{1/2} \quad (18c)$$

where ϵ_0 and ϵ are the MKS conversion factor and the relative dielectric constant, respectively. Integrating between the limits v_0 and ∞, Luo *et al.* [28] found

$$S_I(f) = \frac{4\alpha_0}{3\pi c^2}\frac{e^3(V_{\text{diff}} - V)I_0}{m^* f}\left(\frac{N_d}{\epsilon\epsilon_0 m^*}\right)^{1/2} \cdot\left\{1 - \left[\frac{\pi kT}{4e(V_{\text{diff}} - V)}\right]^{1/2}\right\}. \quad (19)$$

4) $\overline{\Delta v^2}$ Described in Terms of an Energy E [29]–[32]: We finally discuss a set of processes in which $\overline{\Delta v^2}$ can be described in terms of an energy E such that $\overline{\Delta v^2} = 2E/m^*$, where m^* is the effective mass.

In that case α_H may be written

$$\alpha_H = \frac{4\alpha_0}{3\pi}\frac{2E}{m^* c^2}. \quad (20)$$

The question is to find the energy E. We give several examples.

a) Fluctuation in Carrier Injection Across Junction Barriers [29]–[31]: If there are no collisions, $E = e(V_{\text{dif}} - V)$ where $(V_{\text{dif}} - V)$ is the barrier height in electron-volts (the references have an additional term $3kT/2$ that should be removed). If there are collisions, the energy E is lost in steps and v^2 must be replaced by $\Sigma\Delta v^2$, so that (20) might still be valid if $\Sigma\Delta v^2 = \Sigma 2\Delta E/m^* = 2E/m^*$. It is doubtful that this will be the case.

b) Recombination of Electrons and Holes in the Junction Space-Charge Region: This is a two-step process, involving the subsequent capture of an electron (mass m_n^*) and a hole (mass m_p^*). Van der Ziel and Handel [29], [30] find for the collision-free case

$$\alpha_H = \frac{4\alpha_0}{3\pi}\frac{2e(V_{\text{dif}} - V) + 6kT}{[(m_n^*)^{1/2} + (m_p^*)^{1/2}]^2} \quad (20a)$$

if the capture of an electron and a hole are *independent* events. This may need correction if the two events are correlated.

c) Recombination of Electrons in the p-region of an n^+-p Diode: According to van der Ziel [31] $E = 3/2kT$ because the captured electron arrives with an average kinetic energy $3/2kT$. But it might also be argued that an "activation energy" E_a should be added with $E_a = 1/2\,E_{g0}$ for traps at midband and $E_a = E_{g0}$ for direct band-to-band transitions; here E_{g0} is the band gap.

d) Recombination at Surfaces and at Contacts [32]: This problem is similar to case c). Van der Ziel *et al.* [32] added a term $E_a = E_{g0}$ for contact recombination. This should be replaced by $[E_{g0} - |E_f|]$ where E_f is the Fermi level, since the electron drops from the bottom of the conduction band in the semiconductor to the Fermi level in the metal at the contact.

None of the processes a)–d) have been observed so far [33]. It seems that this problem requires further scrutiny, especially the presence of the activation energy E_a [c), d)] and the degree of correlation between subsequent electron and hole captures b).

5) Summary: We have now discussed most of the predictions made by Handel's quantum 1/f noise theory. We shall see in Section V whether these predictions can be refuted or verified by experiment.

The spectra $S_I(f)$ of the collision-free devices are all proportional to the current I. That is a direct consequence of the Hooge equation and comes about because the number N of carriers in the system is proportional to I. We shall see in Section III that the same is true for diffusion-dominated junction devices like n^+-p and n^+-p-n and p^+-n-p BJTs, all at strong forward bias, and we shall also see that it comes about because the minority carrier density $N(x)$ per unit length at x is proportional to the current $I(x)$ at x.

Nevertheless, there are many cases on record where the 1/f noise spectra of junction devices vary as I^γ/f with $\gamma > 1$. It should be clear that in such cases the noise cannot be described by the Hooge equation. It is also found that MOSFETs show a bias dependence $S_I(f)$ different from what is predicted in Section III-A; apparently the Hooge equation is not valid in these cases either. We shall see that in those cases traps in the surface oxide are responsible (Section IV).

Another feature that can be explained by traps is that the $1/f$ noise of many devices may vary strongly from unit to unit and from batch to batch. It comes about because the $1/f$ noise is proportional to the trap density and will therefore vary if that density varies.

On the other hand, the diffusion or mobility $1/f$ noise in BJTs or FETs under comparable conditions (i.e., for comparable interaction processes) all have the same value of α_H. This is not a consequence of Handel's quantum $1/f$ noise theory, but comes about because comparable devices are subjected to identical diffusion or mobility fluctuation processes, even from a classical point of view. As a consequence the α_H's should be identical.

Since the trapping $1/f$ noises in BJTs have usually a different current dependence than the quantum $1/f$ noise, it is often possible to discriminate between the two types of processes.

III. Using Hooge's $1/f$ Equation as a Langevin Noise Source

We saw how for a uniform semiconductor resistor R of length L and cross-sectional area A the parameter α_H could be directly evaluated with the help of definition (2). Things are less simple for nonuniform devices such as JFETs and MOSFETs at arbitrary drain bias, junction diodes, p^+-n-p and n^+-p-n BJTs, and Schottky-barrier diodes. In nearly all these cases a generalized Langevin approach can be used, in which Hooge equation (2), in a distributed form, is used to give an expression for the cross-correlation spectrum of the Langevin (distributed) noise source $H(x, t)$. The approach is in itself not new, but is here applied to the various devices mentioned before, so that the analogy between the various applications becomes obvious. It then also becomes clear why in some cases a modified approach must be used.

For a section Δx at x of a nonuniform device (2) may be written

$$S_{\Delta I}(x, f) = \frac{I^2(x)\alpha_H}{fN(x)\Delta x} \tag{21}$$

where $N(x)$ is the carrier density for unit length at x and α_H is assumed to be independent of x. Usually $I(x)$ is independent of x, but in long n^+-p diodes ("long" means that the length w_p of the p-region is large in comparison with the electron diffusion length $L_n = (D_n\tau_n)^{1/2}$), $I(x)$ depends on x. We shall see that this requires a modification in the method of approach.

Consequently the cross-correlation spectrum of the distributed Langevin noise source is

$$S_H(x, x', f) = \alpha_H \frac{I(x)\, I(x')}{fN(x')} \delta(x' - x). \tag{21a}$$

One can now write down the Langevin equation of the system, linearize it, integrate with respect to x over the device length L, apply the boundary conditions at $x = 0$ and $x = L$, and express the resulting external current fluctuation $\delta I(x, t)$ in terms of the integral of $H(x, t)$ with respect to x. One then transforms to spectra, carries out the integration, and obtains $S_I(f)$.

We give several examples in the following sections.

A. FET (MOSFET and JFET)

The Langevin equation is

$$I_d = g(V)\frac{dV}{dx} + H(x, t) \tag{22}$$

where $I_d = I_{d0} + \Delta I_d(t)$ is the current in the channel, $V = V_0 + \Delta V(x, t)$ the voltage distribution along the channel, and $H(x, t)$ the random source function. Substituting for I_d and V and neglecting second-order terms yields

$$I_{d0} = g(V_0)\frac{dV_0}{dx}$$

$$\Delta I_d(t)\, dx = d[g(V_0)\Delta V(x, t)] + H(x, t)\, dx. \tag{22a}$$

We now h.f. short-circuit the drain to the source, so that $\Delta V(0, t) = \Delta v(L, t) = 0$ for all t, integrate with respect to x for constant t, and divide by L; this yields

$$\Delta I_d(t) = \frac{1}{L}\int_0^L H(x, t)\, dx$$

$$S_{I_d}(f) = \frac{1}{L^2}\int_0^L\int_0^L S_H(x, x', f)\, dx\ dx'. \tag{22b}$$

This holds for both thermal noise [34] and $1/f$ noise [35]. In the latter case

$$S_H(x, x', f) = \frac{I_d^2\alpha_H}{fN(x')}\delta(x' - x)$$

or

$$S_{I_d}(f) = \alpha_H \frac{e_\mu I_d V_d}{L^2 f} \tag{23}$$

for $V_d < V_{ds}$. This result was already obtained by Klaassen [35]. If α_H is independent of $(V_g - V_T)$ the result holds for arbitrary V_d as long as the device is not saturated; if α_H depends on $(V_g - V_T)$, a suitable average α_H must be taken. For small V_d, (23) is always correct because the device acts as a uniform resistor.

B. Short p^+-n Diode ($w_n \ll$ Hole Diffusion Length L_p)

The Langevin equation may be written

$$I_p = -eD_p\frac{dP}{dx} + H(x, t). \tag{24}$$

Substituting $I_p = I_{p0} + \Delta I_p(t)$ and $P = P_0(x) + \Delta P(x, t)$ yields

$$I_{p0} = -eD_p\frac{dP_0}{dx}$$

$$\Delta I_p(t)\, dx = -eD_p\, d\Delta P(x, t) + H(x, t)\, dx. \tag{24a}$$

If the device is h.f. short-circuited $\Delta P(0, t) = \Delta P(w_n, t) = 0$. Carrying out the integration yields

$$\Delta I_p(t) = \frac{1}{w_n}\int_0^{w_n} H(x, t)\, dx$$

$$S_{I_p}(f) = \frac{1}{w_n^2}\int_0^{w_n}\int_0^{w_n} S_H(x, x', f)\, dx\, dx' \tag{24b}$$

in complete analogy with the FET case. But, according to Hooge we have

$$S_H(x, x', f) = \frac{\alpha_H I_{p0}^2}{fP_0(x')}\delta(x' - x). \tag{25}$$

One might argue whether $P_0(x')$ should be replaced by the excess hole concentration $P_0'(x')$. We believe that this should not be done, because one cannot distinguish between "normal" holes and "excess" holes. Moreover, experiments seem to favor $P_0(x')$ rather than $P_0'(x')$ (see Section V).

Carrying out the integrations yields, since $I_{p0} = -eD_p dP_0/dx$,

$$S_{I_p}(f) = \alpha_H \frac{eI_{p0}}{2f\tau_{dp}} \ln\left[\frac{P_0(0)}{P_0(w_n)}\right] \quad (25a)$$

where $\tau_{dp} = w_n^2/2D_p$ is the diffusion time for holes through the n-region. This is the well-known Kleinpenning–van der Ziel result [36]–[38]. Similar equations hold for n^+-p, n^+-p-n, and p^+-n-p devices with slightly different boundary conditions. We come back to that in Section III-D. However, the method breaks down for long diodes ($w_n \gg L_p$), as is shown in Section III-D. It is interesting to note that $S_{I_p}(f)$ is approximately proportional to I_{p0}.

The 1/f noise investigated in this model is diffusion 1/f noise. If the current flow is by generation–recombination in the junction space-charge region, a different approach is needed [29], [30].

C. Diffusion 1/f Noise in n-Type Schottky-Barrier Diodes

The Langevin equation is now, if $x = 0$ at the metal electrode,

$$I = e\mu_n N(x)F + eD_n \frac{dN}{dx} + H(x, t) \quad (26)$$

where μ_n and D_n are the mobility and the diffusion constant, respectively, $N(x)$ the carrier density for unit length, $F = -d\psi/dx$ the field strength, $\psi(x)$ the potential at x, and $H(x, t)$ the random source function.

Multiplying both sides by the integrating factor $\exp[-e\psi(x)/kT]\,dx$, putting $kT\mu = eD_n$ and substituting for $F(x) = -d\psi/dx$ yields

$$I \exp(-\psi/kT)\,dx = D_n d[N(x) \exp\{-e\psi(x)/kT\}] + H(x, t) \exp[-e\psi(x)/kT]\,dx. \quad (26a)$$

If the device is now shortcircuited for h.f., and we put

$$I = I_0 + \Delta I(t) \qquad \psi(x) = \psi_0(x) + \Delta\psi(x, t)$$
$$N(x) = N_0(x) + \Delta N(x, t)$$

then $\Delta N(0, t) = \Delta N(d, t) = 0$ and $\Delta\psi(0, t) = \Delta\psi(d, t) = 0$ for all t. Integrating with respect to x between the limits 0 and d, eliminating second-order terms, and equating the dc terms and the noise yields

$$I_0 I_1 = eD_n\left[-N_0(0) + N_0(d) \exp\left\{-\frac{e(V_{\mathrm{dif}} - V)}{kT}\right\}\right]$$
$$\Delta I(t) = \frac{1}{I_1}\int_0^d H(x, t) \exp\left[-\frac{e\psi_0(x)}{kT}\right] dx \quad (26b)$$

where

$$I_1 = \int_0^d \exp\left[-\frac{e\psi_0(x)}{kT}\right] dx \quad (26c)$$

so that

$$S_I(f) = \frac{1}{I_1^2}\int_0^d \int_0^d S_H(x, x', f) \exp\left[-\frac{e\phi_0(x)}{kT}\right] \cdot \exp\left[-\frac{e\psi_0(x')}{kT}\right] dx\,dx'. \quad (27)$$

This corresponds to the previous cases, except for the weighting factors

$$\exp\left[-\frac{e\psi_0(x)}{kT}\right] \quad \text{and} \quad I_1.$$

According to the Hooge equation we have

$$S_H(x, x', f) = \frac{I_d^2 \alpha_H}{fN_0(x')} \delta(x' - x). \quad (28)$$

Luo *et al.* [28] evaluated the integral and found

$$S_I(f) = \frac{2}{3} \frac{e^3 \alpha_H \mu N_d (V_{\mathrm{dif}} - V) I_0}{\epsilon\epsilon_0 kTf}. \quad (29)$$

Note that this noise spectrum again varies approximately as I_0. Van der Ziel applied this method to evaluate shot noise in Schottky-barrier diodes using the appropriate shot-noise source for $S_H(x, x', f)$ [39].

D. Transfer Function Method for Diode 1/f Noise (Transmission-Line Method)

We consider an n^+-p diode having an arbitrary length w_p of the p-region. We further assume that the device electrodes are h.f. short-circuited; the noise current in the external circuit then equals the noise current at the junction ($x = 0$). However, since the 1/f noise is a distributed noise source, the noise generated at x must propagate to the junction at $x = 0$. If x is comparable to w_p this propagation results in an *attenuation*. As a consequence, the calculation for the diode noise must be redone by another method. We do this first for the diffusion 1/f noise sources.

Let $I_{dn}(x)$ be the electron current in the section Δx at x in the p-region and $\delta\Delta I_{dn}(x, t)$ its fluctuation. If α_{Hnd} is the diffusion Hooge parameter then

$$S_{\Delta In}(x, f) = \frac{I_{dn}^2(x)\,\alpha_{Hnd}}{f[N(x)]\Delta x} \quad (30)$$

where $N(x)$ is the carrier density for unit length at x. The current generator $\delta\Delta I_{dn}(x, t)$ is connected in parallel to Δx.

We now apply the transmission-line method (see [40], [41]). Since $(\Delta x/eD_n)$ is the "equivalent resistance" of the section Δx, as "seen" by $\delta\Delta I_{dn}(x, t)$, the fluctuating carrier density $\delta\Delta N(x, t)$ in the section Δx is

$$\delta\Delta N(x, t) = \delta\Delta I_{dn}(x, t) \frac{\Delta x}{eD_n}. \quad (30a)$$

This, in turn, corresponds to a current generator $\delta\Delta I_{dj}(x, t)$ at the junction, where (see Appendix II)

$$\delta\Delta I_{dj}(x, t) = \frac{\delta\Delta N(x, t)}{Z_{00}} \frac{\cosh \gamma_0(w_p - x)}{\sinh \gamma_0 w_p} = \delta\Delta I_{dn}(x, t)\, \gamma_0 \Delta x \left[\frac{\cosh \gamma_0(w_p - x)}{\sinh \gamma_0 w_p}\right]. \quad (30b)$$

Here $Z_{00} = L_n/(eD_n)$ is the "characteristic impedance" and $\gamma_0 = 1/L_n$ the "propagation constant" of the "equivalent transmission line" describing the diffusion. Moreover, L_n

$= (D_n\tau_n)^{1/2}$ is the electron diffusion length, D_n the electron diffusion constant, and τ_n the electron lifetime, all in the p-region. Hence

$$\Delta S_{I_d}(x, f) = S_{I_{dn}}(x, f)\, \gamma_0^2 \left[\frac{\cosh \gamma_0(w_p - x)}{\sinh \gamma_0 w_p}\right]^2$$
$$= \frac{I_{dn}^2(x)\, \alpha_{Hnd}\gamma_0^2}{fN(x)} \left[\frac{\cosh \gamma_0(w_p - x)}{\sinh \gamma_0 w_p}\right]^2 \Delta x. \quad (31)$$

By integrating with respect to x between the limits 0 and w_p we obtain $S_{I_d}(f)$.

We first consider a *short* diode ($w_p \ll L_n$). Then $\cosh \gamma_0(w - x) = 1$, $\sinh \gamma_0 w_p = \gamma_0 w_p$, and $I_{dn}(x) = I_d$, so that

$$S_{I_d}(f) = \frac{1}{w_p^2}\int_0^{w_p} \frac{\alpha_{Hnd} I_d^2\, dx}{fN(x)}$$
$$= \frac{1}{w_p^2}\int_0^{w_p}\int_0^{w_p} S_H(x, x', f)\, dx\, dx'. \quad (31a)$$

This is identical with the Langevin approach [Section III-B] [37], [38], and that is as expected, because in short diodes there is no attenuation.

We next consider a *long* diode ($w_p \gg L_n$). Then the upper limit of integration may be replaced by ∞, whereas

$$\frac{\cosh \gamma_0(w_p - x)}{\sinh \gamma_0 w_p} = \frac{\exp \gamma_0(w_p - x)}{\exp (\gamma_0 w_p)} = \exp(-\gamma_0 x).$$

Hence

$$S_I(f) = \frac{1}{L_n^2}\int_0^\infty \frac{I_d^2 n(x)\, \alpha_{Hnd}}{fN(x)} \exp(-2\gamma_0 x)\, dx$$
$$= \frac{1}{L_n^2}\int_0^\infty\int_0^\infty [S_H(x, x', f)] \exp(-\gamma_0 x) \cdot \exp(-\gamma_0 x')\, dx\, dx'. \quad (31b)$$

Since these integrals have a built-in attenuation factor $\exp(-2\gamma_0 x)$, they cannot be derived by the Langevin method.

Substituting for $I_{dn}^2(x)$ yields, if $u = \exp(-\gamma_0 x)$

$$S_{I_d}(f) = \alpha_{Hnd} \frac{eI_d}{f\tau_n} \int_0^1 \frac{au^3}{(au + 1)}\, du$$
$$= \alpha_{Hnd} \frac{eI_d}{f\tau_n} f(a) \quad (32)$$

where $I_0 = e(D_n/\tau_n)^{1/2} N_p$ is the back saturation current, N_p the equilibrium concentration of electrons in the p-region, $I_d = I_0 a$, $a = \exp(eV/kT) - 1 = N'(0)/N_p$ and

$$f(a) = \left[\frac{1}{3} - \frac{1}{2a} + \frac{1}{a^2} - \frac{1}{a^3}\ln(1 + a)\right]. \quad (32a)$$

This was already derived by van der Ziel *et al.* [32].

Kleinpenning [37] omitted the attenuation factor $\exp(-2\gamma_0 x)$. Carrying out the integration one then obtains the same equation but with a different factor $f(a)$

$$f(a) = \int_0^1 \frac{au}{du + 1}\, du = \left[1 - \frac{1}{a}\ln(1 + a)\right]. \quad (32b)$$

It is the merit of the transmission-line method that it introduces the attenuation factor $\exp(-2\gamma_0 x)$ automatically.

If we replace $N(x)$ by $N'(x)$ in the expressions for $S_{I_d}(f)$, we must replace $(au + 1)$ by au; $f(a)$ then follows from

$$f(a) = \pm \int_0^1 u^2\, du = \pm \frac{1}{3} \quad (32c)$$

(plus sign for forward bias, minus sign for back bias [32], [41]).

It must be decided by experiment which of these expressions for $f(a)$ is valid.

E. Recombination 1/f Noise in a Long n^+-p Diode [31], [41]

We now turn to recombination $1/f$ noise for the p-region of a long n^+-p diode ($w_p/L_n \gg 1$). If $N'(x)$ is the excess carrier density for unit length at x then $\Delta N'(x) = N'(x)\Delta x$ is the number of excess carriers in the section Δx. Consequently, the recombination current $\Delta I_R(x)$ *disappearing* in the section Δx at x is

$$\Delta I_R(x) = \Delta N'(x)\, e/\tau_n \quad (33)$$

where τ_n is the electron lifetime in that section.

Because the electron capture cross section of the traps in the p-region fluctuates, the lifetime τ_n fluctuates and hence $C_n = 1/\tau_n$ will fluctuate. The current fluctuation *disappearing* in the section Δx at x is therefore

$$\delta\Delta I_R(x, t) = \Delta I_R(x) \frac{\delta C_n(x, t)}{C_n} \quad (33a)$$

where, in analogy with Hooge's equation (2)

$$\frac{S_{cn}(x, f)}{c_n^2} = \frac{\alpha_{Hnr}}{f\Delta N(x)} \text{ (case a)}$$

or

$$\frac{S_{cn}(x, f)}{c_n^2} = \frac{\alpha_{Hnr}}{f\Delta N'(x)} \text{ (case b)} \quad (33b)$$

where $\Delta N(x) = \Delta N'(x) + \Delta N_p$ and ΔN_p is the number of equilibrium minority carriers in the section Δx. We shall prove these relationships in a moment.

Since $N(x) = \Delta N/\Delta x$, we have in case a)

$$\Delta S_{I_R}(f) = \Delta I_R^2(x) \frac{\alpha_{Hnr}}{f\Delta N}$$
$$= \alpha_{Hnr} \frac{e^2}{f\tau_n^2}\left[\frac{N'(x)}{N(x)}\right]^2 \Delta x. \quad (34)$$

But according to the transmission-line model, the fluctuation current disappearing in the section Δx at x corresponds to a current fluctuation $\delta\Delta I(x, t)$ at the junction, where (see Appendix II)

$$\delta\Delta I(x, t) = \delta\Delta I_R(x, t) \exp(-\gamma_0 x) \quad (34a)$$

so that, if $u = \exp(-\gamma_0 x)$

$$S_I(f) = \int_0^\infty S_{\Delta I}(x, f)\, dx = \alpha_{Hnr} \frac{eI}{f\tau_n} f(a) \quad (34b)$$

where $f(a)$ has the same meaning as before. For case b) we find instead $f(a) = \pm 1/3$ (plus sign for forward bias, minus sign for back bias). The total noise is therefore

$$S_I(f) = (\alpha_{Hnd} + \alpha_{Hnr}) \frac{eI}{f\tau_n} f(a). \quad (35)$$

By measuring $S_I(f)$ we can only determine the sum (α_{Hnd} +

α_{Hnr}). According to (40) α_{Hnr} is very small ($\simeq 0.5 \times 10^{-9}$) for electrons in p-type silicon.

We now prove case b). We write (33) as

$$\Delta I_R(x) = e\,\Delta N'(x)\,C_n(x) = e \sum_{i=1}^{\Delta N'(x)} C_{ni}(x) \tag{36}$$

or, by taking averages on both sides,

$$\overline{C_n} = \overline{C_{ni}} \tag{36a}$$

for all i's, since all C_{ni}'s fluctuate independently.

Next we consider fluctuations. Since $\Delta N'(x)$ does not fluctuate

$$\delta\Delta I_R(x, t) = e \sum_{i=1}^{\Delta N'(x)} \delta C_n(x, t) \tag{36b}$$

or

$$\Delta S_{I_R}(x, f) = e^2 \sum_{i=1}^{\Delta N'(x)} S_{cni}(f)$$
$$= e^2 \Delta N'(x)\, S_{cni}(f) \tag{36c}$$

so that

$$\frac{S_{\Delta I_R}(x, f)}{\Delta I_R^2(x)} = \frac{S_{cni}(f)}{(\overline{C_{ni}})^2\, N'(x)} = \frac{\alpha_H}{f \Delta N'(x)} \tag{36d}$$

because $\overline{C_{ni}} = \overline{C}$ and $S_{cni}(f)/(\overline{C_{ni}})^2$ is independent of $\Delta N'(x)$ and hence equal to α_H/f.

Case b) assumes that only the lifetimes of the individual *excess* minority carriers fluctuate. Since one cannot distinguish between *equilibrium* minority carriers and *excess* minority carriers, it is more likely that the lifetime of *each* minority carrier fluctuates. But that corresponds to replacing $\Delta N'(x)$ by $\Delta N(x)$; we then obtain case a).

F. Recombination 1/f Noise in the Base Region of a BJT

This problem can also be solved by the transmission-line model. Consider an n^+-p-n BJT with a base-length w_B. Let the equilibrium electron concentration and the excess electron concentration $N'(w_B)$ at $x = w_B$ be small in comparison with the excess electron concentration $N'(0)$ at $x = 0$. Then

$$N'(x) = N'(0)\,(1 - x/w_B) \tag{37}$$

$$\Delta I_R(x) = \frac{eN'(x)\Delta x}{\tau_n}$$

$$I_{Br} = \int_0^{w_B} dI_R(x)$$
$$= \frac{eN'(0)}{\tau_n} \int_0^{w_B} \left(1 - \frac{x}{w_B}\right) dx$$
$$= \frac{eN'(0)}{2\tau_n}\, w_B \tag{37a}$$

whereas the noise in the section Δx has a spectrum (see (34))

$$S_{\Delta I_R}(x, f) = \alpha_{Hnr} \frac{e^2}{f\tau_n^2} N'(x)\,\Delta x$$
$$= \alpha_{Hnr} \frac{e^2 N'(0)}{f\tau_n^2} \left(1 - \frac{x}{w_B}\right) \Delta x. \tag{37b}$$

According to the transmission-line theory the transfer function is (Appendix II)

$$\frac{\sinh \gamma_0 (w_B - x)}{\sinh \gamma_0 w_B} = \left(1 - \frac{x}{w_B}\right) \tag{37c}$$

for small $\gamma_0 w_B$. Consequently

$$S_{I_{Br}}(f) = \int_0^{w_B} S_{\Delta I_R}(x, f)\, dx$$
$$= \alpha_{Hnr} \frac{e^2 N'(0)}{f\tau_n^2} \int_0^{w_B} \left(1 - \frac{x}{w_B}\right)^3 dx$$
$$= \frac{1}{4}\, \alpha_{Hnr} \frac{e^2 N'(0)}{f\tau_n^2}. \tag{38}$$

Substituting for I_{Br} yields

$$S_{I_{Br}}(f) = \alpha_{Hnr} \frac{eI_{Br}}{2f\tau_n}. \tag{39}$$

This result has not been published before.

According to the end of Section II-B4

$$\alpha_{Hnr} = \frac{4\alpha_0}{3\pi} \left[\frac{3kT}{m_n^* c^2}\right]. \tag{40}$$

Since $kT/e = 25$ mV, $\alpha_{Hnr} = 0.5 \times 10^{-9}$ for electrons in p-type silicon.

The competing 1/f noise mechanism in n^+-p-n BJTs is due to hole injection from the base into the emitter followed by diffusion toward the emitter contact. The base current I_{Bp} is associated with this process. Hence by analogy with (25a)

$$S_{I_{Bp}}(f) = \alpha_{Hp} \frac{eI_{Bp}}{2f\tau_{dp}} \ln \left[\frac{P_0(0)}{P_0(W_E)}\right] \tag{41}$$

where $\alpha_{Hp} \simeq (6\text{–}10) \times 10^{-9}$ (see Section V), $I_{BP} \gg I_{Br}$, and the diffusion time $\tau_{dp} = W_E^2/2D_p \ll \tau_n$. In the base region $\tau_{dn} = W_B^2/2D_n$ and $\tau_{dn}/\tau_n = W_B^2/2\,L_n^2 \ll 1$ (for n^+-p-n BJTs with a large β_F), whereas τ_{dp} and τ_{dn} are comparable. Consequently, $S_{I_{Bp}}(f) \gg S_{I_{Br}}(f)$, so that the recombination effect in the base is generally a negligible source of 1/f noise.

G. Fluctuations in the Contact Recombination Velocity at the Contact to the p-Region of a Short n^+-p Diode

If a short n^+-p diode has a length of w_p of the p-region and the ohmic contact to the p-region has a contact recombination velocity s_{cn} for electrons then the electron current is given by

$$I_{n0} = \frac{eD_n}{w_p} [N'(0) - N'(w_p)] = es_{cn} N'(w_p) \tag{42}$$

where $N'(0)$ and $N'(w_p)$ are the excess electron concentrations at the junction ($x = 0$) at the contact ($w = w_p$), respectively. Solving for $N'(w_p)$ yields $N'(w_p) = (D_n/w_p)N'(0)/(s_{cn} + D_n/w_p)$, and

$$I_{n0} = eN'(0) \frac{D_n}{w_p} \left[\frac{s_{cn}}{s_{cn} + D_n/w_p}\right]$$
$$= eN'(0) \frac{D_n}{w_p} \left[1 - \frac{D_n/w_p}{s_{cn} + D_n/w_p}\right] \tag{43}$$

where $s_{cn} = 6 \times 10^6$ cm/s for electrons in Si, and $s_{cp} = 4 \times 10^6$ cm/s for holes in Si.

If s_{cn} fluctuates, I_{n0} fluctuates and hence

$$\delta I_{n0} = eN'(0)\,\frac{D_n}{w_p}\,\frac{D_n/w_p\ s_{cn}}{[s_{cn} + D_n/w_p]^2}\left(\frac{\delta_{scn}}{scn}\right). \tag{43a}$$

For an n$^+$-p Si diode with $w_p = 1\ \mu$m we have $D_n/w_p \ll s_{cn}$. Hence in first approximation we may replace $(s_{cn} + D_n/w_p)$ by s_{cn}, so that

$$I_{n0} \simeq eN'(0)\,\frac{D_n}{w_p}$$

and

$$\delta I_{n0} = I_{n0}\left(\frac{D_n}{w_p s_{cn}}\right)\cdot\frac{\delta s_{cn}}{s_{cn}}. \tag{43b}$$

Consequently, according to (7a)

$$S_{I_n}(f) = I_{n0}^2\left(\frac{D_n}{w_p s_{cn}}\right)^2\frac{S_{scn}(f)}{s_{cn}^2}$$

$$= I_{n0}^2\left(\frac{D_n}{w_p s_{cn}}\right)^2\frac{\alpha_{Hs}}{fN_{\text{eff}}} \tag{44}$$

where $N_{\text{eff}} \simeq 1/2\,N'(0)w_p$ and α_{Hs} is given by (40). Substituting for N_{eff} yields, if $\tau_{dn} = w_p^2/2\ D_n$

$$S_{I_n}(f) = \alpha_{Hs}\,\frac{eI_{n0}}{f\tau_{dn}}\left[\frac{D_n}{w_p s_{cn}}\right]^2. \tag{44a}$$

This result has not been published before.

For electron diffusion 1/f noise in the p-region (see (25a))

$$S_{I_n}(f) = \alpha_{Hd}\,\frac{eI_{n0}}{2f\tau_{dn}}\ln\left[\frac{N(0)}{N(w_p)}\right]. \tag{44b}$$

Here α_{Hd} and α_{Hs} can have comparable values and $\ln\,[N(0)/N(w_p)] \simeq 3$–$4$ (see next section) so that (44a) is smaller than (44b) by a factor

$$\left(\frac{s_{cn}w_p}{D_n}\right)^2 = \left(\frac{6\times 10^6\times 10^{-4}}{35}\right)^2 \simeq 300$$

for $w_p = 10^{-4}$ cm. Therefore (44a) is negligible in comparison with (44b).

H. Evaluation of [N(0)/N(w_p)] and its Application to Transistor Noise

According to the previous section we have for a short n$^+$-p diode

$$\frac{N'(0)}{N'(w_p)} = 1 + \frac{s_{cn}w_p}{D_n} \tag{45}$$

for large forward bias, $[\exp(eV/kT)] \gg 1$ and $N'(0) \gg N_p$, and $N'(w_p) \gg N_p$. In that case (45) may be written [31]

$$\frac{N(0)}{N(w_p)} = 1 + \frac{s_{cn}w_p}{D_n} \tag{45a}$$

or

$$S_{I_n}(f) = \alpha_{Hn}\,\frac{eI_{no}}{2f\tau_{dn}}\ln\left[1 + \frac{s_{cn}w_p}{D_n}\right] \tag{45b}$$

where $\tau_{dn} = w_p/2D_n$. Here $S_{I_n}(f)$ is the 1/f diffusion noise (25a).

This is easily applied to an n$^+$-p-n transistor. Here the base current I_B is normally due to hole injection from the base into the emitter and diffusion through the emitter region toward the emitter contact. Then

$$P(0)/P(w_E) = 1 + \frac{(s_{cp}w_E)}{D_p} \tag{45c}$$

$$S_{I_B}(f) = \alpha_{Hp}\,\frac{eI_B}{2f\tau_{dp}}\ln\left[1 + \frac{s_{cp}w_E}{D_p}\right] \tag{45d}$$

where $\tau_{dp} = w_E/2D_p$ and w_E is the length of the emitter region.

For the collector current I_C of an n$^+$-p-n transistor the current flow is due to electron diffusion through the base region. It is usually assumed [31] that the electrons leave the base region with the limiting velocity v_{cn} ($\simeq 10^7$ cm/s); more exactly, the velocity in question is the electron velocity in the collector space-charge region. We must then replace s_{cp} by v_{cn}, P by N, p by n, I_C by I_B, and w_E by w_B, so that

$$\frac{N(0)}{N(w_p)} = 1 + \frac{v_{cn}w_B}{D_n} \tag{46}$$

$$S_{I_C}(f) = \alpha_{Hn}\,\frac{eI_C}{f2\tau_{dn}}\ln\left[1 + \left(\frac{v_{cn}w_B}{D_n}\right)\right]. \tag{46a}$$

In short n$^+$-p $Hg_{1-x}Cd_xTe$ photodiodes one needs to known $N(0)/N(w_p)$ for zero near-zero bias or for back bias. Equation (42) must then be rewritten as

$$I_{n0} = eD_n\left[\frac{N(0) - N(w_p)}{w_p}\right] = es_{cn}\,[N(w_p) - N_p] \tag{46b}$$

where $N_p = N(0)\exp(-eV/kT) = An_i^2/N_a$ is the equilibrium hole concentration for unit length. Here A is the cross-sectional area of the diode, n_i the intrinsic carrier concentration, and N_a the acceptor concentration in the p-region. Then by substituting for N_p and solving for $N(w_p)$

$$S_{I_n}(f) = \alpha_{Hn}\,\frac{I_{n0}}{2f\tau_{dn}}\ln\left[\frac{N(0)}{N(w_p)}\right] \tag{46c}$$

$$\frac{N(0)}{N(w_p)} = \frac{1 + D_n/s_{cn}w_p}{1 + (D_n/s_{cn}w_p)\exp(-eV/kT)} \tag{46d}$$

where V is the applied bias, $I_{n0} = I_0[\exp(eV/kT)] - 1$

$$I_0 = eN_p\,\frac{(D_n/w_p)s_{cn}}{s_{cn} + D_n/w_p}. \tag{46e}$$

For large negative bias $I_{n0} = -I_0$ and $N(0)/N(w_p) \simeq \exp[-e|V|/kT]$.

IV. Nonfundamental 1/f Noise Sources

Nonfundamental noise sources are noise sources that involve carrier trapping by and carrier detrapping from traps. These traps may be in a conducting channel, in a space-charge region, or in a surface oxide, and may cause Lorentzian or 1/f type spectra. They are called "nonfundamental," since the magnitude of their spectra is proportional to the trap density; the noise effect can thus be strongly reduced by eliminating most of the traps. On the other hand, the various scattering 1/f noise sources in collision-dominated devices and the Bremsstrahlung 1/f noise in collision-free devices are essential to the operation of the device, and hence should be called "fundamental."

A. The McWhorter Theory

The earliest theory of flicker noise (Schottky, 1926) [42] involved a process governed by a time constant τ. The Langevin equation of the process

$$\frac{dX}{dt} + \frac{X}{\tau} = H(t) \tag{47}$$

yields the spectrum

$$S_X(f) = S_H(0)\frac{\tau^2}{1+\omega^2\tau^2} = 4\overline{X^2}\frac{\tau}{1+\omega^2\tau^2} \tag{47a}$$

where $\overline{X^2} = S_H(0)\tau$. Such a spectrum is called a *Lorentzian* spectrum. A good example is trapping and detrapping of electrons by surface traps, as in the MOS capacitor of MOSFETs, in the surface oxide on the base of a BJT, on the surface of the space-charge region of a p-n junction, or in the bulk space-charge region of a JFET.

A single trap level of time constant τ can be described by (47); hence the number fluctuation spectrum is in analogy with (47a)

$$S_{\Delta N}(f) = 4\overline{\Delta N^2}\cdot\frac{\tau}{1+\omega^2\tau^2} \tag{47b}$$

where $\overline{\Delta N^2}$ is the variance of the fluctuation ΔN in N, the number of carriers in the sample. By analogy we have for discrete, multiple-trap levels

$$S_N(f) = 4\sum_i \overline{\Delta N_i^2}\frac{\tau_i}{1+\omega^2\tau_i^2}. \tag{47c}$$

These spectra are called *generation-recombination* spectra (g-r for short). Such spectra are often found in Si-JFETs, where they can mask the small amount of 1/f noise present in such devices.

It soon became clear that the spectrum was 1/f rather than Lorentzian, and efforts were made to develop models for 1/f spectra. Gradually the idea emerged that a proper distribution in time constants τ might explain the spectra. The idea was first proposed by von Schweidler for the theory of dielectric losses; he could then explain why $\tan\delta$ is nearly independent of frequency over a wide frequency range (1907) [43]. Gevers (1946) applied this theory to his experimental dielectric loss data [44]. Later the idea was applied by du Pře [45] and by van der Ziel [46]. McWhorter [47] applied it to semiconductor devices and made it very popular; for that reason the 1/f model still bears his name.

If one has a time constant distribution $g(\tau)\,d\tau$, then by analogy with (47c)

$$S_N(f) = 4\overline{\Delta N^2}\int_0^\infty \frac{\tau g(\tau)\,d\tau}{1+\omega^2\tau^2} \tag{48}$$

where

$$\int_0^\infty g(\tau)\,d\tau = 1 \text{ (normalization).} \tag{48a}$$

In particular the *normalized* distribution:

$$g(\tau)\,d\tau = \frac{d\tau/\tau}{\ln(\tau_2/\tau_1)}, \quad \text{for } \tau_1 < \tau < \tau_2$$

$$g(\tau)\,d\tau = 0, \quad \text{otherwise} \tag{48b}$$

yields the 1/f spectrum

$$S_N(f) = \frac{\overline{\Delta N^2}}{f\ln(\tau_2/\tau_1)}, \quad \text{for } 1/\tau_2 < \omega < 1/\tau_1 \tag{49}$$

whereas

$$S_N(f) = \frac{2\overline{\Delta N^2}}{\pi f\ln(\tau_2/\tau_1)}\tan^{-1}(\omega\tau_2), \quad \text{for } \omega\tau_1 \ll 1$$

$$S_N(f) = \frac{\overline{\Delta N^2}}{f\ln(\tau_2/\tau_1)}\left[1 - \frac{2}{\pi}\tan^{-1}(\omega\tau_1)\right], \quad \text{for } \omega\tau_2 \gg 1 \tag{49b}$$

so that $S_N(f)$ is constant for $\omega\tau_2 \ll 1$ and $S_N(f)$ varies as $1/f^2$ for $\omega\tau_1 \gg 1$. Equation (49a) was observed on CdHgTe samples in which the surface had been cleaned by sputtering in a mercury discharge; apparently the sputtering removed the surface centers with long time constants [48]. Equation (49b) was observed by Suh and van der Ziel [49] in GaAs MESFETs; this is one of the many different types of spectra observed in these devices [50]–[52] and therefore not much emphasis should be placed on a particular one.

In most cases τ_2 is so long and τ so short that only the 1/f part of the spectrum is observed. How can one then be sure that the 1/f spectrum is really due to a distribution of Lorentzians? By going to samples of very small area ($\simeq$ 1-μm diameter). The effective number of traps in the system then becomes so small that individual Lorentzians become visible.

How can a distribution function of the form (48a) be realized? There are two obvious possibilities: a) excitation from trap levels with activation energies E_a (distribution in E_a) [43]–[46], and b) tunneling to trap levels inside surface oxide at depth z (distribution in z) [47].

Case a): Since the time constant τ depends exponentially on E_a

$$\tau = \tau_0\exp\left(\frac{eE_a}{kT}\right)$$

$$\tau_1 = \tau_0\exp\left(\frac{eE_{a1}}{kT}\right)$$

$$\tau_2 = \tau_0\exp\left(\frac{eE_{a2}}{kT}\right) \tag{50}$$

and the distribution function

$$g(E_a)\,dE_a = \frac{dE_a}{E_{a2}-E_{a1}}, \quad \text{for } E_{a1} < E_a < E_{a2} \tag{50a}$$

and zero otherwise. This is possible for migrating ions with a distribution in activation energies, but is impossible for a distribution of trap levels in the energy gap of a semiconductor.

The reason is simple. All traps a few *kT above* the Fermi level are *empty* and all traps a few *kT below* the Fermi level are *filled*. The spectrum thus consists of two somewhat smeared-out Lorentzians, one due to transitions from the Fermi level to the conduction band and the other due to transitions from the Fermi level to the valence band. *Consequently bulk number fluctuations in semiconductors cannot give 1/f noise.*

Case b): Since the process is due to tunneling,

$$\tau = \tau_1\exp(\gamma z) \qquad \tau_2 = \tau_1\exp(\gamma z_0)$$

$$g(\tau)\,d\tau = \frac{dz}{z_0}, \quad \text{for } 0 < z < z_0 \tag{50b}$$

and zero otherwise. Here γ is the tunneling parameter ($\simeq 10^8\ \text{cm}^{-1}$) and z_0 is the average distance between traps.

Jindal and van der Ziel [53] have proposed a McWhorter model for the interaction of electrons and acoustical phonons. While a $1/f$ spectrum results, it seems unlikely that it can extend to sufficiently low frequencies. The Kousik-van Vliet model, based on Handel's theory, is a more likely candidate, as we shall see in Section V.

B. 1/f Noise in MOSFETs [54]–[58]

In MOSFETs electrons tunnel from traps in the oxide, at a distance z from the interface, to the conducting channel and *vice versa.* As a result, the number of trapped electrons ΔN_t in a volume element $\Delta x \Delta y \Delta z$ in the oxide fluctuates with a mean square value

$$\overline{\delta \Delta N_t^2} = \Delta N_T(E)\, \Delta E\, \Delta x \Delta y \Delta z\, f_t(1 - f_t) \tag{51}$$

where f_t is the Fermi function and $\Delta N_T(E)$ the number of traps per unit volume with an energy between E and ΔE. Since $\tau = \tau_0 \exp(\gamma z)$ is the time constant of a trap at z, where $\gamma(\simeq 10^8/\mathrm{cm})$ is the tunneling parameter of the traps, one has for the spectrum

$$S_{\Delta N_t}(f) = 4N_T(E)\, \Delta E \Delta x \Delta y\, \Delta z f_t(1 - f_t) \frac{\tau}{1 + \omega^2 \tau^2} \tag{51a}$$

and hence by integrating with respect to the trap energy E, the distance z, and y, one obtains [56], [57]

$$S_{\Delta N_t}(x, f) = \frac{N_T(E_f)\, kT}{f} \frac{w \Delta x}{\gamma} \tag{51b}$$

where $N_T(E_f)$ is the trap density per unit energy at the Fermi level.

At arbitrary inversion, according to Jindal and Reimbold [56]–[58] the spectrum of the number fluctuation $\delta \Delta N$ is

$$S_{\Delta N}(x, f) = \frac{N_T(E_f) kT}{f} \frac{w \Delta N}{\gamma} \left(\frac{\delta \Delta N}{\delta \Delta N_t} \right)^2 \tag{52}$$

where

$$\frac{\delta \Delta N}{\delta \Delta N_t} = - \frac{C_n}{C_d + C_{ss} + C_{ox} + C_n}. \tag{52a}$$

Here $C_n = e^2 N/kT$ is the channel charge capacitance per unit area, C_d the depletion capacitance per unit area, C_{ss} the surface state capacitance per unit area; C_{ox} is the oxide capacitance per unit area, and N the electron density in the channel per unit area. Furthermore

$$\Delta S_{I_d}(x, f) = \frac{I_d^2}{\Delta N^2} S_{\Delta N}(x, f)$$

$$S_{I_d}(f) = \frac{1}{L^2} \int_0^L [\Delta S_I d(x, f) \Delta x]\, dx \tag{53}$$

where $[\Delta S_{I_d}(x, f) \Delta f]^{1/2}$ is the noise current generator in parallel to the section Δx at x; $S_{I_d}(f)$ is the spectrum of the current fluctuation in the drain and L is the device length.

At weak inversion $[C_n \ll (C_d + C_{ss} + C_{ox})]$, $N_T(E_f)$ is practically independent of V_g and V_d, and hence $S_{I_d}(f)$ varies as I_d^2. Since $g_m = \partial I_d / \partial V_g$

$$S_v(f) = 4kTR_n = \frac{S_{I_d}(f)}{g_m^2} \tag{53a}$$

is independent of V_g and V_d, the equivalent noise resistance $R_n(f)$ is independent of V_g, V_d, and hence I_d. Measurements indicate that $R_n(f)$ can have turn-over frequencies as low as 1000 Hz at weak inversion [58]. At stronger inversion, $R_n(f)$ increases with increasing saturation current, so that the weak-inversion amplifier is more useful.

At strong inversion, where $C_n \gg (C_d + C_{ss} + C_{0x})$, $N_T(E_f)$ depends on V_g and V_d, because the position of the Fermi level depends on $N(x)$. Reimbold neglected this effect [57], [58], but Klaassen gave an approximate solution [36] by assuming that $N_T(E_f)w$ was proportional to the carrier density $N(x)$ per unit length at x. One can then introduce a Hooge parameter α_H by the definition [31]

$$\alpha_H = \frac{N_T(E_f) kTw}{\gamma N(x)}. \tag{54}$$

This parameter is independent of bias and hence yields

$$S_{I_d}(f) = \alpha_H \frac{e \mu I_d V_d}{L^2 f}, \quad \text{for } V_d < V_{ds}. \tag{54a}$$

It is now possible to evaluate α_H as a function of bias, and so determine whether α_H is indeed independent of bias. In many instances it is [25] but this is not self-evident and needs experimental proof in each case.

We now evaluate the quantum limits for silicon MOSFETs and calculate α_{Hp} and α_{Hn} = 300 K with the help of (15b). Since Θ_D = 645 K, $a = 5.43 \times 10^{-8}$ cm (there are three kinds of holes!), we obtain $\alpha_{Hn} = 2.1 \times 10^{-8}$ and $\alpha_{Hp} = 4.2 \times 10^{-7}$, if they exist. The lowest measured values for n- and p-channel MOSFETs are: $\alpha_{Hn} = 1.0 \times 10^{-6}$ and $\alpha_{Hp} = (3\text{–}9) \times 10^{-7}$. We thus conclude that the surface $1/f$ noise always masks the quantum $1/f$ noise in n-channel MOSFETs, whereas in p-channel MOSFETs the quantum $1/f$ noise may just be observable in the best units. We come back to this problem in Section V.

In MOSFETs with ion-implanted channels, $S_{I_d}(f)$, measured as a function of the drain voltage V_d, has a maximum [60] well before saturation. It comes about because at a given point x in the channel the potential energy $\psi(z)$ has a minimum away from the surface. As a consequence, the electrons must climb a potential barrier before they can reach the surface and interact with oxide traps [62]. The $1/f$ noise is therefore reduced, and this becomes more pronounced near saturation.

We can now understand why most MOSFETs have *surface* $1/f$ noise and most bulk semiconductor resistors have *volume* $1/f$ noise. According to (51b) and (53), $\Delta S_{I_d}(f)$ is proportional to the *surface area*, whereas in resistors N, and hence $S_{I_d}(f)$, are proportional to the device *volume*. Devices with a *small* surface-to-volume ratio have therefore bulk $1/f$ noise whereas devices with a *large* surface-to-volume ratio have *surface* $1/f$ noise, unless the surface is very well-passified.

We finally mention an MOS capacitor experiment by Amberiadis [63]. The channel was p-type and the gate voltage V_g was raised from below flat-band to strong inversion. When V_g passes the flat-band situation, the holes are repelled from the surface, the interaction of holes with traps in the surface oxide diminishes and hence the surface $1/f$ noise due to holes in gradually eliminated. There was some $1/f$ noise left; that can be attributed to hole mobility $1/f$ noise. When an appreciable inversion sets in, the surface part of the channel becomes n-type. The electrons now react with the oxide traps and produce surface $1/f$ noise due to electrons; as a consequence $S_I(f)$ increases by more than one order of magnitude. The merit of this experiment is that it shows how surface $1/f$ noise can be turned off and on.

The trapping of carriers in the surface oxide can also give rise to surface potential fluctuations, and hence to mobility $1/f$ noise. This problem is studied at the University of Rochester (private communication).

C. Trapping Noise in p-n Junctions and BJTs

We saw that in the case of collision $1/f$ noise in BJTs the spectra $S_{I_C}(f)$ and $S_{I_B}(f)$ were proportional to I_C and I_B, respectively. While this occurs in very-low-noise devices, the collision $1/f$ noise is usually masked by trapping noise. Earlier theories (Fonger (1956) [64], Hsu *et al.* (1970) [65]) described this noise by fluctations δs in the surface recombination velocity s of the surfaces. Van der Ziel [66] used this model to reconcile seemingly conflicting data. The noise phenomena can be distinguished from the collision $1/f$ noises with the help of their current dependence (as I_γ, with γ = 1.5 or 2).

Fonger's model can be understood from the McWhorter model as follows: The recombination at the surface goes via fast surface states, but these surface states are *modulated* by the trapping or detrapping of carriers in the surface oxide. The fluctuation δs in s should be proportional to the fluctuation δN_t in the number of trapped electrons in the oxide and hence $S_s(f)$ should be proportional to $S_{N_t}(f)$. But $S_{N_t}(f)$ is proportional to N_t. Since s is also proportional to N_t we obtain

$$S_s(f) = Cs/f \tag{55}$$

which is Fonger's starting point.

We now apply this to the base I_B of a BJT. We write

$$I_B = I_{BE} + I_{BR1} + I_{BR2} \tag{56}$$

where I_{BE} is due to carrier injection from the base into the emitter, diffusion through the emitter, and recombination at the emitter contact; whereas I_{BR1} and I_{BR2} are due to recombination at the base surface and at the surface of emitter–base space-charge region, respectively. In modern devices I_{BE} predominates, but nevertheless most of the *noise* often comes from I_{BR1} and I_{BR2}.

We first consider the noise of I_{BR1}. Since

$$\frac{\delta I_{BR1}}{I_{BR1}} = \frac{\delta s}{s} \quad \text{or} \quad \frac{S_{I_{BR1}}(f)}{I_{BR1}^2} = \frac{S_s(f)}{s^2} = \frac{C}{sf} \tag{57}$$

so that $S_{I_{BR1}}$ varies as I_{BR1}^2; if $I_B \gg I_{BR1}$, $S_{I_{BR1}}(f)$ varies as I_B^2.

Next we turn to I_{BR2}. Here the recombination occurs in a well-defined part of the space-charge region, characterized by the coordinate x_1; at this point, the surface electron and hole densities are comparable. Let for an applied voltage V the potential at x change by V_1($V_1 = 1/2V$ for a symmetric junction, and $V_1 \simeq 3/4V$ for a strongly asymmetric junction), and let $p(x_1)$ and p_1 be the hole concentrations for the applied voltages V and 0, respectively. Then I_{BR2} may be written

$$I_{BR2} = \text{esp}\,(x_1)\,A_{\text{eff}} = \text{es}p_1 \exp\left(\frac{eV_1}{kT}\right)A_{\text{eff}} \tag{58}$$

where A_{eff} is the effective recombination area. If s now fluctuates

$$\delta I_{BR2} = eA_{\text{eff}} \exp\left(\frac{eV_1}{kT}\right) \delta s$$

$$S_{I_{BR2}}(f) = (eA_{\text{eff}})^2 \exp\left(\frac{2eV_1}{kT}\right) S_s(f). \tag{59}$$

Hence $S_{I_{BR2}}(f)$ varies as I_{BR2}, and if I_{BE} predominates, $S_{I_{BR2}}(f)$ varies as $I_{B2}V_1/V$. Here $I = I_0 \exp\,[eV/mkT)]$, where $m = 1$ if diffusion predominates and $m = V/V_1$ if recombination at the surface of the emitter-base space-charge region predominates. In the second case $S_I(f)$ varies as I^2, whereas in the first case ($I \gg I_R$), $S_I(f)$ varies as $I^{2V_1/V}(s/f)$. This explains Hsu *et al.* data [65].

We now discuss a $1/f$ noise mechanism observed by Trippe [67] in small p^+-n^+ tunnel diodes. There was a large amount of $1/f$ noise in the excess current regime at forward bias and at large back bias. In both cases $S_I(f)/I^2$ was constant and the values in the two regimes were nearly equal. The $1/f$ spectrum had some small g–r "bumps," and the h.f. noise was suppressed shot noise, as expected for a two-stage process (van Vliet [68]).

The current flow in each case is caused by tunneling via intermediate states in the space-charge region followed by a transition to the valence or conduction band. The large amount of $1/f$ noise is probably caused by $1/f$ modulation of the transition probabilities due to the fluctuating occupancy of traps near the intermediate states; the deviation from $1/f$ noise may indicate that relatively few traps are involved. According to this interpretation the noise mechanism represents *nonfundamental noise.*

D. Location of 1/f Noise Sources in BJTs

We finally discuss a method of locating $1/f$ noise sources in transistors [69]. There are three noise sources, i_{bc} between base and collector, i_{be} between base and emitter, and i_{ec} between emitter and collector; they come from independent mechanisms and should therefore be uncorrelated. They have spectra $S_{I_{BC}}(f)$, $S_{I_{BE}}(f)$, and $S_{I_{EC}}(f)$, respectively. There are a base series resistance $r_{b'b}$ in series with the base (Fig. 1). It should be borne in mind that i_{bc} is not connected to one of the two endpoints of $r_{b'b}$ but somewhat in between.

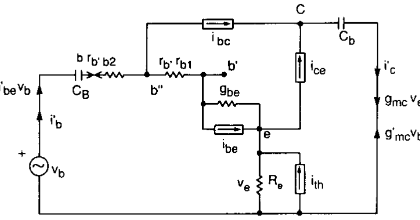


Fig. 1. Equivalent noise circuit of BJT with feedback.

Evaluating the $1/f$ noise in the base and the collector yields

$$S_{I_B}(f) = S_{I_{BE}}(f) + S_{I_{BC}}(f) \tag{60}$$

$$S_{I_C}(f) = S_{I_{BE}}(f)\,[g_{mc}(r_{b'b1} + r_{b'b2})]^2 + S_{I_{BC}}(f)\,(1 + g_{mc}r_{b'b2})^2 + S_{I_{EC}}(f). \tag{60a}$$

Inserting a large resistor $R_e(R_e \gg r_{b'b})$ in the emitter lead yields the feedback spectra

$$S'_{I_B}(f) = S_{I_{BE}}(f) + S_{I_{BC}}(f) + S_{I_{CE}}(f)\left(\frac{g_{be}R_e}{1 + g_{me}R_e}\right)^2 \quad (60b)$$

$$S'_{I_C}(f) = S_{I_{BE}}(f)\left(\frac{g_{mc}R_e}{1 + g_{me}R_e}\right)^2 + S_{I_{BC}}(f) + S_{I_{CE}}(f)\left(\frac{1 + g_{be}R_e}{1 + g_{me}R_e}\right)^2. \quad (60c)$$

The four equations (60)–(60c) may often enable one to locate the three noise sources and identify them by their location and their current dependence.

The h.f. noise is

$$[S_{I_C}(f)_{hf}] = 2eI_C + 4kT\, r_{b'b} g_{mc}^2 \quad (60d)$$

so that $r_{b'b}$ can be evaluated from h.f. noise data.

V. Experiments

We saw in the previous sections how in collision-dominated devices the Hooge parameter α_H could be evaluated. Comparing the experimental data with the theoretical predictions, we apply the following rules:

1) If $(\alpha_H)_{\text{meas}} > (\alpha_H)_{\text{theory}}$, the process in question is *masked* by another noise source.
2) if (α_H) exp $= (\alpha_H)_{\text{theory}}$, theory and experiment *agree.*
3) If $(\alpha_H)_{\text{meas}} < (\alpha_H)_{\text{theory}}$, the process in question is *not present.*

In collision-free processes similar rules apply to $S_I(f)$.

In BJTs and junction diodes, especially at forward bias, $S_I(f)/I$ = constant for the fundamental collision processes, whereas $S_I(f)$ varies as I^γ with $\gamma > 1$ for surface $1/f$ noise processes. In that case one should measure $S_I(f)$ as a function of current, find the low-current regime for which $\gamma = 1$, and then apply the above rules to determine whether α_H agrees with one of the fundamental collision processes.

If surface $1/f$ noise predominates, α_H and $S_I(f)$ vary from unit to unit and from batch to batch. If a particular fundamental collision process predominates, α_H has a characteristic value. In the same way, in collision-free processes $S_I(f)$ is described by a formula without adjusting parameters that should be the same from unit to unit.

In many applications the optimum noise performance is obtained not by minimizing α_H, but rather by minimizing α_H/τ, where τ is the time constant of the carriers or of the system.

A. Collision-Free Processes

1) Vacuum Pentodes: We first consider partition $1/f$ noise in a vacuum pentode. To that end we first apply (8) and the first part of (9) to vacuum diodes and write for the quantum $1/f$ noise

$$S_{I_a}(f) = \frac{4\alpha_0}{3\pi}\frac{v_a^2}{c^2}\frac{I_a^2}{fN_a}\Gamma^4 \quad (61)$$

where I_a is the anode current, v_a the velocity with which the electrons reach the anode, and $N_a = I_a\tau_{ma}/e$ is the number of electrons between potential minimum and anode; here $\tau_{ma} = \tau_{m1} + \tau_{1a}$ and τ_{m1}, and τ_{1a} are the transit times between potential minimum and control grid and between control grid and anode, respectively. The effect cannot be observed since it is masked by *classical* emission $1/f$ noise caused by classical fluctuations of the cathode emission.

The situation is more favorable for a vacuum pentode [12], [18], [70]. Here we have not only cathode $1/f$ noise, distributed between the screen grid g_2 and the anode a, but also partition $1/f$ noise flowing from screen grid to anode [12]. The cathode $1/f$ noise components flowing in the screen grid and the anode leads are *fully* and *positvely* correlated, whereas the partition $1/f$ noise components in both leads are *fully* and *negatively* correlated.

If $S_p(f)$ is the partition $1/f$ noise spectrum, we may thus write (see [14])

$$S_{I_a}(f) = \frac{4\alpha_0}{3\pi}\frac{1}{fc^2}\frac{(v_aI_a)^2}{N_a}\Gamma^4 + S_p(f) \quad (61a)$$

$$S_{I_2}(f) = \frac{4\alpha_0}{3\pi}\frac{1}{fc^2}\frac{(v_2I_2)^2}{N^2}\Gamma^4 + S_p(f) \quad (61b)$$

$$S_{I_a,I_2}(f) = \frac{4\alpha_0}{3\pi}\frac{1}{fc^2}\frac{v_av_2I_aI_2}{(N_aN_2)^{1/2}}\Gamma^4 - S_p(f) \quad (61c)$$

where $N_2 = I_c\tau_{m2}/e$ and $N_a = (I_c\tau_{m2} + I_a\tau_{2a})/e$; τ_{m2} and τ_{2a} are the transit times from the potential minimum to the screen grid and from the screen grid to the anode, respectively, whereas $I_c = I_a + I_2$.

To evaluate $S_p(f)$, we observe that for white shot noise and white partition noise we have by analogy

$$S_{I_a,I_2}(f) = 2e\frac{I_aI_2}{I_c}\Gamma^2 - 2e\frac{I_aI_2}{I_c}. \quad (61d)$$

But this is zero for $\Gamma^2 = 1$; that means that the *total white fluctuations in I_a and I_2 are independent in saturated pentodes.* This should also be valid for $1/f$ noise; hence one would expect $S_{I_a,I_2}(f) = 0$ for $\Gamma^2 = 1$ in (61c), so that

$$S_p(f) = \frac{4\alpha_0}{3\pi}\frac{1}{fc^2}\left[\frac{v_av_2I_aI_2}{(N_aN_2)^{1/2}}\right]. \quad (62)$$

In all pentodes without feedback $S_p(f)$ is masked by classical cathode emission $1/f$ noise. But by inserting a large resistor R_c in the cathode lead, all linear cathode current noises are reduced by the feedback factor $(1 + g_{mt}R_c)$, whereas the partition noise fluctuations are not affected; here g_{mt} is the cathode transconductance. In that manner $S_p(f)$ can be accurately measured, a discrimination between $1/f$ noise and quantum partition $1/f$ noise can be made.

When one does so, one usually finds agreement between theory and experiment within the limit of accuracy (±30 percent) [12], [70]. This is also true for Schwantes's 1960 data [18], [70].

Of particular interest is the dependence of $S_p(10)$ on the anode voltage V_a. This can be measured much more accurately, since only V_a has to be varied. If V_2 was kept constant and only V_a was varied, van der Ziel *et al.* [69] found for a 6AU6 tube

$$\left[\frac{S_p(10)V_a = 454\ \text{V}}{S_p(10)V_a = 134\ \text{V}}\right]\exp = 1.74 \quad (62a)$$

whereas the theoretical ratio was 1.75, in excellent agreement [70]. This effect comes mainly from the v_a term in (62), for $(454)^{1/2}/(134)^{1/2} = 1.84$. On the other hand, in the anode current of a tube *without* feedback, $S_{I_a}(10)$ was independent of V_a [71]. This is easily understood, for that case $S_{I_a}(f)$ is due to emission fluctuations generated at the *cathode*, far away from the screen grid-anode region.

What is missing is a *classical* model of partition $1/f$ noise. Such a model might be constructed as follows. Consider the screen grid and look in a direction parallel to the grid and perpendicular to the grid wires. Noise processes at or near the cathode might now give rise to a fluctuating electron *velocity* component at the screen grid in that direction and this might result in partition $1/f$ noise. Such a model might perhaps explain the anomalous features of the partition $1/f$ noise in 6CE6 pentodes [70].

Some further investigation seems therefore warranted.

2) Secondary Emission Pentodes EFP60: According to (12), the spectrum of secondary emission $1/f$ noise is due to the flow of carriers from the dynode d to the anode a, so that

$$S_{I_a}(f) = \frac{4\alpha_0}{3\pi}\,\delta^2\left(\frac{v_a}{c}\right)^2 \frac{eI_a}{f\tau_{da}} \tag{63}$$

where I_a is the anode current, δ the secondary emission factor

$$v_a = (2e/m)^{1/2}(V_a - V_d)^{1/2}$$
$$\tau_{da} = 2d_{da}/v_a. \tag{63a}$$

Here V_a and V_d are the anode and dynode voltages, respectively, τ_{da} the transit time of the secondary electrons from dynode to anode, v_a the secondary electron velocity at the anode, and d_{da} the path length of the secondary electrons between dynode and anode.

The effect was discovered by Schwantes in 1958 [15], and van der Ziel [16] gave a quantitative quantum interpretation with the help of (63). Fang [17] measured two EFP 60 tubes and showed that $S_{I_a}(10)/(V_a - V_d)^{3/2}$ at constant $I_a\delta^2$, and $S_{I_a}(10)/(\delta^2 I_a)$ at constant $(V_a - V_d)$, were independent of bias [see Table 1]. This verifies the δ^2 term in (63), which indicates a fine structure constant $\delta^2/(137)$, and the v_a^2 term in (63), which implies that acceleration (or Bremsstrahlung) is initiating the secondary emission $1/f$ noise process. From the averages of Table 1 one can evaluate $S_{I_a}(10)/[\delta^2 I_a(V_a - V_d)]^{3/2}$. This yields 1.18×10^{-20} and 1.37×10^{-20} (average 1.27×10^{-20}) A Hz^{-1} V$^{-3/2}$ for device 1 and 0.85×10^{-20} and 0.74×10^{-20} (average 0.79×10^{-20}) A Hz^{-1} V$^{-3/2}$ for device 2. Fig. 2 shows $S_{I_a}(10)/(V_a - V_d)^{3/2}$ as a function of $V_a - V_d$.

Taking the average over devices 1 and 2 yields 1.03×10^{-20} A Hz^{-1} V$^{-3/2}$, and this, in turn, gives $d_{da} = 0.56$ cm, thus verifying the earlier estimate of d_{da} of 0.50 cm [16].

This shows the consistency of the data and indicates that (62) gives a complete description of the secondary emission $1/f$ noise phenomenon. This is no coincidence but represents an established *fact*.

Apparently contradicting this result is a report by Schwantes *et al.* [72], according to which $S_I(f)$ for photomultipliers was white down to 1 Hz. However, since this white noise was not further analyzed, it is not certain that it corresponds to amplified shot and secondary emission

Table 1

EFP 60 Table 1 f = 10 Hz $(V_a - V_d)$ (V)	Device #1 I_a = 10.0 $(V_a - V_d)^{3/2}$	δ = 3.6 $S_{Ia}(f)$ (A^2/Hz)	$S_{Ia}(f)/(V_a - V_d)^{3/2}$ $S_{Ia}(f)/(V_a - V_d)^{3/2}$
50	0.354×10^3	0.48×10^{-18}	1.34×10^{-21}
75	0.650×10^3	1.06×10^{-18}	1.63×10^{-21}
100	1.000×10^3	1.56×10^{-18}	1.56×10^{-21}
125	1.400×10^3	2.32×10^{-18}	1.66×10^{-21}
			Ave. $1.55 \times 10^{-21} \pm 0.09 \times 10^{-21}$

Table 2 f = 10 Hz $V_a - V_d$ = 125 (V) V_2(V)	Device #1 $S_{Ia}(f)$ (A^2/Hz)	δ	$S_{Ia}(f)/\delta^2 I_a$ I_a(mA)	$^{o2}I_a$(mA)	$S_{Ia}/\delta^2 I_a$
250	2.3×10^{-13}	3.60	10.00	129.60	1.77×10^{-17}
225	9.4×10^{-19}	3.04	5.88	54.30	1.73×10^{-17}
200	2.7×10^{-19}	2.43	2.94	17.40	1.56×10^{-17}
175	1.5×10^{-19}	1.97	1.44	5.88	2.60×10^{-17}
					Ave. $1.92 \times 10^{-17} \pm 0.35 \times 10^{-17}$

Table 1 f = 10 Hz $(V_a - V_d)$ (V)	Device #2 I_a = 8.0 mA $(V_a - V_d)^{3/2}$	δ = 3.1 $S_{Ia}(f)$	$S_{Ia}(f)/(V_a - V_d)^{3/2}$ $S_{Ia}(f)/(V_a - V_d)^{3/2}$
50	0.354×10^3	2.60×10^{-19}	7.34×10^{-22}
75	0.650×10^3	4.30×10^{-19}	6.62×10^{-22}
100	1.00×10^3	6.10×10^{-19}	6.10×10^{-22}
125	1.40×10^3	8.70×10^{-19}	6.21×10^{-22}
			Ave. $6.57 \times 10^{-22} \pm 0.41 \times 10^{-22}$

Table 2 f = 10 Hz $V_a - V_d$ = 125 (V) V_2(V)	Device #2 $S_{Ia}(f)$	δ	$S_{Ia}(f)/\delta^2 I_a$ I_a(mA)	$\delta^2 I_a$(mA)	$S_{Ia}(f)/\delta^2 I_a$
250	8.7×10^{-19}	3.1	8.0	76.88	1.13×10^{-17}
225	3.4×10^{-19}	2.6	5.0	33.80	1.01×10^{-17}
200	1.3×10^{-19}	2.1	2.5	11.03	1.18×10^{-17}
175	0.3×10^{-19}	1.7	1.2	3.55	0.87×10^{-17}
					Ave. $1.05 \times 10^{-17} \pm 0.12 \times 10^{-17}$

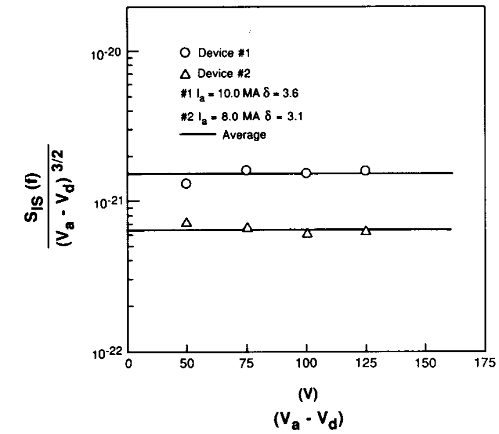


Fig. 2. $S_{I_a}(10)/(V_a - V_d)^{3/2}$ for EFP60 as a function at $(V_a - V_d)$. Cathode 1/f noise removed by feedback. I_a and δ were constant.

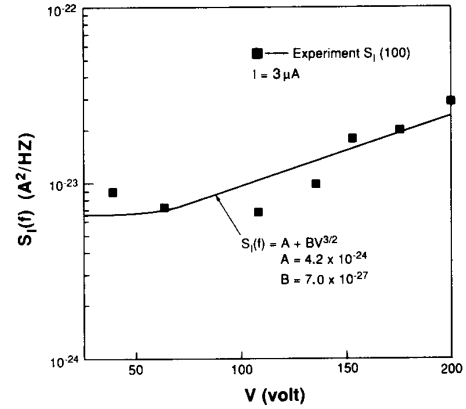


Fig. 3. $S_{I_a}(10)$ versus V_a for vacuum photodiode of 931A photomultiplier. All dynodes connected to d_1 (diode configuration). Full-drawn curve: $A + BV_a^{3/2}$; symbols represent data. A represents work function 1/f noise (adjustable parameter) and B is taken from (64) (theory) with $I_a = 3\mu A$, $d_{ca} = 0.50$ cm, $n = 2$.

noise; there might have been excess white noise present. For that reason a more detailed analysis was carried out by Fang *et al.* (see next section) [73].

The first stage of a photomultiplier is a vacuum photodiode. By tying all the dynodes together and measuring the noise between this "anode" and the photocathode, one can study the white noise and the 1/f noise of this photodiode.

By connecting all the electrodes beyond d_2 to d_2 one has a configuration consisting of a vacuum photodiode and a single secondary-emission stage in series. One can then determine the 1/f noise and the white noise added by the multiplier stage.

3) Vacuum Photodiodes: By analogy with (63)

$$S_{I_a}(f) = \frac{4\alpha_0}{3\pi}\left(\frac{v_a}{c}\right)^2 \frac{eI_a}{f\tau_{ca}} n^2,$$

$$S_{I_a}(\infty) = 2e\, I_a n \qquad (64)$$

where d_{ca} is the cathode-anode distance and

$$v_a = (2e/m)^{1/2} V_a^{1/2} \qquad \tau_{ca} = 2\, d_{ca}/v_a \qquad (64a)$$

and $n > 1$ takes into account that the photoelectrons are emitted in bunches. There are several reasons for this, but we will not go into further detail. $S_{I_a}(\infty)$ is shot noise of charge conglomerates en.

The experimental data agree well with (64); at large V_a $S_{I_a}(f)$ is proportional to I_a and to $V_a^{3/2}$; Fig. 3 shows the latter dependence. The shot-noise data gave $n \simeq 1.5$. Bothersome pickup noise hampered the measurements, and limited their accuracy, but nevertheless the data seem to agree with (64). More work is needed.

Classically, one might expect 1/f noise due to fluctuations in the work function χ. It is easily shown that the leads to a spectrum $S_{I_a}(f) = \text{const.}\ V_a^0 I_a^2$. This has a voltage and a current dependence different from (64). This noise source is present at low V_a (see Fig. 3) [74].

4) Schottky-Barrier Diodes: Early measurements were reported by Hsu [74], [75]. In order to avoid surface 1/f noise, the metal contact was protected by a guard ring. Kleinpenning gave the essential theory in terms of mobility 1/f noise (or diffusion 1/f noise) in 1979 [76] but made a few mathematical errors. For example, his method of approach could not incorporate the integrating factor needed to solve the Langevin equation correctly. Luo [28] has corrected these errors and obtained (29) and (19) for the collision-dominated and for the collision-free models, respectively. Hsu's measurements agreed approximately with (19) but there was one difficulty.

The collision-free model required that there were no collisions in the *whole* space-charge region. But when they compared the free path length of the electrons with the length d of the space-charge region, they found that the electrons made about 7–10 collisions in that region. Any agreement with (19) thus seemed fortuitous.

The dilemma can be resolved by introducing the image-force model [28]. Here the potential energy has a maximum at $x = x_m$; for the bulk space-charge region the current flow is by diffusion, but at $x = x_m$ the current flow can also be considered as being due to thermionic emission (TE-D model). This yields a current

$$I = eN_d \frac{v_d v_r}{v_d + v_r} \exp\left[-e\,\frac{V_{dif} - V}{kT}\right] \qquad (65)$$

$$v_r = \left(\frac{kT}{2\pi m^*}\right)^{1/2}$$

$$v_d = \mu\left[\frac{2eN_d}{\epsilon\epsilon_0}(V_{dif} - V)\right]^{1/2}. \qquad (65a)$$

For $v_r \gg v_d$ the characteristic is said to be *diffusion-limited* and for $v_d \gg v_r$ it is said to be *thermionic-limited* (TE-model); the latter does not imply absence of collisions in the space-charge region.

Luo [28] therefore combined the diffusion model with the image-force model and obtained the general equation (TE-D model)

$$S_I(f) = \frac{e^3\alpha_H I_0}{3kTf}\left(\frac{v_r v_d}{v_r + v_d}\right)\left[\frac{2\, N_d(V_{dif} - V)}{e\epsilon\epsilon_0}\right]^{1/2}. \qquad (66)$$

This leads to the *diffusion model* for $v_r \gg v_d$, and hence to (29), and for $v_d \gg v_r$ to the *approximate TE model* with

$$S_I(f) = \frac{e^3 \alpha_H I_0}{3f} \left[\frac{N_d(V_{dif} - V)}{e\epsilon\epsilon_0 \pi k T m^*}\right]^{1/2} \quad (66a)$$

for $v_d \gg v_r$. Kleinpenning already discussed the TE-D model [76]. For corrected expressions see Luo *et al.* [28].

Luo found that (66a) could be fitted to Hsu's measurements by taking $\alpha_H \approx 1.8 \times 10^{-9}$ (Fig. 4). According to Kousik-van Vliet-Bosman's normal collision model $\alpha_H = 3.3 \times 10^{-9}$ for n-type Si. This is good agreement; the slight discrepancy may be due to the limited number of collisions (7-10) in the space-charge region. Further work is in progress to understand even lower values of α_H.

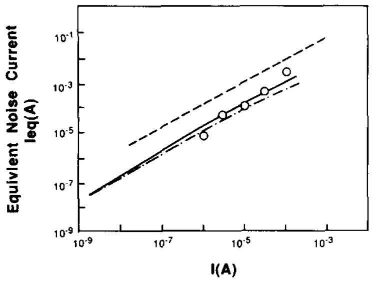


Fig. 4. $I_{eq}(f)$ versus I for Pt-n-Si in Schottky-barrier diodes at 20 Hz. Upper broken line: diffusion model. Lower broken line: ballistic model. Full line: TE model involving image effect; circles: Hsu's data.

Our final conclusion is that the approximate TE model (66a) agrees with Hsu's data, but that the *pure* TE model (19) is never reached.

B. Collision-Dominated 1/f Noise

We now turn to various collision-dominated noise sources (FETs, BJTs, $Hg_{1-x}Cd_xTe$ n^+-p diodes, laser diodes). They all have in common that $\Delta v^2/c^2 \ll 1$. Devices with $\alpha_H = (3\text{-}7) \times 10^{-3}$ (coherent state devices) are covered in Section V-C.

1) Collision 1/f Noise in FETs: The first collision-dominated 1/f noise (quantum 1/f noise) was found by Duh in 1985 [25] in n-channel Si-JFTs supplied by Dr. K. Kandiah. The devices had two gates; by changing the bias voltage of each gate the space-charge regions could be shifted and traps could be made inactive, so that the 1/f noise became observable. Duh found $\alpha_{Hn} = 2.5 \times 10^{-8}$ at $T = 300$ K. Since $a = 5.43 \times 10^{-8}$ cm and $m_n^* = m$, the Umklapp mechanism itself gave

$$\alpha_{Hu} = \frac{4\alpha_0}{3\pi}\left(\frac{h}{m^* ac}\right)^2 = 6.2 \times 10^{-8}. \quad (67)$$

Van der Ziel *et al.* [19] now conjectured that this value should be multiplied by a probability factor $\exp(-\theta_D/2T)$, where θ_D is the Debye temperature (645 K in Si). They thus obtained the heuristic equation

$$\alpha_H = \frac{4\alpha_0}{3\pi}\left(\frac{h}{m^* ac}\right)^2 \exp\left(-\frac{\theta_D}{2T}\right) \quad (67a)$$

yielding $\alpha_{Hn} = 2.1 \times 10^{-8}$ at 300 K, in excellent agreement with Duh's data. See also (15a) and (15b).

Pawlikiewicz [77] found some n-channel Si-JETs (Silicon) with very low g-r noise. He was now able to measure α_H for the 1/f noise in the temperature range 255 K < T < 400 K, but below 255 K the 1/f noise was masked by g-r noise. Pawlikiewicz's data fitted with the formula

$$\alpha_{Hn} = 6.2 \times 10^{-8} \exp\left(-\frac{322.5}{T}\right) \quad (67b)$$

with an accuracy better than 20 percent (Fig. 5).

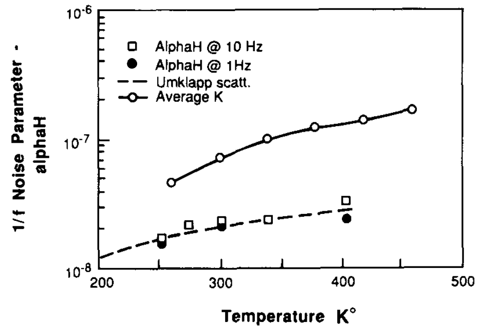


Fig. 5. α_{Hn} versus T for Si n-channel JFET. Comparison of the measured α_H with (67b). Also shown is $\alpha_H(T)$ for the intervalley process.

The accurate determination of α_{Hn} requires knowledge of the temperature dependence $\mu(T)$ of the mobility μ. This can be obtained as follows. The output conductance $g_{d0}(T)$ at zero drain bias can be accurately measured. Since $g_{d0}(T)$ is proportional to the mobility $\mu(T)$

$$\frac{g_{d0}(T)}{g_{dp}(300)} = \frac{\mu(T)}{\mu(300)}. \quad (68)$$

Putting $\mu(300) = 1400$ cm²/V · s, as taken from Sze's book [78], yields a functional dependence of $\mu(T)$ independent of any theory.

However, there remained one difficulty in interpretation. Since the n-channel in each case was rather weakly doped, one would expect the true Umklapp process to have so small a probability that it would be unobservable. On the other hand, intervalley scattering should have a much larger probability, combined with an $\exp(-\theta_D/2T)$ temperature dependence. However, as Fig. 5 shows, the theoretical expression for α_{Hn} lies about a factor 3 above (67a), so that the intervalley scattering model must be discarded also. What is left is the combined effect of intervalley scattering followed by Umklapp into another intervalley. According to van Vliet [79] the combination should be treated as a *single* process, described by $\alpha_H = \alpha_{Hu}(\mu/\mu_u)^2$, where $\mu/\mu_u = \exp(-\theta_D/4T)$. This solves all discrepancies. It is now expected that JFETs or MOSFETs made on different interfaces (100, 110, 111) might show different noise behavior. For example, it could be that the combined process could be forbidden for some interfaces. This problem, which is of fundamental interest, requires further study.

The devices studied by Duh and by Pawlikiewicz had very different structures, but showed the same value of α_H. This is as expected, since they are subjected to *identical* noise processes. Nevertheless, measurements on a few more samples of the same type as well as on a few samples of different types might be useful.

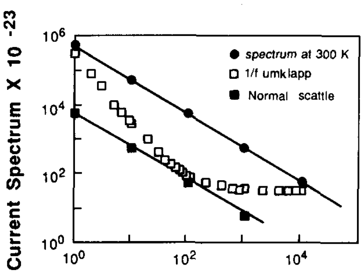
Pawlikiewicz [80] and Birbas [81] measured l.f. noise in p-channel JFETs. They found at first that the l.f. noise varied as $1/f^2$, as expected for a g-r process with a time constant of many seconds. Apparently there was no $1/f$ noise observable.

What kind of $1/f$ noise could be expected? There should, of course, be normal collision $1/f$ noise for holes, with $\alpha_{Hp} \simeq (6\text{–}10) \times 10^{-9}$. But since there are no intervalleys, and the doping of the channel is low, one would not expect noise that is describable by (67a).

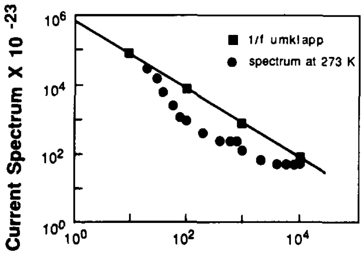
As mentioned in Section II (67a) yields $\alpha_{Hp} = 4.2 \times 10^{-7}$. As before, (23) yields for $V_d < V_{ds}$

$$S_I(f) = \alpha_{Hp} \frac{e\mu I_d V_d}{fL^2}. \quad (69)$$

Substituting for the device parameters and for α_{Hp} yielded a curve lying *above* the measured curve $[S_I(f)]_{\text{meas}}$ (Fig. 6).



(a)



(b)

Fig. 6. $S_I(f)_{\text{exp}}$ versus frequency in Si p-channel JFETs. (a) Theoretical spectrum for $\alpha_{Hp} = 4.2 \times 10^{-7}$ (see (67b)). (b) Theoretical spectrum for $\alpha_{Hp} = 10^{-8}$ (phonon collision model). In cases (a) and (b) the h.f. thermal noise was added.

According to rule 3) at the beginning of Section V, this means that Umklapp $1/f$ noise is *absent*, as predicted in the previous paragraph. Normal collision $1/f$ noise, however, should have $\alpha_{Hp} \simeq (6\text{–}10) \times 10^{-9}$. Substituting $\alpha_{Hp} \simeq 10^{-8}$ into (69) yielded a curve that was usually located *below* the measured curve $[S_I(f)]_{\text{meas}}$, indicating that normal collision $1/f$ noise was now *masked* by g-r noise in those units. Only the lowest noise units showed some $1/f$ noise; then $[S_I(f)]_{\text{meas}}$ and $[S_I(f)]_{\text{theo}}$ coincided at the higher frequencies, indicating that normal collision $1/f$ noise was *compatible* with those data (Fig. 6).

The measurements should be continued to indicate whether Umklapp $1/f$ noise is *always* missing in p-channel JFETs. Moreover, a search should be made for more devices in which normal collision $1/f$ noise is clearly observable. The data obtained so far fit the expected pattern; the best unit gave $\alpha_{Hp} = 9 \times 10^{-9}$, in good agreement with the estimated value $(6\text{–}10) \times 10^{-9}$.

Is Umklapp $1/f$ noise observable in MOSFETs with strongly inverted channels? It might, for in such channels the prohibition against Umklapp $1/f$ noise does not hold. The best n-channels MOSFETs [25] had $\alpha_{Hn} = 10^{-6}$ at $T = 300$ K whereas $(\alpha_{Hn})_{\text{Umk}} = 2.1 \times 10^{-8}$ at $T = 300$ K. The Umklapp $1/f$ noise, if present, would be therefore completely masked by surface $1/f$ noise and a reduction in surface $1/f$ noise by at least a factor 20 would be needed to make it observable. This is not a likely event for the near future.

The situation is much more promising for p-channel MOSFETs. Not only can the surface $1/f$ noise be smaller due to better passivation, but also the value of $(\alpha_{Hp})_{\text{Umk}}$ (4.2×10^{-7}) at $T = 300$ K for holes is a factor 20 larger than the value of $(\alpha_{Hn})_{\text{Umk}}$ (2.1×10^{-8} at $T = 300$ K) for electrons.

Duh [25] measured α_{Hp} in p-channel MOSFETs of 7.5- and 17.5-μm length, respectively; they were test devices obtained from Harris Semiconductor at Melbourne (FL) and had $\alpha_{Hp} = (3\text{–}9) \times 10^{-7}$. A typical plot of α_{Hp} versus $-V_d$ is shown in Fig. 7. We note that α_{Hd} is practically independent

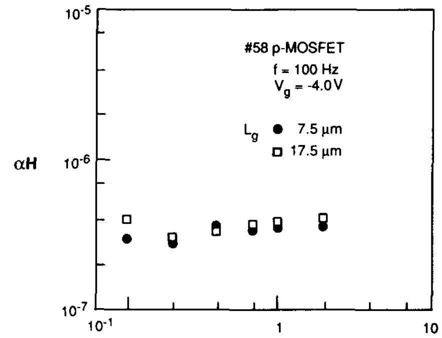


Fig. 7. α_{Hp} versus V_d for p-channel MOSFETs at channel length $L = 7.5$ and 17.5 μm. $V_g = -4.0$ V, $f = 100$ Hz.

of $(-V_d)$ and independent of the device length L, as expected for Umklapp $1/f$ noise. It is therefore quite likely that Duh actually observed Umklapp $1/f$ noise. The direct proof would be to measure the temperature dependence of α_{Hp}. Unfortunately, the devices were lost before tests could be made; a search for similar low-noise devices is in progress.

The noise mechanism of $1/f$ noise in GaAs MESFETs and MODFETs is *most likely due to deep traps in the space-charge regions of the devices*. If one turns the data into an equivalent α_{Hn}, one finds $\alpha_{Hn} \simeq (2\text{–}10) \times 10^{-5}$ [26]. Because the band structure has higher valleys (process d), (intervalley + Umklapp) is possible. With $m_n^* = 0.067m$, $\theta_D = 350$ K, $a = 5.65 \times 10^{-8}$ cm, (67a) then yields $\alpha_{Hn} = 7.1 \times 10^{-6}$.

This verifies again that the Umklapp limit, if any, has not yet been reached. It would be very worthwhile to continue to search for it, especially since the best units are only a factor 3 away from it [82]. Both the presence or the absence of this limit would be important.

2) Collision 1/f Noise in BJTs: The first cases of possible quantum $1/f$ noise in BJTs were published by Kilmer *et al.*

[83], [84] and by Zhu *et al.* [26]. They based their identification on the fact that $S_I(f)/I$ was independent of I and that α_H has values close to the value for Umklapp 1/f noise. For example, in GE 82-185 p^+-n-p transistors α_{Hn} was measured from $S_{I_B}(f)$. Kilmer found $\alpha_{Hn} = 9 \times 10^{-8}$ and Zhu found $\alpha_{Hn} = 6 \times 10^{-8}$, both at 300 K, whereas the Umklapp value for α_H was 2.1×10^{-8}. This agreement is perhaps acceptable, but is by no means spectacular. For microwave n^+-p-n transistors, Zhu found from the measured spectrum $S_{I_B}(f)$ that $\alpha_{Hp} = 1.1 \times 10^{-6}$, whereas the Umklapp value was 4.2×10^{-7}, both at 300 K. We now believe that these devices represent trapping noise with $S_I(f)/I$ = constant.

Later, cases were discovered where $S_I(f)/I$ was constant but α_H did not have a fundamental value; apparently there can be trapping noise where $S_I(f)$ is proportional to I. The clue is that $(\alpha_H)_{\text{meas}}$ is not constant but varies from unit to unit. Pawlikiewicz *et al.* [85] found GE 82-185 transistors in which $S_{I_B}(f)$ yielded $\alpha_{Hn} = 3.5 \times 10^{-9}$, in very reasonable agreement with Kousik's value $\alpha_{Hn} = 3.3 \times 10^{-9}$, calculated for normal collision 1/f noise. To obtain this value he assumed $w_E = 0.5$ μm and $D_n = 15$ cm²/s; this estimate of w_E^2/D_n may be off by ±50 percent. Such low-noise units are hard to find; their α_H values became measurable after extending the spectral observation down to 1 Hz.

Zhu and Kilmer found no measurable collector 1/f noise down to 20 Hz and Zhu therefore concluded that in the collector $\alpha_{Hp} \ll 5 \times 10^{-8}$ for a GE 82-185 p^+-n-p BJT, whereas $\alpha_{Hn} \ll 1.6 \times 10^{-9}$ for a microwave n^+-p-n BJT. At first that did not cause much difficulty, but after the normal collision 1/f noise was identified, it became questionable whether these inequalities even excluded that process too.

To find that out, Pawlikiewicz *et al.* [85] and Fang [86] extended the spectral measurement down to 1 Hz and looked for units in which the collector 1/f component was clearly identifiable and not perturbed by amplified base noise. Pawlikiewicz found $\alpha_{Hp} = 5.3 \times 10^{-9}$ and 6.5×10^{-9} for a GE 85-182 p^+-n-p transistor at T = 300 K at low collector currents, (0.1 and 0.2 mA, respectively) in very reasonable agreement with the estimated theoretical value $(6\text{–}10) \times 10^{-9}$. Fang [86] found $\alpha_{Hn} = 5 \times 10^{-9}$ for an experimental n^+-p-n transistor at 300 K and I_c = 2 mA, in reasonable agreement with the theoretical value $\alpha_{Hn} = 3.3 \times 10^{-9}$. It thus seems clear that the collector 1/f component shows normal collision 1/f noise, just as the base 1/f component in low-noise BJTs does. There was one discrepancy, however, in that Pawlikiewicz found $\alpha_{Hp} = 1.4 \times 10^{-9}$ at I_c = 2 mA. This requires further study.

There is indication that surface 1/f noise may depend on the interface. It seems that (111) surfaces give more noise than the (100) and (110) surfaces [87]. This requires further study. One of the reasons is rather trivial: Many units have very large values at $r_{b'b}$ (e.g., 250 Ω instead of 10 Ω at I_C = 1 mA). As a consequence, the component i_{be} gives a large contribution $S_{I_{BE}}(f)\,(g_{mc}r_{b'b})^2$ to $S_{I_C}(f)$; since $(g_{mc}r_{b'b})^2 \simeq 100$, the amplified base noise predominates by far over the other $S_{I_C}(f)$ components. It should be clear that such devices are unsuitable for collision 1/f noise studies.

The final conclusion is that Si-BJTs always have normal collision 1/f noise but that Umklapp 1/f noise, and intervalley 1/f noise do not show up in the external circuit. This "selection rule" requires a theoretical foundation. Also, it should be investigated whether such a selection rule applies to all interfaces or only to some of them.

3) Collision 1/f Noise in Diodes: In silicon n^+-p diodes at relatively low forward bias the current flow is by recombination at the surface of the space-charge region. This is usually caused by 1/f modulation of surface recombination due to the fluctuating occupancy of oxide traps. This yields a current dependence of $S_I(f)$ of the form I^γ with $\gamma = 1.5$ or 2. The possibility of $\gamma = 1$ needs further study, since it might indicate the presence of another fundamental noise process (see Section V-C).

For GaAs laser diodes at relatively low currents $S_I(f)/I$ is often constant; this might indicate the presence of a fundamental noise source [88]. Since the device is heavily doped ($\simeq 10^{18}$/cm³), the diffusion constant D_n for electrons may be as small as 25 cm²/s. Consequently, since the carrier lifetime τ_n is of the order of 2×10^{-9} s, the diffusion length $L_n = (D_n\tau_n)^{1/2} \simeq 2.2 \times 10^{-4}$ cm. Since the doped regions have a comparable length, the diode is neither long ($w/L_n \gg 1$), nor short ($w/L_n \ll 1$). This case needs further study.

$Hg_{1-x}Cd_xTe$ n^+-p diodes with $x = 0.30$ give $\alpha_H = 5.3 \times 10^{-5}$ at 273 K at back bias, whereas the Umklapp theory gives $\alpha_H = 4.9 \times 10^{-5}$ in excellent agreement [89]. Apparently the *Umklapp process* was responsible. The theoretical value follows from Zhu *et al.* tables [90], whereas the experimental value was obtained from $S_I(f)$ with the help of (32) and (32a). This implies a long diode ($w/L_n \gg 1$) and current flow by diffusion; it furthermore indicates that all carriers contribute to the noise (for proof, see Section V-C). The lifetime τ_n follows from the Honeywell tables ($\tau_n = 1.2 \times 10^{-7}$ s) [91].

C. Coherent State 1/f Noise

We saw that for long resistors the Hooge parameter α_H was 2×10^{-3}. We shall see that there are other cases (long $Hg_{1-x}Cd_xTe$ n^+-p diodes) where comparable values for α_H are found. This makes it plausible that the same principles are involved.

Handel's coherent state theory gives $\alpha_H = 4.6 \times 10^{-3}$; unfortunately, it does not give guidelines for deciding *when* the theory applies. It is the aim of this section to provide clarification.

1) Hooge's experiments [3]: The data speak for themselves, but there is additional information. According to Hooge and Vandamme α_H depends on doping in the following manner [92]:

$$\alpha_H = 2 \times 10^{-3} \left(\frac{\mu}{\mu_{\text{latt}}}\right)^2 \tag{70}$$

where

$$\frac{1}{\mu} = \frac{1}{\mu_{\text{latt}}} + \frac{1}{\mu_{\text{imp}}} \tag{70a}$$

where μ_{latt} and μ_{imp} are the mobilities due to phonon and to impurity scattering, respectively. Applying the Kousik-van Vliet-Bosman approach [20], if μ_{latt} and μ_{imp} both fluctuate

$$\alpha_H = \alpha_{H\text{latt}} \left(\frac{\mu}{\mu_{\text{latt}}}\right)^2 + \alpha_{H\text{imp}} \left(\frac{\mu}{\mu_{\text{imp}}}\right)^2. \tag{70b}$$

This must apply both for normal scattering and for the Hooge-type process.

As an example take normal collisions in n-type silicon. Here $\alpha_{H\text{latt}} = 3.3 \times 10^{-9}$ and $\alpha_{H\text{imp}}$ is not negligible at high doping; as a consequence the first term will predominate at low doping and the second term at high doping. Next take the Hooge process; here $\alpha_{H\text{latt}} \simeq 2 \times 10^{-3}$ and $\alpha_{H\text{imp}}$ must be much smaller, for otherwise (70) would not hold. There is thus an enhancement in $\alpha_{H\text{latt}}$ by a factor 10^6. Of course, one cannot be quite sure that we started from the normal collision process; but if we had started from the Umklapp process the enhancement factor would still be 10^5, so our conclusion about the enhancement factor changes little. This problem requires further study.

In the second place α_H may be field-dependent. According to Bosman *et al.* [7], [8] for n- and p-type silicon resistors

$$\alpha_H = \frac{\alpha_H(0)}{1 + (F_c/F_c')^2} \tag{71}$$

where $\alpha_H(0)$ is the low-field value and $\mu_0 F_c'$ corresponds to the velocity of sound. This suggests that the effect may be due to phonon emission [93]. Bosman used planar geometry; Kleinpenning [94], who used a more complicated point-contact geometry, did not observe the effect.

2) Long $Hg_{1-x}Cd_xTe$ n^+-p Diodes with $x = 0.30$: Wu *et al.* measured long n^+-p diodes with a nonplanar geometry [89]. Assuming a planar approximation, and a diffusion-recombination type current flow one obtains

$$S_I(f) = \alpha_{Hnd} \frac{eIf(a)}{f\tau_n}. \tag{72}$$

Here α_{Hnd} is the (diffusion) Hooge parameter, τ_n the lifetime, and

$$f(a) = f_a(a) = \frac{1}{3} - \frac{1}{2a} + \frac{1}{a^2} - \frac{1}{a^3} \cdot \ln(1 + a), \quad \text{with } a = \exp(eV/kT) - 1 \tag{72a}$$

if all minority carriers contribute to the noise and $f(a) = f_b(a) = \pm 1/3$ if only the excess minority carriers contribute. They plotted $S_I(f)/If_a(a)$ versus eV/kT, used (72a) and found a horizontal line, indicating that *all* minority carriers contribute equally to the noise (Fig. 8).

They then measured the diode admittance and determined τ_n. According to the diffusion theory [95]

$$Y(j\omega) = g(\omega) + jb(\omega) = g_0 (1 + j\omega\tau)^\gamma \tag{73}$$

where $\gamma = 1/2$. This was quite well satisfied for devices operating near zero bias but farther away from zero bias the value of γ was 0.7–0.9, indicating that the lifetime measurement was not reliable in that case. This is probably due to the nonplanar geometry. Otherwise (73) seemed to be valid.

Evaluating α_H/τ_n from $S_I(f)/|If(a)|$ at $T = 193$ K they found that α_{Hnd}/τ_n was nearly independent of bias (Fig. 8). Using the value of τ_n determined by the above method, they found that $\alpha_{Hnd} \simeq (3\text{–}5) \times 10^{-3}$ near zero bias and that τ_n agreed with the Honeywell lifetime tables [91]. Farther away from zero bias, the *measured* values of τ_n were larger than the *theoretical* values obtained from the lifetime tables. As a consequence, α_{Hnd} was now *larger* than 5×10^{-3}, and this was due to the error in the measured values of τ_n (Fig. 9). Finally, they used $\alpha_{Hnd} = 4.6 \times 10^{-3}$, evaluated τ_n from the measured value of α_{Hnd}/τ_n and obtained good agreement with the lifetime tables. They therefore concluded that α_{Hn} had values close to 4.6×10^{-3}, as expected for coherent state noise, and that $\tau_n \simeq (1\text{–}3) \times 10^{-7}$ at $\tau = 193$ K.

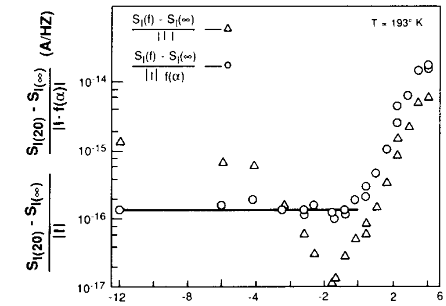


Fig. 8. $[S_I(f) - S_I(\infty)]/|I|$ and $[S_I(f) - S_I(\infty)]/[If(a)]$ versus qV/kT for $Hg_{1-x}Cd_xTe$ n^+-p diodes with $x = 0.30$ at $T = 193$ K and $f = 20$ Hz. This indicates that for back bias and near forward bias all minority carriers contribute to the noise and that α_{Hn}/τ_n is a constant.

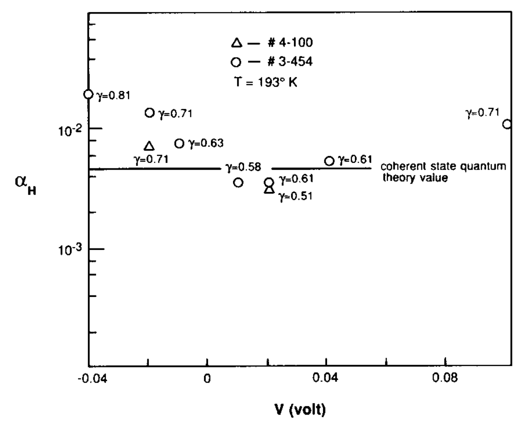


Fig. 9. α_H versus applied voltage V at $T = 193$ K for two units 4.100 and 3.454 indicating that high values of α_H correspond to large values of γ. This means that the admittance method for measuring τ_n breaks down for large values of γ.

Similar results were obtained for all devices at 193 and 113 K whereas for one unit at 273 K they found $\alpha_{Hnd} = 5 \times 10^{-5}$, as expected for the Umklapp process. No transition to $\alpha_{Hnd} = 5 \times 10^{-3}$ was found, however.

Since the coherent state mechanism gives the *highest* fundamental value of α_{Hnd}, considerable improvement could be obtained in the noise performance of these diodes if one could learn to operate them in the Umklapp mode, and still further improvement could be made by operating them in the normal collision mode. This would be an interesting consequence of a systematic noise study of HgCdTe diodes.

3) Noise in Long Si p^+-i-n Diodes: Fang [96] measured noise in long Si p^+-i-n diodes at 410 K. Here the current flow is associated with hole-electron pair recombination (for forward bias) and hole-electron pair generation (for back bias). According to the theory (Section II)

$$S_I(f) = \alpha_H \frac{e|I|}{f\tau} \tag{74}$$

where τ is the time constant associated with the generation or recombination of a single hole-electron pair. He plotted $S_I(f)/|I|$ versus eV/kT at back bias for several diodes and found it to be independent of bias; he could then evaluate

α_H/τ. The values were all very close together and the average value was 3.1×10^2/s. He then evaluated τ from admittance data and found $\tau = 1.4 \times 10^{-5}$ s, so that α_H had an average value of 4.3×10^{-3}, in excellent agreement with the coherent state value $\alpha_H = 4.6 \times 10^{-3}$.

A few words should be said about the device admittance. The equivalent circuit consists of an $R - C$ parallel connection, with a current-dependent series resistance R_d. The current modulation of R_d gives rise to a negative resistance $-R_m = IdR_d/dI$ in series with R_d, plus a parallel combination $(R_m - L)$ where $L/R_m \simeq \tau$. Finally there is a capacitance C_1 from the bottom of the $R - C$ parallel connection to the bottom of the $R_m - L$ parallel connection [95], [96]. This equivalent circuit is quite different from that of the diffusion type diode, which showed no resonances and antiresonances in $g(\omega)$ and $b(\omega)$. Having obtained the circuit elements from $Y(\omega)$ one puts $\tau = CR$.

In p-n junctions and BJTs one often finds modulation-type 1/f noise due to the modulation of recombination centers by the fluctuating occupancy of oxide traps (Section IV). Let it have a Hooge parameter α_{Hm}. The recombination centers themselves should also show Handel-type quantum 1/f noise. If it has a Hooge parameter α_{Hr}, then $\alpha_H = \alpha_{Hr} + \alpha_{Hm}$. If there are no coherent state effects, α_{Hr} is very small and α_{Hm} predominates by far. If the coherent effects are fully developed α_{Hr} is raised to 4.6×10^{-3} and it predominates by far, whereas α_{Hm} is unaffected. This picture is fully equivalent to the one presented in Section IV-C1.

As was already mentioned, the Hooge parameter of semiconductor resistors seems to increase with increasing device length L, reaching the coherent state value for large L. Birbas, Peng, and Amberiadis [97] are studying p-channel MOSFETs with different channel lengths (L = 14 μm to L = 190 μm), made on the same chip, and biased at low drain bias ($V_d = -0.2$ V) so that the device behaves as a linear resistor. Preliminary data seem to indicate that α_H varies as L^2 at intermediate lengths. Much more work is needed, however, to establish beyond doubt that this represents indeed a transition from normal collision 1/f noise to coherent state 1/f noise. Present indications are that this presents a new quantum 1/f noise source caused by carrier acceleration between collisions and coherent generation of 1/f noise along the carrier drift path. An elementary theory allows to calculate the time τ between collisions and to compare with the τ value deduced from the data. Reasonable agreement has been obtained for n-type MOSFETS.

VI. Conclusions

We have shown how a generalized representation of the noisiness in electronic devices can be given with the help of the measured Hooge parameter α_H. In collision-free devices (vacuum pentodes, secondary emission tubes, and the vacuum photodiode part of photomultipliers) α_H is given by fundamental formulas. In collision-dominated devices one observes fundamental noise sources of the normal collision type, the Umklapp type, and the intervalley + Umklapp type scattering processes; pure intervalley type scattering processes have not been found, even in n-type Si at low doping; they cannot occur in p-channel JFETs since they have no intervalleys. Normal collision 1/f noise seems to occur in Si-BJTs (n^+-p-n and p^+-n-p), in p-channel Si JFETs and in Schottky barrier diodes.

Where applicable, Handel's predictions for α_H are usually verified. This is the more remarkable, since there is severe criticism about Handel's derivation. The criticism is not directed against the Bremsstrahlung hypothesis as such, since for collision-free devices it seems to be the only 1/f noise process available. In Appendix I we derive the Handel formula for α_H from semiclassical considerations applied to collision-free devices.

There are several cases of coherent state 1/f noise with $\alpha_H = 5 \times 10^{-3}$ on record, but we have no satisfactory criteria for predicting its occurrence or nonoccurence in advance. Otherwise, the general features of the generalized representation seem to have been established, even though evidence from larger samples would be helpful.

This project started as an attempt to verify or refute the predictions made by Handel's quantum 1/f noise theories; more particularly his theory of the Hooge parameter α_H. This is now practically complete, except for some more work on vacuum photodiodes, on BJTs and on ballistic devices. We see from Section V, that Handel's result, if properly applied to the device under test, agrees with our measurements in nearly all cases. Both the experimental numbers for the various α_H values and their agreement with Handel's predictions represent scientific information that should not be ignored.

Our project cannot check the validity or invalidity of Handel's *derivation* of his predictions for α_H. This is the domain of the theoreticians. They have every right to criticize the derivation and replace it by a better one. In the latter case, they should see to it that *their* prediction for α_H agrees with Handel's prediction for α_H, when the latter has been verified experimentally. Up to now this has not been done.

It is difficult for some scientists to understand how a theory that is in their opinion incorrect can give correct predictions. It must be emphasized that only *experiment* can decide whether a conclusion is correct or incorrect. In our situation experiments decided that the predictions were right, and I see no way to avoid this conclusion.

Since the accuracy of the measurements is ± 30 percent, correction factors close to unity cannot be detected.

Appendix I

Semiclassical Derivation for Handel's Expression of the Hooge Parameter of Collision-Free Devices

According to Hooge's equation the noise spectrum of collision-free devices is

$$S_I(f) = \frac{\alpha_H}{f} \frac{I^2}{N_{\text{eff}}} \tag{A1}$$

where I is the current, $N_{\text{eff}} = I\tau/e$ is the effective number of carriers in the system, τ the appropriate transit time of the electrons, and α_H is the Hooge parameter; (A1) defines α_H.

According to Handel

$$\alpha_H = \frac{4\alpha}{3\pi} \frac{\Delta v^2}{c^2} \tag{A2}$$

where $\alpha = \alpha_0(q/e)^2$ and $\alpha_0 = 2\pi e^2/hc = 1/137$. Here α_0 is the fine structure constant for electrons, and α the fine structure constant for a charge conglomerate q; moreover, Δv is the change in velocity along the electron path.

We now derive (A2) for $q = e$ or $\alpha = \alpha_0$. As is well known, the Bremsstrahlung power emitted by a single electron is [14]

$$P(t) = \frac{2e^2}{3c^3} a^2, \quad \text{for } 0 < t < \tau \tag{A3}$$

and zero otherwise; here τ is the transit time ($\simeq 10^{-9}$ s) and $a = dv/dt$ is the acceleration of the electron.

We now find the spectrum associated with linear pulses

$$P^{1/2} = \left(\frac{2e^2}{3c^3}\right)^{1/2} a, \quad \text{for } 0 < t < \tau. \tag{A3a}$$

If we make the Fourier transform $F(j\omega)$ for $P^{1/2}$, then for $\omega\tau < 1$,

$$F(j\omega) \cong F(0) = \int_0^\tau P^{1/2}\, dt = \left(\frac{2e^2}{3c^3}\right)^{1/2} \Delta v \tag{A3b}$$

where $\Delta v = v(d) - v(0)$ is the change in velocity along the electron path and d is the length of that path. Since $\tau \simeq 10^{-9}$ s, $\omega\tau < 1$ means that $F(j\omega)$ is white up to about 50 MHz.

Since $\lambda = I/e = N_{\text{eff}}/\tau$ is the rate at which pulses occur per second, we have for the $\sqrt{P}$ (Carson's theorem)

$$S_{\sqrt{P}}(f) = 2F(0)^2\lambda = \frac{4e^2\,\Delta v^2}{3c^3}\lambda. \tag{A3c}$$

But we are not interested in the spectrum $S_{\sqrt{P}}(f)$, but rather in the quantum spectrum associated with the pulses P. This spectrum corresponds to the rate of quantum emission and is related to $S_{\sqrt{P}}(f)$ by

$$S_q(f) = \frac{S_{\sqrt{P}}(f)}{hf\tau} = \frac{4e^2\,\Delta v^2}{3c^3}\,\frac{\lambda}{hf\tau} \tag{A4}$$

where hf is the quantum energy; $S_q(f)$ thus has a $1/f$ spectrum.

So far the theory is straightforward and (A4) is a rigorous consequence of (A3). We must now transform from the quantum emission rate spectrum to the current spectrum $S_I(f)$. To that end we observe that the charge transferred by a single electron pulse is (Ramo's theorem)

$$F_e(0) = \frac{e}{d}\int_0^d V(t)\, dt = e. \tag{A5}$$

The shot noise associated with the current I is then

$$[S_I(0)]_s = 2e^2\lambda = 2eI \tag{A5a}$$

as is well known.

We must now connect (A4) and (A5a) to find $S_I(f)$. $S_I(f)$ must be proportional to $S_q(f)$ and must contain the factor $[S_I(0)]_s/\lambda = 2e^2$ (we cannot introduce λ twice!). We thus write

$$S_I(f) = CS_q(f)\cdot 2e^2 = C\,\frac{8e^2\,\Delta v^2}{ec^3\,hf}\,\frac{e^2\lambda}{\tau} \tag{A6}$$

where C is a dimensionless proportionality factor that will be determined. Next we rewrite (A1) as

$$S_I(f) = \frac{\alpha_H}{f}\,e^2\,\frac{N_{\text{eff}}}{\tau^2} = \frac{\alpha_H}{f}\cdot\frac{e^2\lambda}{\tau} \tag{A7}$$

so that comparison with (A6) gives

$$\alpha_H = C\,\frac{8e^2\,\Delta v^2}{3c^3 h} = C\,\frac{4\alpha_0}{3\pi}\,\frac{\Delta v^2}{c^2}. \tag{A7a}$$

For $C = 1$ we obtain Handel's result (A2).

We can also argue as follows. Experimentally (A2) was found to be correct within 20–30 percent and hence $C = 1$ with an accuracy of 20–30 percent. We are now independent of Handel's derivation of α_H.

Equation (A6) is in itself a heuristic expression, but the factor $2Ce^2$ is so chosen that C is dimensionless and that $(\alpha_H)_{\text{theo}}$ fits with $(\alpha_H)_{\text{meas}}$. This requires $0.70 < C < 1.30$.

Equations (A6) and (A7a) are fully equivalent: (A7a) follows from (A6) and by inversion (A6) follows from (A7a).

Our theory is essentially a one-particle theory, since each electron only interacts with its own Bremsstrahlung. With "interaction" we mean that the Bremsstrahlung energy comes from the accelerated electron (energy law).

We now take $C = 1$ and write (A6) as

$$S_q(f) = \frac{S_I(f)}{2e^2}. \tag{A6a}$$

This illustrates energy law stated above.

We now show that (A2) is independent of the model. To that end we consider a collision-free system in which the current is carried by charge conglomerates q that are uniformly accelerated. Taking $P(t)$ from (3) and replacing e^2 by q^2, the average Bremsstrahlung energy emitted per pulse is

$$E = P(t)\tau = \left[\frac{2}{3}\,\frac{e^2}{c}\,\frac{q^2}{e^2}\,\frac{\Delta v^2}{c^2}\right]\frac{1}{\tau} \tag{A8}$$

since

$$a = \frac{dv}{dt} = \frac{\Delta v}{\tau}$$

is the uniform acceleration.

We now consider Handel's equation (A2) and rewrite it as

$$\alpha_H = \frac{4\alpha_0}{3\pi}\,\frac{q^2}{e^2}\,\frac{\Delta v^2}{c^2} = \left[\frac{2}{3}\,\frac{e^2}{c}\,\frac{q^2}{e^2}\,\frac{\Delta v^2}{c^2}\right]\frac{4}{h}. \tag{A9}$$

Note that (A8) and (A9) have the same factor between square brackets in common. The assumption that the $1/f$ noise in collision-free devices is due to Bremsstrahlung already implies the validity of the two terms $(q/e)^2$ and $(\Delta v/c)^2$ that are most easily verified. A detailed analysis of α_H only adds a factor $4/h$.

Where does the factor come from? Comparing with the preceeding we see that the factor 4 comes from applying Carson's theorem twice; each application adds a factor 2. In addition, the factor $1/h$ comes from the change-over from the energy spectrum to a quantum emission rate spectrum. Any derivation of α_H that involves these two steps gives the same result.

The expression of α_H is therefore relatively independent of the detailed electron–photon interaction process.

An exact theory, both for the collision-free and the collision-dominated devices, is urgently needed.

Appendix II

Transmission Line Model for $1/f$ Noise in Diffusion Dominated n^+-p or p^+-n Diodes and n^+-p-n and p^+-n-p BJTS

We first evaluate the response to the series emf e_d (Fig. 10(a)). It is equivalent to Fig. 10(b), with

$$Z_1 = Z_{00}\tanh\gamma_0 x \qquad Z_s = Z_{00}\tanh\gamma_0(w - x) \tag{A10}$$

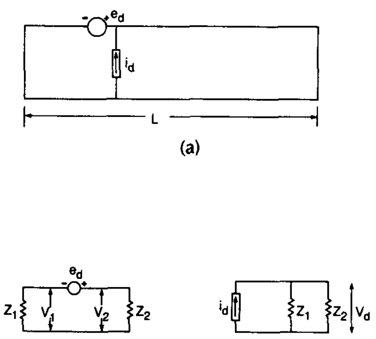


Fig. 10. (a) Equivalent circuit of transmission line of length L. (b) Equivalent circuit for e_d; Z_1 and Z_2 in series. (c) Equivalent circuit for e_d; Z_1 and Z_2 in parallel.

and hence

$$i_0 = \frac{e_d}{Z_1 + Z_2} = \frac{e_d}{Z_{00}}\left[\frac{1}{\tanh \gamma_0 x + \tanh \gamma_0(w - x)}\right]$$

$$= \frac{e_d}{Z_{00}} \frac{\cosh \gamma_0 x \cosh \gamma_0 (w - x)}{\sinh \gamma_0 w}. \quad \text{(A10a)}$$

Try for $0 < y < x$ the expression:

$$i(y) = a \cosh \gamma_0 y + b \cosh \gamma_0(x - y). \quad \text{(A11)}$$

For $y = 0$:

$$i(0) = i_1 = a + b \cosh \gamma_0 x$$

and for $y = x$:

$$i(x) = -i_0 = a \cosh \gamma_0 x + b. \quad \text{(A11a)}$$

For $x \leftarrow 0$ the $b \cosh \gamma_0 x$ term blows up; hence we must take $b = 0$. This means $i_1 = a$; $-i_0 = a \cosh \gamma_0 x$ or

$$i_1 = -\frac{e_d}{Z_{00}} \frac{\cosh \gamma_0(w - x)}{\sinh \gamma_0 w}$$

$$i_2 = \frac{e_d}{Z_{00}} \frac{\cosh \gamma_0 x}{\sinh \gamma_0 w}. \quad \text{(A11b)}$$

We now evaluate the response of the current generator i_d

$$e_d = i_d \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$i_0' = \frac{e_d}{Z_1} = i_d \frac{Z_2}{Z_1 + Z_2} = i_d \frac{\sinh \gamma_0(w - x) \cosh \gamma_0 x}{\sinh \gamma_0 w} \quad \text{(A12a)}$$

$$i_1 = \frac{i_0}{\cosh \gamma_0 x} = i_d \frac{\sinh \gamma_0(w - x)}{\sinh \gamma_0 w};$$

$$i_2 = i_d \frac{\sinh \gamma_0 x}{\sinh \gamma_0 w} \quad \text{(A12b)}$$

so that the two transfer functions for e_d and for i_d have now been evaluated.

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References

[1] F. N. Hooge, "1/f noise is no surface effect," *Phys. Lett.*, vol. A-29, p. 139, 1969.
—, "1/f Noise," *Physica*, vol. 83B, p. 9, 1976.

[2] P. H. Handel, "1/f noise, an 'infrared' phenomenon," *Phys. Rev. Lett.*, vol. 34, p. 1492, 1975.

[3] P. H. Handel, "Quantum approach to 1/f noise," *Phys. Rev.*, vol. 22, p. 745, 1980.

[4] J. Clarke and R. F. Voss, "1/f noise from the thermal fluctuations in metal films," *Phys. Rev. Lett.*, vol. 33, p. 24, 1976.

[5] R. F. Voss and J. Clarke, "Flicker 1/f noise: Equilibrium temperature and resistance fluctuations," *Phys. Rev.*, vol. B-13, p. 556, 1976.

[6] H. Hanafi and A. van der Ziel, "Fluctuation noise in $Hg_{1-x}Cd_xTe$," *Physica*, vol. 94B, p. 351, 1978.

[7] G. Bosman, R. J. J. Zijlstra, and A. van Rheenen, "Flicker noise of hot electrons in silicon at T = 78°K," *Phys. Lett.*., vol. 78A, p. 385, 1980.

[8] G. Bosman, R. J. J. Zijlstra, and A. van Rheenen, "Flicker noise of hot holes in silicon at T = 78°K," *Phys. Lett.*, vol. 80A, p. 57, 1980.

[9] F. N. Hooge, "On expressions for 1/f noise in mobility," *Physica*, vol. 114B, p. 391, 1982.

[10] A. van der Ziel, C. M. van Vliet, R. J. J. Zijlstra, and R. R. Jindal, "Noise in mobility fluctuation and the Boltzmann equation," *Physica*, vol. 121B, p. 470, 1983.

[11] F. N. Hooge, T. G. M. Kleinpenning, and L. K. J. Vandamme, "Experimental studies on 1/f noise," *Rep. Progr. Phys.*, vol. 44, pp. 479–532, 1981.

[12] A. van der Ziel, C. J. Hsieh, P. H. Handel, C. M. van Vliet, and G. Bosman, "Partition 1/f noise in vacuum pentodes," *Physica*, vol. 124B, p. 299, 1984.

[13] A. van der Ziel and P. H. Handel, "Quantum partition 1/f noise in pentodes," *Physica*, vol. 125B, p. 286, 1984.

[14] P. H. Handel, [13, Appendix; written by Dr. Handel].

[15] R. C. Schwantes and A. van der Ziel, "Secondary emission flicker noise," *Physica*, vol. 26, p. 1162, 1960.

[16] A. van der Ziel, "Interpretation of Schwantes's experimental data on secondary emission 1/f noise," *Physica*, vol. 144B, p. 205, 1987.

[17] P. Fang and A. van der Ziel, "Study of secondary emission 1/f noise," accepted by *Phys. Ber.*, in press.

[18] R. C. Schwantes and A. van der Ziel, "Flicker noise in pentodes, flicker partition noise," *Physica*, vol. 26, p. 1157, 1960.

[19] A. van der Ziel, P. H. Handel, X. C. Zhu, and K. H. Duh, "A theory of the Hooge parameters of solid-state devices," *IEEE Trans. Electron Devices*, vol. ED-32, p. 667, 1985.

[20] G. K. Kousik, C. M. van Vliet, G. Bosman, and P. H. Handel, "Quantum 1/f noise associated with ionized impurity scattering and electron-phonon scattering in condensed matter," *Adv. Phys.*, vol. 34, p. 663, 1985.

[21] C. M. van Vliet and A. van der Ziel, "Summary of Second Quantum 1/f Noise Symposium held at Minneapolis, October 24, 1986," *Solid-State Electron.*, vol. 30, p. 777, 1987.

[22] P. H. Handel, "Any particle represented by a coherent state exhibits 1/f Noise," in *Noise in Physical Systems and 1/f Noise*, M. Savelli, G. Lecoy, and J. P. Nougier, Eds. New York, NY: Elsevier, 1983, p. 97.

[23] P. H. Handel, "Coherent state quantum 1/f noise and the quantum 1/f effect," *Noise in Physical Systems and 1/f Noise*, A. d'Amico and P. Mazetti, Eds. New York, NY: Elsevier, 1986, p. 465.

[24] P. H. Handel and A. van der Ziel, "Relativistic correction of the Hooge parameter for Umklapp 1/f noise," *Physica*, vol. 141B, p. 145, 1986.

[25] K. H. Duh and A. van der Ziel, "Hooge parameters in various FET structures," *IEEE Trans. Electron Devices*, vol. ED-32, p. 662, 1985.

[26] X. C. Zhu and A. van der Ziel, "The Hooge parameter of n^+-p-n and p^+-n-p silicon bipolar transistors," *IEEE Trans. Electron Devices*, vol. ED-32, p. 658, 1985.

[27] M. J. Trippe, "Noise measurements of quantum effects at 90 GHz and 2°K and study of the noise of Schottky barrier and tunnel diodes," Ph.D. dissertation, University of Florida, Gainesville, FL, 1986.

[28] M. Y. Luo, G. Bosman, A. van der Ziel, and L. L. Hench, to be published.

[29] A. van der Ziel and P. H. Handel, "1/f noise in n^+-p diodes," *IEEE Trans. Electron Devices*, vol. ED-32, p. 1805, 1985.

[30] A. van der Ziel, and P. H. Handel, "1/f noise in n^+-p junctions calculated with quantum 1/f theory," in *Noise in Physical Systems and 1/f Noise*, A. d'Amico and P. Mazetti, Eds. New York, NY: Elsevier, 1986, p. 481.

[31] A. van der Ziel, *Noise in Solid State Devices and Circuits.* New York, NY: Wiley, 1986.

[32] A. van der Ziel, P. H. Handel, X. L. Wu, and J. B. Anderson, "Review of the status of quantum 1/f noise in n^+-p HgCdTe photodiode detectors and other devices," *J. Vac. Sci. Technol.*, vol. A4, no. 4, p. 2205, 1986, (San Diego MCT workshop 1985).

[33] X. N. Zhang and A. van der Ziel, "Test of the presence of injection-extraction and Umklapp quantum 1/f noise in the collector of silicon transistors," in *Noise in Physical Systems and 1/f Noise*, A. d'Amico and P. Mazetti, Eds. New York, NY: Elsevier, 1986, p. 397.

[34] F. M. Klaassen and J. Prins, "Thermal noise of MOS transistors," *Philips Res. Rep.*, vol. 22, p. 505, 1967.

[35] F. M. Klaassen, "Characterization of low 1/f noise of MOS transistors," *IEEE Trans. Electron Devices*, vol. ED-18, p. 887, 1971.

[36] T. G. M. Kleinpenning, "1/f noise in p-n diodes," *Physica*, vol. 98B, p. 289, 1980.

[37] T. G. M. Kleinpenning, "1/f noise in p-n junction diodes," *J. Vac. Sci. Technol.*, vol. A3, p. 176, 1985.

[38] A. van der Ziel, "Proposed discrimination between 1/f noise in transistors," *Solid-State Electron.*, vol. 25, p. 141, 1982.

[39] A. van der Ziel, "Shot noise in Schottky barrier diodes as diffusion noise," *Physica*, vol. 94B, p. 357, 1978.

[40] A. van der Ziel, "Theory of shot noise in junction diodes and junction transistors," *Proc. IRE*, vol. 43, p. 1639, 1955.

[41] X. L. Wu, J. B. Anderson, and A. van der Ziel, "Diffusion and recombination 1/f noise in long n^+-p $Hg_{1-x}Cd_x$ Te Diodes," *IEEE Trans. Electron Devices*, vol. ED-34, p. 602, 1987.

[42] W. Schottky, "Small-shot effect and flicker effect," *Phys. Rev.*, vol. 28, p. 74, 1926.

[43] Von Egon Ritter von Schweidler, "Studies on the anomalies in the behavior of dielectrics," (Leipzig), vol. 24, p. 711, 1907.

[44] M. Gevers, "The relation between the power factor and the temperature coefficient of the dielectric constant of solid dielectrics," *Philips Res. Rep.*, pp. 1, 197, 279, 362, 467, 1946.

[45] F. K. du Pré, "A suggestion regarding the spectral density of flicker noise," *Phys. Rev.*, pp. 78, 615, 1950.

[46] A. van der Ziel, "On the noise spectra of semi-conductor noise and of flicker effect," *Physica*, vol. 16, p. 359, 1950.

[47] A. L. McWhorter, "1/f noise and related surface effects in germanium," Lincoln Lab Rep. 80, Boston, MA, May 1955.

[48] K. Zheng, K. H. Duh, and A. van der Ziel, "Eliminating low-frequency 1/f noise in HgCdTe by sputtering," *Physica*, vol. 119B, p. 249, 1983.

[49] C. H. Suh and A. van der Ziel, "A noise spectrum described by a distribution of time constants," *Appl. Phys. Lett.*, vol. 37, p. 565, 1980.

[50] J. Graffeuil, "Low-frequency noise spectrum of GaAs FETs," *Electron Lett.*, vol. 17, p. 387, 1981.

[51] K. H. Duh, X. C. Zhu, and A. van der Ziel, "Low frequency diffusion noise in GaAs MESFETs," *IEEE Electron Device Lett.*, vol. EDL-5, pp. 202, 1984.

[52] B. Hughes, N. G. Fernandez, and J. M. Gladstone, "GaAs FETs with a flicker noise corner below 1 MHz," *IEEE Trans. Electron Devices*, vol. ED-33, p. 1852, 1986.

[53] R. P. Jindal and A. van der Ziel, "Phonon fluctuation model for flicker noise in elemental semiconductors," *J. App. Phys.*, vol. 52, p. 2884, 1981.

[54] S. Christensson, I. Lundstrom, and C. Svensson, "Low frequency noise in MOS transistors—I," *Solid-State Electron.*, vol. 11, p. 797, 1968.

[55] S. Christensson and I. Lundstrom, "Low frequency noise in MOS transistors—II," *Solid-State Electron.*, vol. 11, p. 813, 1968.

[56] R. P. Jindal and A. van der Ziel, "Carrier fluctuation noise in MOSFETs due to traps in the oxide," *Solid-State Electron.*, vol. 21, p. 901, 1978.

[57] G. Reimbold, P. Gentil, and A. Chavet, "1/f noise in MOS transistors," in *Noise in Physical Systems and 1/f Noise*, N. Savelli, G. Lecoy, and J. P. Nougier, Eds. New York, NY: Elsevier, 1983, p. 291.

[58] G. Reimbold, "Modified 1/f trapping noise theory and experiments in MOS transistors biased from weak to strong inversion, influence of interface states," *IEEE Trans. Electron Devices*, vol. ED-31, p. 1190, 1984.

[59] K. H. Duh and A. van der Ziel, "Thermal and 1/f noise in weak inversion and limiting 1/f noise in MOSFETs," in *Noise in Physical Systems and 1/f Noise*, M. Savelli, G. Lecoy, and J. P. Nougier, Eds. New York, NY: Elsevier, 1983, p. 291.

[60] H. S. Park, A. van der Ziel, R. J. J. Zijlstra, and S. T. Liu, "Discrimination between two noise models in metal-oxide semiconductors field effect transistors," *J. Appl. Phys.*, vol. 52, p. 296, 1981.

[61] A. van der Ziel, R. J. J. Zijlstra, and H. S. Park, "Discrimination between 1/f noise models of junction field-effect transistors and metal-oxide semiconductors field effect transistors, numerical results," *J. Appl. Phys.*, vol. 52, p. 4300, 1981.

[62] A. van der Ziel, R. J. J. Zijlstra, H. S. Park, and S. J. Liu, "Alternate explanation of 1/f noise in ion implanted MOSFETs," *Solid-State Electron.*, vol. 26, p. 927, 1983.

[63] K. Amberiadis and A. van der Ziel, "1/f noise in diffused and ion implanted MOS capacitors," *Solid-State Electron.*, vol. 26, p. 1009, 1983.

[64] W. Fonger, in *Transistors I. Princeton, NJ:* RCA Labs, 1956, p. 259.

[65] S. T. Hsu, J. T. Fitzgerald, and A. S. Grove, "Surface-state related 1/f noise in p-n junctions and MOS transistors," *Appl. Phys. Lett.*, vol. 12, p. 287, 1968.

[66] A. van der Ziel, "The surface recombination model of p-n diode flicker noise," *Physica*, vol. ED-48, p. 242, 1970.

[67] M. J. Trippe, G. Bosman, and A. van der Ziel, to be published. See also [27].

[68] K. M. van Vliet, "Noise and admittance of the generation-recombination current involving SRH centers in the space-charge region of junction devices," *IEEE Trans. Electron Devices*, vol. ED-23, p. 1236, 1976.

[69] A. van der Ziel, X. N. Zhang, and A. Pawlikiewicz, "Location of 1/f noise source in BJT's and HBJT's-I: Theory," *IEEE Trans. Electron Devices*, vol. ED-33, p. 1371, 1986.

[70] A. van der Ziel, C. J. Hsieh, P. H. Handel, C. M. Van Vliet, and G. Bosman, "Partition 1/f noise in pentodes and its quantum interpretation," *Physica*, vol. 145B, p. 195, 1987.

[71] A. N. Birbas and A. van der Ziel, to be published.

[72] R. C. Schwantes, H. J. Hannam, and A. van der Ziel, "Flicker noise in secondary emission tubes and multiplier phototubes," *J. Appl. Phys.*, vol. 27, p. 573, 1956.

[73] P. Fang, H. Kang, L. He, and A. van der Ziel, to be published.

[74] S. T. Hsu, "Low frequency excess noise in metal-silicon Schottky barrier diodes," *IEEE Trans. Electron Devices*, vol. ED-17, p. 496, 1970.
[75] S. T. Hsu, "Flicker noise in metal-silicon semiconductor Schottky barrier diodes due to multistep tunneling process," *IEEE Trans. Electron Devices*, vol. ED-18, p. 822, 1971.
[76] T. G. M. Kleinpennig, "Low frequency noise in Schottky barrier diodes," *Solid-State Electron.*, vol. 22, p. 121, 1979.
[77] A. Pawlikiewicz and A. van der Ziel, "Temperature dependence of the Hooge parameter in n-channel silicon JFET's," *IEEE Electron Device Lett.*, vol. EDL-6, p. 497, 1986.
[78] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. New York, NY: Wiley, 1981.
[79] C. M. van Vliet and A. van der Ziel, "Summary of Second Quantum 1/f Noise Conference, held at the University of Minnesota, October 24, 1986," *Solid-State Electron.*, vol. 30, p. 777, 1987.
[80] A. Pawlikiewicz, "Origins and temperature dependence of the Hooge parameter α_H in n and p channel JFETs," *IEEE Trans. Electron Devices*, vol. ED-34, 1987, in press.
[81] A. N. Birbas and A. van der Ziel, "1/f noise p-channel SiJFETs," presented at the 9th Int. Conf. on Noise in Physical Systems, Montreal, Canada, May 25, 1987, conference report in press.
[82] B. Hughes *et al.*, see [52].
[83] J. Kilmer, A. van der Ziel, and G. Bosman, "Presence of mobility fluctuations 1/f noise, identified in p^+-n-p transistors," *Solid-State Electron.*, vol. 27, p. 71, 1983.
[84] J. Kilmer, A. van der Ziel, and G. Bosman, "Presence of mobility fluctuations 1/f noise, identified in silicon p-n-p transistors," in *Noise in Physical Systems and 1/f Noise*, N. Savelli, G. Lecoy and J. P. Nougier, Eds. New York, NY: Elsevier, 1983, p. 271.
[85] A. Pawlikiewicz, A. van der Ziel, G. S. Kousik, and C. M. Van Vliet, "Fundamental limit of 1/f noise in p^+-n-p transistors," to be published.
[86] P. Fang, "Collector 1/f noise in a silicon n^+-p-n transistor on a (100) substrate," *Solid-State Electron.*, pps. 30, 777, 1987, presented at the 2nd Quantum 1/f Noise Conference, held at the University of Minnesota, Oct. 24, 1986.
[87] W.-C. Chen, P. Fang, X. L. Wu, A. Pawlikiewicz, and A. van der Ziel, "1/f noise in Si BJTs at different crystal orientations," presented at the 9th Int. Conf. on Noise in Physical Systems, Montreal, Canada, May 25, 1987, conference report in press.
[88] P. Fang and A. van der Ziel, to be published.
[89] X. L. Wu, J. B. Anderson, and A. van der Ziel, "Diffusion and recombination 1/f noise in long n^+-p $Hg_{1-x}Cd_xTe$ diodes," *IEEE Trans. Electron Devices*, vol. ED-34, p. 602, 1987.
[90] X. C. Zhu, X. L. Wu, A. van der Ziel, and E. G. Kelso, "The Hooge parameters for n-type and p-type $Hg_{1-x}Cd_xTe$," *IEEE Trans. Electron Devices*, vol. ED-32, p. 1353, 1985.
[91] G. Hanson and T. Casselman, "Fundamental parameters in mercury cadmium telluride, vol. 3, table of lifetimes in $Hg_{1-x}Cd_x$ Te," Honeywell Corp. Research Center, Minneapolis, MN, Tech. Rep., 1982.
[92] F. N. Hooge and L. K. J. Vandamme, "Lattice scattering causes 1/f noise," *Phys. Lett.*, vol. 66A, p. 315, 1978.
[93] G. Bosman, private communication.
[94] T. G. M. Kleinpennig, "1/f noise of hot carriers in n-type silicon," *Physica*, vol. 103B, p. 340, 1987.
[95] A. van der Ziel, *Solid State Physical Electronics*, 3rd ed. New York, NY: Wiley, 1976, ch. 15.
[96] P. Fang and A. van der Ziel, "Coherent state-quantum 1/f noise in long Si-p-n diodes at elevated temperatures, presented at the 9th Int. Conf. on Noise in Physical Systems, Montreal, Canada, May 24, 1987, conference report in press.
[97] A. N. Birbas, Q. Peng, and K. Ambiriadis, to be published. A preliminary announcement at the 9th Int. Conf. on Noise in Physical Systems, Montreal, Canada, May 25, 1987, conference report in press.

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