NTIA REPORT 82-100

A Guide to the Use of the ITS Irregular Terrain Model in the Area Prediction Mode

G.A. Hufford
A.G. Longley
W.A. Kissick

U.S. DEPARTMENT OF COMMERCE
Malcolm Baldrige, Secretary

Bernard J. Wunder. Jr., Assistant Secretary
for Communications and Information

April 1982

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

A GUIDE TO THE USE OF THE ITS IRREGULAR TERRAIN MODEL IN THE AREA PREDICTION MODE

George A Hufford, Anita G. Longley, and William A. Kissick*

The ITS model of radio propagation for frequencies between 20 MHz and 20 GHz (the Longley-Rice model) is a general purpose model that can be applied to a large variety of engineering problems. The model, which is based on electromagnetic theory and on statistical analyses of both terrain features and radio measurements, predicts the median attenuation of a radio signal as a function of distance and the variability of the signal in time and in space.

The model is described in the form used to make "area predictions" for such applications as preliminary estimates for system design, military tactical situations and surveillance, and land-mobile systems. This guide describes the basis of the model, its implementation, and some advantages and limitations of its use. Sample problems are included to demonstrate applications of the model.

Key words: area prediction; radio propagation model; SHF; statistics; terrain effects; UHF; VHF

1. INTRODUCTION

Radio propagation in a terrestrial environment is an enigmatic phenomenon whose properties are difficult to predict. This is particularly true at VHF, UHF, and SHF where the clutter of hills, trees, and houses and the ever-changing atmosphere provide scattering obstacles with sizes of the same order of magnitude as the wavelength. The engineer who is called upon to design radio equipment and radio systems does not have available any precise way of knowing what the characteristics of the propagation channel will be nor, therefore, how it will affect operations. Instead, the engineer must be content with one or more models of radio propagation--i.e., with techniques or rules of thumb that attempt to describe how the physical world affects the flow of electromagnetic energy.

Some of these models treat very specialized subjects as, for example, microwave mobile data transfer in high-rise urban areas; others try to be as generally applicable as Maxwell's equations and to represent, if not all, at least most, aspects of physical reality. In this report we shall describe one of the latter models. Called "the ITS irregular terrain model" (or sometimes the Longley-Rice

*The authors are with the Institute for Telecommunication Sciences, National Telecommunications and Information Administration, U.S. Department of Commerce, Boulder, Colorado 80303.

model; see Longley and Rice, 1968), it is designed for use at frequencies between 20 MHz and 20 GHz, for a wide variety of distances and antenna heights, and for those problems where terrain plays an important role. It is concerned with the generally available received power and not with the fine details of channel characterization.

On the other hand the model is avowedly statistical. In the physical world received signal levels do vary in what appears to be a random fashion. They vary in time because of changing atmospheric conditions, and they vary in space because of a change in terrain. It is this variability that the model tries to describe, thus providing the engineer estimates of not only the general level of expected received powers but also the magnitude of expected deviations from that general level.

Being a general purpose model, there are many special circumstances it does not consider. In what follows we shall try to describe the general nature of the model, to what uses it may be put, at what points special considerations might enter, and, if we can, what steps might be taken to allow for them. The number of possible special circumstances is so great, however, that we have undoubtedly over looked many important ones. Here, we must depend on the ingenuity of the individual engineer to recognize the circumstance and to determine how to proceed. In general, we expect the user of this Guide to be somewhat familiar with radio propagation and the effects its sometimes capricious behavior will have on radio systems.

The ITS irregular terrain model is specifically intended for computer use. In this regard it is perhaps well to introduce here terminology that makes the distinctions computer usage often requires. A _model_ is a technique or algorithm which describes the calculations required to produce the results. An _implementation_ of a model is a representation as a subprogram or procedure in some specific computer language. An _applications program_ is a complete computer program that uses the model implementation in some way. It usually accepts input data, processes them, passes them on to the model implementation, processes the results, and produces output in some form. In some application programs, radio propagation and the model play only minor roles; in others they are central, the program being but an input/output control. For example, the program QKAREA described and listed in Appendix B is a simple applications program; it calls upon the subprograms LRPROP and AVAR which are listed in Appendix A and which, in turn, are an implementation of version 1.2.1 of the ITS irregular terrain model.

2. AREA PREDICTION MODELS

Most radio propagation models, especially the general purpose ones, can be characterized as being either a "point-to-point" model or an "area prediction" model. The difference is that a point-to-point model demands detailed information about the particular propagation path involved while an area prediction model requires little information and, indeed, may not even require that there be a particular path.

To explain this latter statement, let us consider for what problems a propagation model should help. There seem to be about five areas of concern:

(1) Equipment design. Given specifications of how new radio equipment is to be used and how reliable communications must be, it should be possible to predict the values of path loss (and perhaps other characteristics of the channel) for which the equipment must compensate. Conversely, given the properties of proposed new equipment, one should be able to predict how that equipment will behave in various situations. In particular, one should be able to predict a service range--i.e., a distance at which communications are still sufficiently reliable, under the given conditions.

(2) General system design. This is an extension of the first area. Here, it is the interaction of radio equipment that is to be studied. Often, interference, both between elements of the same system and between elements of one system with another on the same or adjacent frequencies, is an important part of the study. Questions such as the proper co-channel spacing of broadcast stations or the proper spacing of repeaters might be treated.

(3) Specific operational area. In this case one or more radio systems are to be located in one particular area of the world and, perhaps, operated at one particular season of the year or time of day. Within this area, however, all terminals are to be located at random, where "random" may mean not uniformly distributed but according to some predefined selection scheme. These terminals may, for example, be mobile so that they will, indeed, occupy many locations; or they may be "tactical" in that they are to be set up at fixed locations to be decided upon at a later date, perhaps only just before they are put into operation. Questions to be asked might be similar to those in the previous two areas. One technique sometimes used is that of a simulation

procedure in which a Monte Carlo approach is taken in the placement of the terminals or the control of communications traffic.

(4) Specific coverage area. In this case, one of the terminals is at a specific known location while the other (often many others) is located at random somewhere in the same vicinity. The obvious example here is a broadcast station or the base station for a particular mobile system; but other examples might include radars, monitoring sites, or telemetry acquisition base stations. The usual problem is to define a service area within which the reliability of communications is adequate or, sometimes, to find the strength of interference fields within the service area of a second station. If calculations are made before the station is actually set up, one can think of them as part of the decision process to judge whether the station design is satisfactory.

(5) Specific communications link. In this final case, both terminals are at specific known locations, and the problem is to estimate the received signal level. Or more likely, the problem is to characterize the received signal level as it varies in time. Again, calculations made here are often used in the design of the link.

In the last of these areas--the specific communications link--one knows, or presumably can obtain, all the details of the path of propagation. One expects to obtain very specific answers to propagation questions, and therefore one uses a point-to-point model.

In the first two areas, however--the design of equipment and of systems--there is no thought about particular propagation paths. One wants general results, perhaps parametric results, for various types of terrain and types of climate. It is natural to use an area prediction model.

In the case of a specific operational area or a specific coverage area, one is confronted with a different problem. Here one has a large multitude of possible propagation paths each of which can presumably be described in detail. One might, therefore, want to consider point-to-point calculations for each of them. But the sheer magnitude of the required input data makes one hesitant. If a simulation procedure is used to collect statistics of communications reliability, then the point-to-point calculations become lost in the confusion to the point where they seem hardly worth the trouble. An alternative which requires far less input data is to use an area prediction model, particularly if the model provides by itself the required statistics.

Even in the case of a specific communications link, the required detailed information for the propagation path may be unobtainable so that one is forced to use the less demanding area prediction model. Of course, in doing so one expects to lose in precision and in the dependability of the results.

In addition to the ITS area prediction model, other widely used models of this kind include those developed by Epstein and Peterson (1956), Egli (1957), Bullington (1957), the Federal Communications Commission (FCC; Damelin et al., 1966), Okumura et al. (1968), and the International Radio Consultative Committee (CCIR, 1978a). By their nature, all these models use empiricism, by which we mean they depend heavily on measured data of received signal levels. But also, they all depend to a greater or lesser degree on the theory of electromagnetism. In some cases, theory is used only qualitatively to help make sense out of what is always a very wide spread in the measured values. In others of these models theory plays a more important role, and the empirical data serve to provide benchmarks at which the model is expected to agree.

3. THE ITS MODEL FOR THE MID-RANGE FREQUENCIES

Originally published by Longley and Rice (1968), the ITS irregular terrain model is a general purpose model intended to be of use in a very broad range of problems, but not, it should be noted, in all problems. It is flexible in application and can actually be operated as either an area prediction model or as a point-to-point model. We speak here of two separate "modes" of operation. In the point-to-point mode, part of the input one must supply consists of certain "path parameters" to be determined from the presumably known terrain profile that separates the two terminals. In the area prediction mode these same parameters are simply estimated from a knowledge of the general kind of terrain involved. The two modes use almost identical algorithms, but their different sets of input data and their different ranges of application make it inconvenient to discuss them both at once. This report treats only the area prediction mode.

In the present section we shall provide a brief general description of the model including its design philosophy, a list of its input parameters, and a discussion of some of the physical phenomena involved in radio propagation and whether they are or are not treated by the model.

Before continuing, however, we should first consider the units in which received signal levels are to be measured. Here we come upon a confusion, for each discipline of the radio industry seems to have chosen its own separate unit. Examples include electromagnetic power flux, electric field intensity, power available at the terminals of the receiving antenna, and voltage at the receiver input terminals. If one wants to divorce the propagation channel from the equipment, one also speaks of transmission loss, path loss, or basic transmission loss. Most of these quantities are described by Rice et al. (1967; Section 2), but the important property to note is that under normal conditions--when straightforward propagation takes place without near field effects or standing waves and when mismatches are kept to a minimum--all these quantities are easily transformed one to another. Indeed, in this report we use the term "signal level" so as to be deliberately vague about what precise unit is intended, because we feel the question is unimportant.

For each of the quantities that might represent a signal level, it is possible to compute a free space value--a value that would be obtained if the terminals were out in free space unobstructed by terrain and unaffected by atmospheric refraction. This free space value is a convenient reference point for radio propagation models in general and for the ITS model in particular. Our own preference for a measure of signal level is therefore the attenuation relative to free space which we always express in decibels. In what follows we shall use the simple term "attenuation," hoping that the context will supply the reference point. The quantity is sometimes also referred to as an "excess path loss." To convert to any other measure of signal level, one simply computes the free space value in decibels relative to some standard level and then subtracts the attenuation (adds, if one is computing a loss).

3.1 Input Parameters

In Table 1 we list all the input parameters required by the ITS area prediction model. Also indicated there are the allowable values or the limits for which the model was designed. Here we shall try to define the terms involved. As it happens, however, some of the terms are by nature somewhat ambiguous, and we shall defer more complete descriptions to Sections 5 and 6.

The system parameters are those that relate directly to the radio system involved and are independent of the environment. Counting the two antenna heights, there are five values:

Frequency. The carrier frequency of the transmitted signal. Actually, the irregular terrain model is relatively insensitive to the frequency, and one value will often serve for a fairly wide band.

Table 1. Input Parameters for the ITS Model Together With the Original Design Limits

System Parameters	
Frequency	20 MHz to 20 GHz
Distance	1 km to 2000 km
Antenna heights	0.5 m to 3000 m
Polarization	vertical or horizontal
Environmental Parameters	
Terrain irregularity parameter, Δh	
Electrical ground constants	
Surface refractivity	250 to 400 N-units
Climate	one of seven; see Table 4
Deployment Parameters	
Siting criteria	random, careful, or very careful
Statistical Parameters	
Reliability and confidence level	0.1% to 99.9%

Distance. The great circle distance between the two terminals.

Antenna Heights. For each terminal, the height of the center of radiation above ground. This may sound straightforward, and often it is; but neither the center of radiation nor the ground level is always easy to determine. For further discussion see Section 5.

Polarization. The polarization, either vertical or horizontal, of both antennas. It is assumed that the two antennas do have the same polarization aspect.

The environmental parameters are those that describe the environment or, more precisely, the statistics of the environment in which the system is to operate. They are, however, independent of the system. There are five values:

Terrain Irregularity Parameter Δh. The terrain that separates the two terminals is treated as a random function of the distance away from one of the terminals. To characterize this random function, the ITS model uses but a single value Δh to represent simply the size of the irregularities. Roughly speaking, Δh is the interdecile range of terrain elevations--that is, the total range of elevations after the highest 10% and lowest 10% have been removed. Further discussion of

Table 2. Suggested Values for the Terrain Irregularity Parameter

	Δh (meters)
Flat (or smooth water)	0
Plains	30
Hills	90
Mountains	200
Rugged mountains	500
For an average terrain, use Δh=90 m.	

this important parameter will be found in Section 5. Some suggested values are in Table 2.

Electrical Ground Constants. The relative permittivity (dielectric constant) and the conductivity of the ground. Suggested values are in Table 3.

Surface Refractivity N_s. The atmospheric constants, and in particular the atmospheric refractivity, must also be treated as a random function of position and, now, also of time. For most purposes this random function can be characterized by the single value N_s representing the normal value of refractivity near ground (or surface) levels. Usually measured in N-units (parts per million), suggested values are given in Table 4. Further discussion will be found in Section 5.

Climate. The so-called radio climate, described qualitatively by a set of discrete labels. The presently recognized climates are listed in Table 4. Together with N_s, the climate serves to characterize the atmosphere and its variability in time. Further discussion is given in Section 5.

The way in which a radio system is positioned within an environment will often lead to important interactions between the two. Deployment parameters try to characterize these interactions. The irregular terrain model has made provision for one such interaction that is to be applied to each of the two terminals.

Siting Criteria. A qualitative description of the care which one takes to site each terminal on higher ground. Further discussion is given in Section 5.

Table 3. Suggested Values for the Electrical Ground Constants

	Relative Permittivity	Conductivity (Siemens per Meter)
Average ground	15	0.005
Poor ground	4	0.001
Good ground	25	0.020
Fresh water	81	0.010
Sea water	81	5.0

For most purposes, use the constants for an average ground.

Table 4. Radio Climates and Suggested Values for N_s

	N_s (N-units)
Equatorial (Congo)	360
Continental Subtropical (Sudan)	320
Maritime Subtropical (West Coast of Africa)	370
Desert (Sahara)	280
Continental Temperate	301
Maritime Temperate, over land (United Kingdom and continental west coasts	320
Maritime Temperate, over sea	350

For average atmospheric conditions, use a Continental Temperate climate and N_s=301 N-units.

Finally, the statistical parameters are those that describe the kind and variety of statistics that the user wishes to obtain. Very often such statistics are given in the form of quantiles of the attenuation. For a discussion of this subject, and of the meanings we like to give to the terms reliability and confidence, see Section 6.

Aside from the statistical parameters which will vary in number according to the necessities of the problem, there are some twelve parameter values that one must define. Although this seems a rather long list, the user should note that in many cases several of these parameters have little significance and may be replaced by simple nominal values. For example, the only use to which the polarization and the two electrical ground constants are put is to determine in combination the reflectivity of smooth portions of the ground when the incident rays are grazing or nearly so. At high frequencies this reflectivity is nearly a constant.

When both terminals are more than about 1 wavelength above the ground or more than 4 wavelengths above the sea, these three parameters have little significance, and one may as well assume, say, "average" ground constants. At frequencies below about 50 MHz the effect of the conductivity is dominant; otherwise the relative permittivity is the more important.

Similarly, on short paths less than about 50 km, the atmosphere has little effect, and one may as well assume average conditions with a Continental Temperate climate and N_s=301 N-units. And finally, for the siting criteria one will usually assume that both terminals are sited at random. Thus, in a large proportion of practical problems, one is left with only five parameter values to consider: frequency, distance, the two antenna heights, and the terrain irregularity parameter.

3.2 General Description

Given values for the input parameters, the irregular terrain model first computes several geometric parameters related to the propagation path. Since this is an area prediction model, the radio horizons, for example, are unknown. The model uses empirical relations involving the terrain irregularity parameter to estimate their position.

Next, the model computes a reference attenuation, which is a certain median attenuation relative to free space. The median is to be taken over a variety of times and paths, but only while the atmosphere is in its quiet state--well-mixed and conforming to a standard atmospheric model. In continental interiors such an atmosphere is likely to be found on winter afternoons during fair weather. On oversea or coastal paths, however, such an atmosphere may occur only rarely.

As treated by the model, this reference attenuation is most naturally thought of as a continuous function of distance such as that portrayed in Figure 1. As shown there, it is defined piecewise in three regions, called the line-of-sight, diffraction, and forward scatter regions. The "line-of-sight" region is somewhat misnamed; it is defined to be the region where the general bulge of the earth does not interrupt the direct radio waves, but it still may be that hills and other obstructions do so. In other words, this region extends to the "smooth-earth" horizon distance, which is probably farther than the actual horizon distance. In this region the reference attenuation is computed as a combined logarithmic and linear function of distance; then in the diffraction region there is a rather rapid linear increase; and this is followed in the scatter region by a much slower linear increase. Parameters other than distance enter into the calculations by determining where the three regions fall and what values the several coefficients have. But once the system and its deployment (in a homogeneous environment) have been fixed, the notion of attenuation as a function of distance should be a convenient one for many problems.

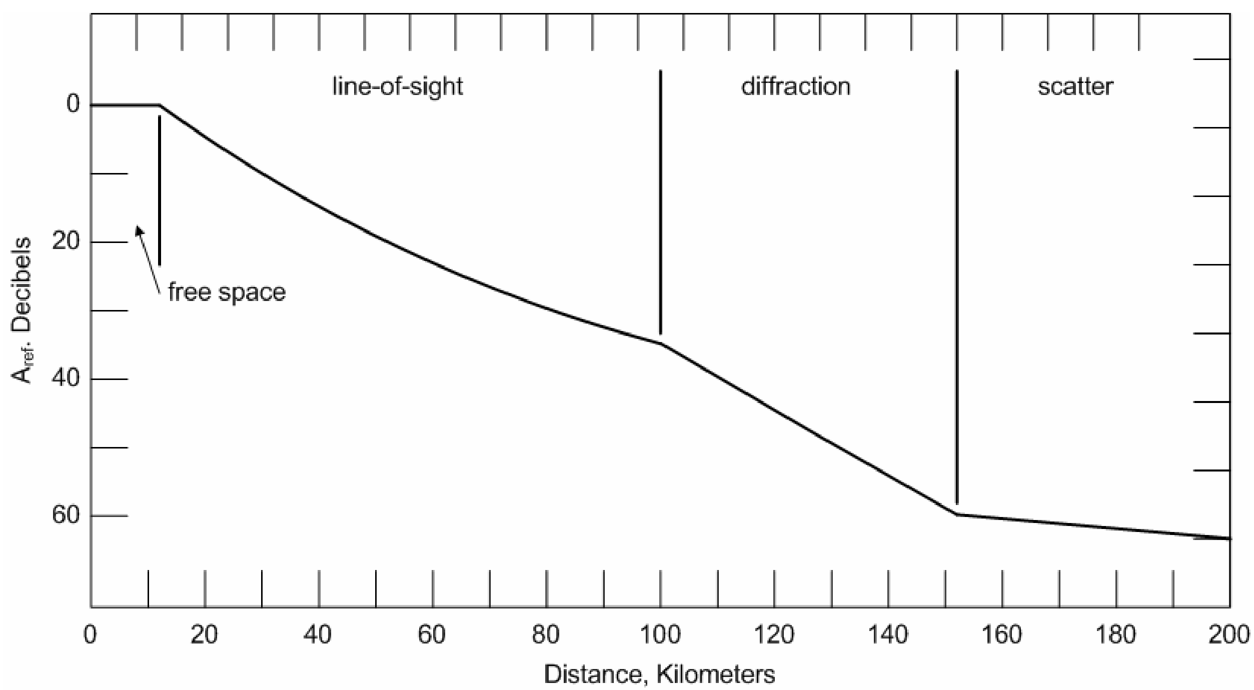


Figure 1. A typical plot of reference attenuation versus distance.

The reference attenuation is a good representative value to indicate to a designer how a proposed system will behave. For some problems, knowing it alone will be sufficient. For most problems, however, one must also obtain the statistics of the attenuation. To do this, the model first subtracts a small adjustment for each climate to convert the reference attenuation to an all-year median attenuation. Then from this median attenuation further allowances are subtracted to account for time, location, and situation variability in the manner described in Section 6.

For its calculations, the model utilizes theoretical treatments of reflection from a rough ground, refraction through a standard atmosphere, diffraction around the earth and over sharp obstacles, and tropospheric scatter. It combines these using empirical relations derived as fits to measured data. This combination of elementary theory with experimental data makes it a semi-empirical model which on the one hand should agree with physical reality at certain benchmark values of the parameters and on the other hand should comply with physical laws sufficiently well to allow us to interpolate between and extrapolate from these benchmark values with a good degree of confidence. Thus the model is a general purpose one that should be applicable under a wide variety of "normal" conditions--particularly those conditions that correspond to the land mobile and broadcast services.

The data used in developing the empirical relations clearly have influenced the model itself. It should then be noted that these data were obtained from measurements made with fairly clear foregrounds at both terminals. In general, ground cover was sparse, but some of the measurements were made in areas with moderate forestation. The model, therefore, includes effects of foliage, but only to the fixed degree that they were present in the data used.

There are several phenomena that the model ignores, chiefly because they occur only in special circumstances. In cases of urban conditions, dense forests, deliberate concealment of the terminals, or concerns about the time of day or season of the year, it is possible to make suitable extra allowances or additions to the basic model. This, of course, requires an engineer who knows the situation involved and the probable magnitude of the consequent effects.

The possibility of ionospheric propagation is what makes us limit the model to frequencies above 20 MHz. Still, there will be occasional cases of ionospheric reflection at frequencies near this lower limit, and scatter from sporadic E will occur at frequencies below about 100 MHz. Such effects, however, will be apparent only on very long paths and only for very small fractions of time.

Atmospheric absorption--particularly the water vapor line at 22 GHz--is what limits the model at the higher frequencies. The effects are measurable above about 2 GHz, but except on very long paths, they can usually be safely ignored below 10 GHz. Since water vapor absorption, which usually dominates below 25 GHz, is directly proportional to the water vapor content in the air, the magnitude of absorption varies in both time and location. If one wants to make extra allowances for this phenomenon, one should keep this variability in mind.

Rainfall attenuation is another phenomenon ignored by the irregular terrain model. This becomes important at frequencies above 6 GHz. However, it is measurable only during heavy rainstorms and therefore influences signal levels only for small fractions of time--usually for less than 1% of the time.

Superrefractive and ducting layers may occur along a path; indeed, in some coastal or oversea paths they may appear for large fractions of the time. The irregular terrain model tries to account for these occurrences, but only in a very general, nonspecific, way. It makes no attempt, for example, to account for the definite differences observed when the terminals lie above, within, or below a layer.

If ionospheric propagation, sporadic E, atmospheric absorption, rainfall, or ducting are important phenomena for a specific problem, the user should turn to other, more specialized, models for guidance. The irregular terrain model is not appropriate for these problems and should not be used.

Another obvious situation where the model should not be used is in predicting the performance of line-of-sight microwave links. With adequate ground clearance, the median received signal level for such links is usually very nearly the free space value modified, perhaps, by atmospheric absorption. There is little, if any, dependence on radio frequency, terrain irregularity, path length, or antenna heights so long as adequate Fresnel zone clearance is maintained. The irregular terrain model, however, will not assume, except on short paths, that there is adequate clearance and may predict a considerable attenuation. In the upper UHF and lower SHF bands, the model should be restricted to such problems as tactical communications and interference.

The model, as presently formulated, is also not suitable for predicting air-to-ground performance for aircraft flying at heights greater than 3 km. Even for heights greater than 1 km, the special conditions that arise makes the model of somewhat questionable usefulness.

In comparing predicted attenuation with measured values, certain additional questions may be encountered. Some of these are illustrated by the example in Section 7.3. In general, we should note that if the terrain varies widely in character within the desired area, then greater variability must be expected. Also, if the terminals are sited in extreme, rather than typical, locations, the calculated attenuation will not represent the median of measurements. An example of such an atypical situation would be propagation along a narrow, steep-sided valley, where the radio signal may be repeatedly reflected from the walls of the valley.

4. DEVELOPMENT OF THE MODEL

During the years prior to 1960, a good deal of information was obtained regarding radio propagation through the turbulent atmosphere over irregular terrain. For paths with fixed terminals a number of prediction models had been developed to describe the power available at the receiver over known profiles by means of line of-sight, diffraction, and forward scatter propagation. A good deal of data had also been accumulated from high-powered broadcast transmitting antennas to rather well-sited receivers. However, land-mobile types of communication systems were becoming increasingly important. In such applications some of the terminals are highly mobile, with randomly changing locations. Little information was available for such systems, especially where low antenna heights and ready mobility are prime requirements.

A theoretical and experimental program was undertaken by the National Bureau of Standards to study propagation characteristics under conditions resembling the operation of army units in the field. Tactical situations may often require that antennas be low and placed as inconspicuously as possible, and that receivers be highly mobile. A report by Barsis and Rice (1963) describes the planned measurement program and proposed terrain analysis. The measurements were to be carried out in various types of terrain, including the open plains of eastern Colorado, the foothills and rugged mountains of Colorado, and the rolling, wooded hills of northeastern Ohio. The report describes terrain profile types in terms of a spectral analysis which depends on a discrete, finite-interval, harmonic analysis of terrain height variations over the great circle path between terminals. Characteristics of terrain profiles of any given length were described relative to a least-squares fit of a straight line to heights above sea level.

As the study progressed, the harmonic analysis of terrain was replaced by a single parameter Δh, which is used to characterize the statistical aspects of terrain irregularity. Terrain statistics were developed for the areas described above by reading a large number of terrain profiles. Each profile was represented by discrete elevations at uniform distances of half a kilometer. Within each region selected for intensive study, 36 profiles 60 km in length were read in each of six directions, providing a total of 216 profiles that form a rather closely spaced grid over a 100 km square area.

Each profile was considered in lengths of 5, 10, 20...60 km to study the effects of path length on the various terrain parameters.

The interdecile range $\Delta h(d)$ of terrain heights above and below a straight line, fitted by least squares to elevations above sea level, was calculated at each of these distances. Usually the median values of $\Delta h(d)$ for a specified group of profiles increase with path length to an asymptotic value, Δh, which is used to characterize the terrain. This definition of Δh differs from that used by the CCIR and by the FCC as noted in Section 5.

An estimate of $\Delta h(d)$ at any desired distance may be obtained from the following empirical relationship:

$$\Delta h(d) = \Delta h[1-0.8 \exp(-d/D_0)] \quad (1)$$

where the scale distance D_0 equals 50 km. For homogeneous terrain, values of $\Delta h(d)$ measured at each distance agree well with those obtained from (1). As the terrain in a desired area becomes less homogeneous, the scatter of measured values of $\Delta h(d)$ increases.

For an area prediction where individual path profiles are not available, median values of terrain parameters to be expected are calculated as empirical functions of the terrain irregularity parameter Δh, the effective earth's radius, the antenna heights, and the siting criteria employed.

Even at first, the model was designed to calculate the reference attenuation below free space as a continuous function of distance. This could be easily converted to basic transmission loss by adding the free-space loss at each distance. These reference values of basic transmission loss, with a small adjustment for climate, represent the median, long-term values of transmission loss predicted for the area.

To provide a continuous curve as a function of distance, this median attenuation is calculated in three distance ranges as shown in Figure 1, Section 3: a) for distances less than the smooth-earth horizon distance d_{Ls}; b) for distances just beyond the horizon from d_{Ls} to d_x; and c) for distances greater than d_x. The model does not provide predictions for distances less than 1 km. For distances from 1 km to d_{Ls} the predicted attenuation is based on two-ray reflection theory and extrapolated diffraction theory. For distances from d_{Ls} to d_x, the predicted attenuation is a weighted average of knife-edge and smooth-earth diffraction calculations. The weighting factor in this region is a function of frequency, terrain irregularity, and antenna heights. For distances greater than d_x, the point where diffraction and scatter losses are equal, the reference attenuation is calculated by means of a forward scatter formulation.

In developing the original model, comparisons with data were made and empirical relationships were established. These include expressions for calculating horizon distances and horizon elevation angles, based on information obtained during the terrain study. The weighting factor, used to obtain the weighted average between rounded earth and knife-edge diffraction calculations, is based on radio data taken from two series of measurements. The first of these provided a large amount of data at 20, 50, and 100 MHz, obtained with low antennas in Colorado and Ohio. The results of these measurements are reported by Barsis and Miles (1965) and by Johnson et al. (1967). The other large body of measurements at VHF and UHF was provided to the Television Allocations Study Organization (TASO). These measurements were made in 1958 and 1959 in the vicinity of several cities in the United States, and the results are summarized by Head and Prestholdt (1960). Signals from television stations at frequencies of about 60 and 600 MHz were measured at uniform distances along radials with 3 and 9 m receiving antenna heights. These measurements were made with both mobile and stationary receivers in terrain that ranged from smooth plains to mountains.

After the model was developed and published (Longley and Rice, 1968), comparisons were made with a large amount of data at frequencies from 20 MHz to 10 GHz. These comparisons are reported by Longley and Reasoner (1970). Further comparisons, reported by Longley and Hufford (1975), were made with data at 172 MHz and 410 MHz taken with very low antennas.

Concerning the question of statistics, recall that the original purpose was to provide an area prediction model for land-mobile applications. Such systems involve low antennas and low transmitter powers with consequently short ranges. For such short paths, over land, the path-to-path variability is considerably greater than the time variability, and therefore the latter was treated rather casually. A Continental Temperate climate was assumed and represented by a cumulative distribution with two slopes--two "standard deviations"--to allow for the observed greater variability of the strong fields than of the weak ones.

As the use of the model was extended to broadcast coverage, with high power radiated from transmitting antennas on tall towers, the effects of differences in climate became more important in terms of possible interference between systems. For such applications, we included sets of mathematical expressions that reproduce the variability curves for various climates defined by the CCIR (1978b) and listed in Table 4. Two other climates, Mediterranean and Polar, are described in the CCIR Report, but curves are not presented for them. For land-mobile services in the United States, the Continental Temperate climate is nearly always chosen.

The original "Longley-Rice" model was published in 1968. Shortly afterward a new version was developed which improved the formulation for the forward scatter prediction, and later the computer implementation was changed to improve its efficiency and increase the speed of operation. Since then, minor but important modifications have been made in the line-of-sight calculations.

To keep track of the various versions, most of which are presently being used at some facility, we have recently begun numbering them in serial fashion. Following the original (which might be called version 0), here is a list of the more important versions, together with approximate dates when they were first distributed:

1.0	January 1969
1.1	August 1971
1.2	March 1977
1.2.1	April 1979
2.0	May 1970
2.1	February 1972
2.2	September 1972

Version 1.2.1 corrects an error in version 1.2; it is the currently recommended version and is the one whose implementation is listed in Appendix A. The second series, beginning with version 2.0, used considerably modified diffraction calculations and tried to incorporate a groundwave at low frequencies. It is not now recommended and is no longer maintained by its developers.

5. DETAILED DESCRIPTION OF INPUT PARAMETERS

The various parameters required as input to the ITS area prediction model were described briefly in Section 3 of this guide. Further description and an explanation of their use is provided here.

The primary emphasis of the model is a consideration of the effects of irregular terrain and the atmosphere on radio propagation at frequencies from 20 MHz to 20 GHz. One of the chief parameters used to describe the atmosphere is the surface refractivity N_s, while the terrain is characterized by the parameter Δh. A discussion of both atmospheric and terrain parameters is presented here.

5.1 Atmospheric Parameters

Atmospheric conditions such as climate and weather affect the refractive index of air and play important roles in determining the strength and fading properties of tropospheric signals. The refractive index gradient of the atmosphere near the earth's surface is the most important atmospheric parameter used to predict a long-term median value of transmission loss. This surface gradient largely determines the amount a radio ray is bent, or refracted, as it passes through the atmosphere. In this model we define an "effective" earth's radius as a function of the surface refractivity gradient or of the mean surface refractivity N_s. This allows us to consider the radio rays as being straight so long as they lie within the first kilometer above the earth's surface. At very much higher elevations, the effective earth's radius assumption over-corrects for the amount the ray is refracted and may lead to serious errors. In this propagation model we use minimum monthly mean values of N_s to characterize reference atmospheric conditions. Since such values are less apt to be influenced by temporary anomalous conditions such as superrefraction or subrefraction, they provide a rather stable reference which is exactly suited to computations of the reference attenuation.

The minimum monthly mean value of N_s, which in the northern hemisphere often corresponds to values measured in February, may be obtained from local measurements or estimated from maps of a related parameter N_O. The refractivity N_O is the value of surface refractivity that has, for convenience, been reduced to sea level. Figure 2 from Bean et al. (1960) is a world-wide map of minimum monthly mean values of N_o. The corresponding value of surface refractivity is then:

$$N_s = N_o \exp(-h_s/H_s) \quad \text{N-units} \qquad (2)$$

where h_s is the elevation of the earth's surface and the scale height H_s equals 9.46 km.

The effective earth's radius is directly defined as an empirical function of N_s, increasing as N_s increases. It is common to set N_s equal to 301 N-units; this corresponds to an effective earth's radius of 8497 km, which is just 4/3 times the earth's actual radius. Values of the effective earth's radius are used in computing the horizon distances, the horizon elevation angles, and the angular distance θ for transhorizon paths.

For short distance ranges the model is not particularly sensitive to changes in the value of surface refractivity. For this reason, in land-mobile systems we may often assume that N_s has the nominal value of 301 N-units. For distances greater than 100 km, changes in N_s have a definite effect on the amount of transmission loss.

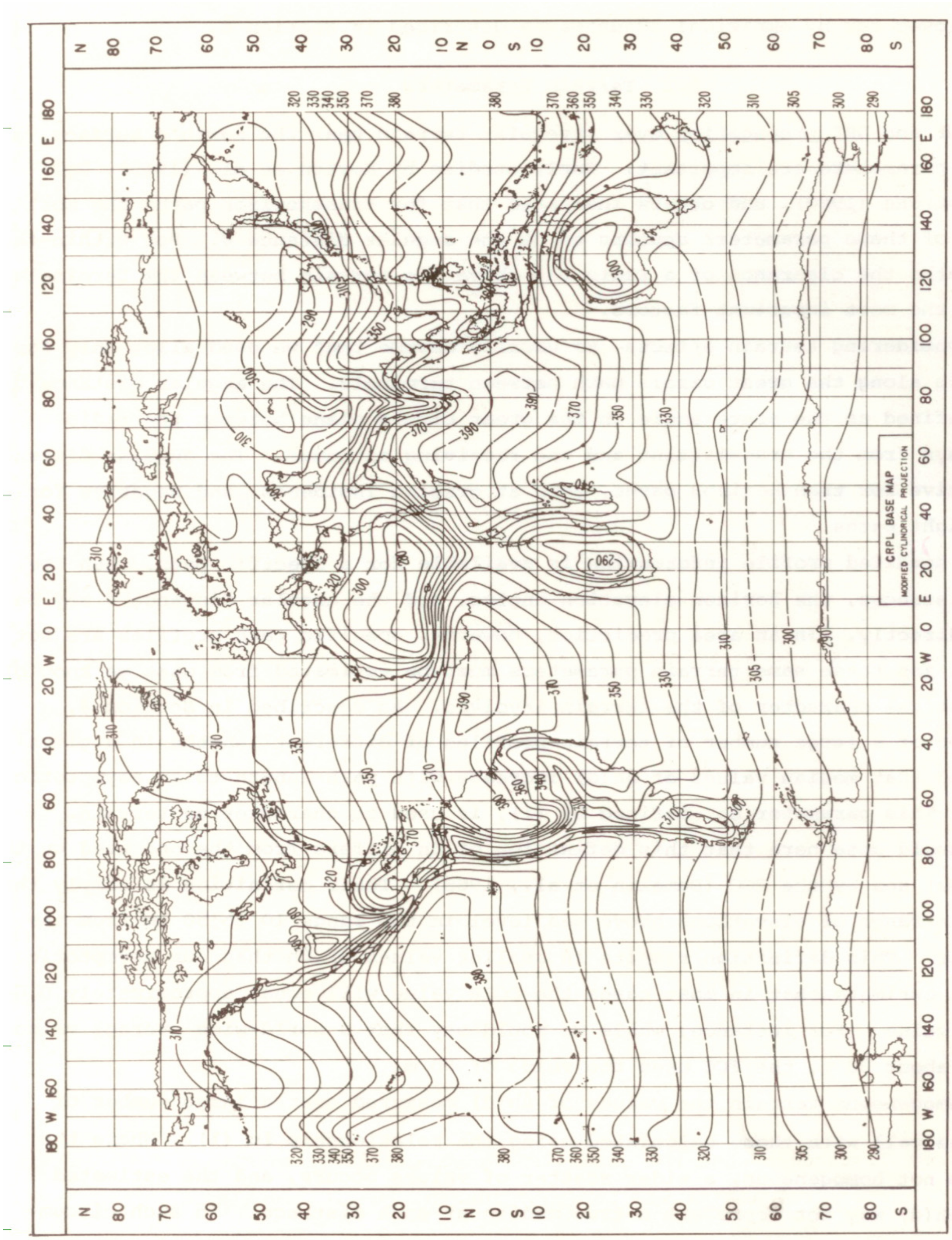


Figure 2. Minimum monthly mean values of surface refractivity referred to mean sea level (from Bean, Horn, and Ozanich, 1960).

Other atmospheric effects, such as changes in the refractive index and changes in the amount of turbulence or stratification, lead to a variability in time that may be allowed for by empirical adjustments described in Section 6.

5.2 Terrain Parameters

In VHF and UHF propagation over irregular terrain near the earth's surface, a number of parameters are important. Early studies by Norton et al. (1955), Egli (1957), LaGrone (1960), and others indicated that for transhorizon paths the most important of these parameters appears to be the angular distance θ. For within-the-horizon paths the clearance of a radio ray above the terrain between the terminals is one of the most important factors.

In considering terrain effects, we usually assume that we need allow only for the terrain along the great circle path between terminals. The angular distance θ is then defined as the acute angle in the great circle plane between the radio horizon rays from the transmitting and the receiving antennas. The angular distance θ is positive for transhorizon paths, zero at grazing incidence, and negative for line-of-sight paths.

When detailed profile information is available for a specific path, then the horizon distances, the horizon elevation angles, and the angular distance θ may be computed directly. In an area prediction, however, specific path profiles are not available, and these same terrain parameters must be estimated from what we know of the statistical character of the terrain involved. As described in Section 4, examination of a large number of terrain profiles of different lengths in a given area showed that median values of $\Delta h(d)$ increase with path length to an asymptotic value Δh. This parameter Δh, defined by (1), is used to characterize terrain.

We should note here that this definition of Δh differs from the one used by the CCIR (1978a) and by the FCC (Damelin et al., 1966). Their definition is simply the interdecile range of elevations above sea level in the range 10 to 50 km from the transmitter. This definition results in smaller values of Δh than our asymptotic value. We estimate that in most cases the CCIR value will equal approximately 0.64 times our value. For instance, while we would say that a world-wide average value for Δh is about 90 m, the FCC uses the value of 50 m.

In homogeneous terrain the values of $\Delta h(d)$ measured over a large number of paths agree well with those calculated using the relationship in (1). Where the terrain is not homogeneous, a wider scatter of values occurs, and the estimated value of $\Delta h(d)$ may not represent a true median at each distance. In such circumstances we may allow for a greater location variability in the prediction, or at times we may consider different sectors of an area and predict for each sector. An example of this would be an area that includes plains, foothills, and mountains. The losses predicted for each sector could be determined for the value of the terrain parameter computed for that sector.

The terrain parameter Δh may be obtained in one of several ways. The method selected will depend on the purpose for which it is used and on the terrain itself. In the original work to determine Δh for an area, a large number of profiles were read at uniform intervals. These profiles criss-crossed the area in such a way as to provide a rather fine grid. The interdecile range $\Delta h(d)$ was obtained for each profile and plotted as a function of distance. The median value at each distance was then used to obtain a smooth curve of $\Delta h(d)$, whose asymptotic value is the desired parameter Δh. This method is quite laborious and may not be necessary for the desired application. One can now use general maps of the terrain irregularity parameter as shown in Figure 3, or one may still go directly to topographic maps of the desired area and from them estimate the proper value. To do this, one may select a random set of paths, compute the value Δh from each path, and use the median of these calculated values to describe the terrain irregularity. With practice and a few elevations read from the map, one can even estimate Δh by eye.

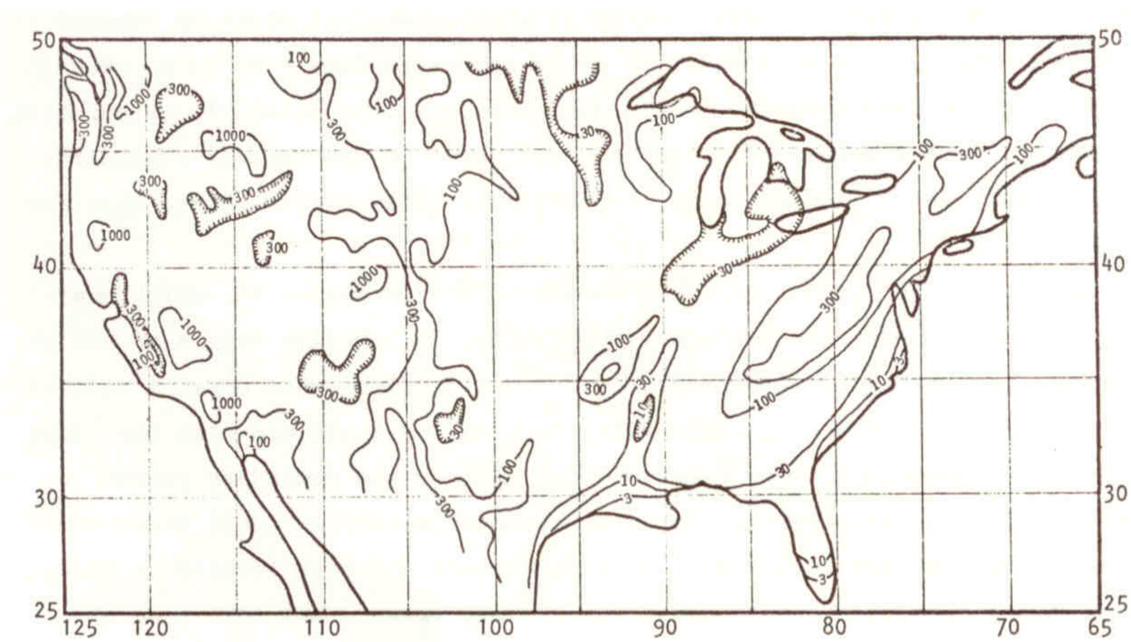


Figure 3. Contours of the terrain irregularity parameter Δh in meters. The derivation assumed random paths and homogeneous terrain in 50 km blocks. Allowances should be made for other conditions.

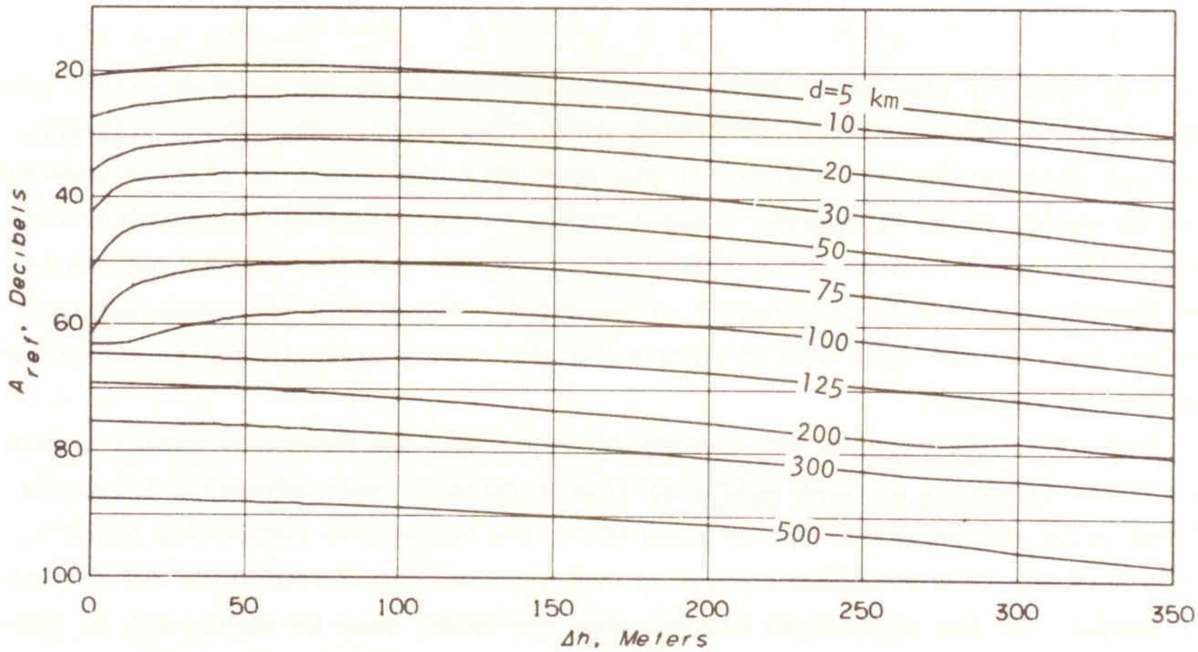
A major problem is that the area of interest is rarely homogeneously irregular. In such a case one must exercise judgment in selecting paths that will be representative of those that will actually be used in a proposed deployment. For example, if the desired paths will always be along or across valleys, one should not choose terrain profiles that cross the highest mountains. When virtually all paths involve terminals on facing hillsides along the same valley, a highly preferential situation exists.

Some qualitative descriptions of terrain and suggested values of Δh are listed in Table 2. Whether or not one needs a better estimate, based on computed values, depends on the sensitivity of the predicted values of transmission loss to changes in Δh. This sensitivity is quite complicated, depending on the value of Δh itself, on the antenna heights, distance range, siting criteria, and the radio frequency. This is probably most readily illustrated by an example. Figure 4 shows plots of attenuation relative to free space as a function of Δh at various distances. These curves are for a land-mobile system over irregular terrain at a frequency of 150 MHz. The upper figure represents base-to-mobile communication with antenna heights of 30 m and 2 m. The lower figure is drawn for mobile-to-mobile units with both antennas 2 m above ground. For small values of Δh the sensitivity to change is quite appreciable, especially at distances in the line-of-sight and diffraction regions. Here the <u>decrease</u> in attenuation (a phenomenon that might be likened to "obstacle gain") may be as much as 10 dB as Δh increases from 0 to 25 m. For larger values of Δh from about 50 to 150 m, there is little change in attenuation while for still larger values of Δh and for distances in the scatter region the <u>increases</u> in attenuation are quite regular and less sensitive to change than for small values of Δh.

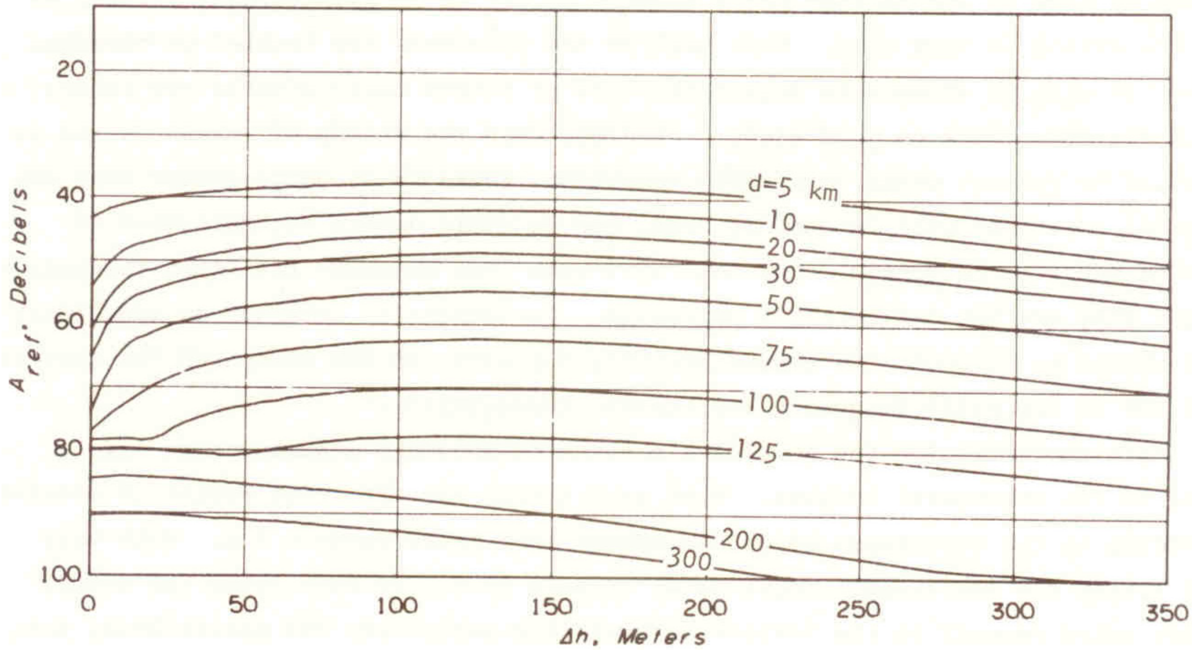
The area prediction model depends heavily on the parameter Δh, which characterizes terrain, and on the surface refractivity, N_s. Median values of all the other terrain parameters are computed from these two values when antenna heights are specified. Estimates of signal variability in time and space are also dependent on these two basic parameters. The relationships between the secondary parameters and the terrain irregularity parameter Δh were developed mainly in rural areas where antenna sites were always chosen with open foreground and were located on or near roads. In these areas the ground cover was usually sparse, but some moderate forestation was present.

5.3 Other Input Parameters

The way a system is deployed--particularly the way the terminal sites are chosen--can have a marked effect on observed signal levels. Unfortunately,



(a) f = 150 MHz, h_{g1} = 30 m, h_{g2} = 2 m



(b) f = 150 MHz, h_{g1} = 2 m, h_{g2} = 2 m

Figure 4. The reference attenuation versus Δh for selected distances.

there have been very few studies of these effects that could provide us with useful guidance. Nevertheless, the area prediction model does require the siting criteria, which are qualitative descriptions of the care with which each of the two terminal sites is chosen so as to improve communications. The effect of these criteria on the model is based on reasonable assumptions, but the validity of the results has been tested in only a limited number of examples. One should therefore exercise caution, both in the selection of values for the siting criteria and in the interpretation of results.

Changes in the value of the siting criterion for one of the terminals affect the assumed effective antenna height of that terminal. This effective height is defined to be the height of the antenna above the "effective reflecting plane" which, in turn, is a characterization of the intermediate foreground. It is actually this height, not the structural height, that the model uses in nearly all of its calculations. When the effective height increases, the model predicts less transmission loss and a greater communication range.

When the terminals of a system are usually sited on high ground and some effort is made to locate them where signals appear to be particularly strong, we say the siting is very good. When most of the terminals are located at elevated sites but with no attempt to select hilltops or points where signals are strong, we would classify these as good sites. Finally, when the choice of antenna sites is dictated by factors other than radio reception, there is an equal chance that the terminal locations will be good or poor, and we would assume the selection of antenna sites to be random. But note that even when antennas are sited randomly we assume they are not deliberately concealed. For concealed antennas an additional loss should be allowed, the amount probably depending on the nature of the concealment and on the radio frequency and terrain irregularity.

With random siting the effective antenna heights are assumed to be simply equal to the structural heights. With good siting the effective height is obtained by adding to the structural height an amount that never exceeds 5 m. With very good siting the additional amount never exceeds 10 m. In both cases the actual amount added depends on the terrain irregularity parameter, the notion being that in more irregular terrain there will be greater opportunity to find elevated ground. In flat areas the effective heights will always equal the structural heights no matter what the siting criteria are. The advantages achieved by good and very good siting are greatest for low antennas with structural heights less than about 10 m. If the antenna is on a high tower, the assumed change in effective height has little significance--but it is definitely significant for antennas located just above the ground.

The effective heights estimated from the siting criteria assume that antennas will be placed on a good site or on the best site within a very limited area. They do not assume that antennas will be placed on the highest mountain top within a total deployment area. But in many special problems, one will actually use just this kind of site selection. One such problem is illustrated in Section 7.3. In that case the receiver site was deliberately chosen at the edge of a high mesa overlooking rather smooth terrain. This is a decidedly atypical situation. One intuitively feels that it should be treated by setting the effective height of the antenna equal to the height above the terrain that lies below the mesa. But in the irregular terrain model it is the structural height that must be adjusted.

We usually define the structural height of an antenna to be the height of the radiation center above ground. But if the antenna looks out over the edge of a cliff, then it seems entirely natural to say that the cliff is really a part of the antenna tower and to include its height in the structural height. Another, more common, example of this same problem occurs in the design and analysis of VHF and UHF broadcast stations. There, it is the usual practice to site the antenna atop a hill or well up the side of a mountain in order to gain a very definite height advantage. While we no longer have an obvious cliff, this height advantage should still be accounted for by including it in the structural height.

There are several rules used by various people to determine what the ground elevation should be above which the antenna height is to be found. The FCC uses the 2 to 10 mi (3 to 16 km) average elevation for the radial of interest. Another rule that has been suggested is that one should not count as "ground" anything that has a depression angle from the center of radiation of more than 45°. In our own work we have sometimes said that consideration of terrain elevations should begin at a point distant about 15 times the tower height.

The choice of a radio climate may be difficult or confusing for the reader. The several climates described by the CCIR have not been mapped out as various zones throughout the world, and there are no hard and fast rules to describe each of the climates. Since our model is intended for use over irregular terrain, our preference is to use the Continental Temperate climate unless there are clear indications to choose another. The CCIR curves showing variability in time are entirely empirical and depend on the climate chosen. The curves for Continental and Maritime Temperate climates are based on a considerable amount of data, while those for the other climates depend on much smaller data samples. The Continental Temperate climate is common to large land masses in the temperate zone. It is characterized by extremes of temperature and pronounced diurnal and seasonal changes in propagation. In mid-latitude coastal areas where prevailing winds carry moist maritime air inland, a Maritime Temperate climate prevails. This situation is typical of the United Kingdom and of the west coasts of the United States and Europe. For paths that are less than 100 km long, there is little difference between the Continental and Maritime Temperate climates, but for longer paths the greater occurrence of superrefraction and ducting in maritime areas may result in much higher fields for periods of 10 percent or less of the year.

In considering time variability, it is important to note that we are concerned only with long-term variability, the changes in signal level that may occur during an entire year. The data on which such estimates are based were median values obtained over short periods of time, an hour or less. The yearly signal distributions are then distributions of these medians. This eliminates much of the short-term variability, which is usually associated with multipath. The rapid, short-term, multipath fading at a mobile receiver depends on many local factors including the type of receiving equipment, reflections from buildings and trees, and the speed at which the recording vehicle travels. In smooth, uncluttered terrain there may be little if any multipath fading, whereas the most severe fast fading is Rayleigh distributed. Even simple diversity techniques will greatly reduce this short-term multipath type of fading.

6. STATISTICS AND VARIABILITY

We come now to a discussion of how the ITS irregular terrain model treats the statistics of radio propagation. As we have mentioned before it seems undeniable that received signal levels are subject to a wide variety of random variations and that proper engineering must take these variations into account. Unfortunately, the problem is considerably more complicated than problems of simple random variables one encounters in elementary probability theory.

The principal trouble is that the population of observed signal levels is greatly stratified--i.e., not only do the results vary from observation to observation (as one would expect) but even the statistics vary. Now it is not surprising that this should be the case when one varies the fundamental system parameters of frequency, distance, and antenna heights; nor is it surprising when one varies the environment from, say, mountains in a continental interior to flat lands in a maritime climate, or from an urban area to a desert. But even when such obvious parameters and conditions are accounted for, there remain many subtle and important reasons why different sets of observations have different statistics.

Our problem here is analogous in many ways to that of taking public opinion polls. There results depend not only on the questions asked but also on many subtleties concerning how, where, and when the questions are asked. If one spends the working day telephoning people at their homes, then one obtains the opinions of those people who own telephones and answer them and who have remained at home that day. This procedure might still be a random sampling and might, indeed, provide acceptable results, if it were not for the fact that public opinion is, again, greatly stratified--i.e., that the opinions of one segment of the population can differ greatly from those of another.

In the case of radio propagation, it is the equipment and how, where, and when it is used that provides an added dimension of variability. Perhaps one or both terminals are vehicle mounted and constrained to streets and roads. Perhaps, instead, one antenna is likely to be mounted on a rooftop. Perhaps it is most probable that both antennas are well removed from trees, houses, and other obstacles; or perhaps it is likely that one of the antennas is close to such an obstacle or even inside a building, whether this be for convenience or because concealment is desirable. It may be that two regions of the world appear, even to the expert's eye, to offer the same set of impediments to radio propagation and yet the differences--whose effects we do not understand--may be important.

In any case, the way in which equipment is deployed has an often important and unpredictable effect on observed signal levels. We propose here to use the word <u>situation</u> to indicate a particular deployment, whether in actual use or simply imagined. In technical terms, a situation is a probability measure imposed on the collection of all possible or conceivable propagation paths and all possible or conceivable moments of time. (A good introduction to the theory of probability measures is given by Walpole and Myers, 1972, Ch. 1.) To choose a path and a time "at random" is therefore to choose them according to this probability measure. Insofar as we want to get below the level at which stratification is important, we would want to restrict a situation (that is, to restrict the set of paths and times where the imposed probability is non-zero) to include only paths with a common set of system parameters, lying within a single, homogeneous region of the world. This is a natural restriction except, perhaps, as it affects the distance between terminals. The distance is a parameter which is difficult to fix while still allowing a reasonable selection of paths.

If we are concerned with a single, well-defined communications link with fixed terminals, then the situation involved has only a single isolated path which is to be chosen with probability one. But the deployment of a land-mobile system in one single area would define a more dispersed situation. Note, moreover, that if the mobile units pass from an urban area to a suburban or rural area, then we would suppose they pass from one situation to another. If one sets out to make a set of measurements of received signal levels, then one will sample from what is, if the measurement program has been properly designed, a situation pre-defined by the program objectives. Often the measurements will be in support of what will become a system deployment. It is then always proper to ask whether the situation from which the data are taken corresponds accurately enough to the situation in which the system will operate.

Once again, all this fussiness would be unnecessary--and radio propagation engineering would long ago have become a finely honed tool--if it were not that the population of received signal levels is a stratified one. The system parameters, the environmental parameters, and the situation in which one is to operate are all important and each of them has some effect on the final statistics. The complexity of nature often forces us to empirical studies of these statistics; but the large number of dimensions involved makes this a difficult task.

6.1 The Three Dimensions of Variability

We turn now to a general discussion of the physical phenomenology involved. First, we should note that there is a very important part of the variability that we do not wish to include. This is the short-term or small displacement variability that is usually attributed to multipath propagation. Although it is probably the most dramatic manifestation of how signal levels vary, we exclude it for several reasons. For one, a proper description of multipath should include the intimate details of what is usually known as "channel characterization," a subject that is beyond our present interests. For another, the effects of multipath on a radio system depend very greatly on the system itself and the service it provides. Often a momentary fadeout will not be of particular concern to the user. When it is, the system will probably have been constructed to combat such effects. It will use redundant coding or diversity. Indeed, many measurement processes are designed so as to imitate a diversity system. On fixed paths, where one is treating the received signal level as a time series, it is common to record hourly medians--i.e., the median levels observed during successive hours (or some comparable time interval). We may liken the process to a time diversity system. If measurements are made with a mobile terminal, one often reports on selected mobile runs about 30 m in length. Then, again, one records the median levels for each run, thus simulating a space diversity system. Under the "frozen-in-space" hypothesis concerning atmospheric turbulence, one expects hourly medians and 30-m run medians to be about the same. (But the analogy becomes rather strained for multipath in urban areas.) To the two measurement schemes above, it would seem reasonable to add a third to correspond to frequency diversity. This would be a "wideband" measurement in which the average or median power over some segment of the spectrum were recorded. In any case, it is only the variation of these local medians that concerns us.

If one still finds it necessary to consider instantaneous values of cw signals, then the usual practice is simply to tack on an additional variability to those we shall describe here. Often, one assumes either that the signal is locally steady (in areas where there is no multipath) or that it is Rayleigh distributed (in areas with extreme multipath). Occasionally one will assume an intermediate case, using the Nakagami-Rice (see, e.g., Rice et al., 1967, Annex V) distributions or the Weibull distributions.

If we set out to measure statistics of local medians, the first step that occurs to us is to choose a particular fixed link and record measurements of hourly median received signal levels for 2 or 3 years. The resulting statistics will describe what we call the <u>time variability</u> on that one path. We could characterize these observations in terms of their mean and standard deviation; but, both because the distribution is asymmetric and not easily classified as belonging to any of the standard probability distributions, and because the practicing engineer seems to feel more comfortable with the alternative, we prefer to use the <u>quantiles</u> of the observations. These are the values not exceeded for given fractions of the time and are equivalent to a full description of the cumulative distribution function as described in the elementary texts on statistics. We would use such phrases as "On this path for 95% of the time the attenuation did not exceed 32.6 dB."

If we now turn our attention to a second path, we find to our dismay that things have changed. Not only are individual values different, as we would expect given the random nature of signal levels, but even the statistics have changed. We have a "path-to-path" variability caused by the fact that we have changed strata in the population of observable signal levels. Suppose, now, that we make a series of these long-term measurements, choosing sample paths from a single situation. In other words, we keep all system parameters constant, we restrict ourselves to a single area of the earth and keep environmental parameters as nearly constant as is reasonable, and we choose path terminals in a single, consistent way. We still find that the long-term time statistics change from path to path and the variation in these statistics we call location variability. Of course, if the situation we are concerned with has to do with a single, well-defined link, then it is improper to speak of different paths and hence improper to speak of location variability. But in the broadcast or mobile services, it is natural to consider such changes. The most obvious reason for the observed variability is the accompanying change in the profile of the terrain lying between the two terminals; although the outward--statistical, so to speak--aspects of the terrain may remain constant, the actual individual profiles, together with other, less obvious, environmental changes, will induce large changes in observed signal level statistics.

If we try to quantify location variability, we must talk of how time variability varies with path location. We have no recourse but to speak of the statistics of statistics. Clinging to the terminology of quantiles, we would speak of quantiles of quantiles and come up with some such phrase as "In this situation there will be 70% of the path locations where the attenuation does not exceed 32.6 dB for at least 95% of the time."

Finally, we must ask what effect there is when one changes from situation to situation. It should be no surprise to be told that the statistics we have so painfully collected following the outline above have changed. If we use like appearing situations--that is, if we change operations from one area to another very similar area or if we merely change the sampling scheme somewhat--then the observed changes in the location variability we call situation variability,

In other contexts this last variability is sometimes referred to as "prediction error," for we may have used measurements from the first situation to "predict" the observations from the second. We prefer here to treat the subject as a manifestation of random elements in nature, and hence as something to be described.

To make a quantitative description however, we must renew our discussion of the character of a "situation." We have defined a situation to be a restricted probability measure on the collection of all paths and times. But if we are to talk of changing situations--even to the point of choosing one "at random"--then we must assume that there is an underlying probability measure imposed by nature on the set of all possible or conceivable situations. And we must assume that at this level we have specified system parameters, environmental parameters, and deployment parameters in sufficient detail so that the variability that remains is no longer stratified--in other words, so that any sample taken from this restricted population will honestly represent that population. It is at this point that "hidden variables" enter--variables whose effects we do not understand or which we simply have not chosen to control. The values of these variables are at the whim of nature and differ between what would otherwise be identical situations. The effects of these differences produce the changes in observed statistics.

We are now at the third level of the statistical description, and evidently we must speak of quantiles of quantiles of quantiles. This produces the phrase, "In 90% of like situations there will be at least 70% of the locations where the attenuation will not exceed 32.6 dB for at least 95% of the time."

In general terms such quantiles would be represented as a function $A(q_T, q_L, q_S)$ of three fractions: q_T, the fraction of time; q_L, the fraction of locations; and q_S, the fraction of situations. The interpretation of this function follows the same pattern as given above: In q_S of like situations there will be at least q_L of the locations where the attenuation does not exceed $A(q_T, q_L, q_S)$ for at least q_T of the time. Note that the inequalities implied by the words "at least" and "exceeds" are important reminders that we are dealing here with cumulative distribution functions. Note, too, that the order in which the three fractions are considered is important. First, one chooses the situation, then the location, and finally the time.

We recall that if a proposition is true with probability q then it is false with probability 1-q. Working our way through all the inequalities involved, we may also say: In $1-q_S$ of like situations there will be at least $1-q_L$ of the locations where the attenuation does exceed $A(q_T, q_L, q_S)$ for at least $1-q_T$ of the time. This is the kind of phrase one uses when trying to avoid interference.

6.2 A Model of Variability

As complicated as it is, the three-fold description of quantiles does not completely specify the statistics. At the first level when we are considering time variability it is sufficient. But at the very next level we have failed to notice that we are trying to characterize an entire function of quantile versus fraction of time q_T. To do this completely, we would need to consider all finite sequences q_{T1}, q_{T2}, ... of fractions of time and to examine the resulting observed quantiles all at once as a multivariate probability distribution. At the third and final level, matters become even worse.

Obviously this becomes too complicated for practical applications; nor would a study following such lines be warranted by our present knowledge. But there are engineering problems that arise which can be aided by a more complete description of these statistics. Implicit within the ITS irregular terrain model is a second model which concerns variability and which can be used to provide such a description. It is a relatively simple model using a combination of simple random variables each of which depends on only one of the three different dimensions of variability. While retaining the features described in the previous paragraphs, it allows the engineer to derive formulas for many needed statistics.

Experience shows us that when signal levels are expressed in decibel notation the observed distributions tend to be normal or at least approximately normal. It is from this fact that inspiration for the model is largely derived. The broad statement of normality does, however, suffer from one important flaw which appears when we discuss signal levels that exceed free space values. Such signal levels are possible and are, indeed, observed; but their occurrence is rare and becomes increasingly more rare as one considers ever higher levels. The distributions we obtain must be truncated or heavily abbreviated at levels above free space.

As it happens, the terminology the ITS irregular terrain model uses to describe the magnitude of variability differs in a slight way from that used above. As in Rice et. al (1967, Annex V), the model considers the positive direction of a deviation as an increase of signal level rather than of attenuation or loss. There is, of course, no real significance to this convention, but the introduction of an extra minus sign does tend to confuse our subsequent arguments. For this one section, therefore, we shall adopt a different posture. Using lower-case letters to refer to random variables, we suppose that the object of concern is the signal level w which we measure in a decibel scale. We leave the precise definition of this signal level deliberately vague, since it is immaterial here whether we speak of power density, field strength, receiver power, or whatever. It would be related to the attenuation a by the formula

$$w = W_{fs} - a \tag{3}$$

where W_{fs}, which is not a random variable, is the signal level that would be obtained in free space.

The above change in convention requires a slight change in our definition of a quantile. To retain the same relations as are used in practice, we now say it is the value which <u>is</u> exceeded for the given fraction. For example, if w were a simple random variable, we would define the quantile W(q) as being the value which w exceeds with probability q. We should perhaps refer to this as a "complementary" quantile, but instead we shall merely depend on the context to determine the implied inequality. The rule to remember here is that we assume the attitude of trying to detect a wanted signal. It must be sufficiently large with a sufficiently high probability.

Our model of variability is a mathematical representation of how one is to view the received signal level as a random variable. First we assume the system parameters, the environmental parameters, and the deployment parameters have been fixed. From the set of all situations with these parameters, we choose at random a particular one s. Then using that situation (which is, remember, a probability measure) we choose at random a location ℓ and a time t. The triple (t,ℓ,s) forms our elementary event, and the corresponding received signal level $w(t,\ell,s)$ becomes a random variable. The model expresses this function of three variables in a more explicit and manageable way. We first define a tentative value of the signal level

$$w'(t,\ell,s) = W_0 + y_S(s) + \delta_L(s)\, y_L(\ell) + \delta_T(s)\, y_T(t), \tag{4}$$

where W_0 is the overall median signal level; y_S, y_L, y_T are three random variables called <u>deviations</u>; and δ_L, δ_T are another two random variables called <u>multipliers</u>. The three deviations are measured in decibels and their median values are 0 dB. The two multipliers are dimensionless, always positive, with medians equal to unity. We now come to the important assumption that the five random variables here are all mutually independent. This enables us to treat each of them separately and then to combine them using standard probability theory.

The final step in our model is to write

$$w(t,\ell,s) = M(w'(t,\ell,s)), \tag{5}$$

where M is a <u>modifying</u> function which corrects values greater than the free space value. For values of w' less than the free space value, we set $M(w')=w'$; but otherwise M puts an upper limit on values or at any rate reduces them considerably. As presently constituted, the ITS irregular terrain model cuts back the excess over free space by approximately a factor of 10. Thus, if W_{fs} is the free space value of received signal level, we have $M(w') \simeq 0.9\, W_{fs} + 0.1\, w'$ when $w' > W_{fs}$.

The statistics of the three deviations and the two multipliers depend on the system parameters, the environmental parameters, and the deployment parameters. Except that the two multipliers must be positive, the five random variables are approximately normally distributed. The deviations have standard deviations on the order of 10 dB, while the multipliers have standard deviations equal to 0.3 or less. The actual values have been derived from empirical evidence and engineering judgment.

Using this model we can, for example, recover the three-dimensional quantiles discussed previously by following the prescribed procedure step by step. At the first step we would assume there is a fixed situation and a fixed location at which we observe the received signal level as a function of time. Now one very useful property of quantiles has to do with the composition of random variables with monotonically increasing functions. If, say, u is a random variable with quantiles U(q) and if F is a monotonically increasing function, then, as one can easily show, the random variable F(u) has the quantiles F(U(q)). Since $\delta_T(s)$ is positive, the right-hand side of (4) is a monotonically increasing function of y_T, and therefore the time variant quantiles are given by

$$W_1'(q_T,\ell,s) = W_o + y_S(s) + \delta_L(s)\, y_L(\ell) + \delta_T(s)\, Y_T(q_T), \qquad (6)$$

where $Y_T(q_T)$ is the q_T quantile of y_T. At the next step we would have a fixed situation and a fixed time variant quantile, and we would look at (6) as a function of location alone. Again, if $Y_L(q_L)$ is the q_L quantile of y_L, we quickly find what is now a twofold quantile

$$W_2'(q_T,q_L,s) = W_o + y_S(s) + \delta_L(s)\, Y_L(q_L) + \delta_T(s)\, Y_T(q_T). \qquad (7)$$

At the third step we must consider (7) as a random variable since the situation s is now to be chosen at random. But here we have a new problem. The right-hand side of (7) is the sum of a fixed number W_o and three mutually independent random variables. The statistics of W_2' must therefore be computed from the convolution of the corresponding three probability distributions. When this has been done, we would pick off the desired quantile and finally come upon the threefold expression $W'(q_T,q_L,q_S)$. In the last step, we recall that the modifying function M is monotonically increasing and so

$$W(q_T,q_L,q_S) = M(W'(q_T,q_L,q_S)) \ . \qquad (8)$$

The only difficult part in this sequence of computations appears when we must find the convolution required by (7). To do this the ITS irregular terrain model uses an approximation sometimes called <u>pseudo</u>-<u>convolution</u>. This is a scheme described by Rice et al. (1967) to treat several applications problems where the sum of independent random variables is concerned. For completeness and because it is useful in many applications of the model, we pause here to provide our own description.

In the general case we would have two independent random variables u and v with corresponding quantiles U(q), V(q). We then seek the quantiles W(q) of the sum w=u+v. We first form the deviations from the medians which we recognize as having quantiles

$$Y_U(q) = U(q) - U(0.5)$$
$$Y_V(q) = V(q) - V(0.5) \quad , \tag{9}$$

and then we simply use a root-sum-square to derive

$$W(q) \approx U(0.5) + V(0.5) + \mathrm{sign}(0.5-q)\ [Y_U(q)^2 + Y_V(q)^2]^{1/2} \ . \tag{10}$$

If u and v are normally distributed, this expression is exact. For other distributions we can only say that results are reasonable and that in our own experience using distributions that arise naturally in the applications the expression is surprisingly accurate. It is, however, meant to be only an approximation and must always be treated as such.

Note that the extension of (10) to more than two summands is straightforward. The expression even shares with actual convolution the property of being associative and commutative so that the order in which summands are combined is immaterial.

6.3 Reliability and Confidence

The use of the three-dimensional quantiles is perhaps best illustrated by its application to the broadcast services. A broadcaster will need to provide an adequate service to an adequate fraction of the locations at some given range. But "adequate service" in turn implies an adequate signal level for an adequate fraction of the time. For television channels 7 to 13, for example, in order to provide Grade A service the broadcaster must deliver (O'Connor, 1968) a field strength 9 m above the ground which exceeds 64 dBμ for more than 90% of the time, and that in at least 70% of the locations. Spectrum managers and also the broadcast industry will in turn want to assure that a sufficient fraction of the broadcasters can meet their objectives. If we assume that each broadcaster operates in a separate "situation," then this last fraction is simply a quantile of the situation variability.

For other services, however, it is often difficult to see how the three-dimensional quantiles fit in, and indeed it is probably the case that they do not. Consider again the broadcast service. A single broadcaster will want to know the probability with which a given service range will be attained or exceeded. Since "service range" involves specified quantiles of location and time, the probability sought concerns situation variability and we return to three-dimensional statistics. On the other hand, consider the same problem from the point of view of an individual receiver. That individual will want to know only the probability at that one location of receiving adequate service--that is, of receiving an adequate signal level for an adequate fraction of the time. The distinction between location variability and situation variability will be of no concern and should not enter into our considerations.

Using our model as in (4) and (5) we quickly note how we can accommodate a new kind of analysis. We can suppose that first both the situation and the location are chosen simultaneously and then, second, the time. The first choice will have said that all four random variables in (4), excepting only y_T, are to be treated at once and are to be combined into a single deviation $y_S + \delta_L y_L$ and a single multiplier δ_T. What we would have remaining is a twofold description of variability involving time variability and a combined situation/location variability, and this is precisely the description that the individual receiver of a broadcast station would find useful.

To continue our discussions, we find it convenient here to introduce the term reliability. This is a quantile of that part of the variability which enters into the notion of "adequate service." For the individual receiver of a broadcast station, reliability is concerned with a fraction of time. For a broadcaster, however, reliability must be expressed as a twofold quantile involving time and location variability separately. For the remaining variability--always at a higher level in the hierarchy--we use the term confidence; and we mean this term in the sense that if one makes a large number of engineering decisions based on calculations that use the same confidence level, then, irrespective of what systems or even what types of systems are involved, that same fraction of the decisions should be correct--and, of course, the remainder should be incorrect. Reliability is a measure of the variability that a radio system will observe during the course of its deployment. Confidence will be measurable only in the aggregate of a large number of radio systems. Clearly, differentiation between the two will depend on the point of view one takes. To a broadcaster, confidence will be a measure of the situation variability; to an individual receiver of a broadcast station, it will be a measure of a combined situation and location variability. But the spectrum planner of the broadcast service will not speak of confidence at all; from that point of view all of the variability is observable and is part of the system.

Remembering that we must retain the order in which the three kinds of variability appear, there are four different ways that one can treat them in combination, all of which have legitimate uses in one kind of service or another.

We call these the four modes of variability, although they are really four different ways of treating the subject of variability.

Two of these four modes we have already discussed. In the broadcast mode we treat all three kinds of variability separately. The typical user of this mode would be the broadcaster for whom reliability would measure both location and time variability and confidence would measure situation variability. In the individual mode situation and location variability are combined so that there remain this combined variability and time variability. Here, the typical user would be the individual receiver of a broadcast station for whom reliability means the time availability, and confidence measures the combined situation/location variability.

It would also be legitimate to combine location and time variability. We call the result the mobile mode, since to a mobile radio unit changes in location translate into changes in time. The typical user of this mode would be a mobile system employing a single base station. Reliability would refer to the combined location/time variability; it would probably translate into fraction of attempts at establishing communications. Confidence would be a measure of the situation variability.

Finally, in the single message mode we combine at once all three of the kinds of variability, thus obtaining the more usual sort of one-dimensional random variable. The statistics to be used here are much simpler than those we have been discussing; but, we think, the useful applications are somewhat limited. One application might be for a communications link that will be used but once. Examples might include a disaster warning system or a radio link attached to a self-destructing device. The statistics involved would then be couched in terms of confidence levels. A more important application, however, would be for a mobile-to-mobile system where the two mobile units are to be deployed worldwide. The statistics would translate into first-try success probabilities (Hagn, 1980) and thereby become expressions of reliability.

6.4 Second Order Statistics

Until now we have been discussing only first order statistics--that is, the statistics of received signal levels for a single path at a single time. But there are many problems in which more needs to be known. These are problems that depend on the relative signal levels on separate paths or at separate times. For example, the problem of interference comes first to mind. Also, there is the question of what happens on successive hops in a chain of communication links, or how to treat the connectivity of a network of repeaters such as has been suggested for military use.

The resolution of such problems depends on second or higher order statistics where one considers the joint probabilities of obtaining given signal levels over two or more paths. The most common statistic employed here is the correlation coefficient, but in the general case one might well be forced to use something more complicated.

Unfortunately, almost nothing is known about the subject. There have been studies concerning diversity systems in which correlation coefficients have been found for the two time series obtained when two receiving antennas are separated by only a few wavelengths or in frequency by only a small fraction of the carrier. But when it is a matter of the local median levels, studies of their possible relationships have been rare and inconclusive.

In attacking problems where higher order statistics are required, we seem forced to devise *ad hoc* approaches. In our own work on interference, for example, we have said that time and location variabilities are independent while situation variabilities are completely dependent. In other words, we have returned to the model in (4) and (5) and supposed two sets of these equations--one for the desired link and one for the undesired link. Thus we find a grand total of ten random variables to consider. Now in each set of five we have assumed these to be mutually independent; but one can still ask about correlations between terms of opposite equations. Our assumption, based on very meager information, has been that terms involving time and terms involving location are again mutually independent. On the other hand, we have argued that the situation involving the receiver is the same, or approximately the same, whether one considers the desired or the undesired transmitter. It would then follow, for example, that the two values of y_S are equal and therefore simply cancel out when one computes the desired-to-undesired signal ratio. Clearly, these assumptions must be viewed suspiciously; they enjoy only the benefit that they appear to give reasonable looking results.

7. SAMPLE PROBLEMS

In this section we give a few examples of how the ITS irregular terrain model can be used to solve engineering problems. They have been selected with an eye towards variety, and because of the opportunities they provide to illustrate different techniques. In none of them do we claim to have carried the solution to its final completion. Our objective has been to set up the problem, to describe how it relates to the model, and to provide only a few illustrative results.

In this section all calculations pertaining to radio propagation have been made by computer using the subprograms LRPROP and AVAR as described and listed in Appendix A. No alterations, however slight, have been permitted. On the other hand, we have not hesitated to prepare short applications programs which process input data into a form that can be used by the model and which process the output of the model in ways that satisfy the requirements of the problem at hand.

We have tried to introduce as little new terminology as possible; what does appear we hope is standard and recognizable throughout the engineering profession.

7.1 The Operating Range of a Mobile-to-Mobile System

Our first problem is a simple one which we shall treat with immediately available tools. We study a particular kind of transceiver meant to operate from vehicles, and we ask to what range they should be able to communicate. More precisely, remembering the many sources of variability within such a system, we must ask to what range they should be able to communicate reliably. The answer, then, will depend on what one will accept as adequate reliability.

On the other hand, we can also turn the problem around and ask, for any given range, what the probability of communications is. Using the terminology of Hagn (1980), upon whose report much of the analysis of this section is based, we speak of "first-try success probability"--the probability that a communications channel is established, disregarding repeated attempts.

We assume that the system will operate worldwide and that the "first-tries" are to be made in many areas. Thus, we assume it is the single message mode of variability that we must use to describe the statistics of propagation. Other parameters of the assumed system are given in Table 5. It is a low band FM system in which both terminals use the same kind of equipment. The required signal-to-noise ratio is the predetection value. To allow for the possibility of multipath fading, we have introduced a safety margin of 6 dB. Assuming Rayleigh statistics, this amount below the median will pick up about 85% of the points in any short run.

There are many other sources of variability in the system besides that due to radio propagation. A particular transceiver will not have exactly the parameters given in Table 1, neither as regards the transmitter output nor on the receiver side as regards the sensitivity factors such as noise bandwidth. The antennas, particularly when one thinks of random orientations, will have variable gains. Man-made noise varies both in location and time.

Table 5. Design Parameters for a Symmetric Mobile-to-Mobile System

Frequency	45 MHz
Polarization	Vertical
Antenna heights	2 m
Transmitter power	16 dBW
Antenna gains	-3 dBi
Line losses	1 dB
kTB (B=25 kHz)	-160 dBW
Rural noise, above kTB	21 dB
Required (rf) signal-to-noise ratio	6 dB
Margin for multipath (Rayleigh) fading	6 dB
Margin for uncertainties	7 dB

In the report cited above, Hagn has carefully estimated the standard deviations of all these factors and, along with the propagation variability, combined all sources of variability into one, using the root-sum-square. Here, however, we have adopted a simpler approach in which we have merely introduced an additional safety margin. Although this approach is often used by designers, we should note how it changes the proper interpretation of the results: For a large fraction of the transceivers deployed, the observed performance should exceed the predicted performance.

If we start with the transmitter power, add to it the two antenna gains, and subtract the two line losses, we find that the power available to the receiver equals 8 dBW less the losses in the propagation channel. For the other side of the ledger, if we start with the noise power and add to it the required signal-to-noise ratio and the two safety margins, we find satisfactory reception if the received power exceeds -120 dBW. It follows that the system will tolerate a basic transmission loss of as much as 128 dB.

To calculate the propagation losses, we have turned to the applications program QKAREA listed in Appendix B. This is a simple program which accepts as input the parameters needed by the ITS irregular terrain model and then lists selected quantiles of basic transmission loss at selected distances. In Figure 5 we have reproduced the output from one such run. The input consisted of the parameters in Table 5 together with values prescribing average terrain characteristics, ground constants, and climate. The principal feature in the figure is the table of quantiles versus distance.

```
OPERATIONAL RANGES FOR A MOBILE-TO-MOBILE SYSTEM

          FREQUENCY          45. MHZ
    ANTENNA HEIGHTS      2.0      2.0 M
  EFFECTIVE HEIGHTS      2.0      2.0 M   (SITING=0,0)
   TERRAIN, DELTA H          90. M

POL=1, EPS=15., SGM=  .005 S/M
CLIM=5, N0=301., NS=301., K= 1.333

SINGLE-MESSAGE SERVICE

ESTIMATED QUANTILES OF BASIC TRANSMISSION LOSS(DB)
```

DIST KM	FREE SPACE	WITH CONFIDENCE 95.0	90.0	80.0	70.0	50.0	20.0
1.0	65.5	113.4	109.7	105.2	102.1	96.9	88.5
2.0	71.5	123.0	119.3	114.8	111.6	106.4	97.9
3.0	75.1	→128.8	125.0	120.5	117.3	112.0	103.6
4.0	77.6	133.1	→129.2	124.7	121.5	116.2	107.7
5.0	79.5	136.5	132.6	→128.0	124.8	119.5	110.9
6.0	81.1	139.3	135.4	130.8	127.6	122.2	113.6
7.0	82.4	141.7	137.9	133.2	→130.0	124.6	116.0
8.0	83.6	143.9	140.0	135.4	132.1	126.7	118.0
9.0	84.6	145.9	142.0	137.3	134.0	→128.6	119.9
10.0	85.5	147.7	143.7	139.1	135.8	130.3	121.6
15.0	89.0	153.4	149.4	144.7	141.4	135.9	127.1
20.0	91.5	157.2	153.2	148.4	145.0	139.5	→130.6
25.0	93.5	160.4	156.3	151.5	148.1	142.6	133.6
30.0	95.1	163.2	159.1	154.3	150.9	145.3	136.3

Figure 5. Output from a run of QKAREA concerning a mobile-to-mobile system. The arrows point to intervals where the quantiles become equal to 128 dB.

The columns are headed by percentages referred to as "confidence levels." For the present problem this is, as we have noted before, a misnomer and should be replaced by "reliability" or "first-try success probability."

Finally, to find the operational ranges we simply read down each column to find the first distance at which the quantile of basic transmission loss exceeds the tolerable limit of 128 dB. Doing this and using simple linear interpolation on the crucial interval, we find the values listed in Table 6. Note that this table really lists the quantiles of operational range as though this latter were a random variable, as indeed it is. If one requires

Table 6. Operational Ranges Under Average Environmental Conditions

Reliability	95%	90%	80%	70%	50%	20%	10%
Range	2.9 km	3.7 km	5.0 km	6.2 km	8.7 km	16 km	24 km

high reliability, one must be content with rather short ranges. On the other hand, there will always be a few locations in a few areas of the world where a first-try succeeds even when the distance is greater than 20 km.

Note that we have first talked about worldwide operations and next about average environmental conditions. What we really mean, of course, is anywhere in the world where conditions are average. For a more complete study of this system, we could continue on to find operational ranges for other than average conditions--for plains, hills, and mountains, for example, and for poor, average, and good ground constants. Then we could display the results as though the operational ranges were a function of the environmental conditions. Or we could package them all together into one truly worldwide table of range quantiles. This last step, however, would require a knowledge of the fraction of attempted communications to be made under each set of conditions.

7.2 Optimum Television Station Separation

In this next sample problem we treat what might be called a system design. We determine possible values for some of the parameters related to the placement of a network of cochannel television stations. Our aim will be to maximize the area that such a network will serve.

It seems clear that we are speaking here of an interference-limited service. If we imagine any arrangement of transmitters, we could require that they all operate at very low powers so that there is no interference. The service is then noise limited. But we can then increase the area covered simply by increasing the transmitter powers; and we can continue to do this until the stations begin to interfere with each other. At this point it is useless to increase powers further since not only do the desired signals increase but also the undesired signals increase.

To fix our ideas we imagine a plane surface infinite in extent which we shall often call "the country" and on which we shall situate the broadcast stations. We assume this plane has homogeneous terrain characteristics and a uniform climate.

To maximize the coverage area, we use the rules of closest packing and assume the stations form a perfect triangular grid and the transmitters all have identical characteristics--that is, that they are all at the same height above the terrain and that they radiate the same power levels from omnidirectional antennas.

For design purposes the United States is divided into three "zones." Zone 1 consists of the urban northeast extending west to include Illinois; Zone 3 includes a narrow region surrounding the Gulf of Mexico; and Zone 2 includes everywhere else. Using Zone 2 as a comparison, stations in Zone 1 are presently packed closer together using lower transmitter heights, while stations in Zone 3 are placed further apart in anticipation of better propagation conditions and higher interference fields. Also for design purposes one speaks of Grade A service and Grade B service, a distinction that refers not so much to the quality of reception as it does to the quality and mobility of the assumed receiving systems. Grade A service pictures an urban environment with relatively inexpensive receiving systems and with relatively tight constraints on where the receiving antennas are located.

For our sample problem we have chosen to examine a grid of Channel 10 stations and to require Grade A service. Channel 10 is in the center of the so-called high VHF band (Channels 7-13), and Grade A service makes the problem a little more interesting. Following O'Connor (1968), we have listed the design parameters in Table 7. These parameters are concerned mostly with the assumed receiving system, but they also include required interference ratios, required reliabilities, and for later comparison with the values we shall obtain, the transmitter parameters presently prescribed for Zone 1.

Adequate service to one location is defined to mean a satisfactory signal for at least 90% of the time. Grade A service then requires adequate service to at least 70% of the locations. In contrast, Grade B service requires the same adequate service to only 50% of the locations.

When we examine the required desired-to-undesired ratios, we find that a new complication is introduced. In television, co-channel stations actually operate on three different frequencies, since by doing so the required interference ratios can be drastically reduced. This has to do with synchronization of the horizontal sweep, which is the first thing to become affected when interference enters. The three frequencies consist of a nominal frequency and two others, 10 kHz above and below the nominal. Precision required is 1 kHz. If two stations are separated in frequency by either 10 or 20 kHz, they are said to operate on offset frequencies; otherwise they are non-offset. As Table 7 shows, the difference in required desired-to-undesired ratios is a full 17 dB.

Table 7. Design Parameters for a Grid of Channel 10 Television Stations

Frequency	193 MHz
Polarization	Horizontal
Receiver Antenna Height	9 m
Receiver Antenna Gain*	2 dBi
Receiver Line Losses*	2 dB
kTB (B=4) MHz	-138 dBW
Urban noise, above kTB*	19 dB
Required signal-to-noise ratio	30 dB
Required D/U ratios:	
Offset frequencies	28 dB
Non-offset frequencies	45 dB
Required reliability*	70% locations, 90% time
Transmitter power, EIRP**	57 dBW
Transmitter antenna height**	305 m
Service range**	64 km
Station separation**	274 km

*These entries define Grade A service.

**These entries are the maximum values presently used in Zone 1. The service range is a calculated value for Grade A service assuming 50% confidence and no interference.

To take advantage of this feature, we should arrange to have neighboring stations on offset frequencies. We superimpose on our triangular grid what is essentially a three-channel network in the manner portrayed in Figure 6. In that figure the zero, plus, and minus signs indicate the positions of stations operating on the normal, high, and low frequencies, respectively. Let s be the "separation distance"--the distance between adjacent nodes of the grid. Then we note from Figure 6 that each station is surrounded by a circle of six offset stations at the distance s and by a second circle of six non-offset stations at the distance $\sqrt{3}$ s. We have succeeded in removing the most grievous interference problems to a further distance.

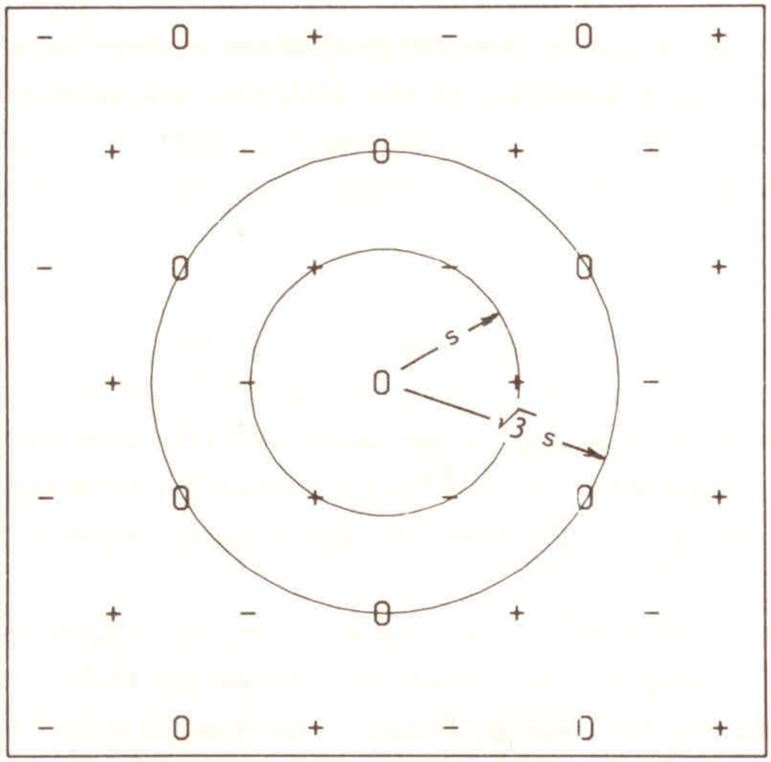


Figure 6. A triangular grid of cochannel television stations showing the arrangement of the three offset frequencies.

At any one location a receiver will attempt to view a desired station while being subject at the same time to interference from all the other co-channel stations. That there are more than one undesired station is called the "multiple interference problem." Presumably the signals from the several undesired stations will add together incoherently to form a total interfering signal; the total power will be the sum of the individual powers. But when two signal levels differ by only a few decibels the corresponding powers differ by a great deal and the smaller contributes very little to the sum. We shall therefore assume that of the many interfering signals only the strongest is of concern. Going a step further, we shall assume that the desired station is the nearest and that the one undesired station of concern is the next nearest.

Suppose a receiver location is on the direct line joining the desired station with an adjacent offset station. If it is distant x from the desired station, it will be distant s-x from the undesired station. We can then compute the desired-to-undesired ratio R(x); or more precisely we can compute the necessary quantile of this ratio. For small x we expect R to be very large; as x increases, R will decrease monotonically, reaching very large negative numbers as x approaches s. Indeed, since we are using quantiles on the high side and thereby favoring the undesired signal, R will reach zero when x is somewhat short of the midpoint at s/2. If R_o is the required desired-to-undesired ratio, there will be a distance r at which

$$R(r) = R_o \quad . \tag{11}$$

Since R_o is positive, r will be less than s/2. For distances greater than r, the ratio R(x) will be less than the required ratio, and interference will be intolerable. But for distances less than r, the observed ratio exceeds the required ratio and interference is tolerable. We say that r is the "interference-free range" although, since we are not really free of interference, the term is somewhat of a misnomer.

If we consider another radial leading out from our desired station, then the interference-free range along that radial will be larger than r since the distance to the undesired station is somewhat larger. However, we must not forget that there are six undesired stations surrounding the desired station at a distance s. When the bearing of the receiver location changes sufficiently it will be subject to interference from another one of these six. In consequence there will be a region surrounding the desired station which we may call the interference-free service area of that station. It will have hexagonal symmetry and an inscribed circle of radius r.

Generally, this region will look very much like a regular hexagon with, however, convex, curvilinear sides. But note that at the vertex of this region there will be two equidistant undesired stations. One may well imagine, therefore, that this is a real case of multiple interference, that the true interference-free range will be somewhat less than indicated by the position of the corner, and that we should suppose these corners somewhat rounded off. Without entering into too many details, we may then suppose that the actual interference-free service area is precisely a circle of radius r.

We must also consider the effects of interference from the six nearest non-offset stations. Ignoring for the moment the offset stations, we find again a circle within which service will suffer only tolerable interference. Thus for any given separation distances we find two service ranges, one concerned with the offset stations and one with the non-offset stations. We shall assume that the interference-free range r which is to be the primary result of our calculations is equal to the smaller of the two. Thus in our final picture surrounding each node of our triangular grid there is a circle within which one can obtain satisfactory service.

The area of a single one of the triangles that form the grid is $(\sqrt{3}/4)s^2$ while the area of that triangle which is covered by an interference-free signal from one of its three vertices is $(\pi/2)r^2$. Thus the fraction of a single triangle (and hence of the entire country) that is served is the number

$$\rho = (2\pi/\sqrt{3})\ (r/s)^2. \qquad (12)$$

It is then our aim to find the optimum value of s--the value of s that maximizes this fraction.

We must hasten to note that this is a narrow use of the term "optimum." While it does seem to be a natural definition, in actual practice there may well be other influences that should be considered. For example, the resulting ranges may be too small to make an individual station economically viable; or the ranges may be too large so that overcoming noise requires more transmitter power than is reasonable. Of course, the most severe criticism of our approach is that we seem to be covering land whereas we should be trying to reach people. It is one of our implicit assumptions that the two problems are equivalent, or, at least, that the solution of one provides useful information for the solution of the other.

It is interesting to see what happens to our problem if we use a simple model of radio propagation. For example, let us assume that free-space calculations suffice. For the offset stations we find

$$R(x) = 20\ \log[(s-x)/x], \qquad (13)$$

and setting $R(r)=R_o$ and solving for r we obtain

$$r/s = (1 + 10^{Ro/20})^{-1}. \qquad (14)$$

Similarly, if R_1 is the required desired-to-undesired ratio for the non-offset stations, we find

$$r/s = \sqrt{3}\ (1 + 10^{R1/20})^{-1}. \qquad (15)$$

In both cases the ratio r/s, and hence also the fraction ρ, is independent of the separation distance s. There is no optimum value, and from this point of view it makes no difference how far apart the stations are placed. Using values from Table 7, we find that the non-offset stations are overpowering and that the final fraction of interference-free service coverage is a paltry

0.03%. Clearly, the bulge of the earth plays a very important role in allowing television service at all.

Turning to the ITS irregular terrain model, a particular question will be how to treat properly the statistics involved. Clearly, we require something very like the broadcast mode of variability modified, however, by the need to find quantiles of a ratio of two signals.

As we have noted in Section 6.4, it will be our assumption that location and time variabilities of the two signals are statistically independent, but that situation variabilities are exactly correlated. Now to each service area there presumably corresponds a separate situation and therefore, one would suppose, a separate interference-free range--the range is a random variable. But because the situation variabilities are exactly correlated, they have a strong tendency to cancel against each other in the ratio; consequently the range should exhibit only a very small variability. In any case, however, since we seek the fraction of the country covered, it is really the average service area we need. This average we may approximate with the median, and then it will follow that we require the medians of the situation variabilities.

Going immediately to median values for the terms involving situation variability, we find from (3) and (4) that at a particular receiver location ℓ and a particular time t the desired-to-undesired ratio becomes

$$R(x) = 20 \log[(s-x)/x] - A_{oD} + A_{oU} + y_{LD}^{(\ell)} - y_{LU}^{(\ell)} + y_{TD}^{(t)} - y_{TU}^{(t)} , \tag{16}$$

where the additional subscripts D and U refer to the desired and undesired transmitters, respectively, and where the A_o's are overall medians of attenuation. Remembering how the required reliability is stated, we first note that satisfactory service is achieved at the location ℓ provided R exceeds the required desired-to-undesired ratio R_o for at least 90% of the time. In other words, we must first compute the 0.9 quantile of R for each fixed location. As one sees, this may be reduced to the simple problem of finding the quantile $Y_{TR}(.9)$ for the difference between the two independent random variables, y_{TD} and y_{TU}. At the next step we must ask whether this 0.9 quantile exceeds R_o at a sufficient number of locations. We must therefore compute the 0.7 quantile of the time-variant quantile. Again, this reduces to the problem of finding the quantile $Y_{LR}(.7)$ for the difference between the two independent random variables, y_{LD} and y_{LU}. In the end we find that the quantile of R(x) required by the problem is given by

$$R(x) = 20 \log[(s-x)/x] - A_{oD} + A_{oU}$$

$$+ Y_{LR}(.7) + Y_{TR}(.9) \quad . \tag{17}$$

To compute the two quantiles of deviations, we use the method of pseudo-convolution to obtain

$$Y_{TR}(.9) = -[Y_{TD}(.9)^2 + Y_{TU}(.1)^2]^{1/2} \tag{18}$$

$$Y_{LR}(.7) = -[Y_{LD}(.7)^2 + Y_{LU}(.3)^2]^{1/2}$$

where the additional Y's are the indicated quantiles of the corresponding random variables. Note that both quantiles of the ratio are negative since both refer to fractions greater than 0.5. Note also that for the undesired station quantiles we use the complementary quantile. This is because to form the convolution we must add the two random variables y_D and $-y_U$; and if y_U has the quantiles $Y_U(q)$ then $-y_U$ has the quantiles $-Y_U(1-q)$.

There remains the problem of how to obtain the four quantiles of individual deviations. We suppose we have available to us the threefold quantiles of attenuation $A(q_T,q_L,q_S)$ as described in Section 6, and from these we shall obtain the required deviations. Always putting q_S=0.5, we may write

$$Y_{TD}(.9) = -A_D(.9,\ .7,\ .5) - A_D(.5,\ .7,\ .5) \tag{19}$$

$$Y_{TU}(.1) = -A_U(.1,\ .3,\ .5) - A_U(.5,\ .3,\ .5)$$

$$Y_{LD}(.7) = -A_D(.5,\ .7,\ .5) - A_D(.5,\ .5,\ .5)$$

$$Y_{LU}(.3) = -A_U(.5,\ .3,\ .5) - A_U(.5,\ .5,\ .5)$$

where, we note, the A_D and A_U are different functions since they refer to different distances.

At this point we have gathered together the parameters and the formulas needed. It remains to make the calculations. To do this we have assembled a short applications program that considers a sequence of proposed station separations s, computes desired-to-undesired ratios, solves (11) for the interference-free range r, and prints it out together with other pertinent data.

The results of one run of this program are shown in Figure 7. To emulate conditions in Zone 1, we have assumed transmitter heights of 300 m, hilly terrain with Δh=90 m, and a continental temperate climate with N_s=301 N-units. The curve in Figure 7 shows the fraction ρ as a function of the separation distance s. The maximum appears when s=210 km at which separation

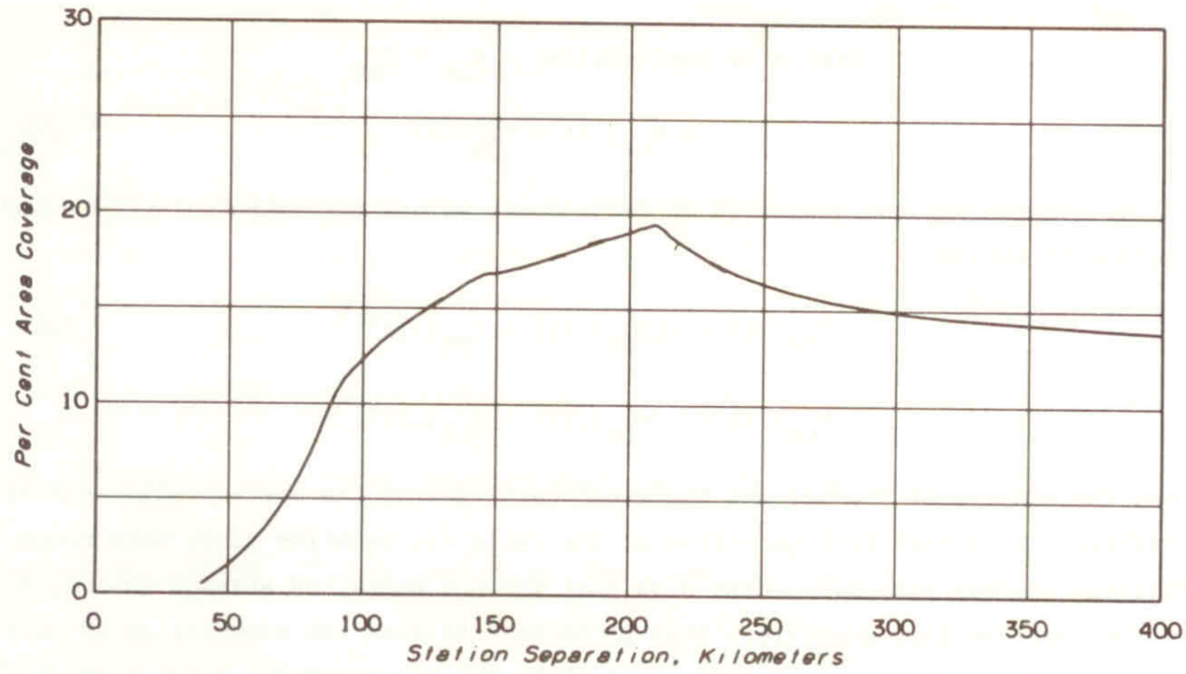


Figure 7. Fraction of the country receiving an interference-free signal versus the station separation. We have assumed transmitting antennas 300 m high and average terrain characteristics.

distance the interference-free range is 48.6 km, implying that 19.5% of the country is covered with an interference-free signal. Note that the curve has two corners, one at the optimum distance of 210 km and another at about 145 km. They appear because for distances between them it is the non-offset stations that determine the interference-free range whereas for other distances the range is determined by the closer offset stations. At the optimum distance, therefore, both offset and non-offset stations are contributing equally to the interference. It may also be interesting to note that for a separation distance of 274 km, which is the distance presently invoked for Zone 1, the interference-free range is 56.5 km and 15.4% of the country is covered.

With the interference-free range determined, we can also find the transmitter power required to overcome noise. Using the receiver characteristics listed in Table 7, and now requiring 90% confidence, we find that for the conditions that obtain at the optimum spacing, we need 53.3 dBW EIRP.

If we were to continue with this problem, we would evaluate these same quantities for varying terrain types, climates, transmitting antenna heights, and frequencies (television channels). The way in which the optimum separation distance varies might then lead to a second phase of the problem which attempts a broader look at how best to provide television service to the country.

7.3 Comparison with Data

As a final example, we want to show a comparison of predictions from the ITS area model with a single set of measured data. This will give us the opportunity to display some of the more obscure aspects of the model and, perhaps, to demonstrate the terrible intransigence of measured data. It should be emphasized at the outset that no single set of data can claim to "verify" the model; that requires a large number of sets of data. Our purpose here can only be to show some of the techniques that might be used in an extensive study.

The set of measured data we have selected is that reported by McQuate et al. (1970). It is familiarly known as the R3 data since it involved the third receiving site in a sequence of special measurement programs. We have chosen this set on a whim, by which we hope to mean "at random." It is immediately available to us; there is no previously published account of its analysis; and it serves to illustrate several problems.

In the measurement program six frequencies were used ranging from 200 MHz to 9 GHz. The transmitter was mobile; it would go to a preselected point, set up there, and begin operations. The receiving antennas were mounted on a carriage which could be continuously moved up and down a 15 m tower; received signal levels were recorded as a function of height.

For this particular set the receiver tower was erected atop North Table Mountain in central Colorado (to be precise, at 39°47'30" N., 105°11'59" W.); it was at the edge of a cliff from where it could look out to the north and east across the plains. The experiment was specifically designed to simulate a low-flying aircraft above the plains, and so the transmitter sites were all in this sector. The plan for choosing these sites involved taking a map of the region, drawing on it circles of convenient radii (5, 10, 20 km, etc.) with centers at the receiver site, and finding where these circles crossed convenient roads. Except for the necessity of keeping to roads, this procedure appears to provide a suitably random selection. We shall have more to say about this below.

Again at whim--and perhaps because the data might also be construed to simulate a UHF television transmitter--we have chosen to use the data for 410 MHz. This leaves us with a specific one of the receiver heights to choose. If we examine the published curves, we note that for increasing receiver height the signal levels tend to start out with rather low values, increase dramatically in the first several meters, and then show a moderate, but not excessive, amount of lobing. Now actually, in order to allow for sufficient guying, the tower was positioned some 8 m back from the edge of the cliff. It is therefore not surprising that at the lower receiver heights we observe ground effects which rapidly disappear at higher heights. The lobing pattern we would attribute to a scattering of energy from the cliff's edge.

In keeping with the purpose of the experiment, we should therefore choose one of the higher receiver heights. And in keeping with the spirit of the kind of radio propagation model we are considering where multipath effects are kept to a minimum, we should choose not a specific height but a smoothed-out average over a range of heights. What we have done is to use signal levels from heights of 10, 11, 12, 13, and 14 m; to find their median value; and to ascribe that value to a height of 12 m above the cliff top.

The resulting data for the 44 different paths on which these measurements were taken are displayed in Figure 8 where we have plotted the observed attenuation relative to free space versus distance. A glance at this plot can only bring dismay. There seems to be no structure at all to this set. Over half of the points are clustered around free space, and if there is any

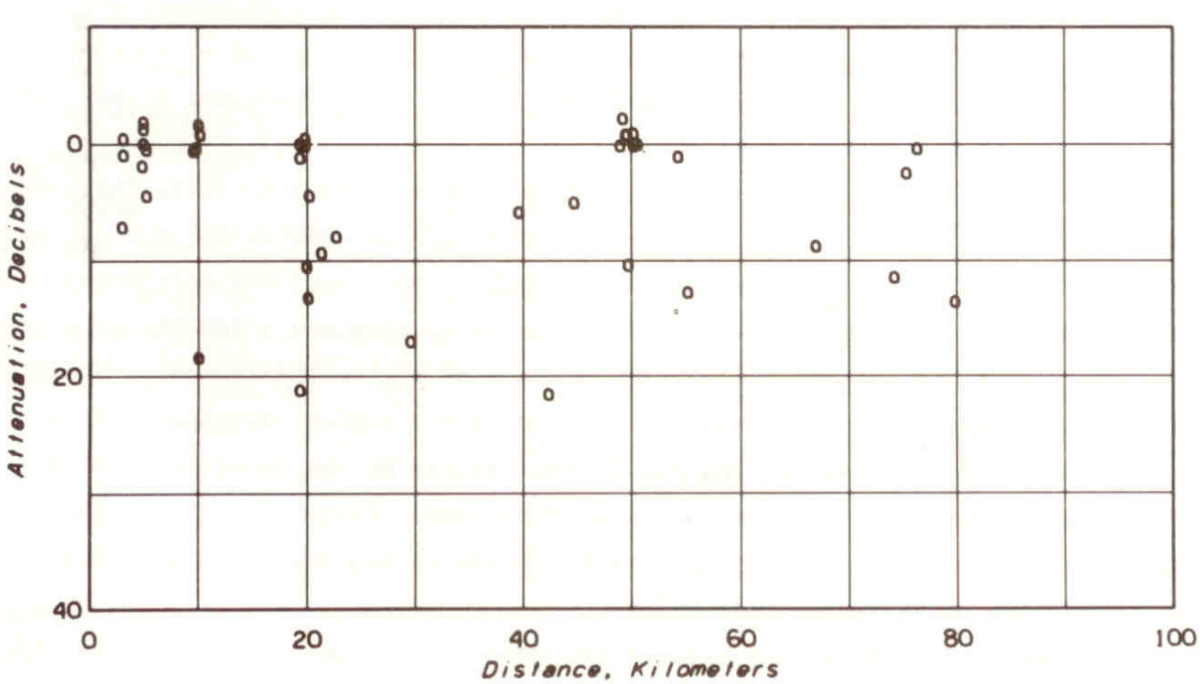


Figure 8. The R3 data at 410 MHz; 44 points.

trend it would almost seem that attenuation decreases with distance. Our first inclination is to abandon this set and to look elsewhere. But remembering our aim is merely to provide an illustrative example, we continue. If one is truly interested in sets of data concerning air-to-ground or UHF television propagation, summaries of others may be found in Johnson and Gierhart (1979) or in Damelin et al. (1966).

In contrast to the previous examples, here we are concerned with a very particular region of the world. We must therefore determine parameter values for the model from that particular region, and, we should emphasize, from that particular situation.

We would assume that the complete set of data represents a sample drawn from one single situation--although the wide range of distances might argue for a different conclusion. As for the kind of statistics involved, we recall that the measurements are spot measurements; that is, the transmitter was dispatched to a random location and at some random time the measurements were recorded. Thus time variability is inextricably entangled with the more naturally expected location variability. Our treatment of statistics should therefore be in terms of the mobile mode of variability. We use the notation $A(q_L,q_C)$ to indicate the quantile of attenuation which, with confidence q_C, is not exceeded for at least q_L of the observations. The subscript L serves to remind us that in reality time variability is small at the short distances we shall consider, and the observed variability will be dominated by location variability. We refer to confidence rather than "fraction of like situations" since we have no set of like situations, nor do we ever expect to obtain one. We can only suppose if we make a large number of (independent) statements with a given confidence q_C, then q_C of them will be correct irrespective of whether the situations involved are like or unlike.

To decide on the proper environmental parameters, our first thought might be to use Figures 2 and 3 together with the coordinates given above. Using digitized versions of these maps, we find values Δh=444 m, N_S=236 N-units, and also an average terrain elevation of 2120 m. But for the situation here these values are wrong. Directly west of North Table Mountain and directly west of the sector in which the measurements were made, there rise abruptly the foothills of the Front Range of the Rocky Mountains. The mountains of this range are majestic, being among the highest in the contiguous United States; and the foothills provide spectacular changes in elevation. Now the values given above assume that it is just as likely that a path goes west into these foothills as that it goes east into the Colorado plains, and thus they reflect the properties of the foothills to a considerable extent. The paths in which we are interested, however, stay entirely clear of the foothills and exhibit quite different characteristics.

In the original data report quoted above, plots are provided of the terrain profiles for each measurement path. From these plots, or digitized versions of them, we can derive directly the parameters we need. For example, a short study of the plots shows an average terrain elevation of about 1700 m. From Figure 2 we find N_o=300 N-units and hence from (2) we derive N_s=250 N-units.

It is clear that the cliff and the mountain below must be included as part of the antenna structure; this is particularly so when we remember the primary purpose of the experiment. We must therefore estimate the height of the mountain above its base as one important parameter. In addition we also need the terrain irregularity parameter Δh.

If we are given a terrain profile leading away from the receiver tower, we can do two things: We can compute the (asymptotic) Δh for that path, and we can find a linear least squares fit to the profile (indeed, that may have been part of the computation for Δh), extend that back beneath the receiver tower, and so find a value for the height of the cliff top above that profile. In the computations here, we should be careful not to use the cliff as part of the profile; we are assuming that it is part of the antenna "tower." And furthermore, at the bottom of the cliff there is a steep talus slope that, in some directions, may extend outwards nearly 1 km. This, too, should not be considered part of the profile. Our computations should use only a portion of the profile, and that portion should begin at least 1 km away from the receiver site.

Using the path profiles of the experiment, we have made such calculations. Restricting ourselves to the 34 paths that are nominally 10 km or more in length, we have found that the individual values of the asymptotic Δh vary from 31 to 212 m with a median of 126 m. It is this latter that we would propose to use in predictions. Similarly, we have found that the top of the cliff lies between 156 and 390 m above the profiles and that the median value is 263 m. Adding the 12 m tower height, we would propose 275 m for the structural height of the receiver.

The calculations on each profile were made by computer using subroutines related to the ITS model in the point-to-point mode and listed in Appendix A. It may seem odd that we use point-to-point techniques to treat an area prediction; but then the determination of the environmental parameters of a specific area is clearly related to the determination of these parameters on individual paths. Still more odd, however, is the fact that we have used precisely those paths on which the measurements were made. To justify this we would argue (1) that the profiles are conveniently available, having been determined as part of the measurement program, and (2) that there seems no better way to assure that we are making our determinations from the same "situation" as that from which the measurements came. An alternative is possible and would have been forced on us if we had been making predictions prior to the measurement program. We simply proceed as before, choosing, however, our own set of paths. While this set is arbitrary, it should imitate as closely as possible the situation of the measurement program. We would suppose a sequence of radials beginning at the receiving site and extending into the sector of interest. And finally we would use terrain profiles for that portion of those radials that extend from 1 km to perhaps 40 or 50 km.

For the final parameters, we note that the transmissions were horizontally polarized, and we would assume average ground and a continental temperate climate. At the high frequency and the short distances involved, these assumptions are not critical.

In Figure 9 we have replotted the data of Figure 8 and superimposed on them predictions from the prediction model using the parameters given above. To be precise, we have plotted five quantiles of the expected attenuation as functions of distance. The central solid curve is the overall median prediction--the "best estimate." For the other four we can interpret them as meaning that with 90% confidence at most 10% of the observations will lie

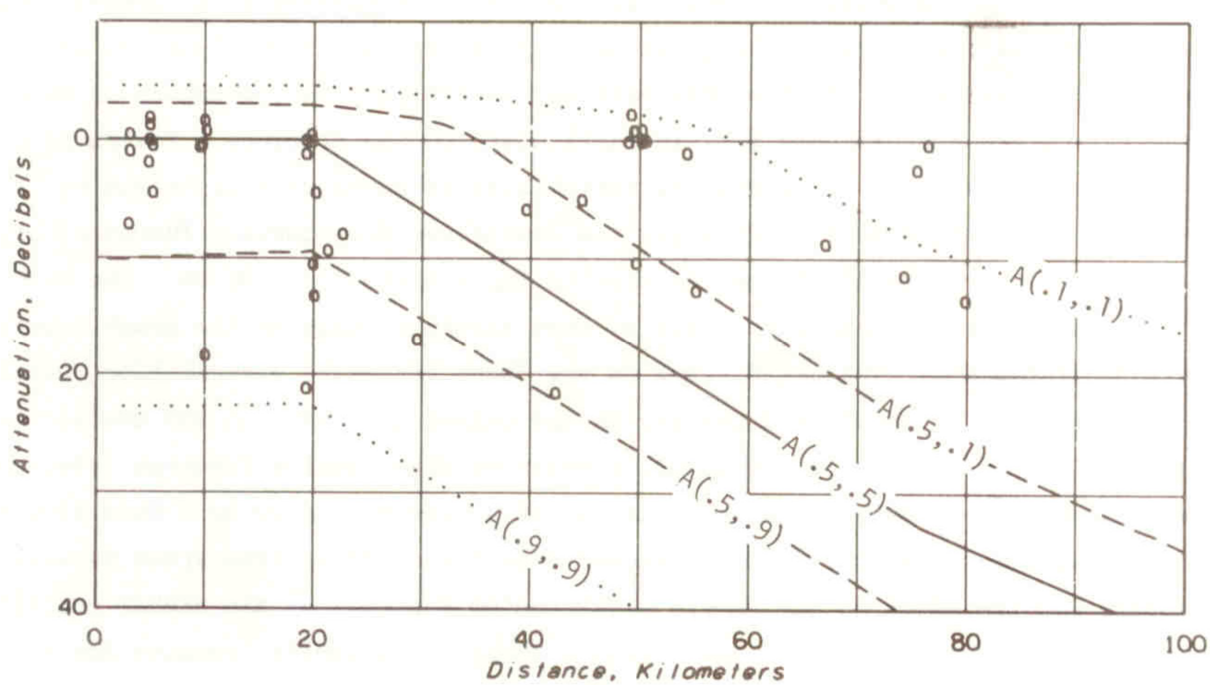
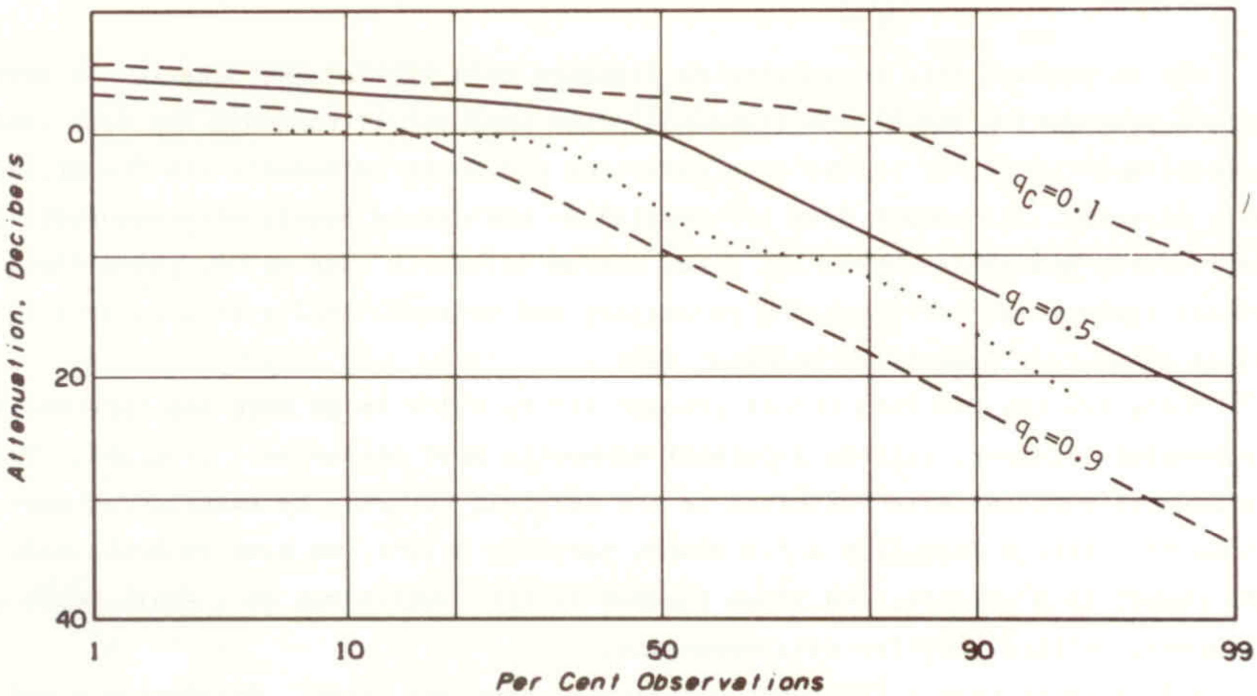


Figure 9. Predicted and observed values of attenuation for the R3 data. Assumed parameters: f=410 MHz, h_{g1}=275 m, h_{g2}=6.6 m, Δh=126 m, N_s=250 N-units.

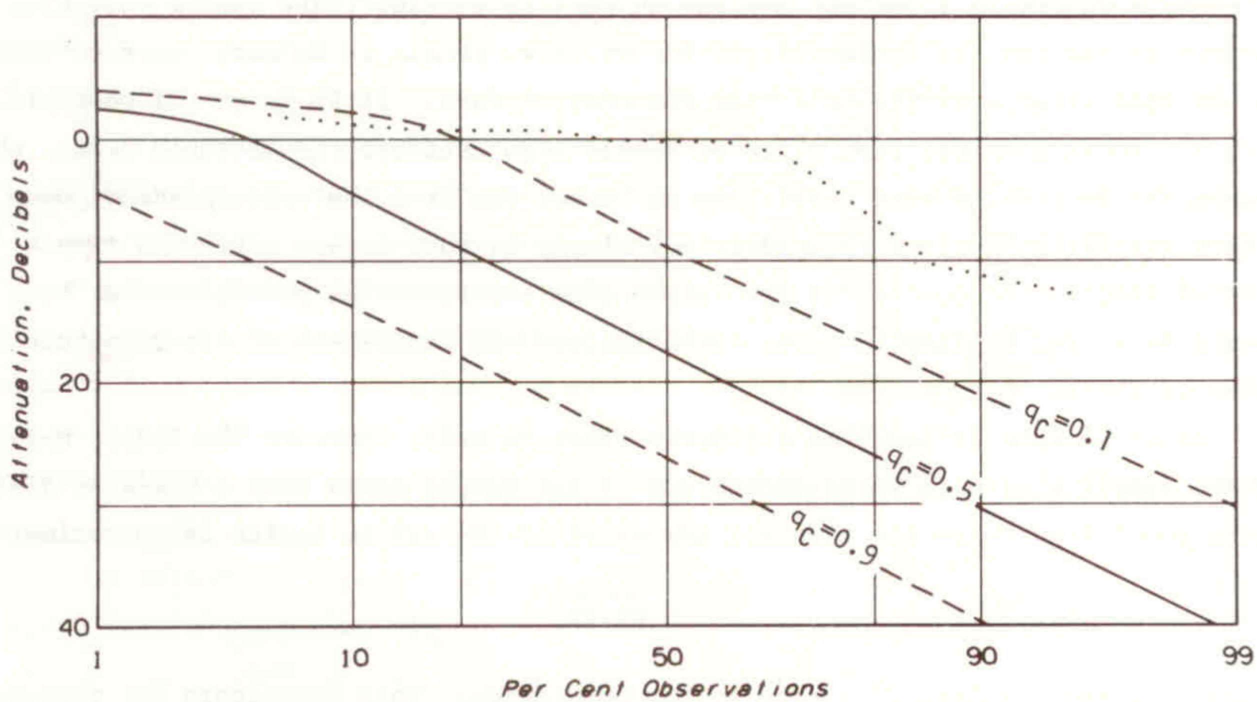
above the upper dotted curve and at most 10% will lie below the lower dotted curve; and with the same confidence at most 50% of the observations will lie above the upper dashed curve and at most 50% below the lower dashed curve. Indeed, of the 44 observations available to us, we find that 2 lie above the upper dotted curve and none below the lower dotted curve; that 14 lie above the upper dashed curve and 5 below the lower dashed curve. Note that for distances greater than 60 km, the difference between the two dotted curves is about 45 dB. According to the ITS irregular terrain model, if we want to consider an interval within which we are fairly confident that a large majority of observed signal levels will lie, then that interval must be very wide.

One notable aspect of Figure 9 is the tendency of both predictions and observations to level out at free space values where the attenuation vanishes. In the case of the predictions, this tendency demonstrates both the "free space" region where the reference attenuation vanishes and the effects of the "modifying" function described in Section 6. The observations, we note, may take on rather large positive values of attenuation but will form tight clusters about zero. Of course, we also note that at large distances the observations still show a stubborn tendency towards free space values that the predictions fail to reproduce.

The wide scatter of attenuation values that one observes in Figure 8 exhibits the "observational variability"--which we are here interpreting as a combined location and time variability. Although it may not be apparent from Figure 8, in principle the statistics of this observational variability should depend on the parameters of the measurements and, in particular, on the distance. Following this stricture we may look at observational variability at particular distances by plotting, as we have done in Figure 10, the cumulative distribution functions of attenuation. In Figure 10(a) are the results for a distance of 20 km. The horizontal axis is a normal probability scale so that straight lines on the graph represent normal distributions. The solid curve is the "best estimate" distribution function as obtained from the model, and the two dashed curves give 10% and 90% confidence ranges. The dotted curve is the sample cumulative distribution function using the 10 measurements made on paths with distances of approximately 20 km. This figure demonstrates again how both predictions and data level off at free space values. Note also how the slope of the sample distribution function at the higher quantiles agrees with that of the predictions. Figure 10(b) gives similar results for a distance of 50 km. The wide disparity here between predictions and observations restates the tendency mentioned above for the observed data to attain free space values even at the longer distances.



(a) Distance, about 20 km; 10 points.



(b) Distance, about 50 km; 11 points.

Figure 10. Predicted and observed curves of observational variability for the R3 data.

Let us examine this dependence on distance in a little more detail. In developing a propagation model, one technique often employed is to group the data into successive intervals of values of a parameter one wants to examine--in the present case, distance. If enough data are available, the use of overlapping intervals is particularly desirable. Then one finds median values in each group, plots these medians against the corresponding parameter, and tries to construct a curve that passes adequately near those medians.

There are too few data in our present set to allow us to make any very profound discoveries. Indeed, this is a paradox common to most measurement programs. There are several thousand data exhibited in the original report. By restricting our selves to a single frequency and a single receiver height, we have suddenly reduced this number to a mere 44; and if we further restrict ourselves to a small range of distances, we find very few data remaining.

All of which brings us to the question of sampling error. Presumably a set of measurements represents a limited sample drawn from the population that comprises a situation. Any statistics we compute from this sample, such as the cumulative distributions of Figure 10 or the medians we want to consider, are random variables subject to the laws of probability. Any value we obtain is no more "correct" than is the face after a single coin toss the correct face. If it is at all possible, when we report a sample statistic, we should also estimate its probable error; that is, we should provide some indication as to how far from the corresponding population statistic it might reasonably be, simply because it was estimated from a limited sample. Of course, as the sample size increases the probable error here should decrease, tending to zero. Nevertheless, an indication of its magnitude is quite valuable.

As an example of how such estimates might be made, consider the sample median. If the sample size n is large enough and if the sample comes from a normally distributed population, then the standard deviation of the sample median is approximately

$$\sigma \sqrt{\pi/2n}$$

where σ is the standard deviation of the population. This represents the probable error of the value we obtain and is the estimate we might provide. The formula comes from the theory of large samples, but we do not require here very great accuracy and the formula is probably adequate even for fairly small sized samples. To particularize, the observational variability predicted by the ITS irregular terrain model for any one distance has a standard deviation of about 10 dB; so if we have a sample of 10 data then the standard deviation of the sample median will be approximately 4 dB; the 10% and 90% confidence levels for the population median will differ from the sample median by about 5 dB. To put this another way, we would then have only 80% confidence that the population median lies within a 10 dB range.

In Figure 11 we have plotted medians for the few groups of distances available to us. The vertical bars are drawn at the median distances for each group; the ticks across them indicate the sample medians of attenuation; and their end points determine an approximately 10% to 90% confidence interval for the population medians. In deciding on these confidence intervals, we have not used the large sample theory described above; instead we have employed the more robust scheme given by Walsh (1962; ch. 6). This scheme depends only on the sample values and is nonparametric in that it assumes very few properties of the population(s) from which the data are drawn. It provides a discrete sequence of exact confidence levels from which we have picked the one lying closest to 10% or 90%. Above each bar in Figure 11 we have indicated what the exact level is. For example, at 20 km 6% of such samples will be drawn from populations whose medians lie above the bar and a like number from populations whose medians lie below the bar. The bar itself thus comprises an 88% confidence interval.

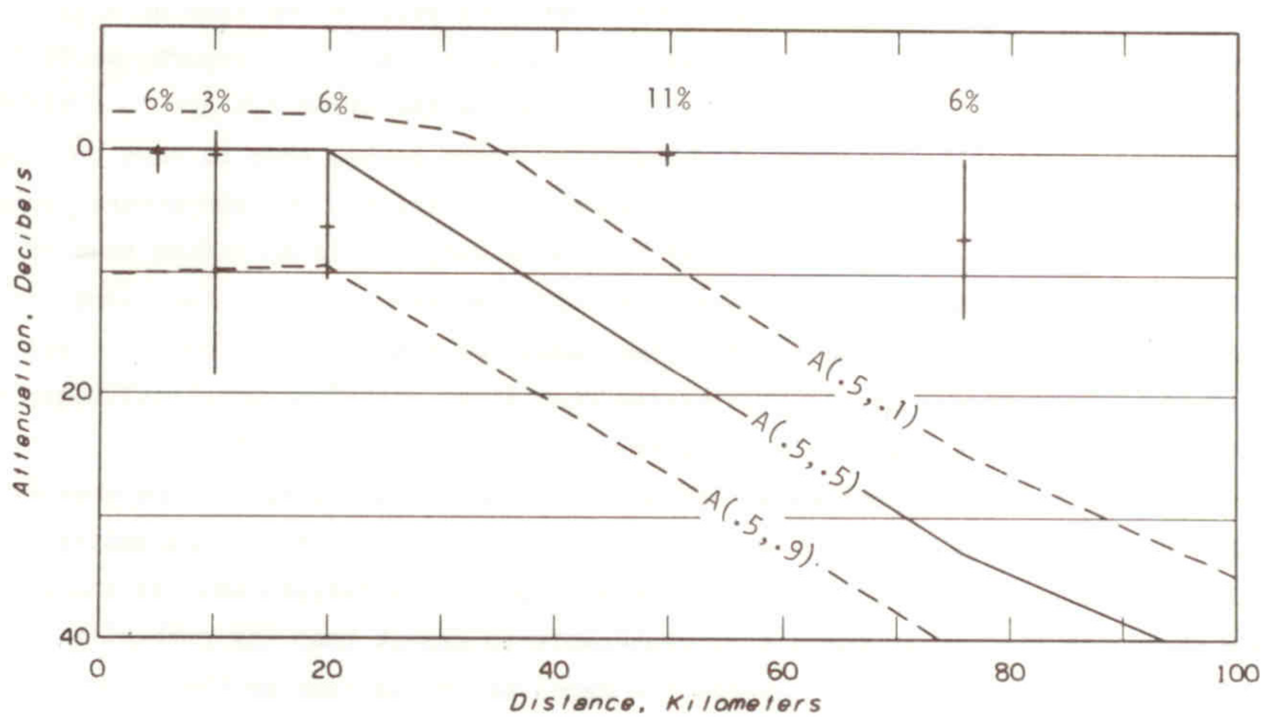


Figure 11. Predicted and observed medians for the R3 data. The bars indicate confidence levels for the sample medians at approximately 10% and 90%.

Note that in Figure 11 the term "confidence" is being used to describe two quite different quantities. The vertical bars indicate confidence intervals pertaining to sampling error arising from a limited sample and induced, presumably, by observational variability at the given distance. On the other hand, the curves which also form a part of Figure 11 delineate a confidence interval which is a prediction of the model and pertains to situation variability. Nevertheless, both confidence intervals refer to the same value--the population median. For example, let us consider the four measurements taken at almost 80 km. As indicated by the vertical bar we have 94% confidence that the sample was drawn from a population whose median attenuation was less than about 14 dB. On the other hand, we predict from the model that with 90% confidence the same population should have a median attenuation greater than about 25 dB. While these two statements are not entirely contradictory, the disparity is certainly considerable.

Very often a comparison, such as we are making here, between model predictions and measured data, is as much a critique of the data as it is of the model. Consider again the four measurements taken at nearly 80 km. As one can tell from the abrupt change in slope of the predicted median attenuation, the smooth earth horizon distance implied by our parameters is about 76 km. If we account also for an irregular earth, paths at this distance should mostly be well beyond line of sight. And yet, looking at the corresponding profiles in the original data report, we find that two of the four are clear line-of-sight paths while the other two are just barely obstructed. The hills, instead of obstructing these paths, seem to have provided platforms which elevate the transmitter above the terrain. Two conjectures come to mind: (1) there is some phenomenon of nature in which hills do indeed tend to elevate more terminals than they obstruct, and that the ITS irregular terrain model fails to recognize this; and (2) for some reason we shall never know there was a tendency in the experiment to pick transmitter sites with favorable positions; in other words, these sites were not, after all, chosen "at random."

For the second conjecture we can devise a test of sorts to examine whether the paths do show a bias. We look at the immediate foreground of the transmitter in the direction of the receiver and ask what the slope of the terrain is. If the sites are chosen at random, it would seem reasonable to expect that the probability with which the ground slopes down towards the receiver is the same as the probability with which it slopes up; we would also expect there will be cases in which the transmitter is at the top of a hill and some in which it is at the bottom of a depression. The test, then, consists of comparing the number of paths which slope down with the number which slope up; of course, there will be sampling error so that we would not expect these numbers to be equal, but we would expect them to be reasonably close. While the test is obviously inspired by the theory of radio propagation, note that it is purely geometric and in itself is entirely independent of what use is to be made of the paths involved.

To carry out such a test for the R3 data, we have simply examined the published terrain profiles and tried in that way to categorize each transmitter site. For the four paths nearly 80 km long we found three of them slope down towards the receiver and the fourth is at the top of a hill. For the 11 paths at approximately 50 km six slope down, one slopes up, two are at the top of hills, one is at the bottom of a depression, and the last is located in a long level stretch that we cannot classify. For all 34 paths whose lengths are nominally 10 km or more, 12 slope down, 3 slope up, 3 are at tops of hills, 2 at bottoms of depressions, and the remaining 14 are on long level stretches. In actual practice we discovered that these classifications are somewhat subjective, and so we would not give too much credence to the exact results. Nevertheless, there is a strong indication here that the paths are indeed biased towards high received signal levels.

Perhaps, then, our choice of parameters is wrong in that we have assumed random siting for the transmitter. Going to the other extreme, we have redrawn Figure 9 using, for the predicted quantiles, all the same parameters except that we have assumed the transmitter is very carefully sited. The results are shown in Figure 12.

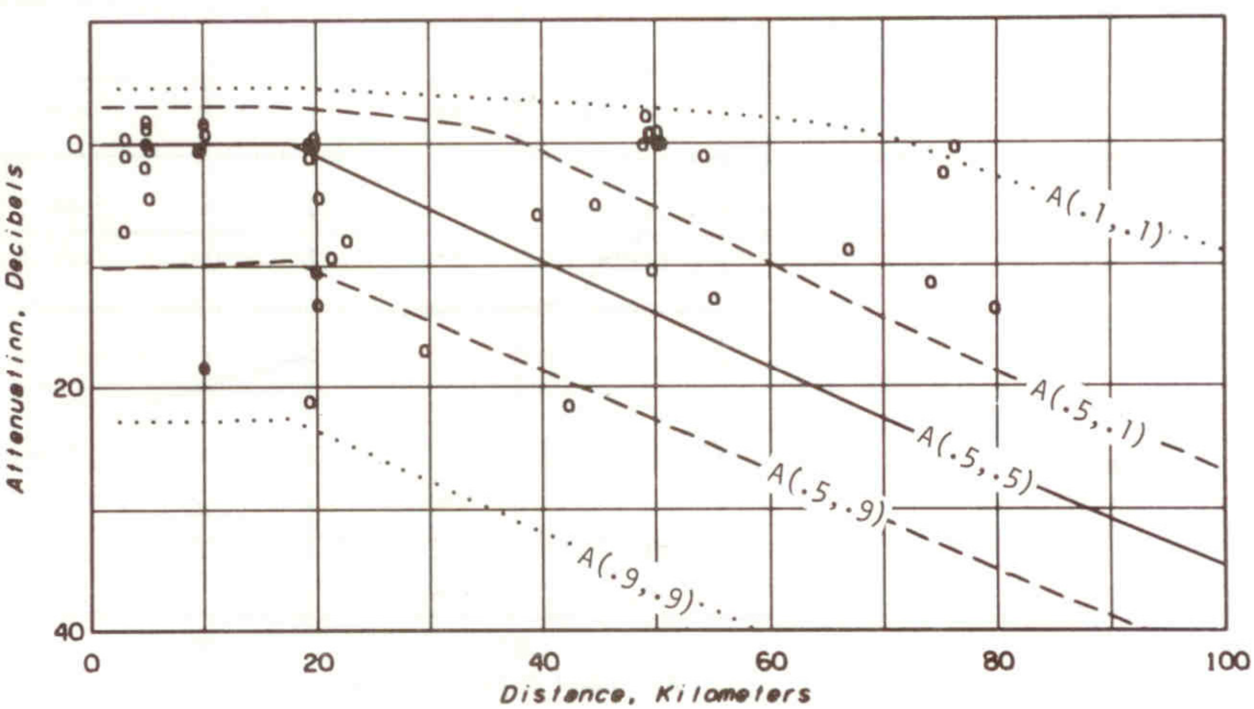


Figure 12. Predicted and observed values of attenuation versus distance for the R3 data. The predictions assumed the transmitters were sited very carefully.

Note that the discrepancy between predictions and observations has decreased, but not dramatically so. In any case, this is "post-act" analysis, carried out after we have discovered a discrepancy. It should be avoided in any serious statistical study.

For many, the only interesting question to ask in a comparison of measurements with predictions is the question of how the totality of measurements compares with the "best estimate" (i.e., median) prediction. The idea is to compute the deviations

$$y = A(.5,.5) - A_{obs} \tag{20}$$

and to treat them as samples of a random variable. Note that the sign here is in keeping with the sign in (4) and that the predicted median must be recomputed for each observation, but only because the distance changes. Note, too, that the random variations in these deviations are a consequence of observational variability, and that, since the locations (and the times) must be assumed to have been independently selected, the deviations should be all mutually independent.

Returning to the randomly sited predictions of Figure 9, we have plotted in Figure 13 the cumulative distribution of the deviations for all 44 observations. But this plot is wrong. It is wrong because the points from which it is made come

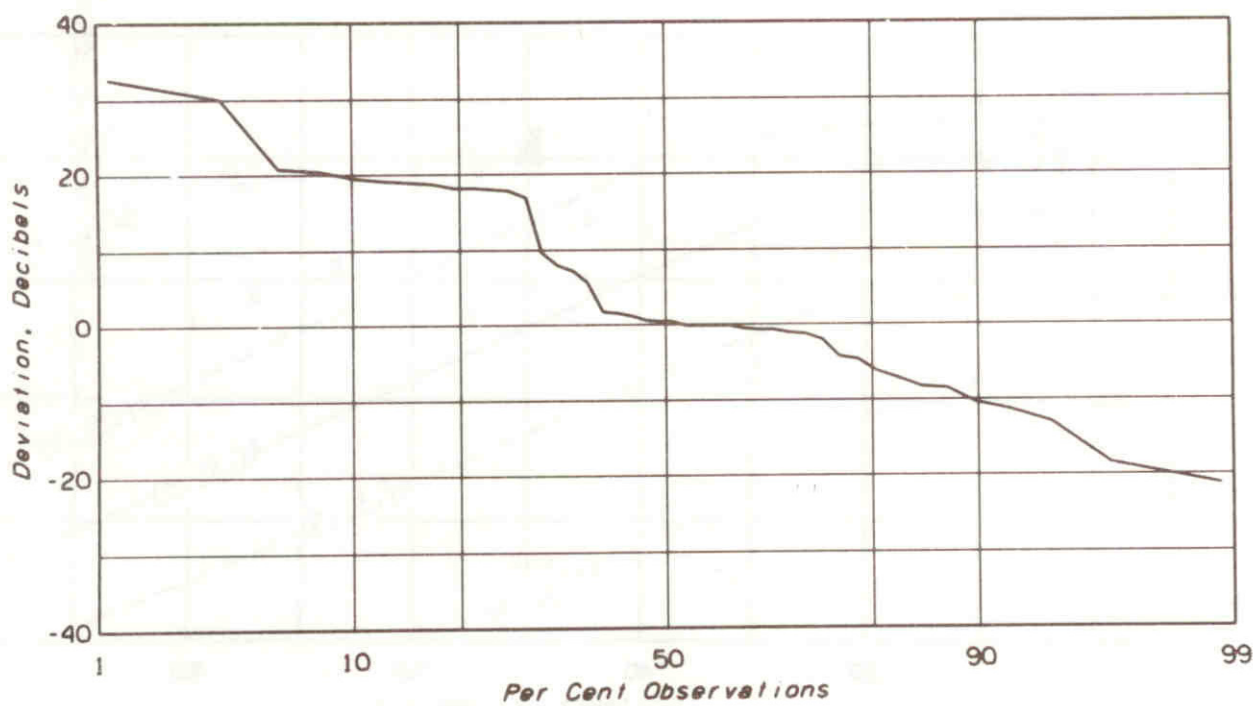


Figure 13. The sample cumulative distribution of deviations. As indicated in the text, this is a misleading plot.

from very different distributions, and they cannot be combined in this way into a single distribution. Now, while it is true that the standard deviation of observational variability changes with distance, the change is very slight and this is not what troubles us. The real difficulty involves the same feature we have noted before--that both data and predictions flatten out at free space values--and at different distances this flattening appears at different deviations from the predicted medians. Thus the fact that in Figure 13 the median deviation is about 0 dB and that this is only one point in a long, flat interval of the curve, merely reflects the fact that a great many of the observations were made at distances less than about 20 km. If the data set had not so emphasized these shorter distances, the long flat interval would not have been so pronounced. Similarly, the flat interval at about 20 dB is due to the accident that several paths had distances clustering at about 50 km. The consequence of such contamination is that the cumulative distribution function of Figure 13 can serve no useful purpose.

There is a way out of the problem. If the high fields represented by free space values had saturated the receiver or sent measuring devices off-scale, we would have been in a similar quandary. We could not then have recorded received signal levels but could only have reported they were too high to measure. Similarly, in the present case we can imagine that the propagation channel itself has become "saturated." Indeed, this is entirely in keeping with the model of variability described in Section 6.3, for the modifying function described there is analogous to the saturation curve of a receiver. We can suppose that what we really want to measure is the unmodified attenuation a', because we expect that the statistics of the corresponding deviations are nearly invariant with distance. However, we must also suppose that when the measured attenuation is nearly zero or less the value of a' cannot be determined.

At this point we have an example of what is known as a sample with censored data (Walsh, 1962; Efron, 1979) where for some of the data we have properly measured values while for others we know only that they lie in certain infinite intervals. The latter are the censored data; they should be neither discarded as useless nor accepted on an equal footing with the remaining data.

Using the method of Kaplan and Meier (1958), we can construct a sample cumulative distribution function for such censored samples. In Figure 14 we have plotted the results for the R3 data when we assume that all data are to be censored if the measured attenuation is less than or equal to 0.5 dB. We feel that this gives a fairly accurate picture of the true statistics of the deviations. Note that a full 19 of the 44 data were censored.

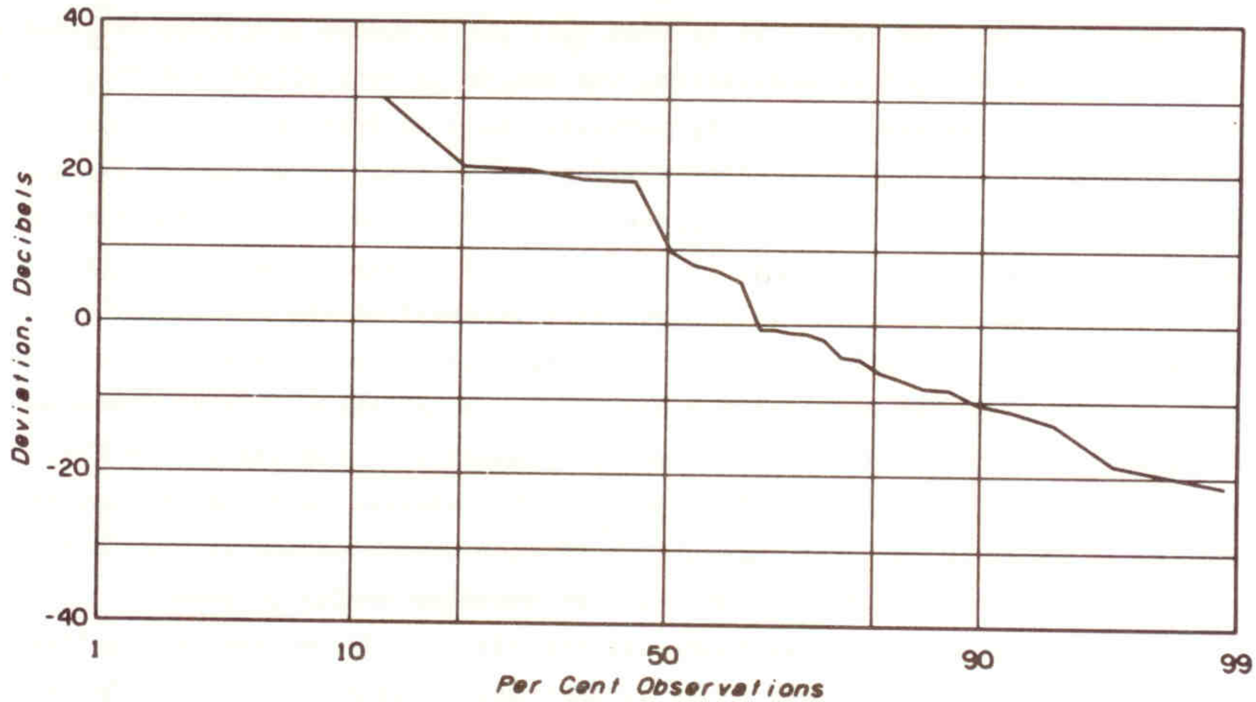


Figure 14. The sample cumulative distribution of deviations assuming the data are censored when A ≤ 0.5 dB.

Another very important example of censored data, which does not appear in the present data set but does in many others, occurs when signal levels fall below the sensitivity of the receiver. Here we truly have attenuations that are unmeasurable, but only because the radio system used is inadequate. Since one important purpose of a measurement program is to discover when it is that signal levels might be below sensitivity, the very fact that we have found such locations is of prime importance to us. Such "data" should never be discarded as being useless; they should simply be treated as censored data. In other words, although we cannot say what the actual attenuation is, we can say that it definitely exceeds some known threshold value. Experimentalists should take note here. It is an important part of the report on system parameters to provide an estimate of the system sensitivity.

From Figure 14 we can see that the sample median of the deviations is 11 dB and that the sample 20% to 80% range is 27 dB. From the latter number we find that the average slope (on the normal probability scale we have used) is 16 dB, a value that can be likened to a standard deviation. These two numbers can be used to describe the discrepancy between the measured data and the "best estimate" predictions.

The ITS irregular terrain model predicts that, barring the possibility of signal levels greater than free space, the observational variability is very nearly normally distributed. Indeed, the curve of Figure 14 appears to the eye to be as nearly linear as one could expect. If we insist that the deviations involved do indeed come from a normal distribution then, despite the fact that we have numerous censored data, we can derive more precise estimates for the underlying mean and standard deviation by resorting to the maximum likelihood estimate.

In general, we would suppose a sample of size n of which m of the deviations have observed values $y_1 \ldots y_m$ and $r(=n-m)$ are known only to exceed threshold values $\eta_1 \ldots \eta_r$. We first define the likelihood function

$$L(\mu,\sigma) = \prod_{i=1}^{m} \frac{1}{\sigma} Z\left(\frac{y_i-\mu}{\sigma}\right) \prod_{j=1}^{r} Q\left(\frac{\eta_j-\mu}{\sigma}\right) \qquad (21)$$

where $Z(x)$ is the probability density function for the standard normal distribution and $Q(x)$ its complementary cumulative distribution function. Then the estimates we seek are the values of μ and σ which maximize L.

Using the same censored data as they appear in Figure 14, we have made these calculations and find an estimated mean of 10.6 dB and an estimated standard deviation of 14 dB. It is interesting to note how these numbers relate to our model of variability. If we rewrite (4) slightly to allow for the assumed mode of variability, we might suppose

$$y = y_S(s) + \delta_L(s)\, y_L(\ell) \qquad (22)$$

and then what we have said is that for the single situation of the R3 data the value of $y_S(s)$ is estimated to be 10.6 dB. Furthermore, since the standard deviation of y_L is about 10 dB, we can also say that, again for this one situation, we estimate $\delta_L(s)$ to be 1.4. Note how, in presenting these sample statistics, we have violated our own stricture to always include an estimate of the probable error. Unfortunately, that must remain a problem for future work.

To summarize this section, we have subjected the ITS irregular terrain model to several tests involving a small subset of data from a single measurement program. Not surprisingly, the model has passed some of these tests and failed others. But recall that our aim here has not been concerned with whether the tests were passed or failed, but rather with illustrating some of the techniques one might use in a more extensive study.

8. REFERENCES

Barsis, A. P., and M. J. Miles (1965), Cumulative distributions of field strength over irregular terrain, using low antenna heights, NBS Report 8891.

Barsis, A. P., and P. L. Rice (1963), Prediction and measurement of VHF field strength over irregular terrain using low antenna heights, NBS Report 7962.

Bean, B. R., J. D. Horn, and A. M. Ozanich, Jr. (1960), Climatic charts and data of radio refractive index for the United States and the World, NBS Monograph 22 (U.S. Government Printing Office, Washington, D.C.).

Bullington, K. (1950), Radio propagation variations at VHF and UHF, Proc. IRE 38, pp. 27-32.

Bullington, K. (1957), Radio propagation fundamentals, BSTJ 36, pp. 593-626.

Causebrook, J. H., B. Davis, and R. S. Sandell (1969), The prediction of co-channel interference to television broadcasting services in the frequency range 470 to 960 MHz, B.B.C. Research Dept Report 1969/33.

CCIR (1978a)*, VHF and UHF propagation curves for the frequency range from 30 MHz to 1000 MHz, Doc. XIVth Plenary V, Recom. No. 370-3.

CCIR (1978b)*, Propagation data required for trans-horizon radio-relay systems, Doc. XIVth Plenary V, Report No. 238-3.

Damelin, J., W. Daniel, H. Fine, and G. Waldo (1966), Development of VHF and UHF propagation curves for TV and FM broadcasting, FCC Report R-6602.

Efron, B. (1979), Computers and the theory of statistics: Thinking the unthinkable, SIAM Review 21, pp. 460-480.

Egli, J. J. (1957), Radio propagation above 40 Mc over irregular terrain, Proc. IRE 45, pp. 1383-1391.

Epstein, J., and D. W. Peterson (1956), A method of predicting the coverage of a television station, RCA Rev. 17, pp. 571-582.

Hagn, G. H. (1980), VHF radio system performance model for predicting communications operational ranges in irregular terrain, IEEE Trans. Commun. COM-28, pp. 1637-1644.

Head, H. T., and O. L. Prestholdt (1960), The measurement of television field strengths in the VHF and UHF bands, Proc. IRE 48, pp. 1000-1008.

Johnson, M. E., M. J. Miles, P. L. McQuate, and A. P. Barsis (1967), Tabulations of VHF propagation data obtained over irregular terrain at 20, 50, and 100 MHz, IER 38-ITSA38, Parts 1, 2, and 3. Available from NTIS, Access No. AD-655-854, AD-662-713, AD 667-530.

* Published by the International Telecommunication Union, Geneva, Switzerland. Also available from the National Technical Information Service, Springfield, VA 22161, with Access. No. PB-298-025.

Johnson, M. E., and G. D. Gierhart (1979), Comparison of measured data with IF-77 propagation model predictions, DOT Rpt. FAA-RD-79-9. Available from NTIS, Access. No. AD-A076-508.

Kaplan, E. L., and P. Meier (1958), Nonparametric estimation from incomplete observations, J. Am. Stat. Assoc. 53, pp. 457-481.

LaGrone, A. H. (1960), Forecasting television service fields, Proc. IRE 48, No.6, pp. 1009-1015.

Longley, A. G., and G. A. Hufford (1975), Sensor path loss measurements analysis and comparison with propagation models, OT Report 75-74. Available from NTIS, Access. No. PB-247-638/AS.

Longley, A. G., and R. K. Reasoner (1970), Comparison of propagation measurements with predicted values in the 20 to 10,000 MHz range, ESSA Tech. Report ERL 148-ITS 97. Available from NTIS, Access. No. AD-703-579.

Longley, A. G., and P. L. Rice (1968), Prediction of tropospheric radio transmission loss over irregular terrain--a computer method 1968, ESSA Tech. Report ERL 79-ITS 67. Available from NTIS, Access. No. AD-676-874.

McQuate, P. L., J. M. Harman, M. E. McClanahan, and A. P. Barsis (1970), Tabulations of propagation data over irregular terrain in the 230 to 9300 MHz frequency range; Part III, North Table Mountain--Golden, ESSA Tech. Report ERL 65-ITS 58-3. Available from NTIS, Access. No. AD-715-753.

Norton, K. A., P. L. Rice, and L. E. Vogler (1955), The use of angular distance in estimating transmission loss and fading range for propagation through a turbulent atmosphere over irregular terrain, Proc. IRE 43, pp. 1488-1526.

O'Connor, R. A. (1968), Understanding television's Grade A and Grade B service contours, IEEE Trans. Broadcasting BC-14, pp. 137-143.

Okumura, Y., E. Ohmori, T. Kawano, and K. Fukuda (1968), Field strength and its variability in VHF and UHF land-mobile radio service, Rev. Tokyo Elec. Commun. Lab. 16, pp. 825-873.

Rice, P. L., A. G. Longley, K. A. Norton, and A. P. Barsis (1967), Transmission loss predictions for tropospheric communication circuits, NBS Tech. Note 101, Vols. I and II. Available from NTIS, Access. Nos. AD-687-820 and AD-687-821.

Walpole, R. E., and R. H. Myers (1972), Probability and Statistics for Engineers and Scientists (The Macmillan Company, New York, N.Y.).

Walsh, J. E. (1962), Handbook of Nonparametric Statistics (D. van Nostrand Co., Princeton, N.J.).