

22. Seminar Optične Komunikacije

Laboratorij za Sevanje in Optiko

Fakulteta za Elektrotehniko

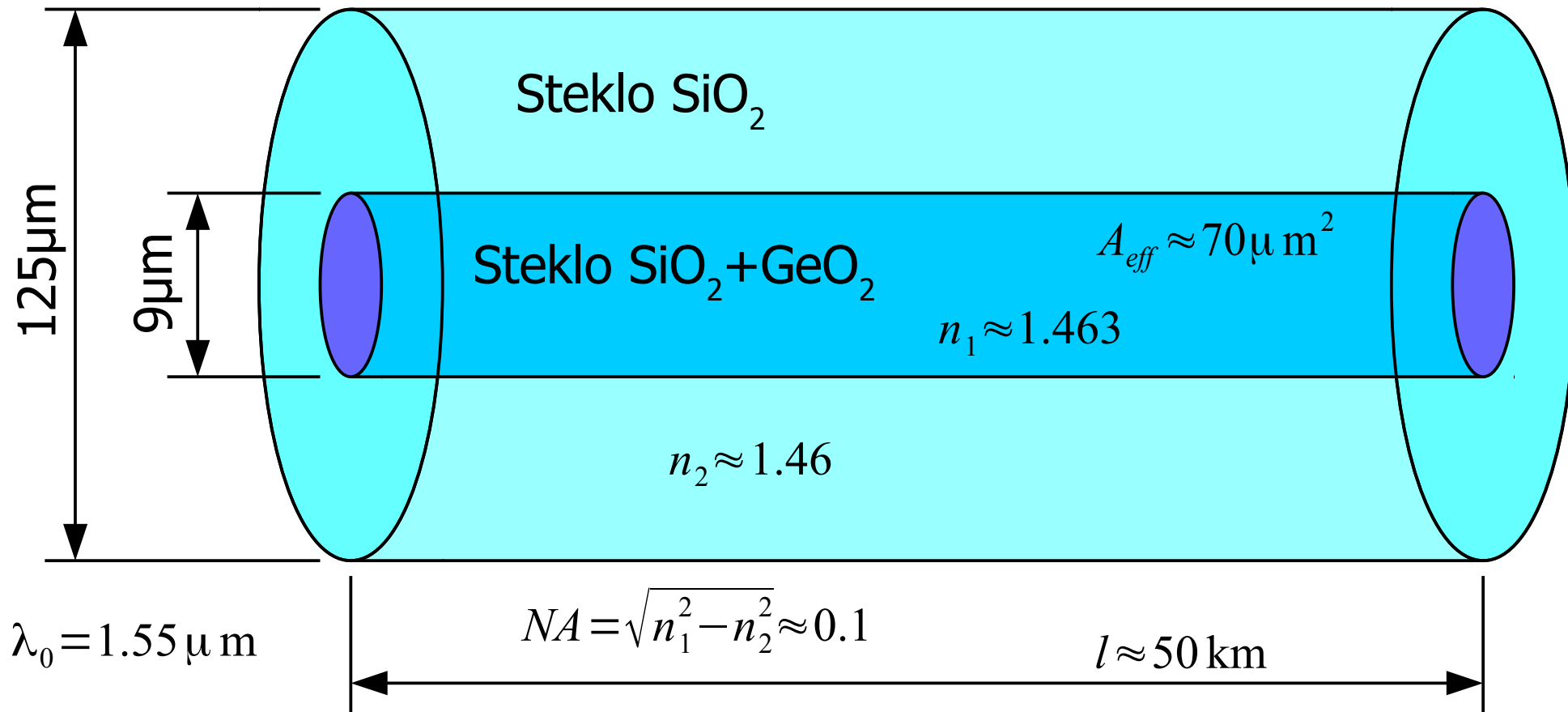
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Nelinearnosti in intermodulacija
v optičnih komunikacijah

Matjaž Vidmar

..... Seznam prosojnic:

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- Slika 3 – Lastna fazna modulacija
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$$a \approx 0.2 \text{ dB/km} = 0.0002 \text{ dB/m}$$

$$\Delta t_g = D \cdot l \cdot \Delta \lambda_0$$

$$D(\lambda_0) \approx 17 \frac{\text{ps}}{\text{nm} \cdot \text{km}} = 17 \cdot 10^{-6} \frac{\text{s}}{\text{m}^2}$$

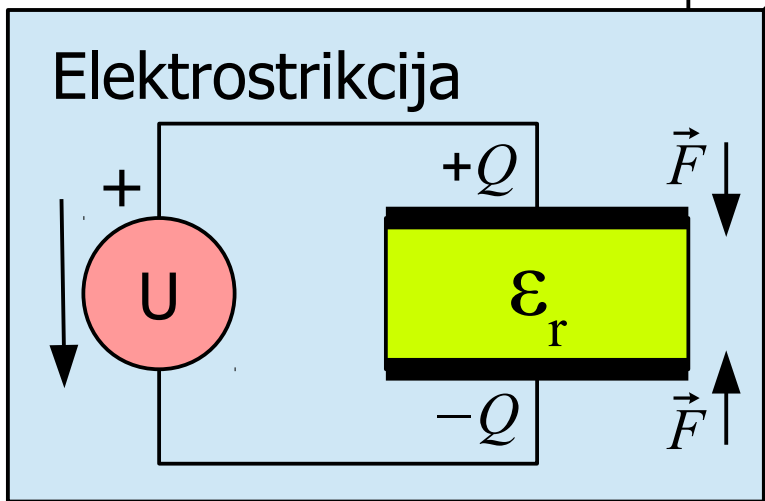
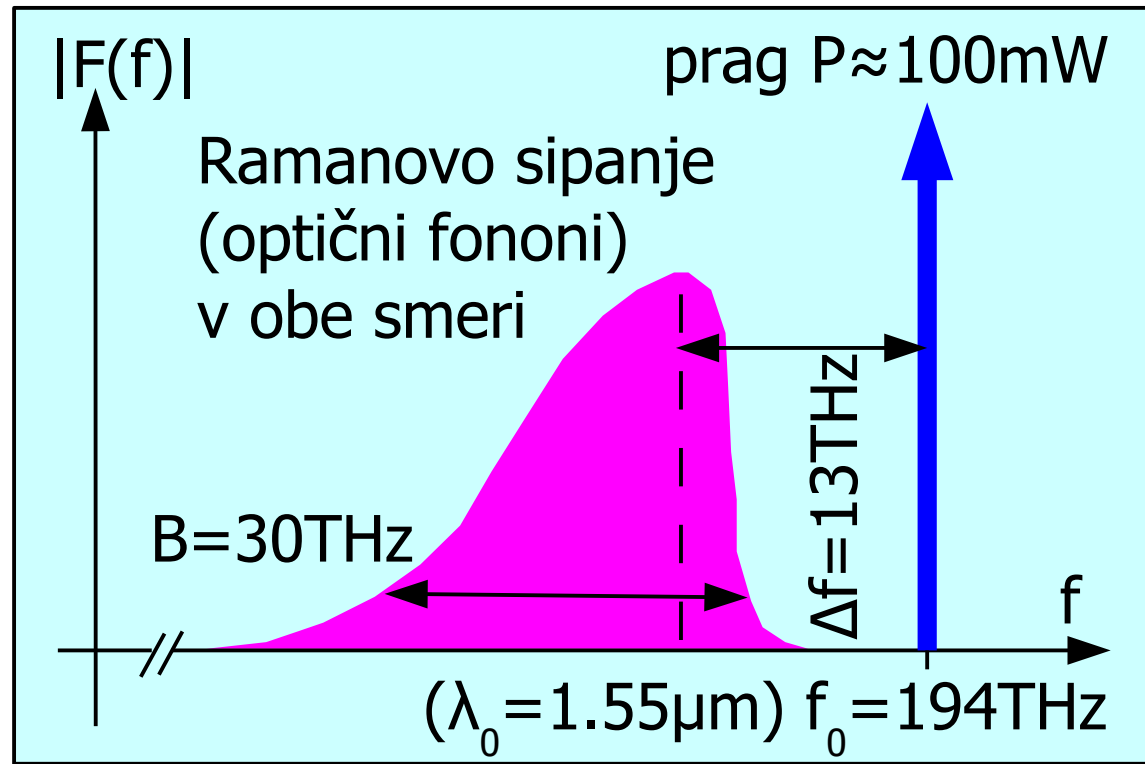
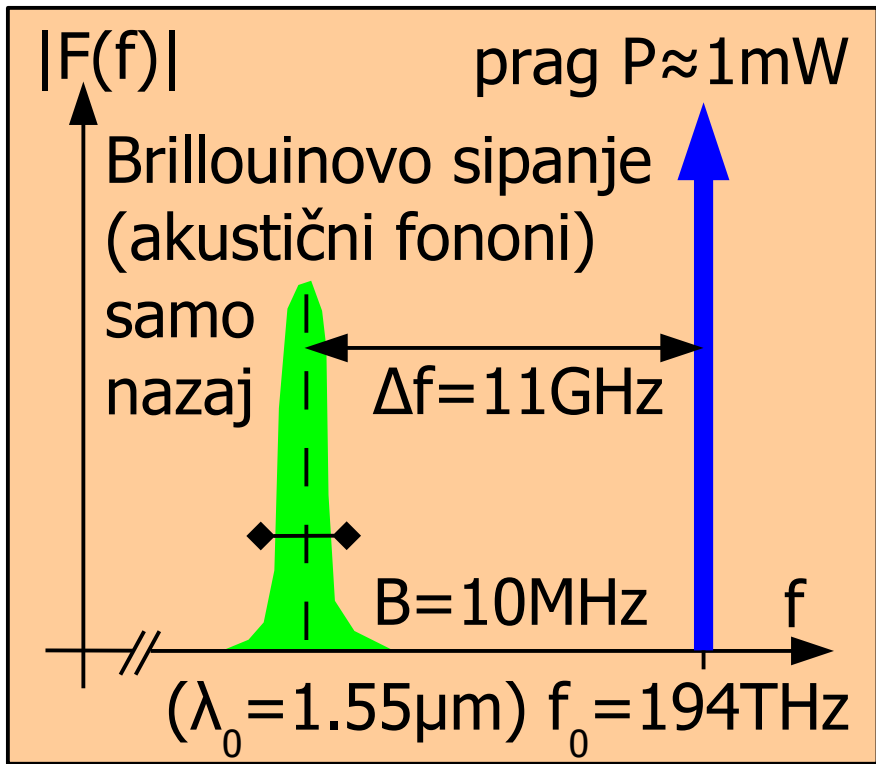
Zgled moči:

$$P = 100 \text{ mW}$$

$$S = \frac{P}{A_{\text{eff}}} = 1.43 \cdot 10^9 \text{ W/m}^2 = 143 \text{ kW/cm}^2$$

$$E = \sqrt{S \cdot 2 Z_0 / n_1} = 8.58 \cdot 10^5 \text{ V/m} = 8.58 \text{ kV/cm}$$

1 – Enorodovno svetlobno vlakno



Nelinearni lomni količnik
(Kerrov pojav)

$$n(S) = n_0 + n_2 \cdot S$$

$$S = \frac{dP}{dA} \left[\frac{\text{W}}{\text{m}^2} \right]$$

Kremenovo steklo SiO_2

$n_0 \approx 1.46$ $n_2 \approx 2.5 \cdot 10^{-20} \text{m}^2/\text{W}$

$$l = 1 \text{ km}$$

Kratko vlakno, približno konstantna moč

$$P = 100 \text{ mW} \quad A_{eff} = 70 \mu\text{m}^2 \quad S = 1.43 \text{ GW/m}^2 \quad \lambda_0 = 1.55 \mu\text{m}$$

$$\Delta n = n_2 \cdot S = 3.58 \cdot 10^{-11} \quad \Delta \varphi = -\Delta n \cdot k_0 \cdot l = -\Delta n \cdot \frac{2\pi}{\lambda_0} \cdot l = -0.145 \text{ rd}$$

$$l = 50 \text{ km}$$

Dolgo vlakno, moč upada zaradi slabljenja vlakna

$$\lambda_0 = 1.55 \mu\text{m} \quad P(0) = 100 \text{ mW} \quad S(z) = \frac{P(z)}{A_{eff}} \quad \Delta n(z) = n_2 \cdot \frac{P(z)}{A_{eff}}$$

$$a = 0.2 \text{ dB/km} = 0.0002 \text{ dB/m} \quad P(z) = P(0) \cdot 10^{-\frac{a \cdot z}{10}} = P(0) \cdot e^{-\frac{\ln 10}{10} \cdot a \cdot z}$$

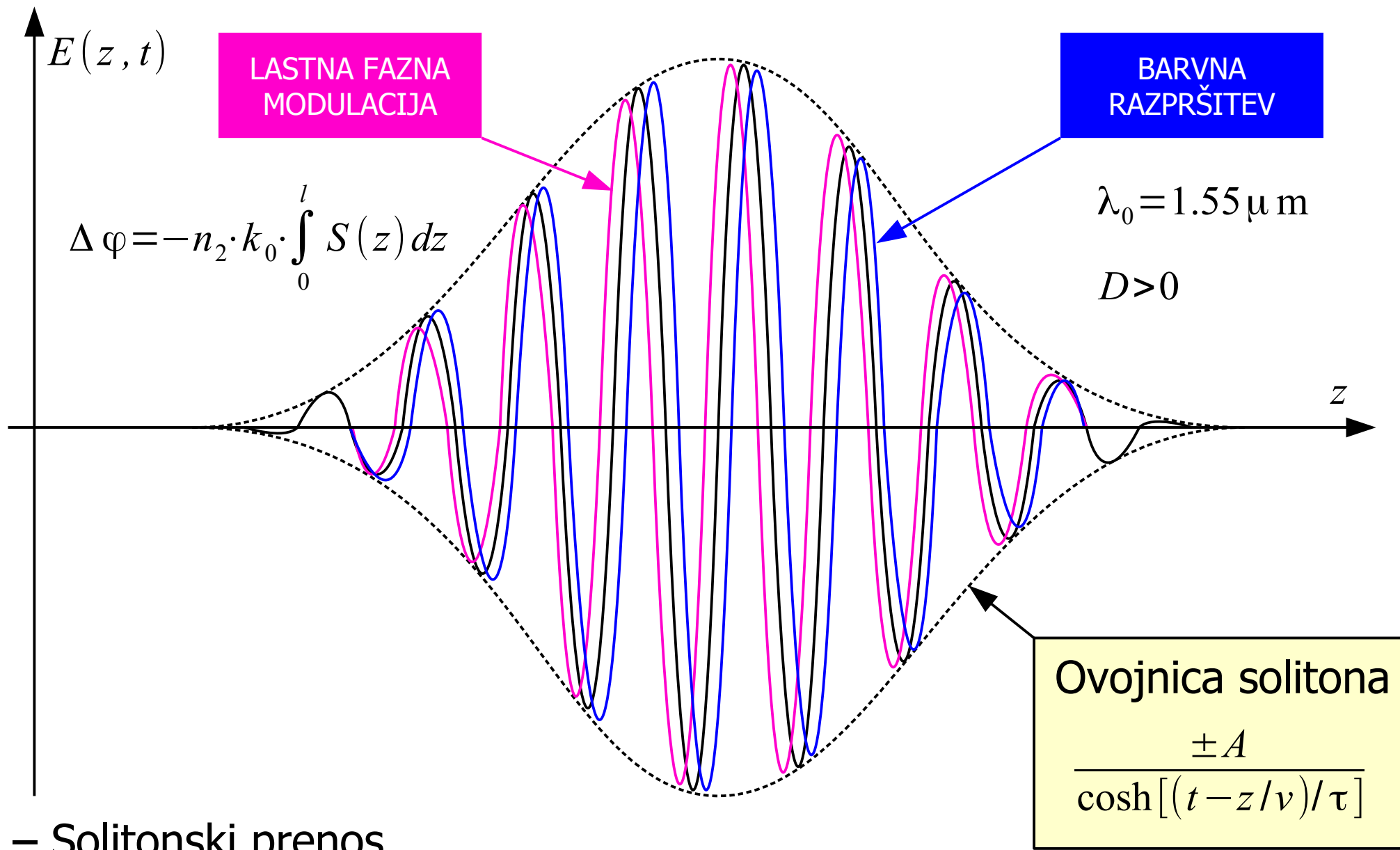
$$\Delta \varphi = -\int_0^l \Delta n(z) \cdot k_0 dz = -\int_0^l \frac{n_2 \cdot P(0)}{A_{eff}} \cdot \left(e^{-\frac{\ln 10}{10} \cdot a \cdot z} \right) \cdot \frac{2\pi}{\lambda_0} dz = -\frac{2\pi \cdot n_2 \cdot P(0)}{A_{eff} \cdot \lambda_0} \cdot \int_0^l e^{-\frac{\ln 10}{10} \cdot a \cdot z} dz$$

$$\Delta \varphi = -\frac{20\pi \cdot n_2 \cdot P(0)}{A_{eff} \cdot \lambda_0 \cdot a \cdot \ln 10} \cdot \left(1 - e^{-\frac{\ln 10}{10} \cdot a \cdot l} \right) = -2.83 \text{ rd}$$

3 – Lastna fazna modulacija



Ojačevana prekooceanska zveza brez regeneratorjev



$$E(t) = E_1(t) + E_2(t) = A_1 \cdot \cos \omega_1 t + A_2 \cdot \cos \omega_2 t$$

Ista polarizacija!

$$p(t) = \alpha \cdot E^2(t) = \alpha \cdot [A_1^2 \cdot \cos^2 \omega_1 t + 2 A_1 A_2 \cdot \cos \omega_1 t \cdot \cos \omega_2 t + A_2^2 \cdot \cos^2 \omega_2 t]$$

$$P_1 = \alpha \cdot \frac{A_1^2}{2}$$

$$P_2 = \alpha \cdot \frac{A_2^2}{2}$$

Hitri členi

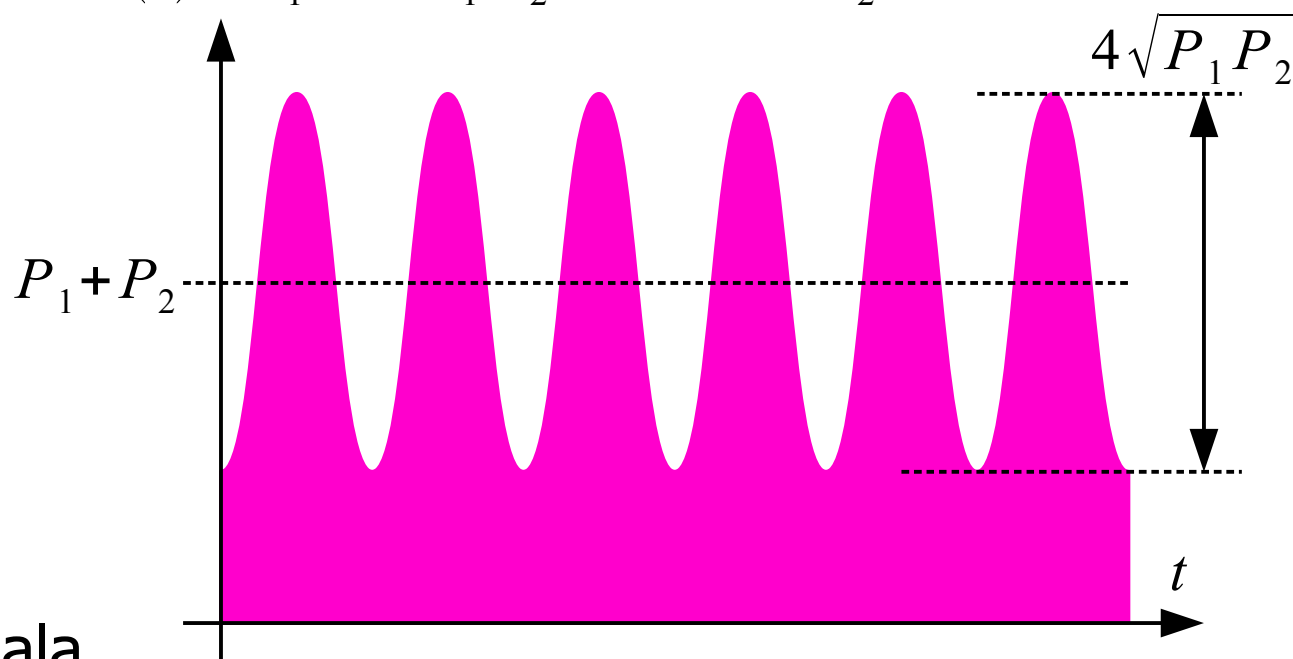
$$2\sqrt{P_1 P_2} = \alpha \cdot A_1 A_2$$

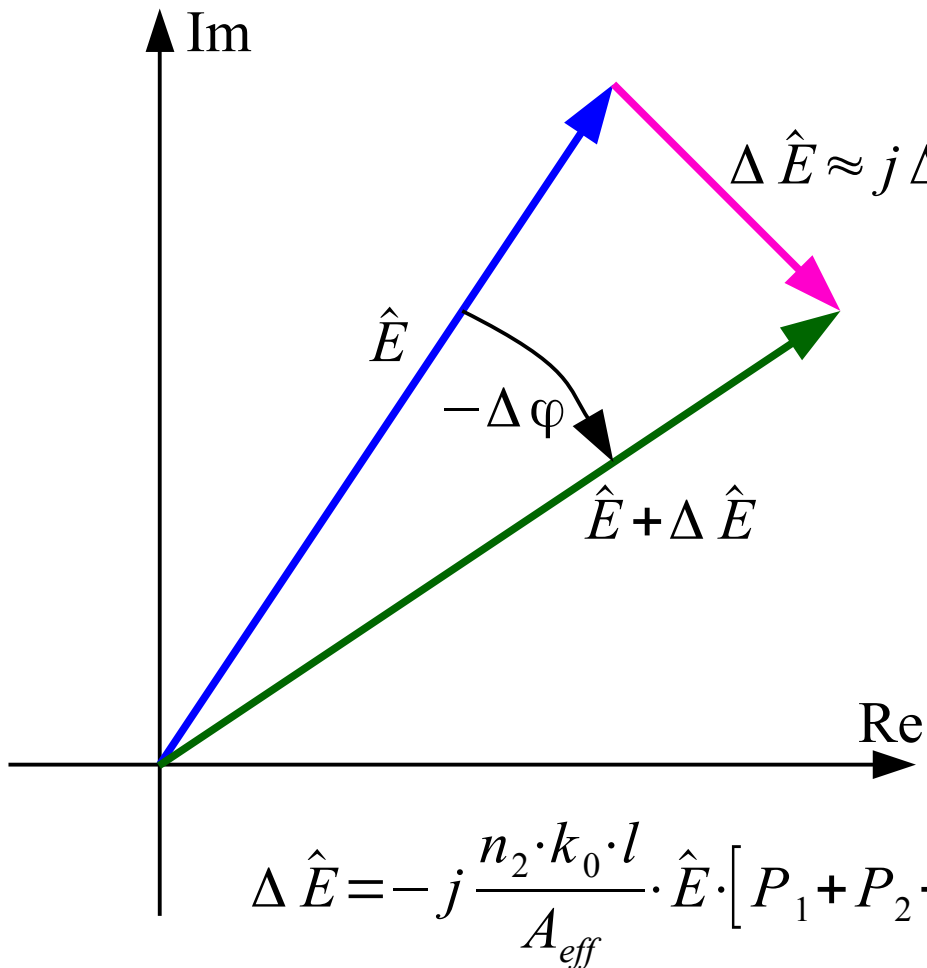
$$p(t) = P_1 \cdot (1 + \cos 2\omega_1 t) + 2\sqrt{P_1 P_2} \cdot [\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t] + P_2 \cdot (1 + \cos 2\omega_2 t)$$

Utripanje moči

$$\omega_1 - \omega_2 = \Delta \omega \ll \omega_1, \omega_2$$

$$P(t) = P_1 + 2\sqrt{P_1 P_2} \cdot \cos \Delta \omega t + P_2$$





$$\Delta \hat{E} \approx j \Delta \varphi \cdot \hat{E} \quad \leftarrow |\Delta \varphi| \ll 1$$

$$\Delta \varphi = -\frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot P(t)$$

$$\Delta \hat{E} = -j \frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot \hat{E} \cdot P(t)$$

$$\Delta \hat{E} = -j \frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot \hat{E} \cdot [P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \Delta \omega t]$$

$$E(t) = A \cdot \cos \omega t$$

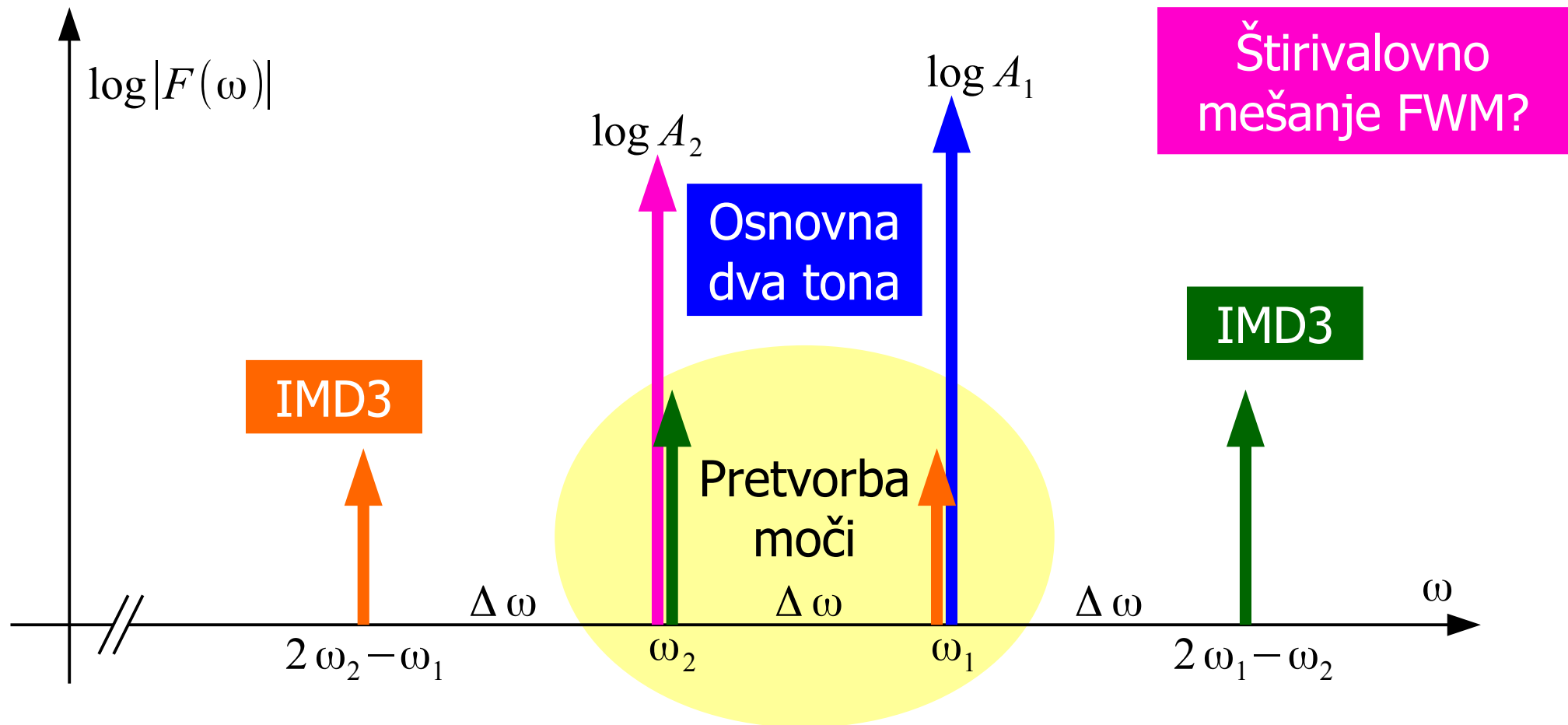
$$\Delta E(t) = \frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot A \cdot \cos(\omega t - \pi/2) \cdot [P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \Delta \omega t] \quad -j = e^{-j\pi/2}$$

$$\Delta E(t) = \frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot A \cdot [(P_1 + P_2) \cdot \sin \omega t + \sqrt{P_1 P_2} (\sin(\omega + \Delta \omega)t + \sin(\omega - \Delta \omega)t)]$$

Dve novi frekvenci!

$$\omega = \omega_1 \quad \Delta E_1(t) = \frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot A_1 \cdot \left[(P_1 + P_2) \cdot \sin \omega_1 t + \sqrt{P_1 P_2} (\sin (2 \omega_1 - \omega_2) t + \sin \omega_2 t) \right]$$

$$\omega = \omega_2 \quad \Delta E_2(t) = \frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot A_2 \cdot \left[(P_1 + P_2) \cdot \sin \omega_2 t + \sqrt{P_1 P_2} (\sin \omega_1 t + \sin (2 \omega_2 - \omega_1) t) \right]$$

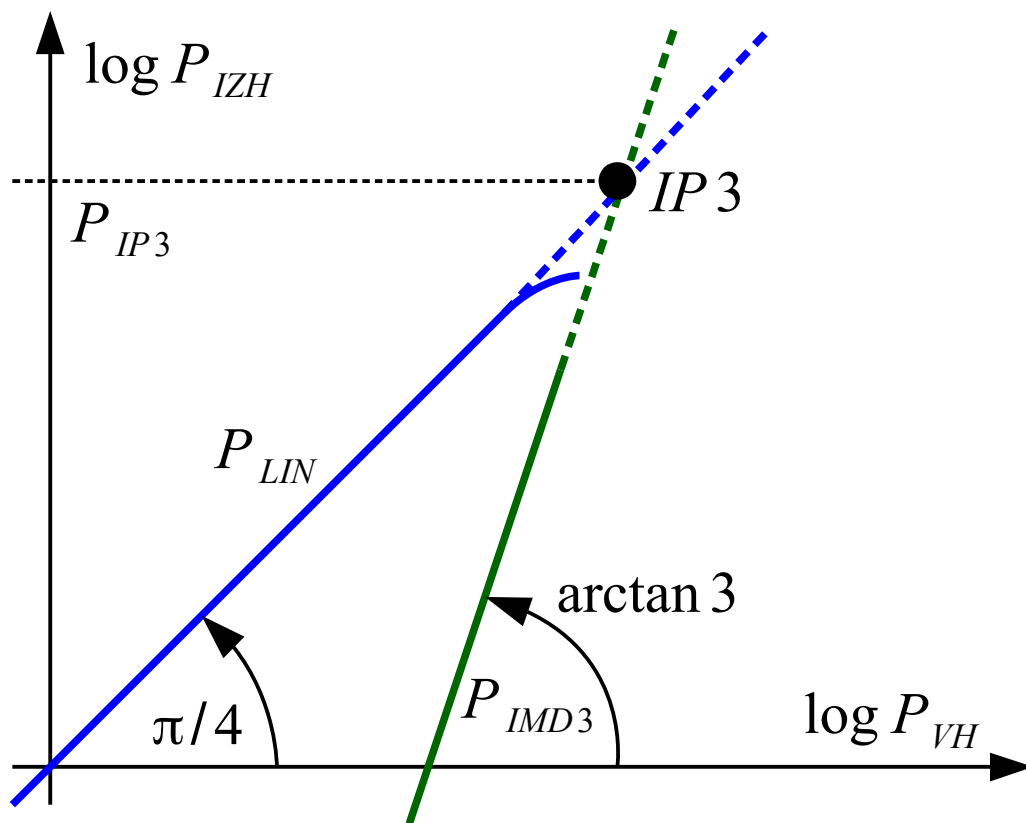


7 – Intermodulacijsko popačenje tretjega reda

$$E_{2\omega_1-\omega_2}(t) = \frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot A_1 \cdot \sqrt{P_1 P_2} \cdot \sin(2\omega_1 - \omega_2)t$$

$$P_1 = \alpha \cdot \frac{A_1^2}{2}$$

$$P_{2\omega_1-\omega_2} = \alpha \cdot \frac{A_{2\omega_1-\omega_2}^2}{2} = \frac{\alpha}{2} \cdot \left(\frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \cdot A_1 \cdot \sqrt{P_1 P_2} \right)^2 = \left(\frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \right)^2 \cdot P_1^2 P_2 = \frac{P_1^2 P_2}{P_{IP3}^2}$$



$$P_{2\omega_2-\omega_1} = \left(\frac{n_2 \cdot k_0 \cdot l}{A_{eff}} \right)^2 \cdot P_1 P_2^2 = \frac{P_1 P_2^2}{P_{IP3}^2}$$

$$P_{IP3} = \frac{A_{eff}}{n_2 \cdot k_0 \cdot l} = \frac{A_{eff} \cdot \lambda_0}{2\pi \cdot n_2 \cdot l}$$

$$P_{IMD3} = \frac{P_{LIN}^3}{P_{IP3}^2}$$

$$\log P_{IMD3} = 3 \cdot \log P_{LIN} - 2 \cdot \log P_{IP3}$$

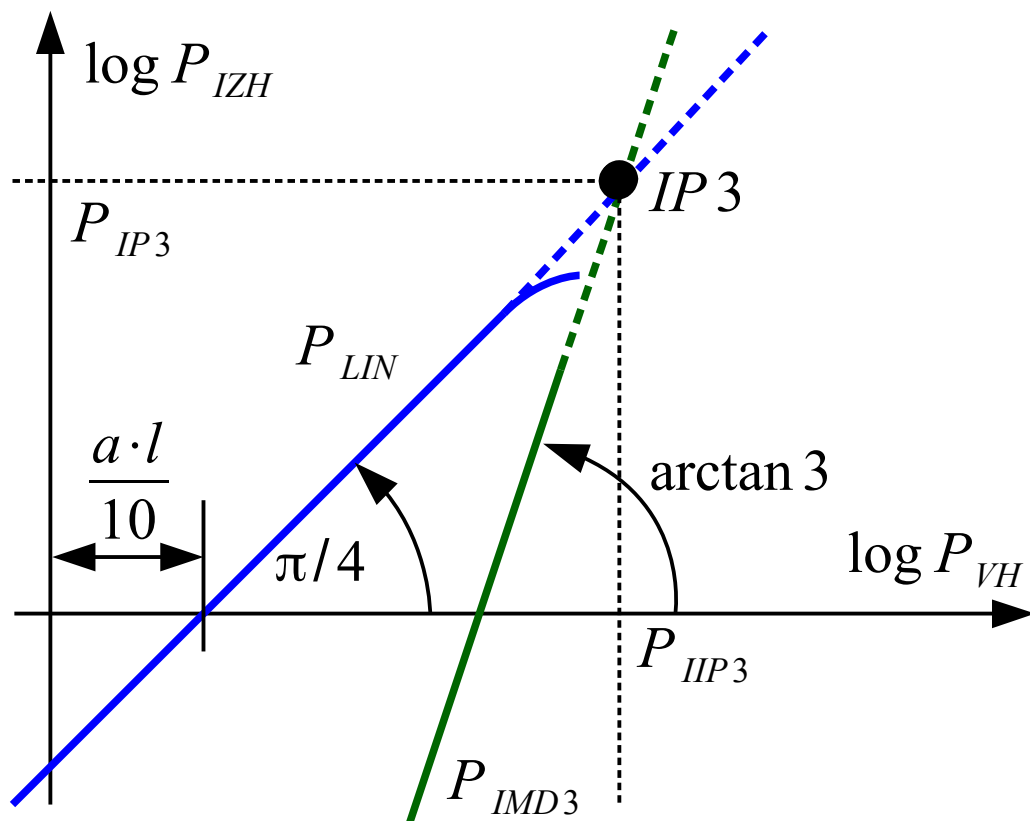
$$\Delta \varphi = -n_2 \cdot k_0 \cdot S(0) \cdot \int_0^l e^{-\frac{\ln 10}{10} \cdot a \cdot z} dz = -\frac{10 \cdot n_2 \cdot k_0 \cdot S(0)}{a \cdot \ln 10} \cdot \left(1 - e^{-\frac{\ln 10}{10} \cdot a \cdot l}\right) = -n_2 \cdot k_0 \cdot S(0) \cdot l_{eff}$$

Efektivna dolžina

$$l_{eff} = \frac{10}{a \cdot \ln 10} \cdot \left(1 - e^{-\frac{\ln 10}{10} \cdot a \cdot l}\right)$$

$$\lambda_0 = 1.55 \mu\text{m} \quad a \approx 0.2 \text{ dB/km}$$

$$l \gg l_{eff} \approx \frac{10}{a \cdot \ln 10} = 21.7 \text{ km}$$



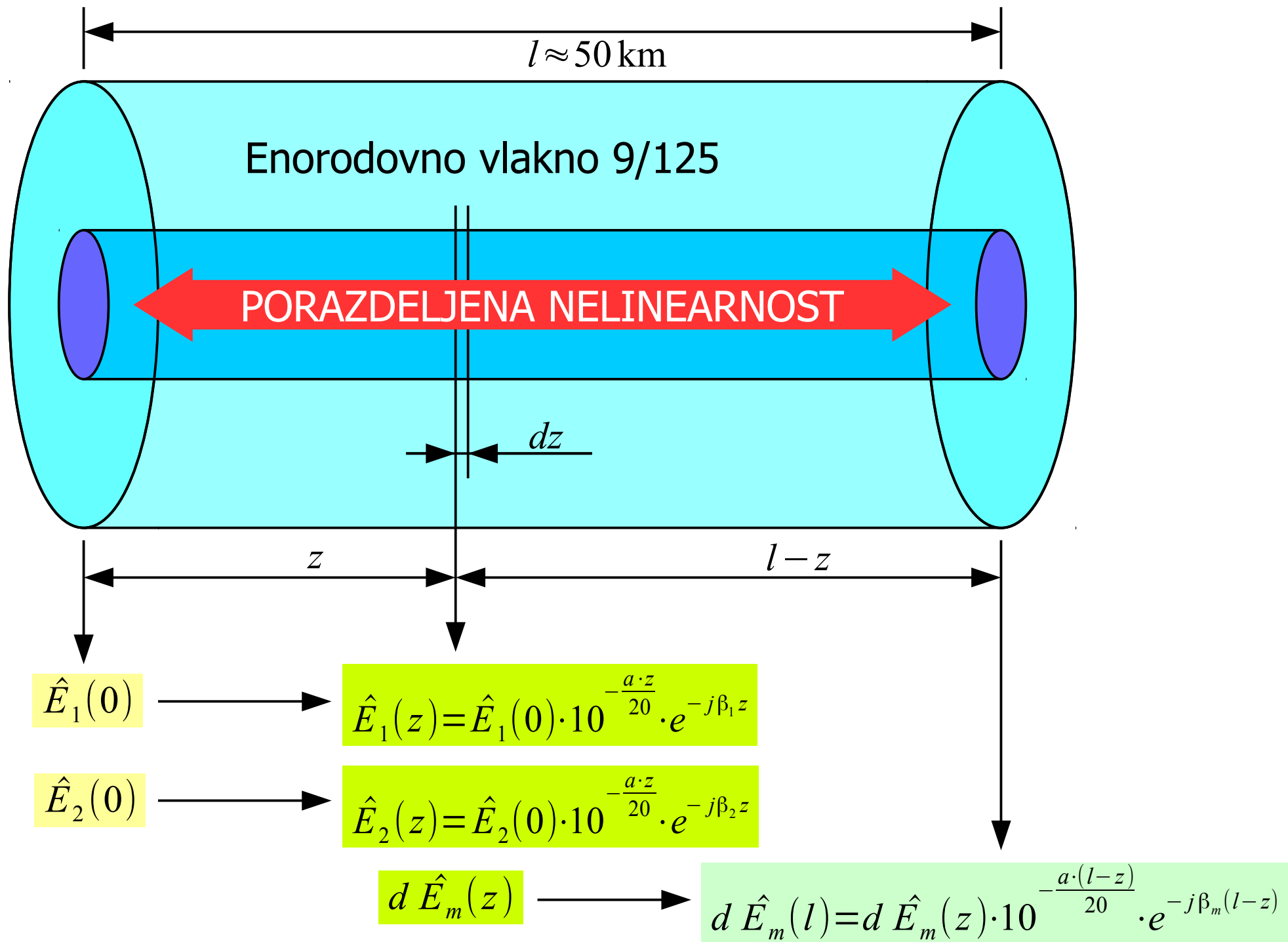
$$P_{IIP3} = \frac{A_{eff}}{n_2 \cdot k_0 \cdot l_{eff}} = \frac{A_{eff} \cdot \lambda_0}{2 \pi \cdot n_2 \cdot l_{eff}}$$

$$n_2 \approx 2.5 \cdot 10^{-20} \text{ m}^2/\text{W} \quad A_{eff} \approx 30 \mu\text{m}^2$$

$P_{IIP3} \approx 13.6 \text{ mW} \approx +11.4 \text{ dBm}$
WDM ni izvedljiv z DSF!

$$P_{LIN} = P_{VH} \cdot 10^{-\frac{a \cdot l}{10}} \quad P_{IP3} = P_{IIP3} \cdot 10^{-\frac{a \cdot l}{10}}$$

$$P_{IMD3} = \frac{P_{LIN}^3}{P_{IP3}^2} = \frac{P_{VH}^3}{P_{IIP3}^2} \cdot 10^{-\frac{a \cdot l}{10}}$$



10 – Fazni zamik do/od porazdeljene nelinearnosti

$$\omega_m = 2\omega_1 - \omega_2$$

$$D \neq 0$$



$$\beta_m \neq \beta_1 \neq \beta_2 \neq \beta_m$$

$$dE_m(z, t) = \frac{n_2 \cdot k_0 \cdot dz}{A_{eff}} \cdot A_1(z) \cdot \sqrt{P_1(z) \cdot P_2(z)} \cdot \sin[(2\omega_1 - \omega_2)t - 2\beta_1 z + \beta_2 z]$$

$$A_1(z) = \sqrt{P_1(z) \cdot 2/\alpha} \quad P_1(z) = P_1(0) \cdot 10^{-\frac{a \cdot z}{10}} \quad P_2(z) = P_2(0) \cdot 10^{-\frac{a \cdot z}{10}}$$

$$d\hat{E}_m(z) = \frac{n_2 \cdot k_0 \cdot dz}{A_{eff}} \cdot \sqrt{P_1^2(z) \cdot P_2(z) \cdot 2/\alpha} \cdot (-j) e^{-j(2\beta_1 z - \beta_2 z)}$$

Fazna neusklajenost

$$\Delta\beta = \beta_m + \beta_2 - 2\beta_1$$

$$\hat{E}_m(l) = \int_0^l 10^{-\frac{a \cdot (l-z)}{20}} \cdot e^{-j\beta_m(l-z)} d\hat{E}_m(z)$$

Kazalčna vsota!

$$\hat{E}_m(l) = \frac{n_2 \cdot k_0}{A_{eff}} \cdot \sqrt{P_1^2(0) \cdot P_2(0) \cdot 2/\alpha} \cdot 10^{-\frac{a \cdot l}{20}} \cdot (-j) e^{-j\beta_m l} \int_0^l 10^{-\frac{a \cdot z}{10}} \cdot e^{j\Delta\beta z} dz$$

$$P_m(l) = \alpha \cdot \frac{|\hat{E}_m(l)|^2}{2} = \left(\frac{n_2 \cdot k_0}{A_{eff}} \right)^2 \cdot P_1^2(0) \cdot P_2(0) \cdot 10^{-\frac{a \cdot l}{10}} \cdot \left| \int_0^l 10^{-\frac{a \cdot z}{10}} \cdot e^{j\Delta\beta z} dz \right|^2$$

$$\int_0^l 10^{-\frac{a \cdot z}{10}} \cdot e^{j \Delta \beta z} dz = \int_0^l e^{\left(-\frac{a \cdot \ln 10}{10} + j \Delta \beta\right)z} dz = \frac{1}{\frac{a \cdot \ln 10}{10} - j \Delta \beta} \cdot \left[1 - e^{\left(-\frac{a \cdot \ln 10}{10} + j \Delta \beta\right)l}\right]$$

$$l_{eff}^2 = \left| \int_0^l 10^{-\frac{a \cdot z}{10}} \cdot e^{j \Delta \beta z} dz \right|^2 \approx \frac{1}{\left(\frac{a \cdot \ln 10}{10}\right)^2 + \Delta \beta^2} \ll l^2$$

$$l_{eff} \approx \frac{1}{\sqrt{\left(\frac{a \cdot \ln 10}{10}\right)^2 + \Delta \beta^2}}$$

$$P_m(l) = \left(\frac{n_2 \cdot k_0 \cdot l_{eff}}{A_{eff}}\right)^2 \cdot P_1^2(0) \cdot P_2(0) \cdot 10^{-\frac{a \cdot l}{10}}$$



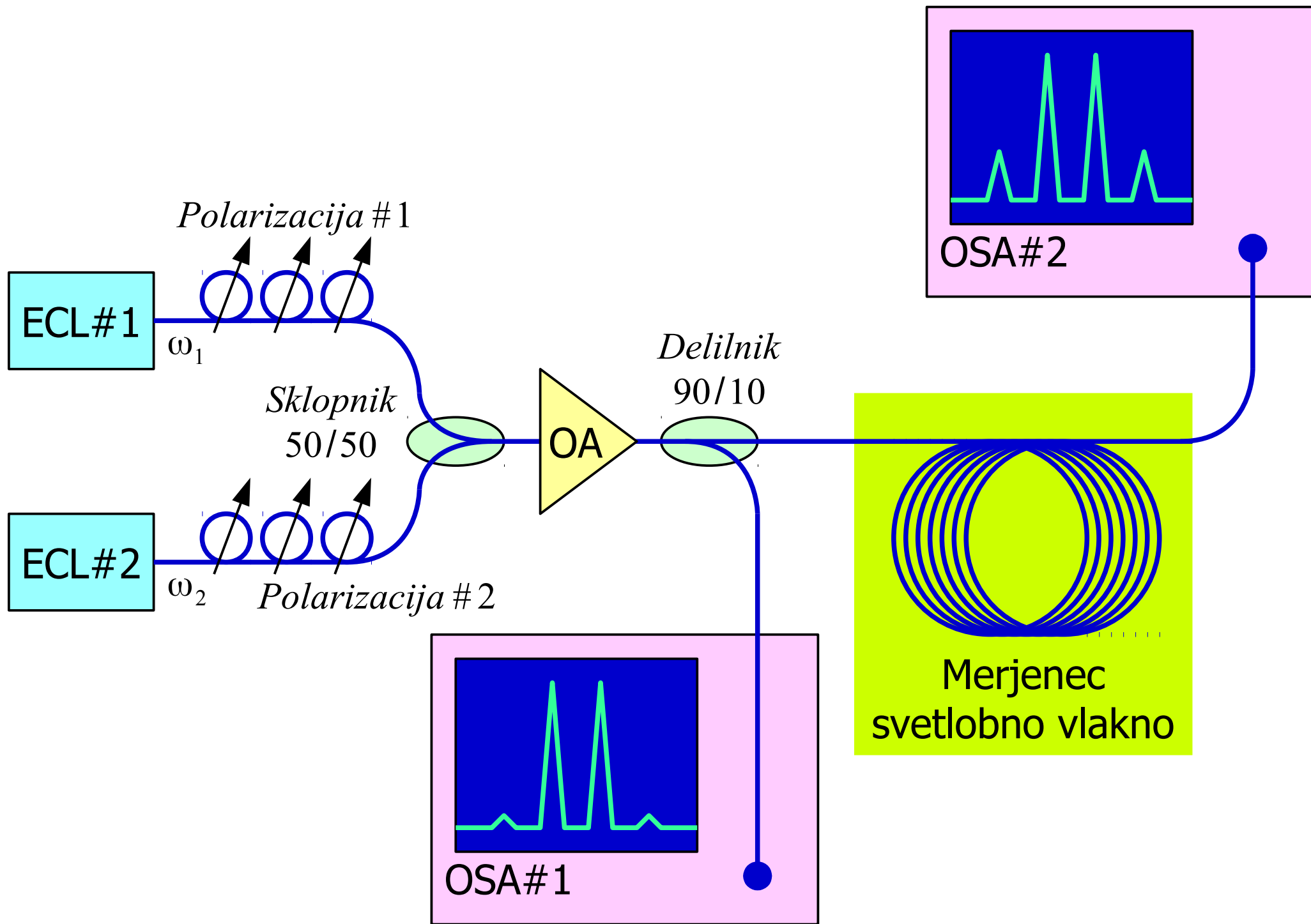
$$P_{IP3} = \frac{A_{eff}}{n_2 \cdot k_0 \cdot l_{eff}} = \frac{A_{eff} \cdot \lambda_0}{2 \pi \cdot n_2 \cdot l_{eff}}$$

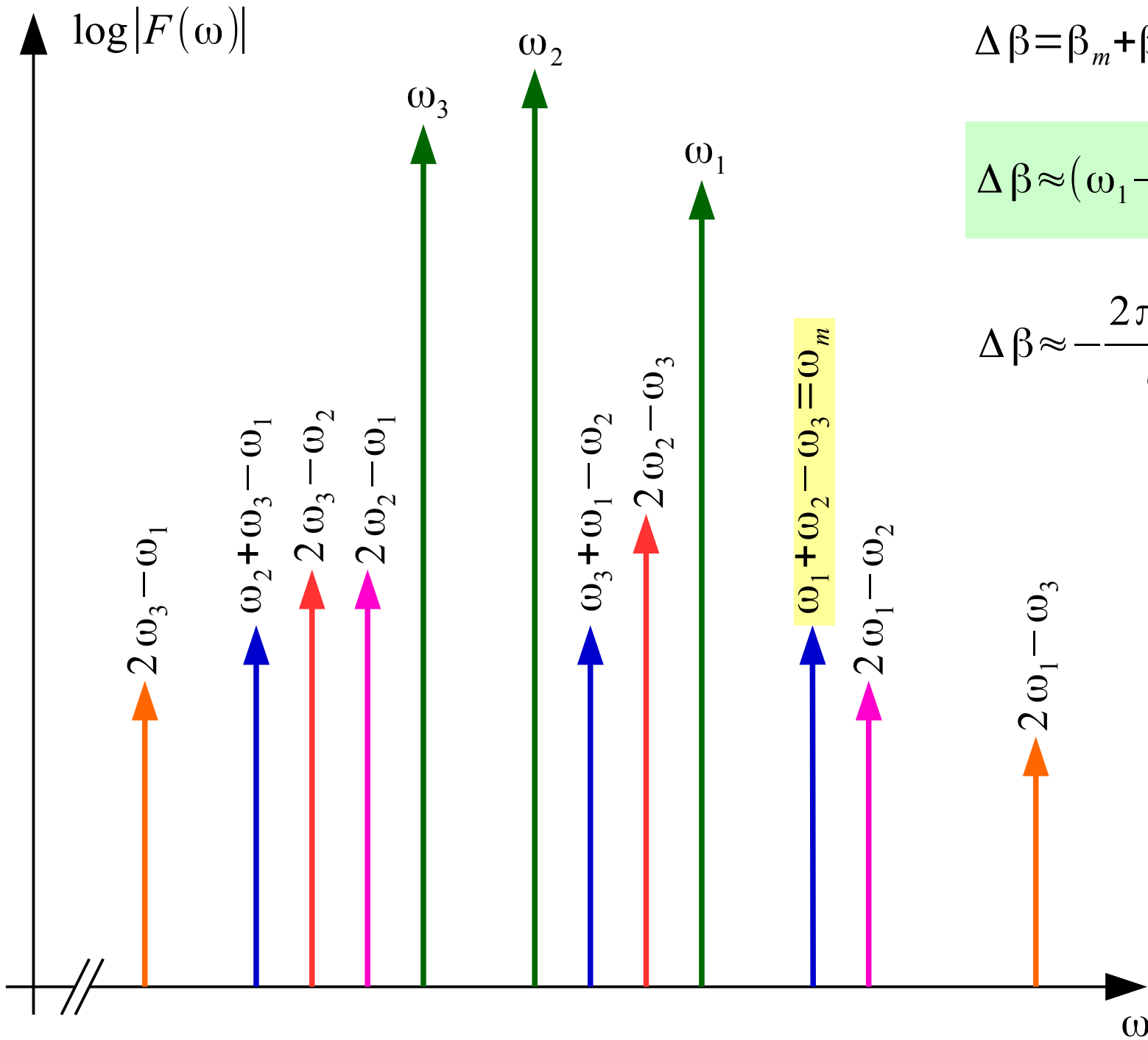
$$D \cdot l = \frac{d t_g}{d \lambda_0} = -\frac{2 \pi \cdot c_0}{\lambda_0^2} \cdot \frac{d t_g}{d \omega} = -\frac{2 \pi \cdot c_0}{\lambda_0^2} \cdot \frac{d}{d \omega} \left(\frac{l}{v_g}\right) = -\frac{2 \pi \cdot c_0}{\lambda_0^2} \cdot l \cdot \frac{d}{d \omega} \left(\frac{d \beta}{d \omega}\right) = -\frac{2 \pi \cdot c_0}{\lambda_0^2} \cdot l \cdot \frac{d^2 \beta}{d \omega^2}$$

$$\Delta \beta = \beta(\omega + \Delta \omega) + \beta(\omega - \Delta \omega) - 2\beta(\omega) \approx \Delta \omega^2 \cdot \frac{d^2 \beta}{d \omega^2} = -\frac{\lambda_0^2}{2 \pi \cdot c_0} \cdot D \cdot \Delta \omega^2 = -\frac{2 \pi \cdot \lambda_0^2}{c_0} \cdot D \cdot \Delta f^2$$

12 – Intermodulacija v vlaknu z barvno razpršitvijo (2)

		Vlakno SMF $a=0.2\text{dB/km}$		Vlakno NZDSF $a=0.2\text{dB/km}$			
		$A_{\text{eff}}=70\mu\text{m}^2$ $D=17\text{ps/nm.km}$		$A_{\text{eff}}=80\mu\text{m}^2$ $D=5\text{ps/nm.km}$			
Δf	$\Delta\beta$	l_{eff}	P_{IIP3}	$\Delta\beta$	l_{eff}	P_{IIP3}	
1GHz	-0.00086rd/km	21.7km	15.0dBm	-0.00025rd/km	21.7km	15.6dBm	
3GHz	-0.0077rd/km	21.4km	15.1dBm	-0.0023rd/km	21.7km	15.6dBm	
10GHz	-0.0855rd/km	10.3km	18.3dBm	-0.0252rd/km	19.1km	16.2dBm	
25GHz	-0.535rd/km	1.86km	25.7dBm	-0.157rd/km	6.1km	21.1dBm	
50GHz	-2.14rd/km	468m	31.7dBm	-0.63rd/km	1.59km	27.0dBm	
100GHz	-8.55rd/km	117m	37.7dBm	-2.52rd/km	397m	33.0dBm	
200GHz	-34.2rd/km	29.2m	43.7dBm	-10.1rd/km	99m	39.0dBm	
400GHz	-137rd/km	7.3m	49.8dBm	-40.3rd/km	25m	45.0dBm	
1THz	-855rd/km	1.2m	57.7dBm	-252rd/km	4m	53.0dBm	

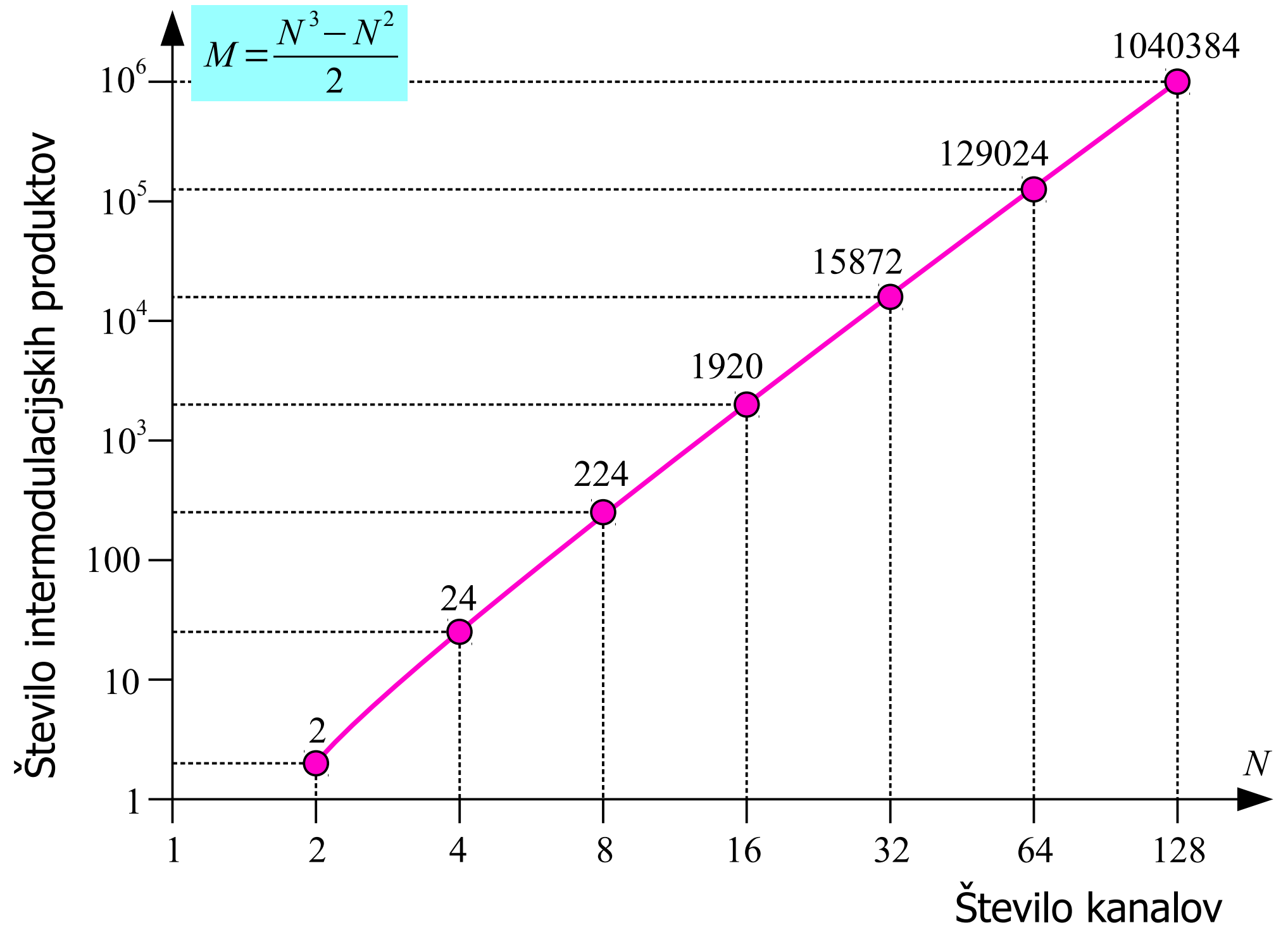




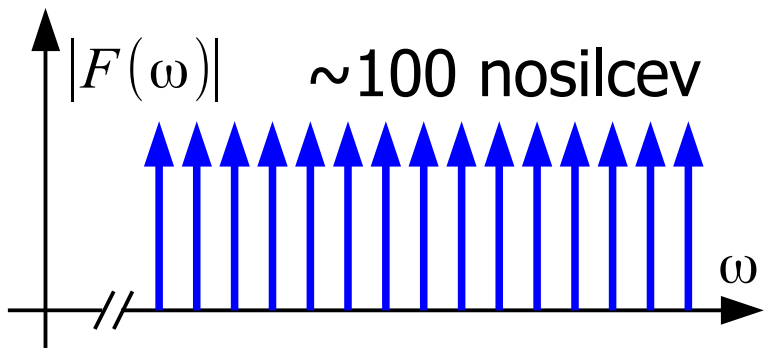
$$\Delta\beta = \beta_m + \beta_3 - \beta_1 - \beta_2$$

$$\Delta\beta \approx (\omega_1 - \omega_3) \cdot (\omega_2 - \omega_3) \cdot \frac{d^2\beta}{d\omega^2}$$

$$\Delta\beta \approx -\frac{2\pi \cdot \lambda_0^2}{c_0} \cdot D \cdot \Delta f_{13} \cdot \Delta f_{23}$$



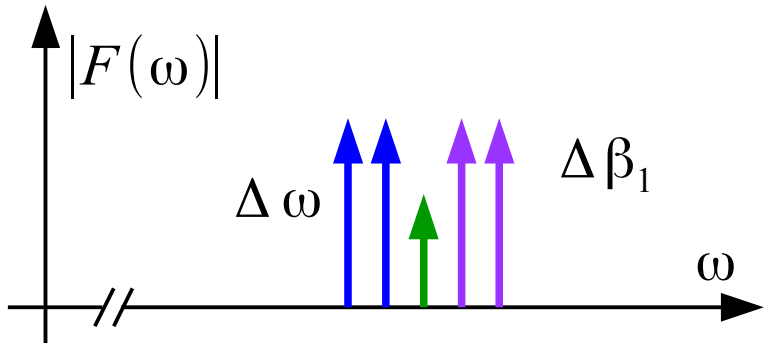
16 – Število intermodulacijskih produktov WDM



$$\Delta f = 50 \text{ GHz} \rightarrow l_{\text{eff}} \approx \frac{1}{\Delta \beta}$$

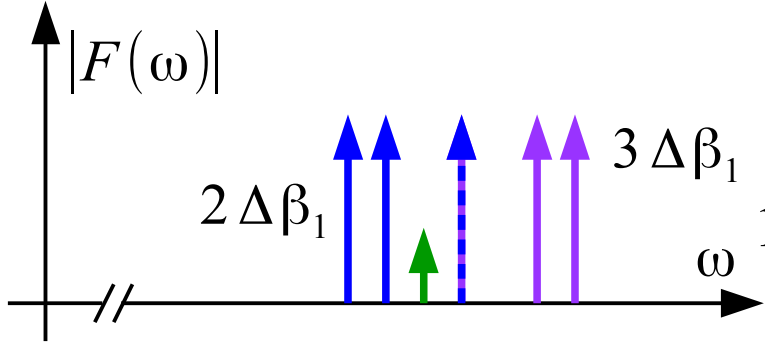
Ista polarizacija?

$$P_{IIMD3} = \frac{P_{VH}^3}{P_{IIP3}^2} \cdot 2 \cdot \sum \left(\frac{\Delta \beta_1}{\Delta \beta} \right)^2$$



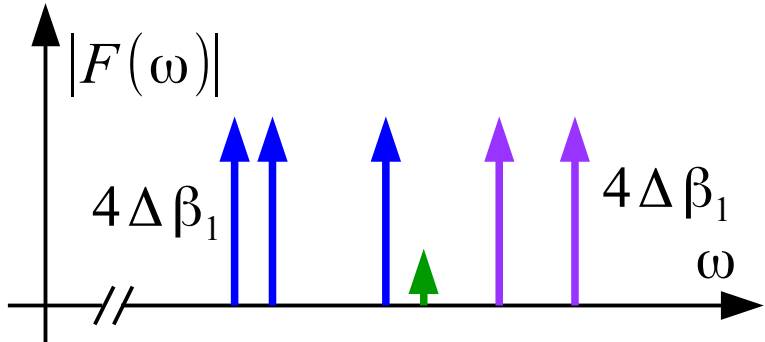
$$\Delta \beta \approx -\frac{2\pi \cdot \lambda_0^2}{c_0} \cdot D \cdot (n \cdot \Delta f) \cdot (m \cdot \Delta f)$$

$$P_{IIMD3} \approx \frac{P_{VH}^3}{P_{IIP3}^2} \cdot 2 \cdot \left(\sum_{n=1}^{\infty} \frac{1}{n^2} \right) \cdot \left(\sum_{m=1}^{\infty} \frac{1}{m^2} \right) = \frac{P_{VH}^3}{P_{IIP3}^2} \cdot 5.4$$



$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \zeta(2) = \frac{\pi^2}{6} \approx 1.6449$$

$$10 \log P_{IIMD3} \approx 3 \cdot (10 \log P_{VH}^3) - 2 \cdot (10 \log P_{IIP3}^2) + 7.3 \text{ dB}$$



$$\text{NZDSF } \Delta f = 50 \text{ GHz} \rightarrow P_{IIP3} \approx +27 \text{ dBm}$$

$$P_{VH} = 1 \text{ mW} = +0 \text{ dBm} \rightarrow P_{IIMD3} = -46.7 \text{ dBm}$$

Prekooceanski 100 odsekov \times 50 km = 5000 km

$$S/N = P_{VH} / (100 \cdot P_{IIMD3}) = 26.7 \text{ dB}$$